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**WAGE DIFFERENTIALS AND LABOR EXTRACTION:  
AN ANALYSIS IN A CLASSICAL GROWTH MODEL**

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Orientador: Prof. Dr. Jaylson Jair  
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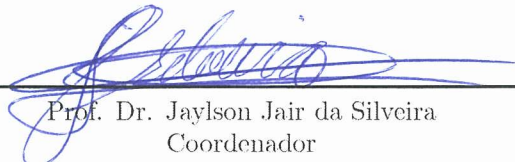
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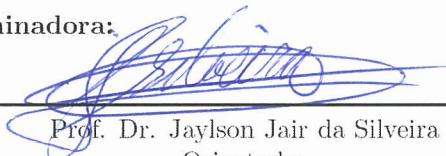
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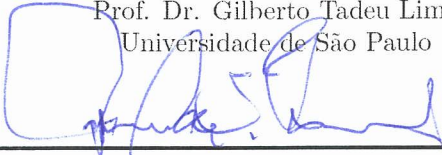
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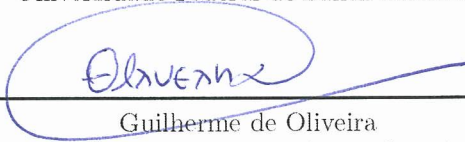
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To my friends and family



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I also want to thank my family and friends. They understood my absence during the period and were there when I needed them the most.





Accumulation of capital is therefore multiplication of the proletariat.

(Karl Marx, 1997)



## RESUMO

Esta dissertação desenvolve um modelo clássico de crescimento econômico no qual a distribuição de renda apresenta diferenciais de salário, visto que as firmas escolhem periodicamente entre duas estratégias de compensação pela extração de trabalho. Trabalhadores são homogêneos em relação à força de trabalho e as firmas escolhem em compensá-los ou com um salário mais alto ou com um salário mais baixo. Evidências empíricas mostram que a produtividade de trabalho varia de acordo com a compensação salarial: salários mais altos extraem mais trabalho dos trabalhadores. A distribuição de frequência das estratégias de compensação dos trabalhadores pela extração de trabalho e a produtividade do trabalho nas firmas varia de acordo com o tempo sendo guiadas por uma dinâmica de imitação 'satisficing' da qual um resultado pode ser extraído: heterogeneidade nas estratégias de compensação pela extração de trabalho, e por consequência desigualdade nos ganhos entre os trabalhadores, pode ser um equilíbrio estável de longo-prazo.

**Palavras-chave:** Crescimento econômico; teoria dos jogos evolucionários; distribuição de renda; diferenciais de salário.



## RESUMO EXPANDIDO

### Introdução

Uma questão que é central para a Economia do trabalho tange a ocorrência de diferenciais de salário na economia. Diferente da proposta walrasiana na qual os salários se ajustam para que ocorra equilíbrio no mercado, o que se vê através das evidências empíricas é que a compensação dos trabalhadores pode ocorrer de diversas maneiras.

Diferenciais na compensação ocorrem tanto quando se faz uma análise intersetores quanto uma análise intrasetorial. Estudos sobre um pagamento não homogêneo intrasetorial se pautam em questões como escolaridade, gênero, raça, localidade da firma, entre outras. Diferenças intersetoriais podem ocorrer, além do mencionado anteriormente, devido a razões como a necessidade de atrair profissionais para profissões mais desgastantes, prestígio da função ou especificidade do nicho.

Entretanto, mesmo quando se controla por estas variáveis observadas, ainda se pode perceber diferenciações salariais na economia. E isto se deve a variáveis não observáveis.

Uma destas variáveis, e aquela que trataremos neste trabalho, é a extração de trabalho. Evidências mostram que o esforço dos trabalhadores é endógeno e relacionado com a compensação salarial que eles recebem. Ou seja, um maior salário é acompanhado de uma maior produtividade - uma maior extração de trabalho do trabalhador.

Questões ainda pairam quando se analisa esta razão para a perpetuação de diferenças salariais ao longo do tempo. O que faz certas firmas pagarem um salário maior enquanto outras se mantêm com uma compensação menor é uma delas. Ocorrendo uma heterogeneidade nas compensações, isto também afeta a taxa de crescimento da economia e o bem-estar dos trabalhadores. Entretanto, o modo e a magnitude deste impacto ainda merece mais investigações teóricas. Tendo estas inquietações em mente, este trabalho busca contribuir teoricamente nessa área.

### Objetivos

O objetivo principal desta dissertação é mostrar, teoricamente, a possibilidade de que a heterogeneidade salarial seja uma característica persistente entre trabalhadores com iguais atributos, mesmo em uma economia clássica, cuja competição entre capitais leva à convergência para um longo prazo caracterizado por homogeneidade das taxas de lucro individuais e, mesmo no curto prazo, não há insuficiência de demanda

efetiva. Para alcançar este objetivo geral, desenvolve-se um modelo Clássico-Marxiano de crescimento com diferenciais de salário e uma dinâmica evolucionária no longo prazo. Para tal, primeiramente é preciso analisar as evidências empíricas e a teoria que embasa o modelo. Em segundo lugar, é necessário explicar a estrutura básica do modelo, como se dá a extração de trabalho e o diferencial de salários, bem como o fechamento macroeconômico dele. Por fim, é essencial mostrar a persistência dos diferenciais de salário no longo prazo e as implicações macroeconômicas disto, fazendo uma análise da dinâmica de ajustamento do curto para o longo prazo regida por uma dinâmica evolucionária guiada por diferenciais de taxas de lucro.

## **Metodologia**

A metodologia empregada nesta dissertação é a de construção de um modelo com dinâmica evolucionária para refletir a adaptação dos agentes ao ambiente macroeconômico que estão inseridos.

O modelo elaborado a frente está baseado em fatos estilizados com forte base empírica e robustez teórica. Isto parte da assertiva de que uma análise econômica não deve se basear apenas em fatos estilizados, mas também em modelos estilizados. O modelo em questão se propõe a analisar a possibilidade de ocorrência de equilíbrios e trajetórias para as variáveis endógenas a partir de um conjunto de estruturas causais correspondente com as evidências empíricas e a teoria macroeconômica de inspiração clássica.

## **Resultados e discussão**

Esta dissertação mostra que a persistência de heterogeneidade salarial em uma população de trabalhadores homogêneos pode emergir como um equilíbrio de longo prazo, no qual duas estratégias salariais coexistem (uma com salário alto e outra com salário baixo) entre firmas formadoras de salários. Isto implica na ocorrência de uma dinâmica macroeconômica interessante.

Se a economia parte de uma situação em que poucas firmas pagam um salário alto em direção ao número de firmas do equilíbrio polimórfico, inicialmente há uma piora na distribuição de renda em favor dos salários, mas no longo prazo ela retorna para o seu valor inicial. Partido de um valor igualmente baixo de firmas que optam pelo salário mais alto em direção ao equilíbrio, vê-se que a taxa média de crescimento econômico inicialmente melhora, mas no equilíbrio ela se estabilizará no mesmo valor que surgiria caso todas as firmas optassem por pagar salários baixos.

No caso da convergência partindo de uma proporção muito alta de

firmas que optam pela estratégia de extração de trabalho através de salários altos, vê-se que a taxa de crescimento econômico média aumenta monotonicamente, enquanto a participação dos trabalhadores na renda piora.

Com estes resultados, é possível ver que os trabalhadores estariam melhor em uma situação em que todas as firmas optassem pela estratégia de salários altos, mas a dinâmica não é guiada pelo bem-estar dos trabalhadores e sim pelo lucro. Do outro lado, os capitalistas estariam em uma situação melhor se não houvesse excesso de firmas que optassem pela estratégia de salários altos, mas as decisões individuais acabam levando a economia para um equilíbrio onde a participação dos lucros na renda é menor.

### **Considerações finais**

As questões levantadas nesta dissertação se mostram relevantes para o entendimento da perenidade de diferenciais de salário ao longo do tempo na economia. A estruturação de um modelo nos moldes clássicos evoca as questões de como o lado da oferta pode afetar a dinâmica macroeconômica e nisto está inserida a questão dos salários.

Os resultados que foram obtidos mostram que ainda há uma agenda de pesquisa ampla a ser explorada no sentido de melhor entender os fatores não observáveis que podem afetar as diferenças salariais. Apesar deste trabalho abordar a questão específica da extração de trabalho, outras questões podem estar envolvidas.

No campo da endogeneidade da extração de trabalho associada aos diferenciais de trabalho, observa-se que ainda se pode fazer estudos neste campo para demonstrar isto empiricamente. Dados brasileiros para isto seriam de suma importância para a construção de uma agenda de políticas públicas voltadas para um melhor bem-estar social.

**Palavras-chave:** Crescimento econômico; teoria dos jogos evolucionários; distribuição de renda; diferenciais de salário.





## ABSTRACT

This dissertation develops a classical model of economic growth in which the distribution of income features wage differentials, since firms choose periodically between two labor-extraction compensation strategies. Workers are homogeneous with regard to labor power, and firms choose to compensate them with either a lower wage or a higher wage. Empirical evidence shows that labor productivity varies according to wage compensation: higher wages extract more labor from workers. The frequency distribution of workers' labor-extraction compensation strategies and labor productivity in firms is time-variant being guided by a satisficing imitation dynamics from which one result can be extracted: heterogeneity in labor-extraction compensation strategies across firms, and consequently inequality in earnings among workers, is a stable long run equilibrium outcome.

**Keywords:** Economic growth; evolutionary game theory; income distribution; wage differentials.



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## INTRODUCTION

A challenge to economists across time is to explain why firms pay different wages in terms of inter-industry and intra-industry analysis. The focus of labor economists is to highlight this behavior and the possible implications to public policy and industry strategies.<sup>1</sup>

Most models assume that agents are rational and selfish. But as shown by some economists, this may not be true.<sup>2</sup> The bounded rationality is a concept that shows that the agents do not act as a Walrasian agent with an omniscience would act. And the evolutionary game theory also shows that fairness (and collaboration) may arise when observing repetitive relations - as the real world seems to have set.

The repetition of our relations in the real world makes it clear that a game's payoff would be better than the suboptimal equilibrium that arises when games are not repetitive or when agents do not base their actions on the previous rounds of the game.

When we put this on the table, the labor relations seems a good field of study to prove . Labor relationships are generally present for a continuous time and the contracts have imperfect information. Based on this, the survival of higher wages in exchange for higher productivity from workers seems plausible. A trade between effort from workers and higher earnings seems fair.

But the wage differentials could be motivated by other factors. Education, genre and race seem to be variables that could also explain why firms pay higher wages or lower wages to their workers.

The question that arises is if when these variables are controlled the differentials persist. Evidence<sup>3</sup> shows that they persist and that unobservable factors from workers may be central to understand the reasons behind it.

The labor productivity associated with each worker seems to be a plausible explanation to this phenomenon. In theory, if a firm pays a higher wage than the average wage in the economy, the worker will be more motivated and, consequently, will produce more.

This can be associated both with a fair compensation for a higher

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<sup>1</sup>See Groshen (1991) for examples; Kerr (1983) and Segal (1986) provide a summary of early studies on employer wage strategies.

<sup>2</sup>Simon (1956) has a good insight about bounded rationality.

<sup>3</sup>The next chapter will show some pieces of empirical studies about the persistence of wage differentials.

productivity and with the price of the job loss. Being fired from a job that pays a higher wage increases the cost of the job loss to the worker.

The development of a model that reflects these assumptions about labor extraction and wage differentials (and their relation) is necessary to better observe the behavior of the economy and how the income distribution will be affected by it.

In this work it will be developed a classical model of growth with wage differentials and endogenous labor extraction to explore the implications for income distribution and economic growth of firms following different strategies to extract labor from workers.

The model and its behavior in the short and the long run shows the evolutionary dynamics of the model and how it behaves according to the tests proposed. The point here is to show how the repetition of the game with the agents learning from their previous experiences influences the model.

The present work will be organized as follows: Chapter 1 will analyze the empirical evidence in wage differentials and labor extraction in its first session and the theory behind the model in the second session, explaining the concepts that support the model. Chapter 2 will describe a classical-Marxian model with wage differentials and will explain its behavior in the short run. Chapter 3 will show the behavior of the model in the long run and the stability properties of the equilibria.



## 1 WAGE DIFFERENTIALS AND LABOR EXTRACTION

When dealing with firms, we can assume that their labor productivity is endogenous to the level of effort of their workers when they are doing their tasks. Even when dealing with workers with similar characteristics (such as education, gender, race, location) , the individual commitment of the workers is different among them.

The reason why some firms can extract more labor from their workers is a question that economists have been trying to answer for a long time. Here, we will assume that wage incentives play a central role in this discussion. The more a worker is rewarded (with earnings) for his job, the more effort he will put into his tasks - and thus will raise his productivity.

As emphasized by Katz and Autor (1999), the wage structure and why there are wage differentials are themes that have been studied by economists since Adam Smith. And inter-industry wage differentials have persisted since then. However, economists have not been able to reach a consensus about the reasons why it is so perennial.

The majority of labor economists focus only on the characteristics of the workers. Higher education will induce to higher earnings. Other economists focus on the characteristics of the workplace. The size of the firm can affect wages of their workers. But recent research, as Katz (1986) and Caju (2010), shows that, even when all these variables are controlled, wage differentials persist.

As proposed before, we will deal with the reasons that are neither related to the workplace nor to worker's characteristics. Here, the reason why there is a persistence in wage differentials will be related directly to the effort that can be extracted from workers based on wage heterogeneity across them. And as will be shown, there is a robust set of empirical and laboratorial evidence that support our proposition.

In a more abstract manner we will also analyze which mechanism is used by firms to revise the strategy to grope for the best wage strategy to improve their profits in a macroeconomic context which coevolve with their strategy choices. Even though a higher wage extracts more labor from workers, this analysis depends on how other firms will act. If there is a high proportion of firms following a strategy of higher wage, the differential of productivity of workers may decline.

This chapter will be divided in two sections. The first section analyzes both empirical and laboratorial evidence of wage differentials

and endogenous labor productivity. The second section analyzes some theories which supply explanations on that and other theoretical traits that are necessary to understand the model proposed in this work, such as the theory of growth used and the satisficing dynamics.

## 1.1 EMPIRICAL EVIDENCE

There is a consistent amount of empirical evidence which corroborates that labor effort is endogenous regarding the wage differential and that there is persistence of wage differentials over time and across firms. These two propositions are essential to the understanding of the model that we are going to develop in this dissertation.

Tarling and Wilkinson (1982) analyze the rank order of the level of hourly earnings in 82 industries in the UK in 1948 and the rank order of the same industries in 1980. The study shows that wage differentials change across time. And the progressive change of wage structure is a result of the difference between the real wages' growth rates of many industries and the average between the 30 years analyzed.

Katz (1986) states that wage efficiency theories have a common proposition that firms in equilibrium tend to pay higher wages than the clearing market one. The set of data used by the author is a sample of private sector workers from 1983 Current Population Survey (CPS) collected in the United States. The author shows that the wage differential remains even when controlling individual characteristics of workers (such as education level, experience, sex and race). The impact of the industry variable is so that a mining industry worker earns near 45 percent more than a worker in the retail industry. Katz's (1986, p. 271) conclusions on the issue of wage differentials are that:

Evidence on industry wage differences indicates that large differentials remain that are quite difficult to explain in terms of differences in labor quality or differences in important nonpecuniary aspects of work requiring compensating differentials. The persistence of industry wage premiums for long time periods implies that they are not just transitory differentials arising to facilitate the sectoral reallocation of labor in a dynamic market economy. Large, persistent wage differentials for similar workers and types of jobs provide strong evidence in favor of the importance of some type of efficiency wage behavior

by many firms. The complex pattern of differentials is difficult to reconcile with individual variants of the efficiency wage argument.

The evidence shown in Katz's (1986) research corroborates to the thesis that the wage differentials are not a transitory factor in the economy. However, the author does not give a proper explanation on which are the benefits for the firms that adopt this strategy, proposing that further empirical research is needed to isolate them.

Dickens and Katz (1987) observe that there are differences between wages in industries even when analyzing similar workers, with similar jobs in the same period and locality. Using data from the US Bureau of Labor Statistics from 1985, they observe that an "entry operator I" in Cleveland could earn from US\$ 160 to US\$ 480 a week in September 1985 depending on which industry the worker was allocated.

When analyzing 1983 CPS micro-data the authors conclude that three variables are important when dealing with possible reasons on wage differentials. The first is education. Average years of education in an industry is positively related to wages. The second is profit. It was positively related with wages of nonunion workers. The last is the establishment size and capital-labor ratios. Those industries with higher average establishment size and higher capital-labor ratios were found to pay wage premiums to their workers.

These studies are important to show that the discussion about wage differences have a large set of empirical evidence and is a recurring theme in Labor Economics. But the wage structure from some OECD countries changed during the 90s, making a new wave of studies about the wage structure arise. Along with this, the amount of micro-data available has become significantly greater, making the empirical research about the theme capable to capture different factors that would not be possible before.

Katz and Autor (1999) show an extensive work about wage structure and the labor market in the US and other OECD countries. The authors use the March CPS database from 1963 to 1995 to analyze how the wage differences behave when focusing on skills, gender, occupation and other variables that other studies use to explain the wage differences. Even when dealing with narrowing differences (such as gender in recent years), the study shows that the difference in wages persists.

Carruth et al. (2004) examine the British Household Panel Study to study the stability of inter-industry wage differences for the UK between 1991 and 1996. The results support that the workers are paid accordingly to their marginal productivity, showing that the higher the

wage, the higher the effort of the worker not to lose his job will be a proposition taken as an assumption in our study. Carruth et al. (2004, p. 38) conclude about the importance of unobserved worker abilities:

To summarise, the headlines of the paper are that the inclusion of unobserved worker ability increases the explanation of individual wages considerably (around 90% of the variation in wages can be explained by observed and unobserved individual characteristics), and that this time-invariant effect is also clearly correlated with the industry specific effects, which leads to a reduction in the inter-industry wage dispersion. This is consistent with some sort of matching story whereby workers with the right traits are sorted into industries which can make the best use of these traits.

This conclusion is important to this dissertation because it shows that the ability - thus productivity - affects the wage of the workers.

Caju et al. (2010) provide a set of data from eight European countries that shows that there is a persistence of inter-industry wage differentials even when controlling observable productive features and characteristics of the workplace. The authors conclude that wage differentials are consistently related with rent sharing, but they cannot exclude other non-competitive explanations such as efficiency wages.

While some authors as the ones cited above don't analyze the relation between effort and wage differentials, there is also robust empirical and laboratorial evidence supporting that higher wages will elicit higher efforts from workers.

Fehr et al. (1998) conducted an experiment with Austrian soldiers with different levels of education and skills to see the effect of gift exchange and effort. The results of the experiment show a strong and permanent positive correlation between effort and wages. Fehr et al.'s (1998, p. 348) results are the following:

The regularities of the data presented in this article show that, under conditions of incomplete labor contracts, reciprocal behavior is a stable phenomenon: there is a strong and permanent positive correlation between effort and wages. Moreover, if we replace, under conditions of incomplete contracts, a bilateral bargaining environment by a competitive market with excess supply of labor, we observe no long-run effect on

wages. Although excess supply of labor creates enormous competition among workers, firms do not take advantage of this fact. It seems that, in the long run, instead of being governed by competitive forces, firms' wage offers are solely governed by reciprocity considerations because the payment of noncompetitive wages generates higher profits. Thus, both firms and workers were better off when they relied on stable reciprocal interactions.

Fehr and Falk (1999) conducted experimental double auctions with complete and incomplete labor contracts to observe the relation between wage levels and effort. Fehr and Falk (1999, p. 131) conclude that effort and wage levels are positively related as well:

It turns out that workers' effort levels are indeed positively related to the wages paid. This positive wage-effort relation prevails when there are weak and when there are strong pecuniary performance incentives. As a consequence, firms face a cost when they reduce their wages, which gives rise to downwardly rigid wages in the market with incomplete contracts. Despite the fact that there is an immense amount of underbidding on the workers' side, firms refuse to accept workers' low wage offers in this market. The positive wage-effort relation also implies that higher wages increase the total gains from trade.

Fehr et al. (2009) conducted experiments in the same line to determine if the wage-efficiency hypothesis could survive the laboratorial examination, being a possible explanation to field results. After the experiments, it could be possible to observe that the hypothesis of wage-efficiency survived the experiment, proving the point of the authors.

Even if laboratorial tests are a good way to show that the theory is related to reality, there is a problem when dealing with the duration of the tests. Usually, they tend to abbreviate the time when comparing to reality (the tests take place in a two-hour period when real labor relations last days, months or years).

To tackle this problem, Leonard and List (2009) did two field tests to observe how the relation between effort and gift exchange takes place. The first test was done in a library and the subjects did not know exactly the surplus of the employer with their work. The second test

was done in a fund-raising action - so the subjects knew how much the employer would be earning with their effort. In both cases, when the subjects earn more than the wage initially proposed, there was a significant higher effort in the first hours and a similar effort in the following hours when comparing to the subjects that did not win a prize.

Empirical evidence can be seen in Goldsmith et al. (2000) and Weisskopf et al. (1983). The first use data from National Longitudinal Survey of Youth (NLSY) in 1992 to show that increased effort has a significant role on wages. Goldsmith et al. (2000, p. 381) also shows that, even with similar workers, inter-industry wage differences still prevail:

The key finding is that effort, and various components of skill-based human capital, significantly affect real wages. Thus, a variable typically treated as part of unobserved individual-specific heterogeneity proves to be an important determinant of hourly compensation when it is included explicitly.

Campbell and Kamlami (1997) make an econometric study with US data since 1960s to show the relation between wages paid and the productivity of workers. The results point that work intensity and effort vary positively with the cost of job loss (the higher the wage, the higher the cost of job loss will be).

Other studies show the positive relation between effort elicitation and wage differentials. Survey evidence on this point can be seen in Campbell and Kamlami (1997, p. 785):

Respondents also gave high scores to the effect of wages on effort as an explanation for wage rigidity. This effect appears to be particularly strong when workers feel that they are being paid less than their fair wage.

Plant-level data from Cappelli and Chauvin (1991, p. 784) show a positive wage-effort elasticity:

Efficiency wage arguments rely on a largely untested relationship between wage premiums and worker productivity. The arguments above find evidence supporting the assertion that wage premiums are associated with reductions in shirking as measured by discipline rates. The wage premium appears to provide incentives to avoid

dismissal rather than leading to the selection of workers less inclined to shirk

Given this evidence on wage differentials and labor productivity both in laboratorial experiences and empirical data from time-series it is possible to conclude that the model that will be built ahead has relevance to the actual economical discussion and has the potential to widen the vision on why wage differentials remain present over time.

## 1.2 THE THEORY BEHIND THE MODEL AND SOME OTHER ASPECTS

As we have shown in the previous section, there is extensive empirical literature about wage differentials and their relation with effort elicitation (and labor extraction). Motivated by the empirical and laboratorial evidence about it, this dissertation will set a classical growth model with wage differentials that will be presented in the next chapter. It will analyze it with the evolutionary dynamics game theory framework in the third chapter.

The classical political economy built the analytical framework necessary for a theory of economic growth of capitalist economic institutions. But on behind this theory, there are different figures, such as Adam Smith, Thomas Malthus, David Ricardo and Karl Marx, with different premises and explanations to economic growth.

The Marxian tradition puts emphasis on capital accumulation and technological change. In this tradition, there will be an unlimited labor supply because capitalists maintain a reserve army of labor to meet their interests to maximize their profits.

From this point of view, we assume a Classical-Marxian model where it is possible to extract labor from workers at will. Workers will respond to the wages set by employers with the expected productivity or they can be replaced with an unemployed one at a low cost for the employer.

Employers choose between two strategies: a higher wage or a lower wage to extract labor. They choose the strategy that better meets their interests and can change it from time to time.

Nonetheless, we need to explain the theory behind it and how it can be related to the objectives of this dissertation. In the next section, the framework and concepts will be pointed out.

### 1.2.1 Wage differentials, labor extraction, and contested exchange

As it has been stressed, the persistence of inter-industry and intra-industry wage differentials even after controlling for observable variables (as schooling, age, gender, etc) of workers is evidence that unobservable characteristics play an important role when dealing with wage compensation.

As the model to be built in the next chapter shows, a polymorphic equilibrium with the coexistence of two different strategies of wage compensation is a stable equilibrium. Therefore, it is possible to understand why the empirical evidence point to the extended persistence of wage differentials across time.

As Silveira and Lima (2016) propose, one of the possible reasons - and the one that we will focus in this study - is that labor effort is endogenous especially to wage compensation. Being the firm's labor productivity endogenous to its workers effort, we see the motivation for a firm to offer a higher wage than the others.

This relationship between wage compensation and labor productivity will lead to different strategies when analyzing different firms. A firm would only be willing to pay a higher wage to its workers if the profit rises in some way.

But to do so, a firm needs to know how much labor it is extracting from workers. A labor extraction function associated to that is, for the purposes of this work, needed to evaluate which strategy the firm will follow.

A labor extraction function, as Bowles (1985) explains, is a function that determines the amount of labor done per hour of labor hired. In other words, it represents how much effective labor is extracted from the hired labor.

Bowles and Gintis (1990) formalize the labor extraction function as follows:

$$e = e(w_\tau), e'(\cdot) > 0, e''(\cdot) < 0. \quad (1.1)$$

This equation needs to exist for two reasons. The first reason is that the amount of labor hired is not necessarily equal to the effective labor. The productivity of each worker is different and this is associated with the effort that each worker puts into his actions. The second reason is that the extraction of labor from workers is not costless. The ways to assure the extraction of labor cost something to a firm. Surveillance, managers and other ways cost money and for this reason, a firm needs



to take this into account when dealing with the effective labor.

Bowles and Gintis (1990) theorize about the contested exchange, which plays a central role to understanding the mechanics of labor exchange. The contested exchange theory drops both assumptions about the constitution of agents and enforcement of claims.

The endogeneity of enforcement of claims is present in a number of cases, such as when the contested attribute is not measured perfectly and when there is a cost to do so, and when there are no means to make a complete contract to address to all the future possibilities of the relation.

The endogeneity of the constitution of agents occurs when the agents are constituted in the process of exchange. Thus, previous decisions and actions can affect how the agent will act in the future.

An example, and the one that concerns us here, of a contested exchange, is the labor exchange. The process here cannot be fully measured by the parties (the effort of the workers) and it needs to have means to enforce it (monitoring and sanctioning the worker). Also, how the worker and the firm will act in the relationship will be determined by the actions taken in the process. If a firm pays a higher wage to the worker, there will be a higher effort by the latter.

For a better understanding of the system as to contested exchanges, six premises are important to be underlined. These six premises formalize a vision of capitalism that differs from the Walrasian one, which supposes an allocative efficiency. (BOWLES; GINTIS, 1993)

The first premise is that the power is allocated to the short-side agents of a non-clearing market. To formalize this premise the labor market with unemployment as equilibrium can be used as an example. Employers exchange money for workers' services. The effort in these services is variable and costly for workers to provide. Employers renew contracts if they are satisfied with the services. The satisfaction probability increases with the effort (and productivity) of the worker. Since there are unemployed people, we can assume that there are other people willing to work and to provide the same service as the worker in the relationship. The employer knows the production function of his workers, both sides know each other's objective functions and the conditions to terminate a contract. For each wage paid by the employer, the worker selects a level of effort that he puts into his activities. The worker has to make a trade-off between the cost of putting higher effort and the cost of job loss. The employer, knowing this, chooses the wage paid to maximize his utility. Being in equilibrium, there will be no advantage for the employer to end the worker's contract and choose an

unemployed one to take his place.(BOWLES; GINTIS, 1993)

In the above situation, we can see that the employer is on the short side and has power over the worker, as he can terminate the relationship and choose another worker at little cost. This proposition alone summarizes the enforcement problem to just one side of the exchange. But to the job market we can use it as it is. The bilateral enforcement may be misleading when dealing with this market.

Even if organized workers can assure part of their rights, when dealing with an individual worker, the threat of the employer is much more feasible than the worker's. An employer hires many employees, and the cost of job loss to a worker is much higher than the costs associated with the dismissal of a worker to a firm.

The second proposition is that cost-minimizing contingent renewal enforcements strategies are inefficient. The two components of these strategies (resource-using monitoring inputs<sup>1</sup> and non-resource-using distributive payments<sup>2</sup>) are costly to the enforcer. They are inefficient because they deviate from the social optimum, generally applying excessive monitoring and suboptimal rents. (BOWLES; GINTIS, 1993)

The third proposition is that the employment relationship is inefficient. The lack of efficiency in this case happens because a redistribution of ownership to workers tend to have Pareto improvements. The employer see wage only as a costly enforcement, but workers can see it as an enforcement instrument and a positive argument in their objective function. This vision of workers makes them set the wage, if in power to do so, more efficiently. (BOWLES; GINTIS, 1993)

The fourth proposition is that the survival of an hierarchical structure in firms may be related to its efficacy in enforcing distributional claims, and not in allocating resources. This happens because there is an advantage to the wealthiest in the credit market. If there was not this assertion, workers would have the capital to buy the firm and apply more efficient enforcement methods. However, this is not a common thing to observe.

"Money talks" is proposition five. As Bowles and Gintis (1993, p. 94) show:

Power-holding is not coextensive with wealth-holding. Some short-siders, such as managers, may not be wealthy, or their wealth may be

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<sup>1</sup>Here we can cite inputs that represents such as surveillance personnel and equipment.

<sup>2</sup>This is the case of enforcement rents: a payment in excess of at least one agent's next best alternative.

a result rather than a source of their power. Moreover, many wealth-holders have no power beyond purchasing power (like passive stockholders). Yet a considerable fraction of top-level decision-making positions in the economy are occupied by wealth-holders. Why are the wealthy not only rich, but powerful? The reason is that offering personal equity or collateral is an effective means of reducing incentive incompatibility in credit markets.

Since money is not an ambiguous claim, such as effort, it is easy to reduce incentive incompatibilities. Because of this difference, there is also a distinct valuation of these assets in contested exchanges. Not being able to fully measure effort without enforcement costs makes it difficult to provide a reduction in incentive incompatibilities.

The last proposition is that "anonymity in market exchange fosters norms hostile to the efficient solution of coordination problems", as Bowles and Gintis (1993) describe. Social norms can facilitate the coordination and thus anonymity can do more harm than good in the real world.

One example that illustrates this is the basic game theory problem: the prisoners' dilemma. The equilibrium strategy of this game that leads to cooperation, and with that a social optimum is possible only if there is a repetition in the game. With more rounds, the retaliation against defection is more tangible. In a single-game situation, we see that the equilibrium is mutual defection, leading to a suboptimal situation.

With these six assumptions we can affirm that there are contested exchanges in the capitalism system and with this, wage differentials and a non-clearing equilibrium will arise.

The relation between wage differentials and labor extraction is vital to understand why this equilibrium occurs. As shown in the previous session, even controlling other factors such as education and gender, the wage differentials still persist. Thus, unobservable variables may explain this - and one of this is the productivity. Firms supporting higher wages arise because they believe in the possibility of higher gains with this, what is proven by the evidence.

### 1.2.2 The Classical-Marxian theory of growth

A classical model is a model that puts a lot of importance in the supply side. The role of the wage is central to understanding how the model behaves. If there are wage differentials and, consequently, a non-clearing market equilibrium, it will affect the model somehow.

When looking at the Marxian tradition, it is important to highlight the concept of extraction in the capitalist economy, as exposed by Gintis (1976, p. 44):

The need to extract a surplus through proper enforcement of the labor exchange will have extensive impact on the capitalist's choice of a job structure, a wage structure and a policy toward staffing and promotion within the enterprise.

Whence the origins of the hierarchical division of labor as an historical phenomenon. Such an organization faces the task of insuring the undistorted transmission of directives downward and of information upward. To this end the capitalist will bureaucratize the social relations of production within the firm. A job will be a position in the organization defined by rules which are merely modified and interpreted by the directives of a single superior. If all are obeyed, we have a paradigm of the rational Weberian bureaucracy: each worker chooses the rules for his or her subordinates by adhering to the rules set for this worker by his or her superiors. There is a perfect transmission of directives from above to below.

The extraction of labor from labor-power, thus, depends on the efficiency of how the workers are fragmented and how the hierarchical superior can control the subordinates. This is important because when wage differentials arise, that could raise questions in unified workers and it could possibly have a mutiny.

The Classical-Marxian growth model will be analyzed, since it will be the "environment" of the model proposed in this dissertation is valid. As Dutt (2011, p. 358) stresses out:

Models of growth in the classical-Marxian tradition continue to take growth to be determined by the growth of aggregate supply. In this they are closer to neoclassical growth theory - in both its old and so-called 'new' incarnations - than to

models of aggregate demand-determined growth as developed by the followers of Kalecki and Keynes, including Harrod (1939), Kahn (1959) and Robinson (1962). According to theories of aggregate demand-determined growth, changing conditions on the demand side, for instance, by exciting animal spirits that induce firms to invest more, can affect the long-run growth rate of the economy. The classical-Marxian and neoclassical approaches, while stressing aggregate supply, disagree on whether capital or labour produces the main supply-side constraint on growth. With endogenous labour supply or unemployment, the former stresses the capital constraint, while the latter, assuming exogenously-growing fully-employed labour, stresses the growth of effective supply of labour through technological change.

It is possible to go deeper into the differences and similarities between the Classical models and the Neoclassical and Keynesian ones.

There are some questionable preconceptions in the Neoclassical tradition in the vision of Classical researchers, Foley and Michl(2010) cite two of these problems:

- They assume that there is full employment, sustained by the use of aggregate production functions similar to a Cobb-Douglas. Any excess of demand or supply of labor can be eliminated changing the wage rate. Classical models with full employment are special cases (such as Kaldor-Pasinetti and Goodwin);
- They rely on the assumption that real wage and profit rate are equal to marginal products of labor and capital. The classical theory does not support that because there may be no mechanism to guarantee that the capital stock is sufficient to maintain full employment and because the marginal productivity theory obscures the relationship of workers and employers by treating the capital as a productive resource.

As the Keynesian tradition, we can see one major difference observed by Foley and Michl (2010), which is the interpretation of the rate of profit. For the classical theory, this rate regulates the rate of growth because the proportion of profits reinvested affects directly the latter rate. For this reason, any technological change or a wage decrease which could improve the profits, is expected to stimulate capital

accumulation. The keynesian theory - according to the stagnationist model - assumes that the profit rate regulates growth (through the investment equation) and reflects growth in an aggregate demand (through the effect of utilization on profitability). For this theory, a cut on wages would affect aggregate demand and, consequently, profitability.

Even with this major difference, there are similarities between the theories. One of them is the skepticism about the definition of the long run by a full employment situation rather than a full utilization of capital stock. There are also similarities in the methodology: they seek to identify macroeconomic foundations in features like social class and corporate form rather than a methodological individualism. Both see the accumulation of capital as an animating force of capitalism and see the level of employment as a result of the amount of capital and its utilization. (FOLEY; MICHL, 2010)

With these similarities, both theories can have a common research program and stimulate debates that could lead to hybrid models. Treating labor extraction as endogenous is a common ground that can be observed in these theories.

But even though the attention is directed to the supply side, a mention of how the demand side affects growth must be analyzed. The Marxian tradition recognizes that general overproduction is a possibility in the economy. If the demand for a product is not sufficient for the capitalist to receive his normal rate of profit, there will be a decline in production and investment in this sector. And as industries are interconnected, it will affect another sector and a general crisis may emerge. Capitalists will attain to their money and there will be a rise in unemployment and capital utilization. (DUTT, 2011)

This is what happens in the short run, but in the long run the scenario is different. There are mechanisms in the economy (such as competitive pressures and fall of wages) that make it return to a growth path where demand does not affect the economy. And again the problem goes back to the supply side.

As Foley and Michl (2010, p. 50) state:

The central regulating factor in the classical political economists' theory of economic growth is the division of value created (or value-added) in production between wages and profits. Economic growth paths on which the wage share in value added continually rises or falls are not sustainable. If wages grow less rapidly than labor productivity, the wage share approaches zero, and the social contradictions of capitalism be-

come unmanageable as workers' contribution to aggregate demand vanishes. If wages grow more rapidly than labor productivity, the wage share approaches unity and the profitability of production vanishes, taking with it the incentives to organize and improve production that drive economic growth.

Based on this assumption, the wage differentials need to be combined, as the empirical evidence shows, with differences in productivity. Higher wages extract more labor from workers and, in consequence, more productivity. Lower wages, on the other hand, are combined with a lower productivity.

The model shown ahead presents an analytical representation to this assumption. A wage differential and a productivity differential play a central role to understanding the behavior of firms in the short run.

But even if this is true, it does not explain why employers choose to follow one strategy of labor extraction with higher wages or one with lower wages. How the other firms act will also affect how one firm acts. A higher proportion of firms extracting labor with higher wages may change the perception of workers of how much extra effort to put into their duties and the strategy may fall short. This can be further analyzed with an evolutionary dynamics.





## 2 STRUCTURE OF THE ECONOMY

In this chapter, we will show the basic structure of the model and its behavior in the short run. It is important to remember that the model is a classical one, so there will be a focus on the supply side of the economy. In the short run, for given values of wage rate differential, the labor productivity differential, the distribution of labor extraction strategies, and individual and average profit shares, it is possible to determine the average rate of economic growth. On the other hand, when moving towards the long run, the distribution of labor extraction strategies changes following an evolutionary dynamics, which will be analyzed in the next chapter.

The first section will approach the basic premises of the model. The labor extraction function and the wage differential will be defined, as well as other important variables to the model. The second section will focus on the macroeconomic closure of the model in the short run.

### 2.1 LABOR EXTRACTION AND WAGE DIFFERENTIAL

The economy is closed and without government activities. There is the production of a single and homogeneous good for both investment and consumption purposes. Output production is carried out by a large and fixed population of firms. These firms combine two factors of production, capital and labor, through a fixed-coefficient technology. They also produce and hire labor without constraint by effective demand which makes them able to sell with profit all output at the prevailing prices.

Following Silveira and Lima (2016), we will assume that firms choose periodically between two strategies of labor extraction compensation, which determines the wage rate they are willing to pay. They can compensate worker by paying a lower wage rate  $w_\ell \in \mathbb{R}_{++}$  or paying a higher wage  $w_h > w_\ell$ . A firm that decides to pay a higher wage rate is called  $h$ -firm and the one that decides to pay a lower wage rate is called  $\ell$ -firm. As the empirical evidence presented in the previous chapter show, an  $h$ -firm is willing to pay a higher wage because it allows potential gains in labor productivity sufficiently high to compensate it. It results, in a  $\lambda_t \in [0, 1] \subset \mathbb{R}$  proportion of  $h$ -firms and a  $1 - \lambda_t$  proportion of  $\ell$ -firms, in a given period  $t$ .

Labor productivity is homogeneous across workers from firms

that play the same strategy. Labor productivity, even if homogeneous between firms of the same type, will be heterogeneous across the two types of firms. Also, labor productivity differential is endogenous, continuously time-varying with the frequency distribution of the strategies played by firms.

After choosing the strategy played, the firm makes an offer to available workers, who are always in excess supply. These workers will accept the offer according to the choice of other firms and deliver the labor effort (and thus labor productivity) expected by firms when they choose the wage compensation to offer. Therefore, workers have a higher productivity when they are receiving higher wages.

Following Silveira and Lima (2016), we simplify the matters by assuming that  $w_\ell$  and  $w_h$  will remain constant over time. This is not to say that they are not important, but that the focus in this work is to analyze the dynamics of the distribution of employee wage compensation strategies and its macroeconomic implications.

For a given distribution of labor-extraction strategies  $(\lambda_t, 1 - \lambda_t)$ , the short run values of wage rate differential, labor productivity differential, income distribution and rate of economic growth are determined. Over time (leading to the long run) there are changes in the frequency distribution of labor-extraction strategies that cause changes in labor productivity differential (and hence in the average wage rate and wage share in income) as well as changes in the short run value of the economic growth rate.

The relation between labor extraction and wage differential across workers, as seen in the previous chapter, will play a major role in our model. As Silveira and Lima (2016, p. 49) well observe:

the process of labor effort elicitation on the part of firms is conceptualized as a contested game, with effort depending both on wage levels and differences. In such contested exchange of labor power, the average wage rate can be perceived by workers as either a conventional measure of their outside opportunities or a conventional reference point to which a given wage offer is to be compared as it embodies workers wage expectations.

Formally, the labor productivity differential in a given period  $t$  can be defined as follows:

$$\alpha_t \equiv \frac{a_h}{a_\ell}, \quad (2.1)$$

where  $a_\tau \equiv \frac{X_{\tau,t}^i}{L_{\tau,t}^i}$  denotes the labor productivity of the  $i$ -th firm of type  $\tau = h, l$ ;  $X_{\tau,t}^i$  is the total output of the  $i$ -th firm of type  $\tau = h, l$  and  $L_{\tau,t}^i$  is the total employment of the  $i$ -th firm of type  $\tau = h, l$ .

We will use the weighted geometric average wage in a given period  $t$  (given by  $w_h^{\lambda_t} w_\ell^{1-\lambda_t}$ ) to express the differential between higher wage and that average wage, which can be written as  $w_h / (w_h^{\lambda_t} w_\ell^{1-\lambda_t})$ , for all  $\lambda_t \in [0, 1] \subset \mathbb{R}$ . In line with the empirical evidence presented in the previous chapter, we assume that the labor extraction increases with the wage differential, such as the labor productivity can be expressed as a function of this differential:

$$\alpha_t = f\left(\frac{w_h}{w_{h,t}^{\lambda_t} w_\ell^{1-\lambda_t}}\right) = f\left(\left(\frac{w_h}{w_\ell}\right)^{1-\lambda_t}\right) \quad (2.2)$$

with  $f'(\cdot) > 0$ , for all  $\lambda_t \in [0, 1] \subset \mathbb{R}$ . Hence, the decision of a firm to adopt the higher wage strategy in a given period  $t$  reduces  $\alpha_t$  and makes it less valuable to workers to be employed by a  $h$ -firm in the next period. consequently, there is a strategic substitutability in the firms' choice of labor-extraction compensation mechanism.

Let  $w \equiv \frac{w_h}{w_\ell}$  be the wage differential between the higher wage and the lower wage and supposing that the labor extraction differential function in (2.2) assumes the linear form  $\alpha_t = Aw^{1-\lambda_t}$ , where  $A > 1$  is a parametric constant. For analytical convenience and without loss of generality, we can set  $A = w^\beta$ , where  $\beta \in (0, 1) \subset \mathbb{R}$  is a parametric constant. Thus, the labor extraction function can be rewritten as:

$$\alpha_t = w^{\beta+1-\lambda_t}. \quad (2.3)$$

We assume that  $w_\ell < a_\ell < a_h$  and  $w_\ell < w_h < a_h$  such that there will be surplus product for any labor-extraction compensation strategy chosen by each firm. With these assumptions it is possible to assure that income shares going to capital and labor remain in the interval  $(0, 1) \subset \mathbb{R}$ .

The real profits of firms are given by:

$$R_{h,t}^i \equiv X_{h,t}^i - w_h L_{h,t}^i = \left(1 - \frac{w_h}{a_h}\right) X_{h,t}^i = (1 - u_{h,t}) X_{h,t}^i \quad (2.4)$$

and:

$$R_{\ell,t}^i \equiv X_{\ell,t}^i - w_\ell L_{\ell,t}^i = \left(1 - \frac{w_\ell}{a_\ell}\right) X_{\ell,t}^i = (1 - u_{\ell,t}) X_{\ell,t}^i, \quad (2.5)$$

where  $u_\tau \equiv \frac{w_\tau}{a_\tau}$  is the wage share of income or the unit labor cost of the  $i$ -th firm of type  $\tau = h, \ell$ . Since  $w_l$  and  $a_l$  are constants by assumption, the wage share of  $\ell$ -firms,  $u_\ell$ , will remain constant over time. For this reason, we will drop the subscript  $t$  of this variable.

Using (2.4) and (2.5) it is possible to obtain the shares of real profit in the short run equilibrium of  $h$ -firms and  $\ell$ -firms, respectively given by:

$$\pi_{h,t} \equiv \frac{R_h^i}{X_h^i} = 1 - \frac{w_h}{a_h} = 1 - u_{h,t} \quad (2.6)$$

and: <sup>1</sup>

$$\pi_\ell \equiv \frac{R_\ell^i}{X_\ell^i} = 1 - u_\ell. \quad (2.7)$$

Remembering that  $w \equiv w_{h,t}/w_\ell$  and using (2.3), we can rewrite (2.6) as follows:

$$\pi_{h,t} = 1 - \frac{w_\ell w}{a_\ell \alpha_t} = 1 - u_\ell w^{\lambda_t - \beta} \equiv \pi_h(\lambda_t). \quad (2.8)$$

Although the share of real profit from  $\ell$ -firms remains constant, the short-run equilibrium value of the share of real profits of  $h$ -firms varies with the proportion of these firms. The intuition behind this assumption is that the greater the proportion of firms adopting the higher wage strategy, the smaller the differential between the higher wage rate and the average wage rate will be, resulting in a smaller additional labor effort extracted.

As presented before, this is called a strategic substitutability in the firms' choice of labor-extraction strategy: the decision of a firm to follow a higher wage strategy in a given period, by reducing the labor productivity differential of the next period, makes it less valuable to a worker to be employed by an  $h$ -firm in the next period - a negative payoff externality on all the other  $h$ -firms.

The conditional expected value of the profit share  $\pi_t$  at a period  $t$ , given the type  $\tau$ , can be expressed as:

$$E(\pi_t|\tau) = \begin{cases} \pi_h(\lambda_t), & \text{if } \tau = h, \\ \pi_\ell, & \text{if } \tau = \ell. \end{cases} \quad (2.9)$$

Based on the law of iterated expectations and the conditional ex-

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<sup>1</sup>As explained before,  $u_l$  is constant over time. For this reason,  $\pi_l$  is also constant and the subscript  $t$  will also be dropped

pectation of the equation above, the short-run average profit share,  $\bar{\pi}_t$ , at period  $t$ , can be established as the expected profit share in the short run for a given frequency distribution of labor-extraction compensation strategies across firms:

$$\begin{aligned}\bar{\pi}_t &= E[E(\pi_t|\tau)] = \lambda_t E(\pi_t|\tau = h) + (1 - \lambda_t)E(\pi_t|\tau = l) \\ &= \lambda_t \pi_h(\lambda_t) + (1 - \lambda_t)\pi_\ell \equiv \bar{\pi}(\lambda_t).\end{aligned}\quad (2.10)$$

Drawing on Silveira and Lima (2017), we assume that there is heterogeneity in individual stocks of capital across firms. Let  $k^i \equiv \frac{X_\tau^i}{K_\tau^i}$  be the individual output to capital ratio of the  $i$ -th firm of type  $\tau = h, l$ , where  $K_\tau^i$  is the respective capital stock. It is possible to assume that they remain constant when firms switch labor-extraction compensation strategy. Using (2.4), (2.5), (2.7) and (2.8) it is possible to formalize the profit rates of firms following higher wage and lower wage strategies in the short run:

$$r_{\tau,t}^i \equiv \frac{R_{\tau,t}^i}{K_{\tau,t}^i} = \begin{cases} \pi_h(\lambda_t)k^i = \left(1 - u_\ell \frac{w}{\alpha_t}\right)k^i, & \text{if } \tau = h, \\ \pi_\ell k^i = (1 - u_\ell)k^i, & \text{if } \tau = l. \end{cases}\quad (2.11)$$

We assume that the individual output to capital ratios given by  $k^i$  are randomly distributed across the population of firms around the average value  $k \in \mathbb{R}_{++}$ , which is taken as an exogenous constant. The conditional expected value of the profit share  $r_t$  given the type  $\tau$  is given by:

$$E(r_t|\tau) = \begin{cases} \pi_h(\lambda_t)k, & \text{if } \tau = h, \\ \pi_\ell k, & \text{if } \tau = l. \end{cases}\quad (2.12)$$

Based on the law of iterated expectations and the conditional expectation of (2.12), the short run average profit rate at a period  $t$ ,  $\bar{r}_t$ , can be determined using the frequency distribution of strategies across firms  $(\lambda_t, 1 - \lambda_t)$ :

$$\begin{aligned}\bar{r}_t \equiv E[E(r_t|\tau)] &= \lambda_t E[r_t|\tau = h] + (1 - \lambda_t)E[r_t|\tau = \ell] \\ &= [\lambda_t \pi_h(\lambda_t) + (1 - \lambda_t)\pi_\ell]k = \bar{\pi}(\lambda_t)k.\end{aligned}\quad (2.13)$$

Therefore, comparing (2.10) and (2.13) it shows that in the short

run the average profit rate is a multiple of the average profit share. With this, the two measures of profitability move in the same direction.

From (2.10) it is possible to observe the response of  $\lambda_t$  over the short-run average profit share to a change in the frequency distribution of labor extraction strategies:

$$\frac{\partial \bar{\pi}(\lambda_t)}{\partial \lambda_t} = [\pi_h(\lambda_t) - \pi_\ell] + \lambda_t \frac{\partial \pi_h(\lambda_t)}{\partial \lambda_t}. \quad (2.14)$$

We know that, for all  $\lambda_t \in (0, 1] \subset \mathbb{R}$ , we have

$$\lambda_t \frac{\partial \pi_h(\lambda_t)}{\partial \lambda_t} = -\lambda_t u_\ell w^{\lambda_t - \beta} \ln w < 0. \quad (2.15)$$

As  $\pi_h(\beta) - \pi_\ell = 0$ , we can infer from (2.14) and (2.15) that  $\partial \bar{\pi} / \partial \lambda_t < 0$  for all  $\lambda_t \in (\beta, 1] \subset \mathbb{R}$ . However, in the interval  $[0, \beta) \subset \mathbb{R}$  we have  $\pi_h(\lambda_t)(\lambda_t) - \pi_\ell > 0$ . Therefore, the sign of (2.14) in such interval requires further investigation.

Based on (2.7) and (2.8) we can rewrite (2.14) as follows:

$$\frac{\partial \bar{\pi}(\lambda_t)}{\partial \lambda_t} = u_\ell [(1 - w^{\lambda_t - \beta}) - \lambda_t w^{\lambda_t - \beta} \ln w]. \quad (2.16)$$

And considering (2.16) we have:

$$\frac{\partial \bar{\pi}(0)}{\partial \lambda_t} = u_\ell (1 - w^{-\beta}) > 0 \quad (2.17)$$

and

$$\frac{\partial \bar{\pi}(\beta)}{\partial \lambda_t} = -u_\ell \beta \ln w < 0. \quad (2.18)$$

It follows from (2.17) and (2.18) and the continuity of (2.16) that there is a frequency distribution of labor extraction strategies  $\bar{\lambda} \in (0, \beta) \subset \mathbb{R}$  where the following condition holds:

$$\frac{\partial \bar{\pi}(\bar{\lambda})}{\partial \lambda_t} = u_\ell [(1 - w^{\bar{\lambda} - \beta}) - \bar{\lambda} w^{\bar{\lambda} - \beta} \ln w] = 0. \quad (2.19)$$

Moreover, it follows from (2.16) that:

$$\frac{\partial^2 \bar{\pi}(\lambda_t)}{\partial \lambda_t^2} = u_\ell [-2w^{\lambda_t - \beta} (\ln w) - \lambda_t w^{\lambda_t - \beta} (\ln w)^2] < 0 \quad (2.20)$$

for all  $\lambda_t \in [0, 1] \subset \mathbb{R}$ . It follows from (2.20) that  $\bar{\lambda} \in (0, \beta) \subset \mathbb{R}$  is unique, with the straightforward implication that  $\partial \bar{\pi}(\lambda_t) / \partial \lambda_t > 0$

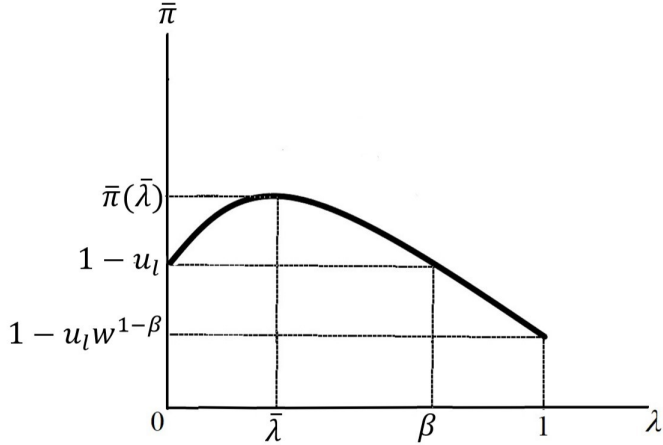


Figure 1 – Behavior of income distribution along transitional dynamics

for all  $\lambda_t \in [0, \bar{\lambda}) \subset \mathbb{R}$ ,  $\partial \bar{\pi}(\bar{\lambda}) / \partial \lambda_t = 0$ , and  $\partial \bar{\pi}(\lambda_t) / \partial \lambda_t < 0$  for all  $\lambda_t \in (\bar{\lambda}, 1] \subset \mathbb{R}$ . These properties can be seen in Figure 1.

## 2.2 A CLASSICAL MACROECONOMIC CLOSURE

As a Classical-Marxian model with no government and a closed economy, we assume that there are two classes in the economy: capitalists and workers. Workers provide labor and earn a wage income (higher or lower depending on the firm's strategy). Workers spend all their earnings on consumption. Capitalists, who own the firms, homogeneously save a fraction  $\gamma \in (0, 1) \subset \mathbb{R}$ , of their profit income. We also assume that capitalists save it in order to fully finance their investment decisions - everything that is not consumed will turn into investment. For further simplification, individual capital stocks will be assumed as not depreciating, so the growth rate of the capital stock of the  $i$ -th firm of type  $\tau$  is expressed as:

$$g_{\tau,t}^i \equiv \frac{S_{\tau,t}^i}{K_{\tau,t}^i} = \begin{cases} \frac{\gamma R_{\tau,t}^i}{K_{\tau,t}^i} = \gamma \pi_h(\lambda_t) k^i, & \text{if } = h, \\ \frac{\gamma R_{\tau,t}^i}{K_{\tau,t}^i} = \gamma \pi_\ell k^i, & \text{if } = l, \end{cases} \quad (2.21)$$

where  $S_{\tau,t}^i$  represents the savings of the  $i$ -th firm of type  $\tau$ .

As individual output of capital ratios are randomly distributed

across firms around the average value  $k$ , the conditional expected value of the growth rate  $g_t$  at the period  $t$  for a type  $\tau$  firm is:

$$E(g_t|\tau) = \begin{cases} \gamma\pi_{h,t}k, & \text{if } \tau = h, \\ \gamma\pi_{\ell}k, & \text{if } \tau = l. \end{cases} \quad (2.22)$$

Using the law of iterated expectations and considering individual growth rates of (2.21), the short-run average growth rate  $\bar{g}_t$  at each period  $t$  is given by the expected growth rate in the short run for a given frequency distribution of employee compensation strategies across firms  $(\lambda_t, 1 - \lambda_t)$ :

$$\begin{aligned} \bar{g}_t &= E[E(g_t|\tau)] = \lambda_t E(g_t|\tau = h) + (1 - \lambda_t) E(g_t|\tau = l) \\ &= \gamma[\lambda_t\pi_h(\lambda_t) + (1 - \lambda_t)\pi_\ell]k \equiv \bar{g}(\lambda_t), \end{aligned} \quad (2.23)$$

which can be rewritten based on (2.10) as:

$$\bar{g}(\lambda_t) = \gamma k \bar{\pi}(\lambda_t). \quad (2.24)$$

From (2.24) it is possible to observe that  $\bar{g}(\lambda_t)$  will have the same behavior as  $\bar{\pi}(\lambda_t)$ , summed up in Figure 1. And with this equation it is possible to notice that the average growth rate depends on parametric constants along with labor productivity differential and frequency distribution of labor compensation strategies, which are predetermined in the short run and co-evolve in the transition to the long run. As in the classical theory, the economic growth is driven by capital accumulation from aggregate saving of capitalists, which is shown by the positive variation of the average growth rate with the saving propensity of capitalists and the average profit share in aggregate income.



### 3 PERSISTENCE OF WAGE DIFFERENTIALS AND MACROECONOMIC IMPLICATIONS

The distribution of labor-extraction compensation strategies across firms,  $(\lambda_t, 1 - \lambda_t)$ , which is given in the short run as a result of previous dynamics, changes beyond the short run according to an evolutionary dynamics - a satisficing evolutionary dynamics. We will better define this dynamics in this chapter.

Changes in the aggregate stock of capital,  $K$ , due changes in the individual stocks of capital,  $\{K^i\}_{i \in [0,1] \subset \mathbb{R}}$ , the supply of available labor,  $N$ , the labor productivity differential,  $\alpha_t$ , and the frequency distribution of labor-extraction compensation strategies,  $\lambda_t$ , change when the economy is moving towards the long run.

In order to focus on the analysis of the relationship between the frequency distribution of strategies and the productivity differential, we will assume that the aggregate growth of labor force is endogenous and varies at the same rate of the average growth rate of capital stock given in (2.23).

When we set  $\lambda_t = 1$  (all firms adopting the higher wage strategy), then  $w^{\beta+1-\lambda_t} = w^\beta$ . Being the labor productivity uniform among all firms and knowing that it should be higher than the labor productivity when  $w_\ell$  is paid, then  $w^\beta > 1$ .

On the other hand, when assuming that  $\lambda_t = 0$  (all firms adopting the lower wage strategy), then  $w^{\beta+1-\lambda_t} = w^{\beta+1}$  will assume its maximum value. With the potential wage rate differential being at the maximum, a firm that decides to change the strategy to a higher wage one will be able to extract the largest possible labor productivity, since the labor extraction function will be given by  $w^{\beta+1}$ . Figure 2 exposes these properties.

Three intuitions can be taken as possible to justify a higher extraction of labor:

- The average wage can be seen by the worker as a measure of gains outside the firm. If a worker is offered a higher wage rate than the average one, then this worker puts an additional effort when comparing to the effort if a lower wage rate was offered.
- The average wage can also be seen as the conventional reference point with which the higher wage rate will be compared when determining how much above-normal effort will be provided. Above-normal wage rate then extract above-normal effort.

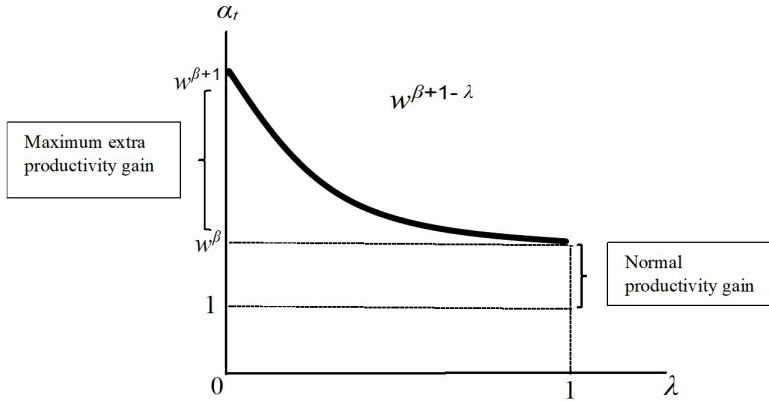


Figure 2 – Extra labor extraction function

- This conventional reference point can also be seen as composing the worker's expectation on wage. Therefore, a higher wage will justify an above-normal effort. There is evidence that wage expectations interfere in the effort of a worker. The study of Abeler et al. (2011) is an example of experimental evidence.

The first section of this chapter will be dedicated to showing the satisficing evolutionary dynamics of the model. The second section will deal with the long run as an evolutionary equilibrium, showing the existence of equilibria in the long run, their stability properties and the long-run macroeconomic implication of the evolutionary dynamics.

### 3.1 TRANSITION BETWEEN SHORT AND LONG-RUN EQUILIBRIA: A SATISFICING EVOLUTIONARY DYNAMICS

The contributions of Herbert Simon (1955, 1956 and 2017) will be used to describe the evolutionary 'satisficing' imitation dynamics of the model, producing the law of motion of the proportion of higher wage strategy firms,  $\lambda_t$ . The satisficing theory, as put by Simon (1997, p. 295):

Faced with a choice situation where it is impossible to optimize, or where the computational cost of doing so seems burdensome, the decision maker may look for a satisfactory, rather than an optimal alternative. Frequently, a course of action satisfying a number of constraints, even a sizeable number, is far easier to discover than a course of action maximizing some function.

The example has been given of searching for a needle in a haystack. Given a probability density distribution of needles of varying degrees of sharpness throughout the haystack, searching for the sharpest needle may require effort proportional to the size of the haystack. The task of searching for a needle sharp enough to sew with requires an effort that depends only on the density of needles of the requisite sharpness, and not at all on the size of the stack. The attractiveness of the satisficing criterion derives from this independence of search cost from the size and complexity of the choice situation.

An  $h$ -firm takes its current profit rate given by (2.11) and compares it to the profit rate it considers acceptable, denoted by  $\rho^i = \pi^i k^i$ , being  $\pi^i$  the profit share associated with the acceptable profit rate, for a given  $k^i$ . Let  $t$  be the current period, if  $r_{h,t}^i \geq \rho^i = \pi^i k^i$ , this  $h$ -firm does not consider changing its strategy in  $t+1$ . On the other hand, if  $\rho^i > r_{h,t}^i$ , the  $h$ -firm  $i$  in question then becomes a strategy reviser. The acceptable profit rate of a firm depends, among other things, on idiosyncratic features, so it will be considered as randomly and independently determined across firms and over time.

The profit share  $\pi^i$  will be assumed as a random variable with cumulative distribution  $F : \mathbb{R} \rightarrow [0, 1] \subset \mathbb{R}$  which is continuously differentiable and strictly increasing. With this assumption, the probability of randomly choosing a firm  $i$  in the subpopulation of  $h$ -firms which consider the current profit rate as unacceptable is:

$$Pr(\rho^i > r_{h,t}^i) = Pr(\pi^i > \pi_{h,t}) = 1 - F(\pi_{h,t}). \quad (3.1)$$

As derived in Vega-Redondo (1996), when such satisficing behavior transforms an  $h$ -firm into a potential strategy reviser it will change to the other labor-extraction compensation strategy (here this strategy is the lower wage one) with probability given by the fraction of firms which adopted the alternative strategy before. The imitation effect

here is triggered by the concept of satisficing. With this premise and assuming that random variables related to satisficing and imitation effects are independent from each other, the measure of  $h$ -firms changing to  $\ell$ -firms is:

$$\lambda_t[1 - F(\pi_h(\lambda_t))](1 - \lambda_t). \quad (3.2)$$

On the other hand, the efflux from the population of  $\ell$ -firms changing to  $h$ -firms is:

$$(1 - \lambda_t)Pr(\rho^i > r_{i,t}^i)\lambda_t = (1 - \lambda_t)Pr(\pi^i > \pi_\ell)\lambda_t = (1 - \lambda_t)[1 - F(\pi_\ell)]\lambda_t. \quad (3.3)$$

Subtracting (3.2) from (3.3) results in the evolutionary satisficing imitation dynamics below:

$$\lambda_{t+1} - \lambda_t = \lambda_t(1 - \lambda_t)[F(\pi_h(\lambda_t)) - F(\pi_\ell)]. \quad (3.4)$$

As  $F(\cdot)$  is a strictly increasing function, a rise in the profit rate related to the higher wage strategy in the current period leads to a higher proportion of firms choosing this strategy in the next period. The inverse occurs when the relative profit rate related to the lower wage strategy rises (a higher proportion of firms choosing this strategy in the next period). The evolutionary dynamics in (3.4) shows a mechanism of selection where the proportion of firms playing a strategy varies positively with the 'relative fitness' (profit rate) of the strategy in question.

The result is that the state transition of the economy is guided by the differential equation of (3.4). The state transition of the economy is driven by this satisficing evolutionary dynamics in whose state space is given by  $\Theta = \{\lambda_t \in \mathbb{R} : 0 \leq \lambda_t \leq 1\}$ .

### 3.2 THE LONG RUN AS AN EVOLUTIONARY EQUILIBRIUM

We will show that satisficing dynamics (3.4) has two long-run equilibria, which are characterized by the survival of only one of the labor-extraction compensation strategy in each of them. These pure-strategy equilibria can also be called monomorphic equilibria. It will also be shown the possibility of existence of a mixed strategy equilibrium in the long run characterized by the survival of both strategies, also called polymorphic equilibrium.

The situation where  $\lambda_{t+1} = \lambda_t = 0$  for any  $t \in \{0, 1, 2, \dots\}$  satisfies (3.7). In this equilibrium all firms play the lower wage strategy. Based on (2.3) it follows that labor productivity differential assumes the value  $\alpha_t = w^{\beta+1} > 1$  for all  $t \in \{0, 1, 2, \dots\}$ .

The situation where  $\lambda_{t+1} = \lambda_t = 1$  for any  $t \in \{0, 1, 2, \dots\}$ , the satisficing evolutionary dynamics (3.7) is also satisfied. In this equilibrium all firms play the higher wage labor-extraction compensation strategy. In that situation, the labor productivity differential (2.3) assumes the value  $\alpha_t = w^\beta \in (1, w^{\beta+1}) \subset \mathbb{R}$  for all  $t \in \{0, 1, 2, \dots\}$ .

The last possible equilibrium is defined by the condition  $\lambda_{t+1} = \lambda_t = \lambda^* \in (0, 1) \subset \mathbb{R}$  for all  $t \in \{0, 1, 2, \dots\}$ . As  $F(\cdot)$  is continuous and strictly increasing, the profit share must be equalized. Then, using (2.7) and (2.8) we are able to obtain the mixed strategy equilibrium as follows:

$$\begin{aligned} \frac{w_h}{w_\ell} = w^{\lambda^* - \beta - 1} &\iff 1 = w^{\lambda^* - \beta} \\ &\iff \lambda^* = \beta. \end{aligned} \quad (3.5)$$

When  $\lambda_t = \lambda^* = \beta$ , then (2.3) assumes the value  $\alpha_t = w$ . As expected, the equilibrium with the highest value of labor productivity is the monomorphic equilibrium with the survival of only the lower wage strategy. In second place comes the polymorphic equilibrium. And in the last place comes the monomorphic equilibrium where all firms play the higher wage strategy.

If we assume that  $F(\cdot)$  follows an uniform distribution, we can rewrite (3.4) as:

$$\lambda_{t+1} - \lambda_t = \lambda_t(1 - \lambda_t)(\pi_h(\lambda_t) - \pi_\ell). \quad (3.6)$$

Using (2.7) and (2.8) we are able to rewrite this equation:

$$\begin{aligned} \lambda_{t+1} &= \lambda_t + \lambda_t(1 - \lambda_t) \left( \left( 1 - u_\ell \frac{w}{\alpha_t} \right) - (1 - u_\ell) \right) \\ &= \lambda_t + \lambda_t(1 - \lambda_t)(1 - w^{\lambda_t - \beta})u_\ell \equiv h(\lambda_t; w, \beta, u_\ell). \end{aligned} \quad (3.7)$$

The stability properties of the long-run equilibria can be studied using the first-order derivative of (3.7):

$$\frac{\partial h(\lambda_t; w, \beta, u_\ell)}{\partial \lambda_t} = 1 + (1 - 2\lambda_t)(1 - w^{\lambda_t - \beta})u_\ell - \lambda_t(1 - \lambda_t)w^{\lambda_t - \beta}(\ln w)u_\ell. \quad (3.8)$$

In the long-run equilibrium with  $\lambda_t = 0$  we have:

$$\frac{\partial h(0; w, \beta, u_\ell)}{\partial \lambda_t} = 1 + (1 - w^{-\beta})u_\ell. \quad (3.9)$$

This derivative is greater than one because  $(1 - w^{-\beta})u_\ell > 0$  since  $\beta \in (0, 1) \subset \mathbb{R}$  and  $u_\ell \in (0, 1) \subset \mathbb{R}$ . Thus, the monomorphic equilibrium with the survival of only the lower wage strategy ( $\lambda = 0$ ) is unstable.

The same analysis must be done to the other monomorphic equilibrium. When  $\lambda_t = 1$  the following result appears:

$$\frac{\partial h(1; w, \beta, u_\ell)}{\partial \lambda_t} = 1 + (1 - w^{\beta - 1})u_\ell. \quad (3.10)$$

The derivative is also greater than one because  $(1 - w^{\beta - 1})u_\ell > 0$  since  $\beta \in (0, 1) \subset \mathbb{R}$  and  $u_\ell \in (0, 1) \subset \mathbb{R}$ . Therefore, the monomorphic equilibrium with the higher wage strategy ( $\lambda = 1$ ) is also unstable.

Finally, analyzing the polymorphic equilibrium, there is a different scenario. When  $\lambda_t = \lambda^* = \beta$  the derivative (3.8) becomes:

$$\frac{\partial h(\beta; w, \beta, u_\ell)}{\partial \lambda_t} = 1 - \beta(1 - \beta)(\ln w)u_\ell. \quad (3.11)$$

This derivative can assume a value  $|\frac{\partial h(\beta; w, \beta, u_\ell)}{\partial \lambda_t}| \leq 1$ . So it is possible to assume that the equilibrium  $\lambda_t = \lambda^* = \beta$  is locally stable.

If this derivative is inside the unit circle, then the polymorphic equilibrium  $\lambda_t = \lambda^* = \beta$  is locally asymptotically stable. By direct calculation, we can conclude that this assertion will be true if  $w \in (1, \bar{w}) \subset \mathbb{R}$ , where:

$$\bar{w} \equiv e^{\frac{2}{(1-\beta)\beta u_\ell}} \quad (3.12)$$

### 3.2.1 Existence of a flip bifurcation in the polymorphic equilibrium and its empirical irrelevance

Bifurcations may occur if (3.11) assumes the value  $-1$  or  $1$ . As it has been shown, the derivative is strictly less than one for any  $\beta \in (0, 1) \subset \mathbb{R}$ ,  $w \in (1, \infty) \subset \mathbb{R}$ , and  $u_\ell \in (0, 1) \subset \mathbb{R}$ , it is impossible to

have  $\partial h(\lambda_t; w, \beta, u_\ell) / \partial \lambda_t = 1$  because  $\beta(1 - \beta)(\ln w)u_\ell > 0$ . Thus, this category of bifurcation will not occur.

However, the derivative (3.11) can assume the value -1. By direct calculation, we can show that there is a unique value of bifurcation of the wage differential given by  $w = \bar{w}$ . Actually, in this value of bifurcation, there is not only a change of stability, but also the emergence of cycles, as it will be presented ahead.

In fact, as we have shown before, at  $w = \bar{w}$ , we have that 3.11 assumes the value  $-1$ , which is a necessary condition to attest the existence of a flip bifurcation. Medio and Lines (2003, p. 156) define a flip bifurcation, in a map  $G$ , as:

The bifurcation of a fixed point of  $G$  occurring when its eigenvalue passes through minus one, the nonzero fixed point loses its stability and a stable period-2 cycle is born, is called a flip bifurcation.

Thus, analyzing with plausible values of the other parameters of the model, it is possible to see that the value of wage differential which a flip bifurcation will occur in the system is not economically relevant. To illustrate the flip bifurcation and its irrelevance to economical analysis, we assume the parameters of the model as  $\beta = 0.5$ , the wage share of income of the  $\ell$ -firms,  $u_\ell = 0.8$ ; the wage differential,  $w = 1.1$ , the capitalists' propensity to save,  $\gamma = 0.4$ , the output to capital ratio,  $k = 0.3$  and an initial share of  $h$ -firms,  $\lambda_0 = 0.5$ . With these values, then the relation between higher wages and lower wages is given by:

$$\bar{w} = 22026, 5. \quad (3.13)$$

Figure 3 illustrates this behavior.<sup>1</sup>

As exemplified, a flip bifurcation will only occur in extremely high values of the wage differential. Even if this theoretical possibility of a flip bifurcation exists, it is not economically plausible. The next subsection will analyze the relevant scenario, in other words, where the wage differential assumes a value below the bifurcation value. Summing up, the satisficing dynamics (3.7) shows a polymorphic equilibrium globally asymptotically stable for economically plausible values of the wage differential.

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<sup>1</sup>The numerical simulations and the respective graphical representation were obtained with the software E&F Chaos, version 1.03, available at <http://cendef.uva.nl/software/ef-chaos/ef-chaos.html>. The source code used to generate the given results is obtained upon request to the author.

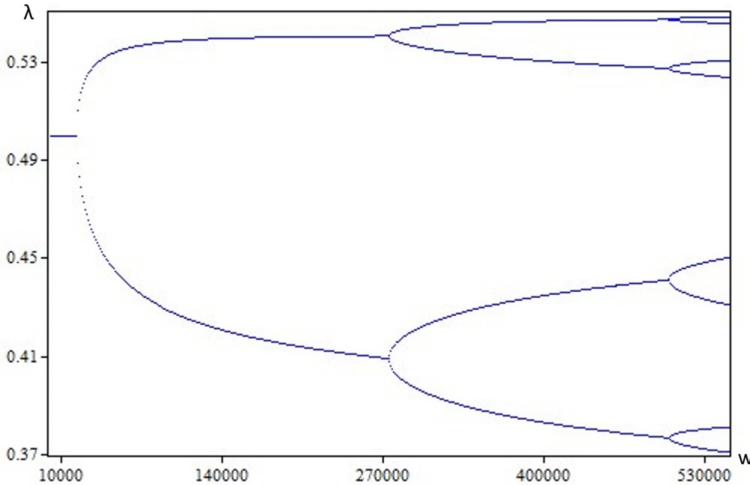


Figure 3 – Existence of flip bifurcation

### 3.2.2 Long-run macroeconomic features

As explained in the previous section, for any  $\lambda_t \in (0, 1) \subset \mathbb{R}$ , the system converges to the polymorphic equilibrium for economically plausible values of the wage differential. Following the conclusion of Section 2.1, summed up in Figure 1, and knowing that the equilibrium given by  $\lambda_t = \lambda^* = \beta$  is asymptotically stable, we can express the income distribution behavior during the transitional dynamics toward the long-run equilibrium in Figure 4.

If we observe this graphics it is possible to obtain two major conclusions. When the initial value of  $\lambda_t < \bar{\lambda}$ , the convergence to the long-run equilibrium is given by an increasing value of  $h$ -firms. Being the economy in this situation, although the average wage rate is also increasing, the average wage share in the income distribution initially decreases (for  $\lambda_t < \bar{\lambda}$ ). The economy reaching the critical number of  $h$ -firms ( $\bar{\lambda}$ ), the income distribution behavior changes, and the average profit share decreases until it reaches the value that would emerge if all firms played the lower wage strategy.

Using (2.14) we can observe the reason for this behavior. We can split the behavior when there is a variation of  $\lambda_t$  in two opposite effects. The first is the profit differential effect in the income. The second is



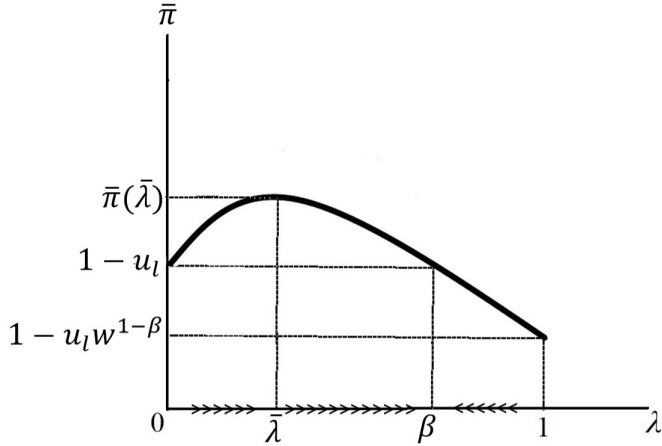


Figure 4 – Behavior of income distribution along transitional dynamics

the compression in the fall of productivity over  $\pi_h(\lambda_t)$  induced by the increasing number of  $h$ -firms. The first effect tends to overcome the second when the fraction of  $h$ -firms is low ( $\lambda_t < \bar{\lambda}$ ). But when it hits the critical value  $\bar{\lambda}$ , this balance shifts.

This shift occurs because of the behavior of the productivity differential. Figure 5 sums up how this behavior affects the economy.

Until the critical value of  $\lambda_t = \bar{\lambda}$ , there is a sharp decline in the productivity differential. After it reaches this fraction of  $h$ -firms, it declines in a decreasing rate.

We can observe that the shift in the income distribution behavior does not lead the economy to a better situation for the workers when comparing to the situation where all firms play the lower wage strategy. However, during the transition where the fraction of  $h$ -firms increases, even though there is an increasing value of the average wage, the workers' share in the income initially decreases. We can call that a "higher wage fetishism".

The second insight occurs when the initial fraction of  $h$ -firms is  $\lambda_t > \beta$ . The satisficing evolutionary dynamics leads the economy to a gradual reduction of  $h$ -firms. During this convergence towards the equilibrium, there is an increasing participation of profits in the distribution of income, while the workers' share in the income gets worse.

Workers would be in a better situation if all firms played the

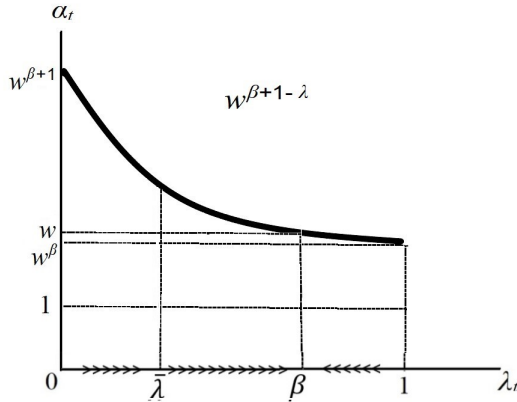


Figure 5 – Behavior of the productivity differential along transitional dynamics

higher wage strategy ( $\lambda_t = 1$ ), but the dynamics is guided by the profit. In the long run we have  $\lambda_t = \beta < 1$ , where the individual profit rates are equalized, as expected in a classical setting. On the other hand, the capitalists would be better if an excess of  $h$ -firms did not exist ( $\lambda_t = \bar{\lambda}$ ). In this situation, however, there is an heterogeneity of profit rates across firms so that the satisficing evolutionary dynamics leads to the uniformity of profit shares, ( $\pi_h = \pi_\ell = 1 - u_\ell$ ), which is expected in a classical framework.

Concerning the average growth rate, we can observe that its behavior follows the average profit rate behavior, as explained in Chapter 3. The convergence to the polymorphic equilibrium when the fraction of  $h$ -firms is lower than the critical value  $\bar{\lambda}$  results in an initial increase in the growth rate. After reaching the critical value of  $h$ -firms, the behavior shifts towards the same average growth rate that would happen if all firms played the lower wage strategy. On the other hand, when the fraction of  $h$ -firms is higher than the equilibrium value,  $\lambda_t = \beta$ , the average wage rate monotonically increases until the equilibrium value is reached.

## CONCLUSION

The motivation of this study was the extensive list of empirical evidence showing the persistence of wage differentials across time. Firms seem to adopt different strategies of wage compensation (either with higher wages or other mechanisms) and the observable factors of workers (such as education) do not fully explain why this happens.

Unobservable factors seem to complete this explanation. And labor productivity is one of the main factors in this category to be analyzed. Laboratorial and survey evidence show that workers are more motivated in their tasks when receiving higher wages and employers pay higher wages because they expect a higher productivity in return.

The theory also sustains that labor productivity and wages are related. And to verify the effects of this relation, it was proposed a classical-Marxian model of growth to analyze the impacts of wage differentials in the economy.

The model follows an evolutionary satisficing dynamics which assumes that a number of employers may revise their strategy if the acceptable profit rate is superior to their own.

The results suggest that an evolutionary dynamics will sustain three possible equilibriums in the economy. Two monomorphic equilibriums where either all firms play the lower wage strategy or all firms play the higher wage strategy. And one polymorphic equilibrium where there is a frequency distribution of workers' labor-extraction compensation strategies that equalizes the profit of the firms playing the lower wage strategy and the higher wage strategy.

When focusing on the stability of the model, it is possible to assume that both monomorphic equilibriums are unstable, which means that if there is a small change, the economy will not be attracted to them.

On the other hand, when the polymorphic equilibrium is analyzed, it seems that - with plausible values of the variables in the economy - there is an asymptotically stable equilibrium, so the economy is attracted to it. If the values extrapolate the plausible ones, it is possible that there is a flip bifurcation in this dynamics or stable fluctuations.

The results show that, if we consider the polymorphic equilibrium as a stable equilibrium, the economy would follow a path where the income distribution is equal to the one when all firms play the lower wage strategy. Although it seems that there would be no changes

to workers and capitalists, during the transition towards the long-run equilibrium, the wage participation in the income distribution declines until a certain fraction of  $h$ -firms arises. This shows that, during the transition, workers would be in a worse situation than before.

Even though derived in a specific economic setting, this result is important to understand why wage differentials remain present in the economy across times and this differentiation of homogeneous workers leads to inequality in the income.

Experiments using computational programs may have a potential to analyze the behavior of the system with different values of variables and possible patterns. Even if unrealistic, the possibility of a  $w$  so high that a flip bifurcation occurs may be a sign of possible chaos in the model.

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