94. livro

95. a) $\log_2 x = \log_2 3 \Rightarrow x = 3$

b) $\log_3 (x-2) = \log_3 4 \Rightarrow (x-2) = 4 \Rightarrow x = 4+2 = x = 6$

c) $\log_{\frac{1}{2}} (2x-1) = \log_{\frac{1}{2}} 3 \Rightarrow (2x-1) = 3 \Rightarrow 2x = 3+1 = 2x = 4 \Rightarrow$
$x = 2$

96. a) $\log_3 x > \log_3 1/2$ p/ $\boxed{x > \frac{1}{2}}$

b) $\log_{\frac{1}{2}} x < \log_{\frac{1}{2}} 3$ p/ $\boxed{x > 3}$

c) $\log_2 (x-1) > \log_2 5$ p/ $x - 1 > 5$
$x > 5 + 1$
$\boxed{x > 6}$

21/03

97. a) $f(x) = \log_2 (2x-1)$
por definição o logaritmando tem que ser > 0.

$2x - 1 > 0$
$2x > 1$
$x > \frac{1}{2}$

$D = \{x \in \mathbb{R} / x > 1/2\}$

b) $f(x) = \log_3 (x^2 - 3x + 2)$

$x^2 - 3x + 2 > 0$

$+ \quad 1 \quad - \quad 2 \quad +$

$D = \{x \in \mathbb{R} / x > 2 \text{ e } x < 1\}$

98. a) $f(x) = \log_3 (x+1)$

$x + 1 > 0$

$\boxed{x > -1}$

$D = \{x \in \mathbb{R} / x > -1\}$

b) $f(x) = \log_{\frac{1}{2}} (1 - 3x)$

$1 - 3x > 0$

$-3x > -1$

$3x < 1$

$x < \frac{1}{3}$

$D = \{x \in \mathbb{R} / x < \frac{1}{3}\}$

c) $f(x) = \log_4 (x^2 - x - 2)$

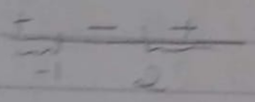
$x^2 - x - 2 > 0$

$\Delta = 1 + 8$

$\Delta = 9$

$x = \frac{1 \pm 3}{2}$ $x' = 2$

$x'' = -1$



$D = \{x \in \mathbb{R} / x < -1 \text{ e } x > 2\}$

d) $f(x) = \log_x (-2x^2 + 5x - 2)$

$-2x^2 + 5x - 2 > 0$

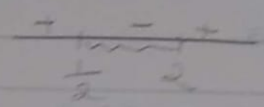
$2x^2 - 5x + 2 < 0$

$\Delta = 25 - 16$

$\Delta = 9$

$x = \frac{5 \pm 3}{4}$ $x' = 2$

$x'' = \frac{1}{2}$

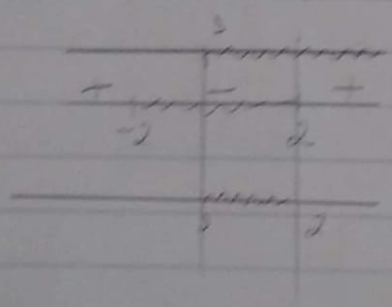


$D = \{x \in \mathbb{R} / \frac{1}{2} < x < 2\}$

99. $f(x) = \log_2 (x-1) + \log_2 (4 - x^2)$

$x - 1 > 0 \Rightarrow x > 1$

$4 - x^2 > 0 \Rightarrow -x^2 > -4 \Rightarrow x^2 < 4 \Rightarrow x < \pm 2$



$D = \{x \in \mathbb{R} / x > 1 \text{ e } x < 2\}$

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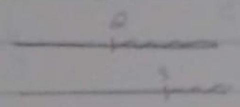
100. a) $f(x) = \log_{2}^{(x-2)} + \log_{2}^{(x-3)}$

$x - 2 > 0 \Rightarrow x > 2$

$x - 3 > 0 \Rightarrow x > 3$

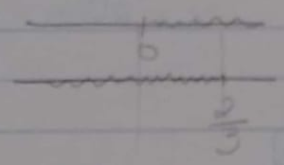


$D = \{x \in \mathbb{R} / x > 3\}$

b) $f(x) = \log_{3}^{x} - \log_{3}^{(2-3x)}$

$x > 0$

$2 - 3x > 0 \Rightarrow -3x > -2 \Rightarrow 3x < 2 \Rightarrow x < \frac{2}{3}$



$D = \{x \in \mathbb{R} / x > 0 \text{ e } x < 2/3\}$

c) $y = \log_{2}^{x} - \log_{2}^{(x^2-5x+6)}$

$x > 0$

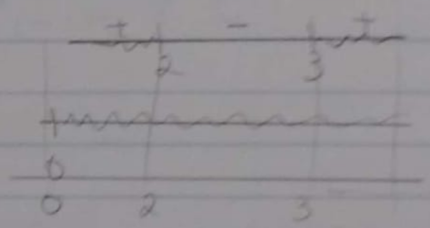
$x^2 - 5x + 6 > 0$

$\Delta = 25 - 24$

$\Delta = 1$

$x = \frac{5 \pm 1}{2}$ $x' = 3$

$x'' = 2$



$D = \{x \in \mathbb{R} / x < 2 \text{ e } x > 0 \text{ ou } x > 3\}$

Extras

- Construir os gráficos:

a) $f(x) = \log_{2}^{|x|}$

b) $f(x) = |\log_{2}^{x}|$

c) $f(x) = |\log_{2}^{|x|}|$

Equações logarítmicas

101.

a) $\log_3 (x^2-x) = \log_3 2$

$x^2 - x = 2$

$x^2 - x - 2 = 0 \rightarrow x' = -1$

$\rightarrow x'' = 2$

Verificação

$x^2 - x > 0$

$(-1)^2 - (-1) > 0$

$1^2 + 1 > 0$ (V)

$x^2 - x > 0$

$2^2 - 2 > 0$

$4 - 2 > 0$ (V)

$S = \{-1, 2\}$

b) $\ln (2x) = \ln (x^2-3)$

$2x = x^2 - 3$

$x^2 - 2x - 3 = 0$

$\Delta = 4 + 12$

$\Delta = 16$

$x = \dfrac{2 \pm 4}{2}$ $\quad x' = 3$

$x'' = -1$

Verificação

$2x > 0$

$2.(-1) > 0$

$-2 > 0$ (F)

$2.3 > 0$

$6 > 0$ (V)

$S = \{3\}$

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102. a) $\log_{10}^{(3x-1)} = \log_{10}^{5}$

$3x - 1 = 5$

$3x = 6$

$x = \frac{6}{3}$

$x = 2$

Verificação

$3x - 1 > 0$

$3.2 - 1 > 0$

$6 - 1 > 0$

$S = \{2\}$

Exercícios Extras

a) $f(x) = \log_2^{|x|}$

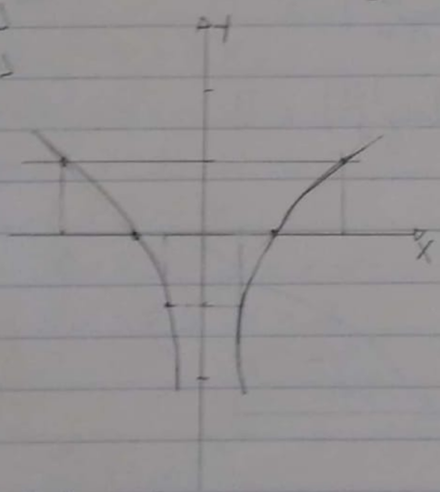
x	y
-1	0
-2	1
1	0
2	1
$-1/2$	-1

$\log_2^{|-1|} = \log_2^{1} = 2^x = 1 \Rightarrow 2^x = 2^0 \Rightarrow x = 0$

$\log_2^{|-2|} = \log_2^{2} = 1$

$\log_2^{|1|} = \log_2^{1} = 0$

$\log_2^{|2|} = \log_2^{2} = 1$



b) $f(x) = |\log_2^{x}|$

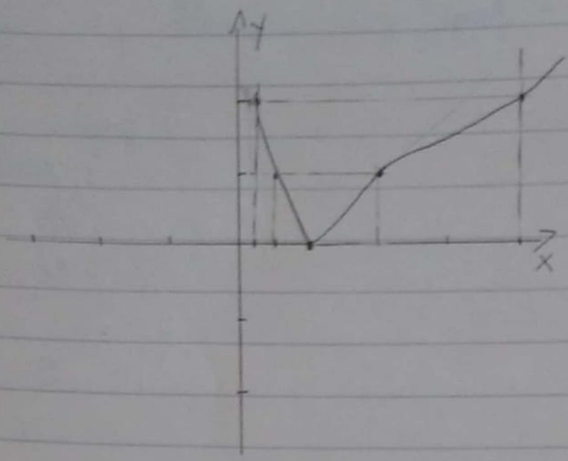
x	y
$1/4$	2
$1/2$	1
1	0
2	1

$|\log_2^{1/4}| \Rightarrow |2^x = \frac{1}{4}| \Rightarrow 2^x = 2^{-2} \Rightarrow |-2| = 2$

$|\log_2^{1/2}| \Rightarrow |2^x = \frac{1}{2}| \Rightarrow 2^x = 2^{-1} \Rightarrow |-1| = 1$

$|\log_2^{1}| \Rightarrow |2^x = 2^0| \Rightarrow |0| \Rightarrow 0$

$|\log_2^{2}| \Rightarrow |1| \Rightarrow 1$



c) $f(x) = \left|\log_2^{|x|}\right|$

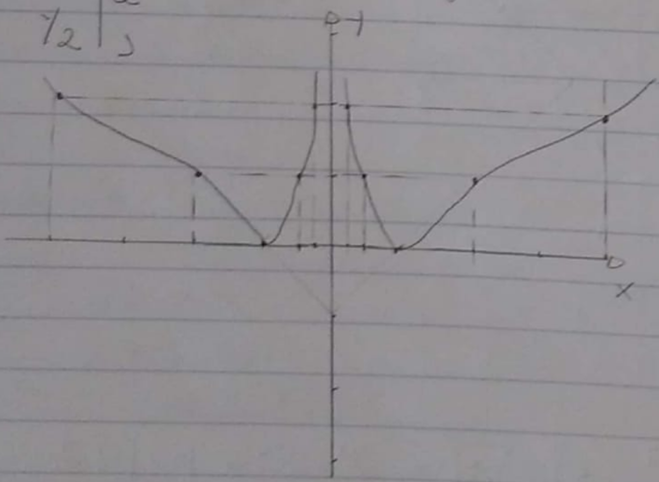
x	y
-2	1
-1	0
1	0
2	1
$-1/4$	2
$-1/2$	1
-4	2
4	2
$1/2$	1

$\left|\log_2^{|-2|}\right| = \left|\log_2^{2}\right| \Rightarrow 1$

$\left|\log_2^{|-1|}\right| = \left|\log_2^{1}\right| \Rightarrow 0$

$\left|\log_2^{|1|}\right| = 0$

$\left|\log_2^{|2|}\right| = 1$



24/03 102. b) $\log_5 x = \log_5 (2x-3)$

$x = 2x-3$

$x - 2x = -3$

$-x = -3$

$x = 3$

Verificação

$\log_5 3 = \log_5 3$

(V)

$S = \{3\}$

c) $\ln(3x+5) = \ln(2x)$

$3.-5$
$-15+5=-10$

$3x + 5 = 2x$

$5 = 2x - 3x$

$5 = -x$

$x = -5$

Verificação

$3.(-5)+5>0$

$-15+5>0$

$-10>0$ (F)

$S = \emptyset$

103. $\log_2 (2x^2-1) = \log_2 (3x+1)$

$2x^2-1 = 3x+1$

$2x^2-3x-1-1=0$

$2x^2-3x-2=0$

$\Delta = 9+16$

$\Delta = 25$

$x = \frac{3\pm 5}{4}$ $x' = 2$

$x'' = -1/2$

$S = \{2\}$

Verificação

$2.(2^2)-1>0$

$2.4-1>0$

$8-1>0$ (V)

$2.\left(-\frac{1}{2}\right)^2-1>0$

$2.\frac{1}{4}-1>0$

$\frac{2}{4}-1>0$

$\frac{2-4}{4}>0$ F

104- $\log_2 (2x^2-5x+1) = 2$ definição

$2x^2-5x+1 = 2^2$

$2x^2-5x-3=0$

$\Delta = 25+24$

$\Delta = 49$

$x = \frac{5 \pm 7}{4}$ $x' = 3$ $x'' = -\frac{1}{2}$

$S = \{-\frac{1}{2}, 3\}$

Verificação

$2.(3^2)-5.3+1 > 0$

$2.9-15+1>0$

$18-15+1>0$

$3+1>0$

(V)

$2.\left(-\frac{1}{2}\right)^2-5\left(-\frac{1}{2}\right)+1>0$

$2.\frac{1}{4}+\frac{5}{2}+1>0$

$\frac{2}{4}+\frac{5}{2}+1>0$

$\frac{2+10+4}{4}>0$

$\frac{16}{4}>0$ (V)

105- a) $\log_{1/3}(2x-1) = -1$ definição

$(2x-1) = \left(\frac{1}{3}\right)^{-1}$

$2x-1=3$

$2x=4$

$x = \frac{4}{2} = 2$

Verificação

$\log_{1/3} 3 : \frac{1}{3}^x = 3 \Rightarrow$

$3^{-x} = 3 \Rightarrow -x=1$

$x=-1$

(V)

b) $\log_2 (x^2-3x) = 2$

$2^2 = x^2-3x$

$4-x^2+3x=0 \Rightarrow x^2-3x-4=0$

$\Delta = 9+16$

$\Delta = 25$

$x = \frac{3 \pm 5}{2}$ $x' = 4$ $x'' = -1$

$16-3.4=$

$16-12=4$

$1-3(-1)$

$1+3=4$

Verificação

$\log_2 4 = 2$ (V) | $\log_2 4 = 2$

(V)

$S = \{4, -1\}$

c) $\log_x 16 = 2$ $\qquad$ $\log_4 16 = 2$ (V)

$x^2 = 16$

$x = 4$ $\qquad$ $S = \{4\}$

106- $5^{\log_3 (2x^2-5x)} = 5^1$

$\log_3 (2x^2-5x) = 1$

$2x^2-5x = 3^1$

$2x^2-5x-3=0$ $\quad x' = -1/2$ $\quad x'' = 3$

Verif

$2x^2-5x > 0$

$2\left(-\frac{1}{2}\right)-5\left(-\frac{1}{2}\right) > 0$

$2 \cdot 3^2 - 5 \cdot 3 > 0$

$S = \{-1/2, 3\}$

107- $\log_{10} x + \log_{10} (2x-1) = \log_{10} 3$

$\log_{10} [x(2x-1)] = \log_{10} 3$

$2x^2 - x = 3$

$2x^2 - x - 3 = 0$

$\Delta = 1 + 24$

$\Delta = 25$

$x = \frac{1 \pm 5}{4}$ $\quad x' = 3/2$ $\quad x'' = -1$

Verif

$x > 0$ (V)

$2x - 1 > 0$

$2 \cdot 3/2 - 1 > 0$

$\frac{6}{2} - 1 > 0$

$\frac{6-2}{2} > 0$ (V)

$2(-1) - 1 > 0$

$-2 - 1 > 0$ (F)

$S = \left\{\frac{3}{2}\right\}$

108- a) $\log_3 x + \log_3 (3x-5) = \log_3 2$

$\log_3 [x.(3x-5)] = \log_3 2$

$3x^2 - 5x = 2$

$3x^2 - 5x - 2 = 0$

$x' = 2$

$\cancel{x'' = -1/3}$

Verif

$x > 0$

$3x - 5 > 0$

$3 \cdot 2 - 5 > 0$ (V)

$\ln = \log_e \quad \lg = \log_{10}$

b) $\log_{10}^{(x+1)} - \log_{10}^{(3x-2)} = \log_{10}^{2}$

$\log_{10}^{\frac{x+1}{3x-2}} = \log_{10}^{2}$

$\frac{x+1}{3x-2} = 2 \Rightarrow x+1 = 6x-4$

$-5x = -5$

$\boxed{x = 1}$

Verif.

$x+1 > 0$

$x > -1$

$3x-2 > 0$

$3x > 2$

$x > 2/3$

$S = \{1\}$

109. $\begin{cases} \log_5^{x} - \log_5^{y} = \log_5^{3} \Rightarrow \log_5^{x/y} = \log_5^{3} = \frac{x}{y} = 3 \Rightarrow \boxed{x = 3y} \\ x - 2y = 3 \end{cases}$

$\begin{cases} x = 3y \\ x - 2y = 3 \Rightarrow 3y - 2y = 3 \Rightarrow \boxed{y = 3} \end{cases}$

$x = 3y = 3 \cdot 3 = 9$

$S = \{(9, 3)\}$

110. a) $\ln^{(x+2)} + \ln^{(5x-1)} = \ln^{(26x-8)}$

$\ln[(x+2)(5x-1)] = \ln^{(26x-8)}$

$(x+2)(5x-1) = (26x-8)$

$5x^2 - x + 10x - 2 = 26x - 8$

$5x^2 + 9x - 2 - 26x + 8 = 0$

$5x^2 - 17x + 6 = 0$

$x' = 3$

$x'' = \frac{2}{5}$

Verif.

$x + 2 > 0$

$3 + 2 > 0$

$2/5 + 2 > 0$

$5x - 1 > 0$

$5 \cdot 3 - 1 > 0$

$5 \cdot 2/5 \cdot 1 > 0$

$26x - 8 > 0$

$26 \cdot 3 - 8 > 0$

$26 \cdot 2/5 \cdot 8 > 0$

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b) $\log_{10}(x^2+x) - \log_{10}(x+1) = \log_{10} 3$

$\log_{10}\left[\frac{(x^2+x)}{(x+1)}\right] = \log_{10} 3$

$\frac{x^2+x}{x+1} = 3$

$3x+3 = x^2+x$

$S=\{3\}$ $3x+3-x^2-x=0$

$-x^2+2x+3=0$

$x^2-2x-3=0$

$\Delta = 4+12$

$\Delta = 16$

$x = \frac{2 \pm 4}{2}$ $x' = 3$ $x'' = -1$

Verificação

$x^2+x > 0$

$3^2+3>0$

$9+3>0$ (V)

$x^2+x>0$

$(-1)^2+(-1)>0$

$1-1>0$ (F)

$x+1>0$

$3+1>0$ (V)

$-1+1>0$ (F)

111- $\begin{cases} \log_2 x + \log_2 y = \log_2 12 \\ x+y=7 \end{cases}$ $\Rightarrow \log_2 x\cdot y = \log_2 12$

$\begin{cases} x+y=7 \\ xy=12 \end{cases}$ $\rightarrow x = \frac{12}{y}$

$\frac{12}{y} + y = 7$

$12+y^2=7y$

$y^2-7y+12=0$ $\rightarrow y'=4$

$\rightarrow y''=3$

$y=4 \rightarrow x=3$

$y=3 \rightarrow x=4$

$S=\{(3,4)(4,3)\}$

112- $\log_2 x + \log_2 (x-1) = 1$

$\log_2 [x(x-1)] = 1$

$x(x-1) = 2^1$

$x^2-x-2=0$

$\Delta = 1+8=9$

$x = \frac{1\pm 3}{2}$ $x'=2$ $x''=-1$

Verif

$x>0$

$-1>0$

(F)

$x-1>0$

$1>0$

$S=\{2\}$

113- a) $\log_2(x-3) + \log_2(x-2) = 1$

$\log_2[(x-3)\cdot(x-2)] = 1$

$(x-3)(x-2) = 2^1$

$x^2-2x-3x+6-2=0$

$x^2 - 5x + 4 = 0$

$\Delta = 9$

$x' = 4$

$x'' = 1$

Verif

$x - 3 > 0$

$1 > 0$

$-2 > 0$

$x - 2 > 0$

$2 > 0$

$S = \{4\}$

b) $\log_3 (x-1) + \text{colog}_3 (x-2) = 1$

$\log_3 (x-1) - \log_3 (x-2) = 1$

$\log_3 \left(\frac{x-1}{x-2}\right) = 1$ definição

$\frac{x-1}{x-2} = 3^1$

$x - 1 = 3x - 6$

$2x = 5$

$x = \frac{5}{2}$

Verif

$x - 1 > 0$ | $x - 2 > 0$

$\frac{5}{2} - 1 > 0$ | $\frac{5}{2} - 2 > 0$

$\frac{3}{2} > 0$ | $\frac{1}{2} > 0$

(V)

$S = \{5/2\}$

114. $\log_3 x^3 + \log_3 (2x+1)^3 = 3$

$\log_3 [x^3(2x+1)^3] = 3$

$x^3(2x+1)^3 = 3^3$

$[x(2x+1)]^{\cancel{3}} = 3^{\cancel{3}}$

$2x^2 + x = 3$

$2x^2 + x - 3 = 0$

$x' = \cancel{\frac{-3}{2}}$ $\quad x'' = 1$

Verif

$x^3 > 0$

$(2x+1)^3 > 0$

$S\{1\}$

28/03 Equações Exponenciais

$5^x = 4 \Rightarrow x = \log_5 4$

Exercícios - 2ª lista de exercícios

1) Resolver as equações

a) $3^x = \frac{1}{2} \Rightarrow x = \log_3 \frac{1}{2}$ $\quad S = \{\log_3 \frac{1}{2}\}$

b) $7^{\sqrt{x}} = 2 \Rightarrow (\sqrt{x})^2 = (\log_7 2)^2$

$x = (\log_7 2)^2$

c) $3^{(x^2)} = 5$

$x^2 = \log_3 5 \Rightarrow x = \pm\sqrt{\log_3 5}$ $\quad S = \{\pm\sqrt{\log_3 5}\}$

d) $5^{4x-3} = 0,5$

$\frac{5^{4x}}{5^3} = 0,5$

$\frac{625^x}{125} = 0,5$

$625^x = 62,5$

$x = \log_{625} 62,5$

$S = \{\log_{625} 62,5\}$

f) $3^{2x+1} = 2$

$3^{2x} \cdot 3^1 = 2$

$9^x = \frac{2}{3}$

$x = \log_9 \frac{2}{3}$ $\quad S = \{\log_9 \frac{2}{3}\}$

2) Resolver as equações:

a) $2^x = 3^{x+2}$

$2^x = 3^x \cdot 3^2$

$\frac{2^x}{3^x} = 9 \Rightarrow \left(\frac{2}{3}\right)^x = 9 \Rightarrow x = \log_{2/3} 9$ $S = \{\log_{2/3} 9\}$

b) $7^{2x-1} = 3^{3x+4}$

$\frac{7^{2x}}{7} = 3^{3x} \cdot 3^4$

$\frac{49^x}{7} = 27^x \cdot 81$

$\frac{49^x}{27^x} = 81 \cdot 7$

$\left(\frac{49}{27}\right)^x = 567$

$x = \log_{\frac{49}{27}} 567$ $S = \{\log_{\frac{49}{27}} 567\}$

2º lista de exercícios

1) a) $5^x = 4$

$x = \log_5 4$ $S = \{\log_5 4\}$

b) $3^x = \frac{1}{2}$

$x = \log_3 \frac{1}{2} \Rightarrow x = \log_3 2^{-1} \Rightarrow x = -\log_3 2$ $S = \{-\log_3 2\}$

c) $7^{\sqrt{x}} = 2$

$(\sqrt{x})^2 = (\log_7 2)^2$

$x = (\log_7 2)^2$ $S = \{(\log_7 2)^2\}$

d) $3^{(x^2)}$

3) Resolver as equações:

a) $3^x = 2^x + 2^{x+1} \Rightarrow 3^x = 2^x + 2^x \cdot 2^1$

$3^x = 2^x(1+2)$

$\frac{3^x}{2^x} = 1+2$

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$\left(\frac{3}{2}\right)^x = 3 \Rightarrow \log_{3/2} 3$ $\quad S = \{\log_{3/2} 3\}$

b) $5^x + 5^{x+1} = 3^x + 3^{x+1} + 3^{x+2}$

$5^x + 5^x . 5 = 3^x + 3^x . 3^1 + 3^x . 3^2$

$5^x(1+5) = 3^x(1+3+9)$

$\frac{5^x}{3^x} = \frac{1+3+9}{1+5}$

$\frac{5^x}{3^x} = \frac{13}{6}$ $\quad S = \{\log_{5/3} 13/6\}$

$\left(\frac{5}{3}\right)^x = \frac{13}{6} \Rightarrow x = \log_{5/3} 13/6$

1/04 4) $2^{3x+2} . 3^{2x-1} = 8$

$(2^{3x} . 2^2) . \left(\frac{3^{2x}}{3}\right) = 8$

$(8^x . 4) . \left(\frac{9^x}{3}\right) = 8$

$8^x . 9^x = \frac{8.3}{4}$

$8^x . 9^x = 6$

$72^x = 6 \Rightarrow x = \log_{72} 6$ $\quad S = \{\log_{72} 6\}$

5) a- $4^x - 5 . 2^x + 6 = 0$

$2^{2x} - 5 . 2^x + 6 = 0$ $\quad 2^{2x} = y^2$

$y^2 - 5y + 6 = 0$ $\quad 2^x = y$

$\Delta = 25 - 24$

$\Delta = 1$

$y = \frac{5 \pm 1}{2}$ $\quad y' = 3$, $y'' = 2$

se $2^x = y \Rightarrow 2^x = 3 \Rightarrow x = \log_2 3$

$\Rightarrow 2^x = 2 \Rightarrow x = \log_2 2 = 1$

$S = \{\log_2 3, 1\}$

b) $4^x - 6.2^x + 5 = 0$

$2^x = y$

$2^{2x} = y^2$

$2^{2x} - 6.2^x + 5 = 0$

$y^2 - 6y + 5 = 0$

$\Delta = 36 - 20$

$\Delta = 16$

$y = \frac{6 \pm 4}{2}$ $y' = 5$ $y'' = 1$

$S = \{\log_2 5, 0\}$

se $2^x = 5$ $x = \log_2 5$

$2^x = 1$ $\log_2 1 = 0$

c) $9^x - 3^{x+1} - 4 = 0$

$3^{2x} = y^2$

$3^x = y$

$3^{2x} - 3^x . 3 - 4 = 0$

$y^2 - 3y - 4 = 0$

$\Delta = 9 + 16$

$\Delta = 25$

$y = \frac{3 \pm 5}{2}$ $y' = 4$ $y'' = -1$

$S = \{\log_3 4\}$

se $3^x = 4 \Rightarrow \log_3 4$

$3^x = -1 \Rightarrow \log_3 -1$

d) $3^{2x+1} - 3^{x+1} + 2 = 0$

$3^{2x} = y^2$

$3^x = y$

$3^{2x} . 3 - 3^x . 3 + 2 = 0$

$3y^2 - 3y + 2 = 0$

$\Delta = 9 - 24$

$\Delta < 0$

$S = \emptyset$

c) $4^{x+1} - 2^{x+4} + 15 = 0$

$4^x \cdot 4^1 - 2^x \cdot 16 + 15 = 0$

$2^{2x} \cdot 4 - 2^x \cdot 16 + 15 = 0$

$4y^2 - 16y + 15 = 0$

$\Delta = 256 - 240$

$\Delta = 16$

$2^{2x} = y^2$

$2^x = y$

$y = \dfrac{16 \pm 4}{8}$ $y' = 5/2$ $y'' = 3/2$

$2^x = \dfrac{5}{2} \Rightarrow x = \log_2^{5/2}$

$2^x = \dfrac{3}{2} \Rightarrow x = \log_2^{3/2}$

$S = \{\log_2^{5/2}, \log_2^{3/2}\}$

(4)

6. a) $4^x + 6^x = 9^x$

$4^x = 9^x - 6^x$

$2^{2x} = 3^{2x} - (2 \cdot 3)^x$

$2^{2x} = 3^{2x} - (2^x \cdot 3^x)$

$2^{2x} = 3^{2x} - 2^x\, 3^x$

$\dfrac{2^{2x}}{2^{2x}} = \dfrac{3^{2x}}{2^{2x}} - \dfrac{2^x \cdot 3^x}{2^{2x}}$

$1 = \dfrac{3^{2x}}{2^{2x}} - \dfrac{3^x}{2^x}$

$\left(\dfrac{3}{2}\right)^x = y$

$1 + \left(\dfrac{3}{2}\right)^x = \left(\dfrac{3}{2}\right)^{2x}$

$1 + y = y^2$

$-y^2 + y + 1 = 0$

$y^2 - y - 1 = 0$

$\Delta = 1 + 4$

$\Delta = 5$

$y = \dfrac{1 \pm \sqrt{5}}{2}$ $y' = \dfrac{1 + \sqrt{5}}{2}$ $y'' = \dfrac{1 - \sqrt{5}}{2}$

$\left(\dfrac{3}{2}\right)^x = \dfrac{1 + \sqrt{5}}{2}$

$x = \log_{\frac{3}{2}}^{\frac{1+\sqrt{5}}{2}}$

$\left(\dfrac{3}{2}\right)^x = \dfrac{1 - \sqrt{5}}{2}$

$\cancel{x = \log_{\frac{3}{2}}^{\frac{1-\sqrt{5}}{2}}}$ → negativo

$S = \{\log_{\frac{3}{2}}^{\frac{1+\sqrt{5}}{2}}\}$

b) $4^x = 2 \cdot 14^x + 3 \cdot 49^x$

$\dfrac{2^{2x}}{2^{2x}} = \dfrac{2 \cdot 2^x \cdot 7^x}{2^{2x}} + \dfrac{3 \cdot 7^{2x}}{2^{2x}}$

$1 = 2 \cdot \dfrac{7^x}{2^x} + 3 \cdot \dfrac{7^{2x}}{2^{2x}}$

$1 = 2 \left(\dfrac{7}{2}\right)^x + 3 \left(\dfrac{7}{2}\right)^{2x}$

$\left(\dfrac{7}{2}\right)^x = y$

$1 = 2y + 3y^2$

$3y^2 + 2y - 1 = 0$

$\Delta = 4 + 12$

$\Delta = 16$

$y = \dfrac{-2 \pm 4}{6}$ $y' = 1/3$ $y'' = -1$

$\left(\dfrac{7}{2}\right)^x = \dfrac{1}{3}$

$x = \log_{\frac{7}{2}}^{1/3}$

$S = \{\log_{\frac{7}{2}}^{1/3}\}$

7) $a^{4x} + a^{2x} = 1$ $\qquad a^{2x} = y$

$y^2 + y = 1$

$y^2 + y - 1 = 0$

$y' = \frac{-1+\sqrt{5}}{2}$

$y'' = \frac{-1-\sqrt{5}}{2}$

$a^{2x} = \frac{-1+\sqrt{5}}{2} \Rightarrow x = \log_{a^2} \frac{-1+\sqrt{5}}{2} \Rightarrow x = \frac{1}{2} \log_a \frac{-1+\sqrt{5}}{2}$

8) $\begin{cases} 64^{2x} + 64^{2y} = 40 \\ 64^{x+y} = 12 \end{cases}$

$\begin{cases} 64^{2x} + 64^{2y} = 40 \\ 64^{x} \cdot 64^{y} = 12 \end{cases}$ $\qquad 64^x = m$, $64^y = n$

$\begin{cases} m^2 + n^2 = 40 \\ m \cdot n = 12 \Rightarrow m = \frac{12}{n} \end{cases}$

$\left(\frac{12}{n}\right)^2 + n^2 = 40$

$\frac{144}{n^2} + n^2 = 40$

$\frac{144}{n^2} + \frac{n^4}{n^2} = \frac{40n^2}{n^2}$

$n^4 - 40n^2 + 144 = 0$ $\qquad n^2 = z$

$z^2 - 40z + 144 = 0$

$\Delta = 1600 - 576$

$\Delta = 1024$

$n^2 = 36 \qquad n^2 = 4$

$n = \pm 6 \qquad n = \pm 2$

$z = \frac{40 \pm 32}{2}$ $\qquad z' = \frac{72}{2} = 36$

$z' = \frac{8}{2} = 4$

se $n = 6 \Rightarrow m = \frac{12}{n} \Rightarrow m = \frac{12}{6} \Rightarrow m = 2$

se $n = -6 \Rightarrow m = \frac{12}{-6} \Rightarrow m \cancel{=} -2$

se $n = 2 \Rightarrow m = \frac{12}{2} \Rightarrow m = 6$

se $n = -2 \Rightarrow m = -\frac{12}{2} \Rightarrow m \cancel{=} -6$

Concluindo:

$64^x = m$

$64^x = 2 \Rightarrow x = \log_{64} 2$

$64^x = 6 \Rightarrow x = \log_{64} 6$

$64^y = n$

$64^y = 6$

$y = \log_{64} 6$

$64^y = 2$

$y = \log_{64} 2$

$S = \{(\log_{64} 2, \log_{64} 6), (\log_{64} 6, \log_{64} 2)\}$

8/04

Inequações logarítmicas

Para resolvermos uma inequação logarítmica, além da definição devemos lembrar as propriedades:

a) Quando $a > 1$ a função logarítmica $y = \log_a x$ é crescente

$$\log_a x_2 > \log_a x_1 \Rightarrow x_2 > x_1$$

b) Quando $0 < a < 1$ a função $y = \log_a x$ é decrescente

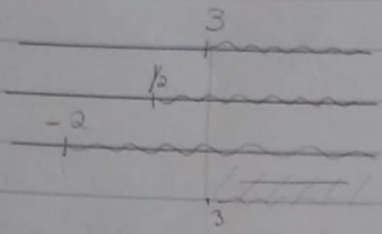
$$\log_a x_2 > \log_a x_1 \Rightarrow x_2 < x_1$$

115) a- $\log_{3}^{(2x-1)} > \log_{3}^{(x+2)}$

$2x-1 > x+2 \Rightarrow 2x - x > 2+1 \Rightarrow \boxed{x > 3}$

$2x-1 > 0 \Rightarrow 2x > 1 \Rightarrow \boxed{x > \frac{1}{2}}$

$x+2 > 0 \Rightarrow \boxed{x > -2}$



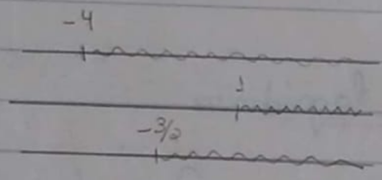
$S = \{x \in \mathbb{R} / x > 3\}$

b- $\log_{1/3}^{(x-1)} > \log_{1/3}^{(2x+3)}$

$x-1 < 2x+3 \Rightarrow x - 2x < 3+1 \Rightarrow -x < 4 \Rightarrow \boxed{x > -4}$

$x-1 > 0 \Rightarrow \boxed{x > 1}$

$2x+3 > 0 \Rightarrow 2x > -3 \Rightarrow \boxed{x > -\frac{3}{2}}$

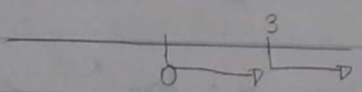


$S = \{x \in \mathbb{R} / x > 1\}$

116- a) $\log_{2}^{x} > \log_{2}^{3}$

$x > 3$

$x > 0$ e $3 > 0$ (V)



$S = \{x \in \mathbb{R} / x > 3\}$

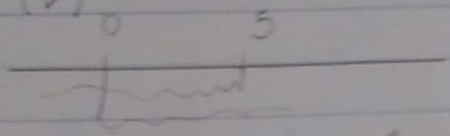
b) $\log_{3}^{x} < \log_{3}^{5}$

$\log_{3}^{5} > \log_{3}^{x}$

$5 > x$

$x > 0$

$5 > 0$ (V)



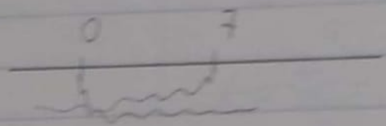
$S = \{x \in \mathbb{R} / 0 < x < 5\}$

c) $\log_{1/2} x > \log_{1/2} 7$

$x < 7$

$x > 0$

$7 > 0$ (V)



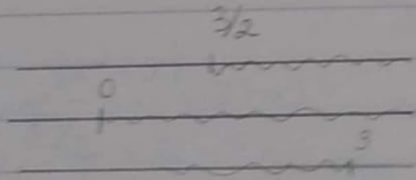
$S = \{x \in \mathbb{R} / 0 < x < 7\}$

d) $\log_{30} x > \log_{30} (3-x)$

$x > 3 - x \Rightarrow x + x > 3 \Rightarrow 2x > 3 \Rightarrow x > \frac{3}{2}$

$x > 0$

$3 - x > 0 \Rightarrow -x > -3 \Rightarrow x < 3$



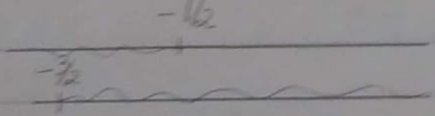
$S = \{x \in \mathbb{R} / \frac{3}{2} < x < 3\}$

e) $\log_{1/2} (2x+3) > \log_{1/2} 2$

$2x + 3 < 2 \Rightarrow 2x < 2 - 3 \Rightarrow x < -\frac{1}{2}$

$2x + 3 > 0 \Rightarrow 2x > -3 \Rightarrow x > -\frac{3}{2}$

$2 > 0$ (V)



$S = \{x \in \mathbb{R} / -3/2 < x < -1/2\}$

117- $\log_3 (x^2+x) \leq \log_3 6$

$\log_3 6 \geq \log_3 (x^2+x)$

$6 \geq x^2+x \Rightarrow x^2+x-6 \leq 0$

$\Delta = 1+24$

$\Delta = 25$

$x = \frac{-1 \pm 5}{2}$ $\quad x' = 2$ $\quad x'' = -3$

$-3 \quad 2$

$+ \quad - \quad +$

$x^2+x > 0$

$\Delta = 1$

$x = \frac{-1 \pm 1}{2}$ $\quad x' = 0$ $\quad x'' = -1$

$-1 \quad 0$

$+ \quad - \quad +$

$-3 \quad 2$

$-1 \quad 0$

$S = \{x \in \mathbb{R} \,/\, -3 \leq x < -1$ ou $0 < x \leq 2\}$

118- $\log_{1/3} (x^2-x) < \log_{1/3} (x+3)$

$\log_{1/3} (x+3) > \log_{1/3} (x^2-x)$

$x+3 < x^2-x \Rightarrow -x^2+x+x-3 < 0$

$-x^2+2x-3 < 0$

$x^2-2x+3 > 0$

$\Delta = 4+12$

$\Delta = 16$

$x = \frac{2 \pm 4}{2}$ $\quad x' = 3$ $\quad x'' = -1$

$-1 \quad 3$

$+$

$x+3 > 0$

$\underline{x > -3}$

$x^2-x > 0$

$\Delta = 1$

$x' = 0$

$x'' = -1$

$-1 \quad 0$

$+ \quad - \quad +$

$S = \{x \in \mathbb{R} / 3 < x < -1$ ou $x > 3\}$

113. a) $\log_3 (x^2-2x) > 1$

$\log_3 (x^2-2x) > \log_3 3$

$x^2 - 2x > 3$

$x^2 - 2x - 3 > 0$

$\Delta = 4 + 12$

$\Delta = 16$

$x = \frac{2 \pm 4}{2}$ $\quad x' = 3$ $\quad x'' = -1$

$x^2 - 2x > 0$

$\Delta = 4 - 0$

$x = \frac{2 \pm 2}{2}$ $\quad x' = 2$ $\quad x'' = 0$

$S = \{x \in \mathbb{R} / x < -1$ ou $x > 3\}$

b) $\log_{1/2} (x^2+x) > -1$

$\log_{1/2} x^2+x > -1 \log_{1/2} 1/2 \quad = \left(\frac{1}{2}\right)^{-1} = 2$

$\log_{1/2} x^2+x > \log_{1/2} 2$

$x^2 + x < 2$

$x^2 + x - 2 < 0$

$\Delta = 1 + 8$

$\Delta = 9$

$x = \frac{-1 \pm 3}{2}$ $\quad x' = 1$ $\quad x'' = -2$

$x^2 + x > 0$

$x' = 0$

$x'' = -1$

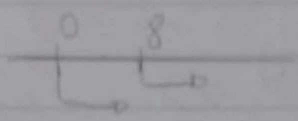
$S = \{x \in \mathbb{R} / -2 < x < -1$ ou $0 < x < 1\}$

120. a) $\log_2 x > 3$

$\log_2 x > \log_2 8$

$x > 8$

$x > 0$



$S = \{x \in \mathbb{R} / x > 8\}$

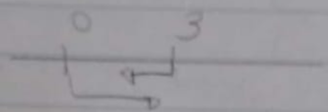
b) $\log_{1/3} x > -1$

$\log_{1/3} x > -1 \log_{1/3} 1/3$

$\log_{1/3} x > \log_{1/3} 3$

$x < 3$

$x > 0$



$S = \{x \in \mathbb{R} / 0 < x < 3\}$

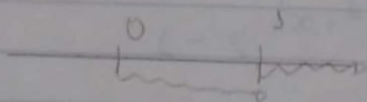
c) $\log_{1/2} x < 0$

$\log_{1/2} x < \log_{1/2} 1$

$\log_{1/2} 1 > \log_{1/2} x$

$1 < x$

$x > 0$



$S = \{x \in \mathbb{R} / x > 1\}$

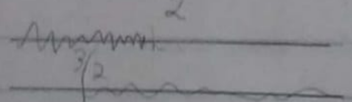
d) $\log_5 (2x-3) < 0$

$\log_5 (2x-3) < \log_5 1$

$\log_5 1 > \log_5 (2x-3)$

$1 > 2x - 3 \Rightarrow 1 + 3 > 2x \Rightarrow \frac{4}{2} > x \Rightarrow 2 > x$

$2x - 3 > 0 \Rightarrow 2x > 3 \Rightarrow x > \frac{3}{2}$



$S = \{x \in \mathbb{R} / 3/2 < x < 2\}$

12) a) $\log_2 (3x^2-5x) > 1$

$\log_2 (3x^2-5x) > \log_2 2$

$3x^2-5x > 2$

$3x^2-5x-2 > 0$

$\Delta = 25+24$

$\Delta = 49$

$x = \frac{5 \pm 7}{6}$ $x' = 2$ $x'' = -\frac{1}{3}$

$3x^2-5x > 0$

$\Delta = 25$

$x = \frac{5 \pm 5}{6}$ $x' = 0$ $x'' = 5/3$

$S = \{x \in \mathbb{R} / x < -1/3$ ou $x > 2\}$

b) $\log_{1/4} (x^2-3x) > -1$

$\log_{1/4} (x^2-3x) > -1 \log_{1/4} 1/4$

$\log_{1/4} (x^2-3x) > \log_{1/4} 4$

$x^2-3x < 4$

$x^2-3x-4 < 0$

$\Delta = 9+16$

$\Delta = 25$

$x = \frac{3 \pm 5}{2}$ $x' = 4$ $x'' = -1$

$x^2-3x > 0$

$\Delta = 9$

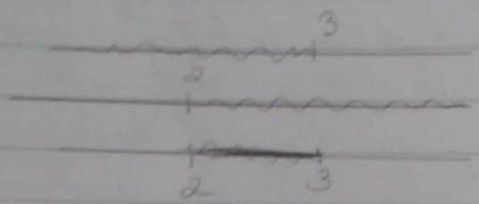
$x = \frac{3 \pm 3}{2}$ $x' = 3$ $x'' = 0$

$S = \{x \in \mathbb{R} / -1 < x < 0$ ou $3 < x < 4\}$

122.

a) $f(x) = \log_2 [\log_{1/3}(x-2)]$

$\begin{cases} \log_{1/3}(x-2) > 0 \Rightarrow \log_{1/3}(x-2) > \log_{1/3} 1 \Rightarrow x-2 < 1 \\ \qquad \boxed{x < 3} \\ x - 2 > 0 \Rightarrow x > 2 \end{cases}$



$D = \{x \in \mathbb{R} / 2 < x < 3\}$

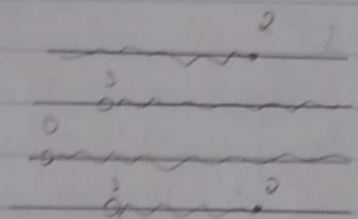
15/04 b) $f(x) = \sqrt{\log_{1/2}(\log_2 x)}$

$\log_{1/2}(\log_2 x) \geq 0 \Rightarrow \log_{1/2}(\log_2 x) \geq \log_{1/2} 1 \Rightarrow \log_2 x \leq 1 \Rightarrow$

$\log_2 x \leq \log_2 2 \Rightarrow \boxed{x \leq 2}$

$\log_2 x > 0 \Rightarrow \log_2 x > \log_2 1 \Rightarrow \boxed{x > 1}$

$\boxed{x > 0}$



$V = \{x \in \mathbb{R} / 1 < x \leq 2\}$

123- a) $f(x) = \log_e [\log_{10}(x-3)]$

$\log_{10}(x-3) > 0 \Rightarrow \log_{10}(x-3) > \log_{10} 1 \Rightarrow x - 3 > 1 \Rightarrow \boxed{x > 4}$

$x - 3 > 0 \Rightarrow \boxed{x > 3}$

$S = \{x \in \mathbb{R} / x > 4\}$

b) $f(x) = \sqrt{\log_{1/2}^{(x-1)}}$

$\log_{1/2}^{(x-1)} \geq 0 \Rightarrow \log_{1/2}^{(x-1)} \geq \log_{1/2}^{1} \Rightarrow x-1 \leq 1$
$\boxed{x \leq 2}$

$x - 1 > 0 \Rightarrow \boxed{x > 1}$

$S = \{x \in \mathbb{R} / 1 < x \leq 2\}$

c) $f(x) = \sqrt{\log_{1/3}^{[\log_2^{(x+1)}]}}$

$\log_{1/3}^{[\log_2^{(x+1)}]} \geq 0 \Rightarrow \log_{1/3}^{[\log_2^{(x+1)}]} \geq \log_{1/3}^{1} \Rightarrow \log_2^{(x+1)} \leq 1$
$\log_2^{(x+1)} \leq \log_2^{2} \Rightarrow x+1 \leq 2$
$\boxed{x \leq 1}$
$\log_2^{(x+1)} > 0 \Rightarrow \log_2^{x+1} > \log_2^{1} \Rightarrow x+1 > 1 \Rightarrow \boxed{x > 0}$

$x + 1 > 0 \Rightarrow \boxed{x > -1}$

$S = \{x \in \mathbb{R} / 0 < x \leq 1\}$

124. $\log_e^{x} + \log_e^{(3x-5)} > \log_e^{2}$

1) $\boxed{x > 0}$
2) $3x - 5 > 0 \Rightarrow 3x > 5 \Rightarrow \boxed{x > \frac{5}{3}}$

3) $\log_e^{x.(3x-5)} > \log_e^{2} \Rightarrow x(3x-5) > 2$
$3x^2 - 5x - 2 > 0$
$x' = -1/3$
$x'' = 2$
$x > 2$ e $x < -1/3$

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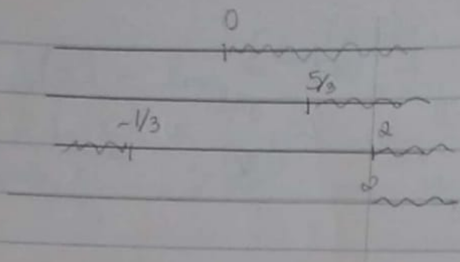


$S = \{x \in \mathbb{R} / x > 2\}$

125 - a) $\log_{10}(2x-1) + \log_{10} x \leq \log_{10} 3$

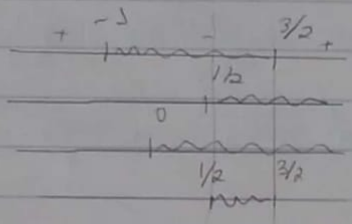
1) $2x - 1 > 0 \Rightarrow 2x > 1 \Rightarrow x > 1/2$

2) $x > 0$

3) $\log_{10}[(2x-1).x] \leq \log_{10} 3$

$(2x-1)x \leq 3$

$2x^2 - x - 3 \leq 0$



$S = \{x \in \mathbb{R} / x > 1/2 \text{ e } x \leq 3/2\}$

126. $\log_{1/2} x + \log_{1/2}(x-1) \leq -1$

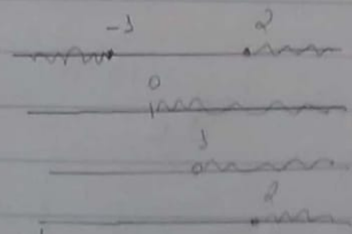
$\log_{1/2} x + \log_{1/2}(x-1) \leq \log_{1/2} 2$

1) $x > 0$

2) $x - 1 > 0 \Rightarrow x > 1$

3) $\log_{1/2}[x.(x-1)] \leq \log_{1/2} 2$

$x^2 - x \geq 2 \Rightarrow x^2 - x - 2 \geq 0$



$S = \{x \in \mathbb{R} / x \geq 2\}$

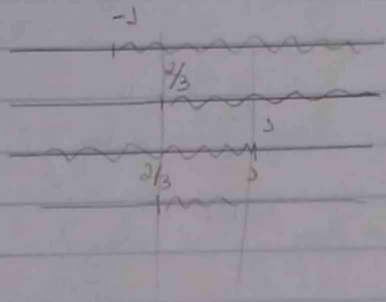
125. b) $\log_3 (x+1) - \log_3 (3x-2) > \log_3 2$

1) $x+1>0 \Rightarrow x>-1$

2) $3x-2>0 \Rightarrow x>2/3$

3) $\log_3 \left(\frac{x+1}{3x-2}\right) > \log_3 2 \Rightarrow \frac{x+1}{3x-2} > 2 \Rightarrow x+1 > 6x-4$

$x-6x > -4-1 \Rightarrow -5x > -5 \Rightarrow 5x < 5 \Rightarrow \boxed{x<1}$



$S = \{x \in \mathbb{R} / 2/3 < x < 1\}$

126. b) $\log_2 (2x) + \text{colog}_2 (x-1) > 2$

$\log_2 2x - \log_2 (x-1) > \log_2 4$

1) $2x>0 \Rightarrow x>0$

2) $x-1>0 \Rightarrow x>1$

3) $\log_2 \left(\frac{2x}{x-1}\right) > \log_2 4$

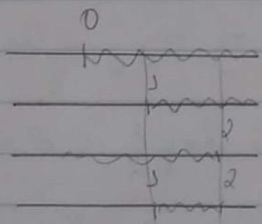
$\frac{2x}{x-1} > 4$ mmc: $x-1$ (ñ pode multiplicar em cruz)

$2x > 4x-4$

$2x-4x > -4$

$-2x > -4$

$\boxed{x<2}$



$S: \{x \in \mathbb{R} / 1 < x < 2\}$

16/04 127) $\log_3 (x-2) - \log_9 (x+1) = \log_3 2$

$x-2>0 \Rightarrow x>2$

$x+1>0 \Rightarrow x>-1$

$\log_3 (x-2) - \frac{\log_3 (x+1)}{\log_3 9} = \log_3 2$

$\log_3 (x-2) - \frac{\log_3 (x+1)}{2} = \log_3 2$

$2\log_3 (x-2) - \log_3 (x+1) = 2\log_3 2$

$\log_3 (x-2)^2 - \log_3 (x+1) = \log_3 4$

$\log_3 \left[\frac{(x-2)^2}{x+1}\right] = \log_3 4$

$\frac{x^2-4x+4}{x+1} = 4$

$x^2-4x+4 = 4x+4$

$x^2-8x=0$

$x'=0$

$x''=8$ $\qquad S=\{8\}$

128) $\log_4 (x+8) + \log_4 x = \log_2 3$

$\log_{2^2} (x+8) + \log_{2^2} x = \log_2 3$

$\frac{1}{2}\log_2 (x+8) + \frac{1}{2}\log_2 x = \log_2 3$

$\log_2 (x+8) + \log_2 x = 2\log_2 3$

$\log_2 [(x+8)\cdot x] = \log_2 3^2$

$x^2+8x=9$

$\Rightarrow x^2+8x-9=0 \rightarrow \cancel{x'=-9}$

$x''=1$

$\begin{cases} x+8>0 \Rightarrow \boxed{x>-8} \\ \boxed{x>0} \end{cases}$

$S=\{1\}$

129) $\log_2 (x+2) + \log_2 (2x+3) + \log_{1/2} (x+3) = 1$

$\log_2 (x+2) + \log_2 (2x+3) - \log_2 (x+3) = 1$

$\log_2 \frac{[(x+2)(2x+3)]}{x+3} = \log_2 2$

$\frac{(x+2)(2x+3)}{x+3} = 2$

$\frac{2x^2+3x+4x+6}{x+3} = 2$

$2x^2+7x+6 = 2x+6$

$2x^2+7x-2x = 0$

$2x^2+5x = 0$

$x' = 0$

$\cancel{x'' = -5/2}$

$S = \{0\}$

$x+2 > 0$
$x > -2$

$2x+3 > 0$
$2x > -3$
$x > -3/2$

$x+3 > 0$
$x > -3$

-2

$-3/2$

-3

$x > -3/2$

130) $\log_2 x + \log_4 (x+1)^2 - 3 \log_8 (2x-1) = 1$

$\log_2 x + 2 \log_{2^2} (x+1) - 3 \log_{2^3} (2x-1) = 1$

$\log_2 x + 2 \cdot \frac{1}{2} \log_2 (x+1) - 3 \cdot \frac{1}{3} \log_2 (2x-1) = 1$

$\log_2 x + \log_2 (x+1) - \log_2 (2x-1) = 1$

$\log_2 \frac{x \cdot (x+1)}{(2x-1)} = \log_2 2$

$\frac{x(x+1)}{(2x-1)} = 2$

$x^2 + x = 4x - 2$

$x^2 + x - 4x + 2 = 0$

$x^2 - 3x + 2 = 0$

$\Delta = 9 - 8$

$\Delta = 1 \quad x = \frac{3 \pm 1}{2} \quad x' = 2$

$x'' = 1$

$\boxed{x > 0}$

$x+1 > 0$
$\boxed{x > -1}$

$2x-1 > 0$
$x > \frac{1}{2}$

$-1 \quad 0 \quad 1/2$ $\quad \boxed{x > 1/2}$

$S = \{1, 2\}$

131) $4^x - 5 . 2^x + 6 = 0$

$(2^2)^x - 5 . 2^x + 6 = 0$

$2^x = y$

$y^2 - 5y + 6 = 0$

$\Delta = 25 - 24$

$\Delta = 1$

$y = \frac{5 \pm 1}{2}$ $y' = 3$

$y'' = 2$

$2^x = 3$ $2^x = 2$

$x = \log_2^3$ 1

$S = \{1, \log_2^3\}$

132) a- $3^{2x} - 7 . 3^x + 12 = 0$

$3^x = y$

$y^2 - 7y + 12 = 0$

$\Delta = 49 - 48$

$\Delta = 1$

$y = \frac{7 \pm 1}{2}$ $y' = 4$

$y'' = 3$

$3^x = 3$ $3^x = 4$

$x = 1$ $x = \log_3^4$

$S \{1, \log_3^4\}$

b- $2 . e^{2x} - 5 . e^x + 2 = 0$

$e^x = y$

$2y^2 - 5y + 2 = 0$

$\Delta = 25 - 16$

$\Delta = 9$

$y = \frac{5 \pm 3}{4}$ $y' = 2$

$y'' = \frac{1}{2}$

$e^x = y$

$e^x = 2$

$x = \log_e^2$

$e^x = \frac{1}{2}$

$x = \log_e^{1/2}$

$x = \log_e^{(2)^{-1}}$

$x = -\log_e^2$

$S = \{\log_e^2, -\log_e^2\}$

133_ $\log_{10}(2^x-1) - \log_{10} 2 = \log_{10}(2^x+3) - \log_{10}(2^x-1)$

$\log_{10} \frac{2^x-1}{2} = \log \frac{(2^x+3)}{2^x-1}$

$\frac{2^x-1}{2} = \frac{2^x+3}{2^x-1}$ $\quad 2^x = y$

$\frac{y-1}{2} = \frac{y+3}{y-1}$ $\quad 2^x = 5 \quad \cancel{2^x = -1}$

$x = \log_2 5$

$y^2 - 2y + 1 = 2y + 6$

$y^2 - 4y - 5 = 0$

$y' = 5$

$y'' = -1$

$S = \{\log_2 5\}$

134_ $f(x) = (\log_2 x)^2 - 6 \cdot \log_2 x + 8$

a) $f(x) = 0$

$(\log_2 x)^2 - 6 \log_2 x + 8 = 0$ $\quad (\log_2 x) = y$

$y^2 - 6y + 8 = 0$

$y' = 2$

$y'' = 4$

$\log_2 x = 2 \Rightarrow x = 2^2 = 4$

$\log_2 x = 4 \Rightarrow 2^4 = x \Rightarrow 16$

$S = \{4, 16\}$

$\log_e^2 = \ln_2^x$ $\ln = \log_e$

$\ln_e^x = -2 \Rightarrow e^{-2} = x$

b) $f(x) > 0$

$$(\log_2^x)^2 - 6\log_2^x + 8 > 0 \qquad (\log_2^x) = y$$

$y^2 - 6y + 8 > 0$

$y' = 2$

$y'' = 4$

$y > 4 \quad e \quad y < 2$

$\log_2^x < 2 \Rightarrow \log_2^x < \log_2^4$

$x < 4$

$\log_2^x > 4 \Rightarrow \log_2^x > \log_2^{16}$

$x > 16$

$\boxed{x > 0}$

$S = \{x \in \mathbb{R} / 0 < x < 4 \text{ ou } x > 16\}$

135. a) $3(\ln x)^2 + 5 \cdot \ln x = 2$ $\qquad \ln x = y$

$3y^2 + 5y - 2 = 0$

$y' = -2$

$y'' = +1/3$

$\ln x = -2 \Rightarrow x = e^{-2}$

$\ln x = \frac{1}{3} \Rightarrow x = e^{1/3}$ $\qquad S = \{e^{-2}; e^{1/3}\}$

Verif. $x > 0$

b) $4(\log_2^x)^2 = 1 + 3\log_2^x$ $\qquad \log_2^x = y$

$4y^2 - 3y - 1 = 0$

$y' = 1$

$y'' = -1/4$

$\log_2^x = 1 \Rightarrow x = 2$

$\log_2^x = -\frac{1}{4} \Rightarrow x = 2^{-1/4} \Rightarrow x = \frac{1}{2^{1/4}} \Rightarrow x = \frac{1}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} =$

$\frac{\sqrt[4]{2^3}}{\sqrt[4]{2^4}} = \frac{\sqrt[4]{2^3}}{2}$

136. $3(\log_8^x)^2 + 3 = \log_8 x^{10}$

$3(\log_8^x)^2 + 3 = 10\log_8^x$ $\qquad \log_8^x = y$

$3y^2 + 3 = 10y$

$3y^2 - 10y + 3 = 0$

$\Delta = 100 - 36$

$\Delta = 64$

$\log_8^x = 3$

$8^3 = x$

$x = 512$

$y = \frac{10 \pm 8}{6}$ $\qquad y' = 3$

$y'' = \frac{2}{6} = \frac{1}{3}$

$\log_8^x = \frac{1}{3}$

$8^{1/3} = x$

$x > 0$

$x = \sqrt[3]{8}$

$x = 2$

$S = \{512, 2\}$

137 - a) $\frac{2 - \log_{10}^x}{1 - \log_{10}^x} = 3$ $\qquad \log_{10}^x = y$

$\log_{10}^x = \frac{1}{2}$

$\frac{2 - y}{1 - y} = 3 \Rightarrow 3 - 3y = 2 - y$

$-2y = -1$

$2y = 1$

$y = 1/2$

$10^{1/2} = x$

$x = \sqrt{10}$

$S\{\sqrt{10}\}$

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b) $\dfrac{1}{\log_{10}^{x}} + \dfrac{1}{1-\log_{10}^{x}} = 1$

$\dfrac{1}{y} + \dfrac{1}{1-y} = 1$ mmc $y(1-y)$

$1-y+y = y-y^2$

$y^2-y+1=0$
$\Delta = -3$ $S=\emptyset$

138- a) $(\log_{10}^{x})^2 + 2.\log_{10}^{x} - 3 < 0$

$\log_{10}^{x} = y$

$y^2-2y-3<0$
$y' = -3$
$y'' = 1$

$y<1$ $y>-3$

$\log_{10}^{x} < 1$

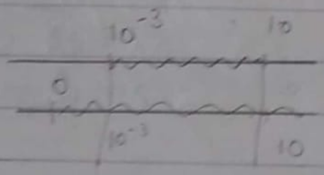
$\log_{10}^{x} > -3$

$\log_{10}^{x} < \log_{10}^{10}$

$\log_{10}^{x} > \log_{10}^{10^{-3}}$

$\boxed{x<10}$

$\boxed{x>10^{-3}}$



$S = \{x \in \mathbb{R} / 10^{-3} < x < 10\}$

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b) $2(\log_e x)^2 - \log_e x > 6$

$\log_e x = y$

$2y^2 - y - 6 > 0$

$\Delta = 1 + 48$

$\Delta = 49$

$+ \quad -\frac{3}{2} \quad - \quad 2 \quad +$

$y = \frac{1 \pm 7}{4} \qquad y' = 2 \qquad y'' = -\frac{3}{2}$

$y < -\frac{3}{2}$ ou $y > 2$

$\log_e x < -\frac{3}{2} \qquad \log_e x > 2$

133- a) $\log_4 (x-3) - \log_{16} (x-3) = 1$

$\log_4 (x-3) - \log_{4^2} (x-3) = 1$

$\log_4 x-3 - \frac{1}{2}\log_4 x-3 = 1$

$\frac{1}{2}\log_4 x-3 = 1$

$\log_4 x-3 = 2$

$x - 3 = 4^2$

$x = 16+3$

$x = 19$

Verif.

$x - 3 > 0$

$19 - 3 > 0$ (V)

$S = \{19\}$

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140) $\log_3 (x+2) - \log_9 (x+2) \leq 1$

$\log_3 (x+2) - \frac{1}{2} \log_3 (x+2) \leq 1$

$\frac{2}{2} \log_3 (x+2) - \frac{1}{2} \log_3 (x-2) \leq 1$

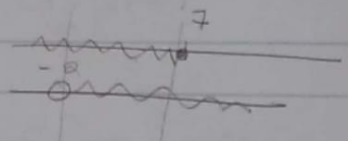
Verif.

$\frac{1}{2} \log_3 (x+2) \leq \log_3 3$

$x + 2 > 0$

$x > -2$

$\log_3 (x+2)^{1/2} \leq \log_3 3$



$(x+2)^{1/2} \leq 3$

$\left[(x+2)^{1/2}\right]^2 \leq 3^2$

$x + 2 \leq 9$

$\boxed{x \leq 7}$

$V = \{x \in \mathbb{R} / -2 < x \leq 7\}$

139) b- $\log_2 x \cdot \log_4 x = 8$

$\log_2 x \cdot \frac{1}{2} \log_2 x = 8$

$\log_2 x = y$

$y \cdot \frac{1}{2} \cdot y = 8$

$y^2 = 16$

$y = \pm 4$

$\log_2 x = 4 \quad x = 2^4 = 16$

$S = \{16, \frac{1}{16}\}$

$\log_2 x = -4 \quad x = 2^{-4} = \frac{1}{16}$

141- $\log_{1/2} (3x+1) + 2 \cdot \log_4 (x+2) > 0$

$\log_{2^{-1}} (3x+1) + 2 \cdot \log_{2^2} (x+2) > 0$

$-\log_2 (3x+1) + \log_2 (x+2) > 0$

$\log_2 (x+2) - \log_2 (3x+1) > 0$

$$\log_2 \left(\frac{x+2}{3x+1}\right) > \log_2 1$$

$$\frac{x+2}{3x+1} > 1 \qquad \text{mmc: } (3x+1)$$

$x+2 > 3x+1$

$x-3x > 1-2$

$-2x > -1$

$2x < 1$

$x < 1/2$

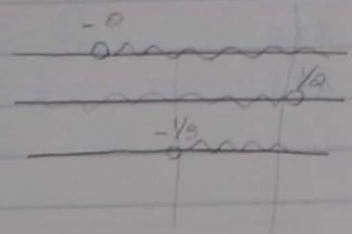
$x+2 > 0$

$x > -2$

$3x+1 > 0$

$3x > -1$

$x > -1/3$



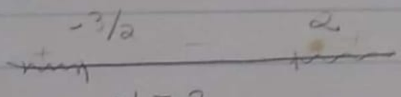
$V = \{x \in \mathbb{R} / -1/3 < x < 1/2\}$

138) b. $2\left(\log_e x\right)^2 - \log_e x > 6$ $\qquad \log_e x = y$

$2y^2 - y - 6 > 0$

$y' = -3/2$

$y'' = 2$



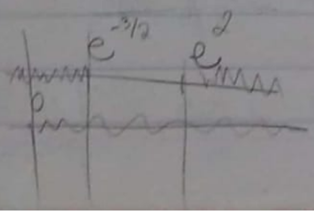
$y > 2$

$y < -3/2$

$\log_e x > 2 \Rightarrow \log_e x > \log_e e^2 \Rightarrow x > e^2$

$\log_e x < -3/2 \Rightarrow \log_e x < \log_e e^{-3/2} \Rightarrow x < e^{-3/2}$

$x > 0$



$S = \{x \in \mathbb{R} / 0 < x < e^{-3/2}$ ou $x > e^2\}$

29/04

3º lista de exercícios

1) Resolva as equações

a) $x^{\log_x (x-5)^2} = 9$

$(x-5)^2 = 9$

$x^2 - 10x + 25 - 9 = 0$

$x^2 - 10x + 16 = 0$

$x' = 8$

$x'' = 2$

$S = \{8, 2\}$

$x > 0$

$x \neq 1$

Verif.

$(x-5)^2 > 0$

$(8-5)^2 > 0$ (V)

$(2-5)^2 > 0$

$(-3)^2 > 0$

$9 > 0$ (V)

b) $(\sqrt[3]{x})^{\log_x (x^2+2)} = 2 \log_3 \sqrt{27}$

$x^{1/3 \log_x (x^2+2)} = 2 \log_3 27^{1/2}$

$x^{\log_x (x^2+2)^{1/3}} = 2 \cdot \frac{1}{2} \log_3 27$

$(x^2+2)^{1/3} = 3$

$[(x^2+2)^{1/3}]^3 = 3^3$

$x^2 + 2 = 27$

$x^2 = 25$

$x = \pm 5$

$x^2 + 2 > 0$

$x > 0$

$x \neq 1$

$S = \{5\}$

2) Resolver as equações

a) $\log_{10} x \, (\log_{10} x - 1) = 6$

$y(y-1) = 6$

$y^2 - y - 6 = 0$

$\Delta = 1 + 24$

$\Delta = 25$

$x > 0$ (V)

$y = \frac{1 \pm 5}{2}$

$y' = 3$

$y'' = -2$

$\log_{10} x = -2$

$10^{-2} = x$

$x = 1/100$

$S = \{1/100, 1000\}$

$\log_{10} x = y$

$\log_{10} x = 3$

$x = 10^3$

$x = 1000$

Matéria Matemática

Professor Otília

Anotações

$\log_2^x = y$

b) $\dfrac{3+\log_2^x}{\log_2^x} + \dfrac{2-\log_2^x}{3-\log_2^x} = \dfrac{5}{2}$

$\dfrac{3+y}{y} + \dfrac{2-y}{3-y} = \dfrac{5}{2}$

$\dfrac{(3+y)\,2(3-y) + (2-y)\,2y}{2y(3-y)} = \dfrac{5y(3-y)}{2y(3-y)}$

$(6+2y)(3-y) + 4y - 2y^2 = 15y - 5y^2$

$18 - \cancel{6y} + \cancel{6y} - 2y^2 + 4y - 2y^2 - 15y + 5y^2 = 0$

$y^2 - 11y + 18 = 0$

$\Delta = 121 - 72$

$\Delta = 49$

$y = \dfrac{11 \pm 7}{2}$ $\quad y' = 9$ $\quad y'' = 2$

$\log_2^x = 9$

$x = 2^9$

$x = 512$

$\log_2^x = 2$

$x = 4$

$S = \{512, 4\}$

Verif

$x > 0$ (V)

c) $\log_x^{(3x^2-13x+15)} = 2$

$\log_x^{3x^2-13x+15} = \log_x^{x^2}$

$3x^2 - 13x + 15 - x^2 = 0$

$2x^2 - 13x + 15 = 0$

$\Delta = 169 - 120$

$\Delta = 49$

$x = \dfrac{13 \pm 7}{4}$ $\quad x' = 5$ $\quad x'' = 3/2$

$S = \{5, 3/2\}$

Verif

$3x^2 - 13x + 15 > 0$

$3 \cdot 5^2 - 13 \cdot 5 + 15 > 0$

$75 - 65 + 15 > 0$ (V)

$3 \cdot \left(\frac{3}{2}\right)^2 - 13 \cdot \left(\frac{3}{2}\right) + 15 > 0$

$\dfrac{27}{4} - \dfrac{39}{2} + 15 > 0$

$\dfrac{27 - 78 + 60}{4} > 0$ (V)

$x > 0$ e $x \neq 1$

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d) $\log_{x}^{(4-3x)} = 2$

$x \neq 1$

$\log_{x}^{4-3x} = \log_{x}^{x^2}$

$4 - 3.(-4) > 0$

$4 - 3x = x^2$

$4 + 12 > 0$

$-x^2 - 3x + 4 = 0$

(V)

$x^2 + 3x - 4 = 0$

mas $\boxed{x > 0}$

$\Delta = 9 + 16$

$\Delta = 25$

$S = \emptyset$

$x = \frac{-3 \pm 5}{2}$ $\quad \cancel{x' = 1}$ $\quad x'' = -4$

e) $\log_{x}^{(4x-3)} = \log_{x}^{(2x+1)}$

$x \neq 1$

$4x - 3 = 2x + 1$

$2x = 4$

$4.2 - 3 > 0$

$x = 2$

$5 > 0$ (V)

$2.2 + 1 > 0$

$5 > 0$ (V)

$S = \{2\}$

f) $\log_{(x+5)}^{(3x^2-5x-8)} - \log_{(x+5)}^{(2x^2-3x)} = 0 \Rightarrow \log_{(x+5)}^{(3x^2-5x-8)} = \log_{(x+5)}^{(2x^2-3x)}$

$3x^2 - 5x - 8 = 2x^2 - 3x$

$x^2 - 2x - 8 = 0$

Verif

$x' = 4 \qquad x'' = -2$

$3x^2 - 5x - 8 > 0$

$3.4^2 - 5.4 - 8 > 0$ (V)

$3.(-2)^2 - 5.(-2) - 8 > 0$ (V)

$S = \{4, -2\}$

$x + 5 > 0 \qquad x \neq 1$

$4 + 5 > 0$ (V)

$-2 + 5 > 0$ (V)

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6/05 g) $\log_x^2 (5x-6) - 3\log_x (5x-6) + 2 = 0$

$\log_x (5x-6) = y$

$y^2 - 3y + 2 = 0$

$\Delta = 9 - 8$

$\Delta = 1$

$y = \frac{3 \pm 1}{2}$ $y' = 2$ $y'' = 1$

$\log_x (5x-6) = 2$

$x^2 = 5x - 6$

$x^2 - 5x + 6 = 0$

$\Delta = 25 - 24$

$\Delta = 1$

$x = \frac{5 \pm 1}{2}$ $x' = \boxed{3}$ $x'' = \boxed{2}$

$\log_x (5x-6) = 1$

$x = 5x - 6$

$-4x = -6$

$x = \frac{6}{4} = \boxed{3/2}$

$x \neq 1$ 3(V) 2(V) 3/2(V)

$S = \{3, 2, 3/2\}$

$5x - 6 > 0$

$5.3 - 6 > 0$ (V)

$5x - 6 > 0$

$5.2 - 6 > 0$ (V)

$5x - 6 > 0$

$5.\frac{3}{2} - 6 > 0$

$\frac{15}{2} - 6 > 0 \Rightarrow \frac{15-12}{2} > 0 \Rightarrow \frac{3}{2} > 0$

h) $2\log_{(3x-2)}^2 (4-x) - 5\log_{(3x-2)} (4-x) + 2 = 0$

$\log_{(3x-2)} (4-x) = y$

$2y^2 - 5y + 2 = 0$

$\Delta = 25 - 16$

$\Delta = 9$

$y = \frac{5 \pm 3}{4}$ $y' = 2$ $y'' = 1/2$

$\log_{(3x-2)} (4-x) = \frac{1}{2}$

$(3x-2)^{1/2} = 4 - x$

$(\sqrt{3x-2})^2 = (4-x)^2$

$3x - 2 = 16 - 8x + x^2$

$3x - 2 - 16 + 8x - x^2 = 0$

$+x^2 - 11x + 18 = 0$

$\Delta = 121 - 72$

$\Delta = 49$

$x = \frac{11 \pm 7}{2}$ $\boxed{x' = 9}$ $\boxed{x'' = 2}$

$\log_{(3x-2)} (4-x) = 2$

$(3x-2)^2 = (4-x)$

$9x^2 - 12x + 4 = 4 - x$

$9x^2 - 12x + \cancel{4} + x - \cancel{4} = 0$

$9x^2 - 11x = 0$

$x(9x - 11) = 0$

$\boxed{x = 0}$

$9x - 11 = 0$

$9x = 11$

$\boxed{x = 11/9}$

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$4 - 0 > 0$ (V)

$4 - x > 0$
$4 - 9 > 0$ (F)

$4 - x > 0$
$4 - 2 > 0$ (V)

$4 - 11/9 > 0$
$\frac{36-11}{9} > 0$ (V)

$3x - 2 > 0$
$3x > 2$
$x > 2/3$

$11/9$ (V)
2 (V)

$S = \{ 11/9 , 2 \}$

i) $\log_2 (x-3) + \log_2 (x+8) = 4$

$\log_2 (x-3) + \log_2 (x+8) = \log_2 16$

$(x-3) \cdot (x+3) = 16$
$x^2 \cancel{+3x} \cancel{-3x} - 9 - 16 = 0$
$x^2 - 25 = 0$
$x^2 = 25$
$x = \pm 5$

$x - 3 > 0$
$5 - 3 > 0$ (V)
$-5 - 3 > 0$ (F)

$S = \{ 5 \}$

j) $\log_{10} x + \log_{10} (x-21) = 2$

$\log x + \log (x-21) = \log_{10} 100$

$x \cdot (x - 21) = 100$
$x^2 - 21x - 100 = 0$
$\Delta = 441 + 400$
$\Delta = 841$

$x = \frac{21 \pm 29}{2}$ $x' = 25$ $x'' = -4$

Verif.
$x > 0$
$x > 21$

$S = \{ 25 \}$

3) Resolver as equações:

a) $(0,4)^{\log_{10}^2 x+1} = 6,25^{2-\log x^3}$

$\left(\frac{4}{10}\right)^{\log^2 x+1} = \left(\frac{625}{100}\right)^{2-\log x^3}$

$\left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\frac{25}{4}\right)^{2-\log x^3}$

$\left(\frac{2}{5}\right)^{\log^2 x+1} = \left[\left(\frac{5}{2}\right)^2\right]^{2-\log x^3}$

$\left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\frac{5}{2}\right)^{4-2\log x^3}$

$\left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\frac{2}{5}\right)^{-(4-2\cdot 3\log}$

$\log^2 x+1 = -4+6\log^x$

$y^2+1 = -4+6y$

$y^2-6y+5=0$

$y' = 5$

$y'' = 1$

$\log x = 1 \Rightarrow x = 10^1 = 10$

$\log_{10} x = 5 \Rightarrow x = 10^5$

$S = \{10, 10^5\}$

b) $\dfrac{\log_3 (2x)}{\log_3 (4x-15)} = 2$

$\log_3 (2x) = 2\log_3 (4x-15)$

$\log_3 (2x) = \log_3 (4x-15)^2$

$2x = (4x-15)^2$

$2x = 16x^2 - 120x + 225$

$16x^2 - 120x + 225 - 2x = 0$

$16x^2 - 122x + 225 = 0$

$x' = \frac{100}{32} = \frac{25}{8}$

$x'' = \frac{144}{32} = \frac{9}{2}$

Verificação

$2x > 0 \Rightarrow x > 0$

$4x - 15 > 0 \Rightarrow 4x > 15$

$x > \frac{15}{4}$

$S = \{9/2\}$

4-a) $\log_2 (9^{x-1}+7) - \log_2 (3^{x-1}+1) = 2$

$\log_2 \frac{(9^{x-1}+7)}{(3^{x-1}+1)} = \log_2 4$

$\frac{(9^{x-1}+7)}{(3^{x-1}+1)} = 4$

$4(3^{x-1}+1) = (9^{x-1}+7)$ $\quad 3^{x-1}=y$

$4(y+1) = (y^2+7)$

$4y+4 = y^2+7$

$-y^2+4y-3=0$

$y^2-4y+3=0$

$\Delta = 16-12$

$\Delta = 4$

$y = \frac{4\pm 2}{2}$ $\quad y' = 3$ $\quad y'' = 1$

$3^{x-1} = 3$

$x-1 = 1$

$\boxed{x = 2}$

$3^{x-1} = 1$

$3^{x-1} = 3^0$

$\boxed{x = 1}$

Verif

$9^{x-1}+7 > 0$

$9^{2-1}+7 > 0$

$9+7 > 0$ (V)

$3^{x-1}+1 > 0$

$3^1+1 > 0$ (V)

$9^x+7 > 0$

$9^{0-1}+7 > 0$ (V)

$3^0+1 > 0$ (V)

$S = \{2, 1\}$

c) $2\log(\log^x) = \log(7-2\log^x) - \log^5$

$\log_{10}(\log^x)^2 = \log_{10}\frac{(7-2\log^x)}{5} \Rightarrow (\log^x)^2 = \frac{7-2\log^x}{5} \Rightarrow$

$\log^x = y$

$y^2 = \frac{7-2y}{5} \Rightarrow 5y^2 = 7-2y \Rightarrow 5y^2+2y-7=0$

$\Delta = 4+140$

$\Delta = 144$

$y = \frac{-2 \pm 12}{10}$ $y' = 1$ $y'' = -7/5$

Verif.

$\log_{10} x = y$

$\log_{10} x = 1 \Rightarrow 10^1 = x \Rightarrow \boxed{x=10}$

$\log_{10} x = -7/5$

$10^{-7/5} = x$

$\left(\frac{1}{10}\right)^{7/5} = x$

$7 - 2\log_{10} 10 > 0$

$7 - 2.1 > 0$ (V)

$S = \{10\}$

d. $x + \log_{10}(1+2^x) = x\log_{10} 5 + \log_{10} 6$

$\boxed{\log_{10} 10^x} = x\log_{10} 10 \Rightarrow$

$x.1 = \boxed{x}$

$x + \log_{10}(1+2^x) = \log_{10} 5^x + \log_{10} 6$

$x + \log_{10}(1+2^x) = \log_{10} 5^x.6$

$\log_{10} 10^x + \log_{10}(1+2^x) = \log_{10} 5^x.6$

$10^x.(1+2^x) = 5^x.6$

$\frac{10^x}{5^x} + \frac{10^x.2^x}{5^x} = \frac{5^x.6}{5^x}$

$2 + 2.2^x = 6$

$2.2^x = 4$

$2^x = 2$

$\boxed{x=1}$

e) $\log^{-1} x = 2 + \log x^{-1}$

$\frac{1}{\log x} = 2 - \log x$

$\log x (2 - \log x) = 1$

$2\log x - \log x^2 - 1 = 0$ $\log x = y$

$V = \{10\}$

$2y - y^2 - 1 = 0$

$y^2 - 2y + 1 = 0$

$\Delta = 4 - 4$

$\Delta = 0$

$y = \frac{2 \pm 0}{2}$ $y' = 1$ $y'' = 1$

$\log_{10} x = 1$ $\boxed{x=10}$ / $\log_{10} x = 1$ $\boxed{x=10}$

5) a) $\begin{cases} 2x^{y} - x^{-y} = 1 \\ \log_2 y = \sqrt{x} \end{cases}$ $y \to$ se o x é fica o logaritimando ou base, então coloca $x = 0$, $x = 1$

$y = 2^{\sqrt{x}}$

$2x^{(2^{\sqrt{x}})} - x^{-(2^{\sqrt{x}})} = 1$

$2x^{(2^{\sqrt{x}})} - \frac{1}{x^{(2^{\sqrt{x}})}} = 1$

$2z - \frac{1}{z} = 1$

$2z^2 - 1 = z$

$2z^2 - z - 1 = 0$

$\boxed{z' = -\frac{1}{2}}$ $\boxed{z'' = \frac{1}{2}}$

$x^{(2^{\sqrt{x}})} = z$

$x^{(2^{\sqrt{x}})} = 1$

$x^{2^{\sqrt{x}}} = x^0$

$2^{\sqrt{x}} = 0$ impossível

$\cancel{x^{(2^{\sqrt{x}})} = -\frac{1}{2}}$

Se: $x = 0$

$0^{(2\sqrt{0})} = 1$

$0^1 = 1$ (F)

$x = 1$

$1^{2^{\sqrt{1}}} = 1$ (V)

$y = 2^{\sqrt{x}} \Rightarrow y = 2^1 = 2$

$S = \{(1, 2)\}$

b) $\begin{cases} x + y = 6 \\ \log_2 x + \log_2 y = \log_2 8 \end{cases} \Rightarrow \log_2 x.y = \log_2 8 \Rightarrow x.y = 8 \Rightarrow$

$x = \frac{8}{y}$

$\frac{8}{y} + y = 6$

$\frac{8 + y^2}{\cancel{y}} = \frac{6y}{\cancel{y}}$

$y^2 - 6y + 8 = 0$

$\Delta = 36 - 32$

$\Delta = 4$

$y = \frac{6 \pm 2}{2}$ $y' = 4$ $y'' = 2$

$x = \frac{8}{4} \Rightarrow 2$

$x = \frac{8}{2} \Rightarrow 4$

$V = \{(2, 4)(4, 2)\}$

c) $\begin{cases} 4^{x-y} = 8 \Rightarrow 2^{2(x-y)} = 2^3 \Rightarrow 2(x-y) = 3 \\ \log_2^x - \log_2^y = 2 \Rightarrow \log_2^{\frac{x}{y}} = \log_2^4 \Rightarrow \frac{x}{y} = 4 \end{cases}$

$\boxed{x = 4y}$

$2(4y - y) = 3$
$2(3y) = 3$
$6y = 3$
$y = 1/2$

$x = 4 \cdot \frac{1}{2}$

$x = \frac{4}{2} = 2$

$S = \{(2, 1/2)\}$

d) $\begin{cases} 2^{\sqrt{x}+\sqrt{y}} = 512 \Rightarrow 2^{\sqrt{x}+\sqrt{y}} = 2^9 \Rightarrow \sqrt{x}+\sqrt{y} = 9 \\ \log_{10}^{\sqrt{xy}} = 1 + \log_{10}^2 \Rightarrow \log_{10}^{\sqrt{xy}} = \log_{10}^{10} + \log_{10}^{2} \Rightarrow \end{cases}$

$\sqrt{xy} = 10 \cdot 2$

$\begin{cases} \sqrt{x} + \sqrt{y} = 9 \\ (\sqrt{xy})^2 = (20)^2 \Rightarrow xy = 400 \Rightarrow x = \frac{400}{y} \end{cases}$

$\sqrt{\frac{400}{y}} + \sqrt{y} = 9$

$\frac{20}{\sqrt{y}} + \sqrt{y} = 9$

$20 + \sqrt{y}^2 = 9\sqrt{y}$
$\sqrt{y}2 - 9\sqrt{y} + 20 = 0$
$y - 9\sqrt{y} + 20 = 0$
$(y + 20)^2 = (9\sqrt{y})^2$
$y^2 + 40y + 400 = 81y$
$y^2 - 41y + 400 = 0$
$y' = 25$
$y'' = 16$

$S = \{(16, 25)(25, 16)\}$

$y = 25 \Rightarrow x = \frac{400}{25} = 16$

$y = 16 \Rightarrow x = \frac{400}{16} = 25$

14/05 **Logaritmos decimais**

$$10 < 20 < 100$$
$$10 < 20 < 10^2$$
$$\log 10^1 < \log 20 < \log 10^2$$
$$1 < \log 20 < 2$$

$$\log 20 = \underbrace{1}_{\text{parte inteira}},\overbrace{\ldots\ldots}^{\text{parte decimal}}$$

Genericamente /

$$10^c < x < 10^{c+1}$$
$$\log 10^c < \log x < \log 10^{c+1}$$
$$c < \log x < c+1$$

$$\log x = \underbrace{c}_{\text{parte inteira}} + \overbrace{m}^{\text{parte decimal}}$$

$$0 \leq m < 1$$

parte inteira (c) chama-se "característica"
" decimal (m) " "mantissa"

$\log 9$ mantissa
$m = 0,954243$

$\log 317$ $m = 0,501059$

$\log 50 = \log 50.000$

Regras da característica

1º caso) $x > 1$
A "característica" do logaritmo decimal de um número $x > 1$ é igual ao número de algarismos da sua parte inteira

menos 1.

Ex: $10^0 < 2,3 < 10$

$\log 10^0 < \log 2,3 < \log 10$

$0 < \log 2,3 < 1$

$\log \overset{1\text{ nº na parte inteira } -1 = 1}{\widetilde{2},3} = 0,3617\ldots$

$1 < \log \overset{2\text{ nos } -1 = 1}{\widetilde{23}} < 2 \qquad \log 23 = 1,3617\ldots$

Exercícios:

- Calcule:

a) $\log 62 = 1,792392$

$C = 1$

b) $\log 6,2 = 0,792392$

$C = 0$

c) $\log 421 = 2,624282$

$C = 2$

d) $\log 42,1 = 1,624282$

$C = 1$

e) $\log 4,21 = 0,624282$

$C = 0$

2º caso) $0 < x < 1$

A característica do logarítimo decimal $0 < x < 1$ é o oposto da quantidade de 0 (zeros) que precedem o 1º algarismo significativo

$10^{-1} < 0,2 < 10^0$

$\log 10^{-1} < \log 0,2 < \log 10^0$

$-1 < \log \overset{1\text{ zero}}{\widetilde{0},2} < 0$

$\log 0,2 \; -1 + \text{mantissa}$

$C = -1$

$\boxed{\text{se fosse } \underset{1\,2\,3}{\widetilde{0,00}2} \text{ então seria } -3}$

Calcule:

a) $\log 0{,}02 = c + m = -2 + 0{,}301030 = -1{,}698970$

$c = -2$

$m = +0{,}301030$

outra maneira:

$\log 0{,}02 = \bar{2},301030$ significa que a característica é negativa

Chama-se forma mista ou preparada

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142. a) $\log 31 = 1{,}491362$

$c = 1$

$m = 0{,}491362$

b) $\log 7 = 0{,}845098$

$c = 0$

$m = 0{,}845098$

c) $\log x = \bar{1},698970$

$c = -1$

$m = 0{,}698970$

d) $\log x = \bar{2},477121$

$c = -2$

$m = 0{,}477121$

143. a) $\log 600 =$

$c = 2$

$m = 0{,}778151$

b) $\log 60 =$

$m = 0{,}778151$

c) $\log 6 =$

$m = 0{,}778151$

d) $\log^{5} = 0,698970$

e) $\log^{50} = 0,698970$

f) $\log^{500} = 0,698970$

144. $\log_{10}^{x} = 3,167317$

$c = ? = 6$

$m = ? = 0,167317$

$\log^{(1000x)} = \log^{10^3} + \log^{x}$

$3 + 3,167317 = 6,167317$

$\log^{(0,01x)} = \log^{10^{-2}} + \log^{x}$

$-2 + 3,167317 = 1,167317$

$c = 1$

$m = 0,167317$

145. $\log^{x} = \bar{1},396189$

a) $\log^{10x} = \log^{10} + \log^{x}$

$1 + \bar{1},396189$

$x - \cancel{1} + 0,396189$

$0,396189$

$c = 0$

$m = 0,396189$

b) $\log^{10^4 x} \quad \log^{10^4} + \log^{x}$

$4 + \bar{1},396189$

$4 - 1 + 0,396189$

$3 + 0,396189 = 3,396189$

$c = 3$

$m = 0,396189$

c) $\log\left(\frac{x}{1000}\right) = \log x - \log 10^3$

$\bar{1},396183 - 3$

$-1 + 0,396183 - 3$

$-4 + 0,396183 = \bar{4},396183$

d) $\log\frac{x}{10} = \log\frac{x}{10} = \log x - \log 10$

$\bar{1},396183 - 1$

$-1 + 0,396183 - 1$

$-2 + 0,396183$

$\bar{2},396183$

146. a) $\log x = -4,361728$

$-4 - 0,36178$

$\underbrace{-4-1}+\underbrace{1-0,36178}$

$-5 + 0,638272$

$\bar{5},638272$

b) $\log x = -0,397940$

$-0 - 0,397940$

$\underbrace{-0-1}+1-0,397940$

$-1 + 0,602060$

$-1,602060 =$

$\bar{1},602060$

$$\begin{array}{r} 1,000000 \\ -\ 0,397940 \\ \hline 0,602060 \end{array}$$

c) $\log x = -1,875061$

$-1 - 0,875061$

$\underbrace{-1-1}+1-0,875061$

$-2 + 0,124939$

$-2,124939$

$\bar{2},124939$

$$\begin{array}{r} 1,000000 \\ -\ 0,875061 \\ \hline 0,124939 \end{array}$$

147. $\operatorname{colog} x$

$\log x = 2,301030$

$\operatorname{colog} x = -\log x$

$\operatorname{colog} x = -2,301030$

$-2-1+1-0,301030$

$-3 + 0,698970$

$\operatorname{colog} x = \bar{3},698970$

Calcular o logaritmando sabendo que

a) $\log x = 2,488551$ $\underline{\underline{308}}$

$c = 2$

$m = 0,488551$

$\log^{x} = \log^{308}$

b) $\log x = 3,488551$

$c = 3$

$m = 0,488551$

$\log^{x} = \log^{3080}$

c) $\log^{x} = 1,488551$

$c = 1$

$m = 0,488551$

$\log^{x} = \log^{30,8}$

d) $\log^{x} = \bar{1},488551$

$c = -1$

$m = 0,488551$

$\log^{x} = \log^{0,308}$

e) $\log^{x} = \bar{2},488551$

$c = -2$

$m = 0,488551$

$\log^{x} = \log^{0,0308}$

Calcular

a) antilog $3,863917 = x$

731

$\log^{x} = 3,863917$

$x = 7310$

b) antilog $\bar{1},803457$

$\log x = \bar{1},803457 \Rightarrow x = 0,636$

c) antilog $0,917506$

$\log x = 0,917506 \qquad x = 8,27$

d) antilog $\bar{1},514548$

$\log x = \bar{1},514548 \qquad x = 0,327$

e) antilog $\bar{2},12705$

$\log x = \bar{2},12705 \qquad x = 0,0134$

4ª lista de Exercícios

1) $\dfrac{-(-2)^2 - \sqrt[3]{-27}}{(-3+5)^0 - \log_2 4} = \dfrac{-4+3}{1-2} = \dfrac{-1}{-1} = 1$

2) $\log_{1/3}\left(\log_4 (x^2-5)\right) > 0$

$\log_{1/3}\left(\log_4 (x^2-5)\right) > \log_{1/3} 1$

$\log_4 (x^2-5) < 1$

$\log_4 (x^2-5) < \log_4 4$

$x^2 - 5 < 4$

$x^2 - 9 < 0$

$x' = 3$

$x'' = -3$

$-3 < x < 3$

$\log_4 (x^2-5) > 0 \Rightarrow \log_4 (x^2-5) > \log_4 1$

$(x^2-5) > 1 \Rightarrow x^2 > 6$

$x^2 - 6 > 0$

$-\sqrt{6} \quad \sqrt{6}$

$-3 \quad 3$

$-\sqrt{6} \quad \sqrt{6}$

$x^2 - 5 > 0$

$-\sqrt{5} \quad \sqrt{5}$

$x^2 > 5$

$-3 \quad -\sqrt{6} \quad \sqrt{6} \quad 3$

$V = \{x \in \mathbb{R} \mid -3 < x < -\sqrt{6} \text{ ou } \sqrt{6} < x < 3\}$

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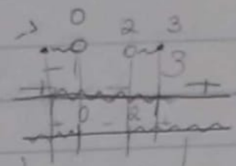
3) $\log_{1/3}(x^2-2x) \geq -1$

$x^2-2x > 0$
$x(x-2) > 0$
$x > 0$
$x > 2$

$\log_{1/3}(x^2-2x) \geq \log_{1/3}(1/3)^{-1}$

Obs. a base é menor que 1

$x^2-2x \leq 3$
$x^2-2x-3 \leq 0$
$\Delta = 4+12$
$\Delta = 16$
$x = \dfrac{2 \pm 4}{2}$ $\quad x' = 3$ $\quad x'' = -1$



$V = \{x \in \mathbb{R} \,/\, -1 \leq x \leq 0$ ou $2 < x \leq 3\}$

4) $3^{x+y} = 1$
$2^{x+2y} = 2$

$x - y = ?$

$3^{x+y} = 3^0 \Rightarrow x+y=0$
$x+2y=1$

$\begin{cases} x+y=0 \Rightarrow x=-y \\ x+2y=1 \end{cases}$

$(-y)+2y=1$
$\boxed{y=1}$
$x=-y$
$\boxed{x=-1}$

$x-y = -1-1 = -2$

5) $\log x^2 + \log x = 1$

$\log x^2 \cdot x = \log 10 \Rightarrow x^2 \cdot x = 10$
$x^3 = 10$
$x = \sqrt[3]{10}$
$x = 10^{1/3}$

6) a) $\log_x 4 = 2 \Rightarrow x^2 = 4 \Rightarrow x^{\cancel{2}} = 2^{\cancel{2}} \Rightarrow x = 2$

b) $\log_{10} \sqrt{x} + \log_{100} x = 2 \Rightarrow \log_{10} x^{1/2} + \log_{10^2} x = 2 \Rightarrow$

$\frac{1}{2}\log_{10} x + \frac{1}{2}\log_{10} x = 2$

$\frac{2}{2}\log_{10} x = 2 \Rightarrow 10^2 = x \Rightarrow x = 100$

20/03 7)

$$4^{\log_2 A} + 2A - 2 = 0$$

$$2^{2\log_2 A} + 2A = 2$$

$$A^2 + 2A - 2 = 0 \qquad A = X$$

$$X^2 + 2X - 2 = 0$$

$$\Delta = 4 + 8$$

$$\Delta = 12$$

$$X = \frac{-2 \pm \sqrt{12}}{2} \qquad X' = \frac{-\cancel{2} + \cancel{2}\sqrt{3}}{\cancel{2}} = -1 + \sqrt{3}$$

$$\cancel{X'' = -1 - \sqrt{3}} \qquad A > 0$$

8) $$\frac{x - \log 2}{x + \log 2} = \frac{\log 8}{\log 4}$$

$$\frac{X - \log 2}{X + \log 2} = \frac{\log 2^3}{\log 2^2} \to \frac{X - \log 2}{X + \log 2} = \frac{3\log 2}{2\log 2} \Rightarrow$$

$$2(X - \log 2) = 3(X + \log 2)$$

$$2X - 2\log 2 = 3X + 3\log 2$$

$$2X - 3X = 3\log 2 + 2\log 2$$

$$-X = 5\log 2$$

$$X = -5\log 2$$

9) $$\log_2 x = \log_{\sqrt{x}} x^2 + \log_x 2$$

$$\log_2 x = 2\log_{x^{1/2}} x + \log_x 2$$

$$\log_2 x = 2.2\log_x x + \log_x 2$$

$$\log_2 x = 4.1 + \log_x 2$$

$$\frac{\log_x x}{\log_x 2} = 4 + \log_x 2$$

$$\frac{1}{\log_x 2} = 4 + \log_x 2 \qquad \log_x 2 = y$$

$$\frac{1}{y} = 4 + y$$

$$1 = 4y + y^2$$

$$y^2 + 4y - 1 = 0$$

$$\Delta = 16 + 4$$

$$\Delta = 20$$

$$y = \frac{-\cancel{4}^{2} \pm \cancel{2}\sqrt{5}}{\cancel{2}_1}$$

$$y' = -2 + \sqrt{5}$$

$$y'' = -2 - \sqrt{5}$$

$\log_x 2 = -2+\sqrt{5}$

$x^{(-2+\sqrt{5})} = 2$

$\log_x 2 = -2-\sqrt{5} \Rightarrow x^{(-2-\sqrt{5})} = 2$

10- $\begin{cases} 2^x = \dfrac{1}{2^{4+y}} \Rightarrow 2^x = 2^{-(4+y)} \Rightarrow x = -4-y \\ \log_2 (2x+y) = 1 \Rightarrow \log_2 (2x+y) = \log_2 2 \Rightarrow 2x+y=2 \end{cases}$

$2x = 2-y$

$x = \dfrac{2-y}{2}$

$2(-4-y)+y=2$

$-8-2y+y=2$

$-2y+y=2+8$

$-y=10$

$\underline{y=-10}$

$x=-4-(-10)$

$x=-4+10$

$\underline{x=+6}$

$x-y$

$6-(-10)=16$

11- $\log_2(4x-4) - \log_2(x-1) \leq 2+2\log_2 3$

$\log_2(4x-4) - \log_2(x-1) \leq \log_2 4 + \log_2 9$

$\log_2 \dfrac{4x-4}{x-1} \leq \log_2 4.9$

$\dfrac{4x-4}{x-1} \leq 36$

$\dfrac{4x-4}{\cancel{x-1}} \leq \dfrac{36(x-1)}{\cancel{x-1}}$

$4x-4 \leq 36x-36$

$4x-4-36x+36 \leq 0$

$-32x+32 \leq 0$

$-32x \leq -32$

$32x \geqslant 32$

$x \geqslant 1$

Verif

$4x-4>0$

$4x>4$

$\underline{x>1}$

$x-1>0$

$\underline{x>1}$

12 - $x^{\log_3 x} \leq 9x$

$x^{\frac{\log_x x}{\log_x 3}} \leq 9x$ $\qquad x > 0$

$x^{\frac{1}{\log_x 3}} \leq 9x$

$\frac{1}{3} \leq 9x$

$\frac{1}{3} \cdot 9 \leq x$

$\boxed{3 \leq x}$

13 -

$\log_{1/2}(3x-1) + 2\log_4(x+2) > 0$

$\log_{2^{-1}}(3x-1) + 2 \cdot \frac{1}{2}\log_2(x+2) > \log_2 1$

$-\log_2(3x-1) + \log_2(x+2) > \log_2 1$

$\log_2(3x-1)^{-1} + \log_2(x+2) > \log_2 1$

$\log_2 (3x-1)^{-1} \cdot (x+2) > \log_2 1$

$(3x-1)^{-1} \cdot (x+2) > 1$

$\frac{1}{3x-1} \cdot (x+2) > 1$

$\frac{x+2}{3x-1} > 1$

$\frac{x+2}{\cancel{3x-1}} > \frac{3x-1}{\cancel{3x-1}}$

$x + 2 - 3x + 1 > 0$

$-2x + 3 > 0$

$-2x > -3$

$2x < 3$

$x < 3/2$ $\quad 1{,}5$

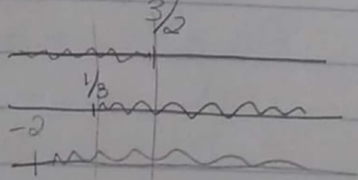
$3x - 1 > 0$

$3x > 1$

$x > 1/3$

$x + 2 > 0$

$x > -2$



$V = \{x \in \mathbb{R} \mid \frac{1}{3} < x < 3/2\}$

14 - $a > 1$

$$\log_a \left[\log_{1/a} \left(\log_a x\right)\right] \geq 0$$

$$\log_a \left[\log_{1/a} \left(\log_a x\right)\right] \geq \log_a 1$$

$$\log_{1/a} (\log_a x) \geq 1$$

$$\log_{1/a} (\log_a x) \geq \log_{1/a} \tfrac{1}{a}$$

$$\log_a x \leq \frac{1}{a}$$

Verificação

$x > 0$

$\log_a x > 0 \Rightarrow x > 1$

$\log_{1/a} (\log_a x) \geq \log_{1/a} 1$

23/05

- Calcule (~~o valor de~~ /)

a) $\log_3 2 = \frac{\log_{10} 2}{\log_{10} 3} = \frac{0,301030}{0,477121}$

b) $\log_6 4 = \frac{\log_{10} 4}{\log_{10} 6} = \frac{0,602060}{0,778151} \simeq 0,77$

0,301030 | 0,477121

2862726 0,6309

0147574

143 1363

0044 3770

34089

- Calcule o valor de x

a) $5^x = 100 \quad x = \log_5 100 = \frac{\log 100}{\log 5} \Rightarrow x = \frac{2}{0,698970} \simeq 2,86$

b) $3^x = 20 \quad x = \log_3 20 \Rightarrow \frac{\log 20}{\log 3} \Rightarrow x = \frac{1,301030}{0,477121} \simeq 2,72$

c) $3^{2x} - 3^x \cdot 5 + 4 = 0$

$y^2 - 5y + 4 = 0$

$y' = 4$

$y'' = 1$

$3^x = 4$

$x = \log_3 4 = \frac{\log 4}{\log 3} = \frac{0,602060}{0,477121} \simeq 1,26$

$3^x = 1$

$3^x = 1 \Rightarrow 3^x = 0 \Rightarrow x = 0$

$S = \{1{,}26 ; 0\}$

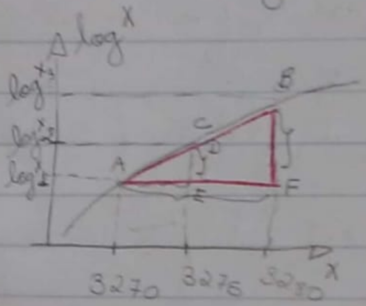
Calcular o $\log 3.275$

$X_1 = 3270 = (3{,}24) \Longrightarrow \log^{3270} = 3{,}514548$

$X_2 = 3275 \Rightarrow y$

$X_3 = 3280 \Rightarrow$

3,515874



$\Delta ABF \approx \Delta ADE$

$$\frac{DE}{BF} = \frac{3275 - 3270}{3280 - 3270}$$

$$\frac{y - \log^{x_1}}{\log^{x_3} - \log^{x_1}} = \frac{5}{10}$$

$$\frac{y - \log^{x_1}}{3{,}515874 - 3{,}514548} = \frac{5}{10}$$

$$\frac{y - \log^{x_1}}{0{,}001326} = \frac{5}{10}$$

subst: $y - \log^{x_1} = d$

$$\frac{d}{0{,}001326} = \frac{5}{10}$$

$10d = 0{,}00663$

$d = \frac{0{,}00663}{10}$

$\boxed{d = 0{,}000663}$

$y - 3{,}514548 = 0{,}000663$

$y = 0{,}000663 + 3{,}514548$

$y = 3{,}515211$

19) $\log_2 (a-b) = m$ $\qquad a+b=8$

$$\log_2 (a^2-b^2) = \log_2 (a+b)(a-b) \Rightarrow \log_2 (a+b) + \log_2 (a-b)$$

$$\Rightarrow \log_2 8 + m \Rightarrow \boxed{3+m}$$

Continuação ... 4º lista de Exercícios

15) $\log_a b \cdot \log_b c \cdot \log_c d \cdot \log_d x = 1$

$$\frac{\cancel{\log_d b}}{\log_d a} \cdot \frac{\cancel{\log_d c}}{\cancel{\log_d b}} \cdot \frac{\cancel{\log_d d}}{\cancel{\log_d c}} \cdot \log_d x = 1$$

$$\frac{1}{\log_d a} \cdot \log_d x = 1$$

$$\frac{\log_d x}{\log_d a} = 1 \Rightarrow \log_d x = \log_d a$$

$$\underline{X = a}$$

16) $\log_3 (\log_9 x) = \log_9 (\log_3 x)$

$\log_3 (\log_{3^2} x) = \log_{3^2} (\log_3 x)$

$\log_3 (\frac{1}{2} \log_3 x) = \frac{1}{2} \log_3 (\log_3 x)$ $\qquad \log_3 x = y$

$\log_3 (\frac{1}{2} y) = \frac{1}{2} \log_3 y$

$\log_3 \frac{y}{2} = \log_3 y^{1/2}$

$\frac{y}{2} = y^{1/2}$

$\left(\frac{y}{2}\right)^2 = \left(y^{1/2}\right)^2$

$\frac{y^2}{4} = y$

$y^2 = 4y$

$y^2 - 4y = 0$

$y' = 0$

$y'' = 4$

$\log_3 x = 0 \to x = 3^0 \Rightarrow \boxed{x = 1}$

$\log_3 x = 4 \to x = 3^4 \to \boxed{x = 81}$

$\boxed{x > 0}$

$\log_9 x > 0 \Rightarrow \boxed{x > 1}$

$S = \{81\}$

$3^x = 1 \Rightarrow 3^x = 0 \Rightarrow x = 0$

$S = \{1{,}26 ; 0\}$

Calcular o $\log 3.275$

17) $\log a/b + \log b/c + \log c/d - \log \frac{ay}{dx}$

$\log a/b + \log b/c + \log c/d - [\log ay - \log dx]$

$\log a/b + \log b/c + \log c/d - \log ay + \log dx$

$\log \left[\frac{\frac{a}{b} \cdot \frac{b}{c} \cdot \frac{c}{d} \cdot dx}{ay}\right] = \log \left[\frac{\cancel{a}}{\cancel{b}} \cdot \frac{\cancel{b}}{\cancel{c}} \cdot \frac{\cancel{c}}{\cancel{d}} \cdot \cancel{d}x \cdot \frac{1}{\cancel{a}y}\right] = \log \frac{x}{y}$

alternativa b

18) $\log m = b - \log n$

$\log m = \log 10^b - \log n$

$\log m = \log \frac{10^b}{n}$

$m = \frac{10^b}{n}$

n.r.a

19) $\log_2 (a-b) = m \qquad a+b=8$

$\log_2 (a^2-b^2) = \log_2 (a+b)(a-b) = \log_2 (a+b) + \log_2 (a-b)$

$\Rightarrow \log_2 8 + m \Rightarrow \boxed{3+m}$

20) $\log x = \log b + 2\log c - \frac{1}{3}\log a$

$\log x = \log b + \log c^2 - \log a^{1/3}$

$\log x = \log \frac{b \cdot c^2}{a^{1/3}}$

$x = \frac{b \cdot c^2}{\sqrt[3]{a}}$

21) $x^{\log x} = \frac{x^3}{100}$

$\log x^{\log x} = \log \left(\frac{x^3}{100}\right)$

$\log x \cdot \log x = \log x^3 - \log 100$

$(\log x)^2 = 3\log x - 2 \qquad \log x = y$

$y^2 = 3y - 2$

$y^2 - 3y + 2 = 0$

$y' = 2$

$y'' = 1$

$\log x = 2 \Rightarrow x = 100$

$\log x = 1 \Rightarrow x = 10$

$S = \{10, 100\}$

22) $\log_x (x+1) = \log_{x+1} x$

$\log_x (x+1) = \frac{\log x}{\log_x (x+1)}$

$\log_x (x+1) = \frac{1}{\log_x (x+1)} \qquad \log_x (x+1) = y$

$y = \frac{1}{y} \Rightarrow y^2 = 1$

$y = \pm 1$

$\log_x (x+1) = 1$	$\log_x (x+1) = -1$
$x+1 = x$	$x+1 = \frac{1}{x}$
impossível	$x^2 + x - 1 = 0$

$x' = \frac{-1+\sqrt{5}}{2}$ $\qquad$ $x'' = \frac{-1-\sqrt{5}}{2}$

<u>Verif</u>

$x+1>0 \to \boxed{x>-1}$

$\boxed{x>0}$

$x \neq 1 \to x>0$

$x+1 \neq 1 \Rightarrow \boxed{x \neq 0}$

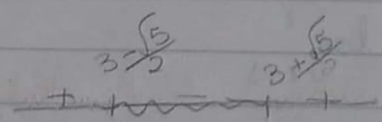
$x+1>0 \Rightarrow \boxed{x>-1}$

23- $\frac{1}{\log_e^x} + \frac{1}{\log_e^{x-1}} > 1 \qquad \neq \log_e^{(x-1)}$

$\log_e^x = y$

$$\frac{1}{y} + \frac{1}{y-1} > 1$$

$$y-1+y > y(y-1)$$
$$y-1+y > y^2-y$$
$$-y^2+y+y-1+y>0$$
$$-y^2+3y-1>0$$
$$y^2-3y+1<0$$
$$y' = \frac{3+\sqrt{5}}{2}$$
$$y'' = \frac{3-\sqrt{5}}{2}$$



$$\frac{3-\sqrt{5}}{2} < y < \frac{3+\sqrt{5}}{2}$$

$$\frac{3-\sqrt{5}}{2} < \log_e^x < \frac{3+\sqrt{5}}{2}$$

$$\log_e x < \frac{3+\sqrt{5}}{2} \Rightarrow \log_e x < \log_e e^{\frac{3+\sqrt{5}}{2}}$$

$$\boxed{x < e^{\frac{3+\sqrt{5}}{2}}}$$

$$\log_e x > \frac{3-\sqrt{5}}{2} \Rightarrow \log_e x > \log_e e^{\frac{3-\sqrt{5}}{2}} \Rightarrow \boxed{x > e^{\frac{3-\sqrt{5}}{2}}}$$

$\boxed{x > 0}$ $\qquad e^{\frac{3-\sqrt{5}}{2}} < x < e^{\frac{3+\sqrt{5}}{2}}$

Fuvest 84 – 1ª fase

Se $x = \log_4 7$ e $y = \log_{16} 49$ então

$x - y$ é igual a

a) $\log_4 7$ $\qquad$ b) $\log_{16} 7$

4/06 Sequências

(2, 4, 6, 8, 10 ...) números pares

genericamente

$(a_1, a_2, a_3, a_4 \ldots)$

Exercícios pág 3

1- 2, 3, 5, 7, 11, 13, 17, $\boxed{19}$

2- $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

5º $a_n = \frac{n}{n+1}$ $\qquad$ 100º $\quad a_{100} = \frac{100}{100+1} = \boxed{\frac{100}{101}}$

$a_5 = \frac{5}{5+1} = \boxed{\frac{5}{6}}$

3. $a_n = n^3$

$a_1 = 1^3 = 1$
$a_2 = 2^3 = 8$
$a_3 = 3^3 = 27$ $(1, 8, 27, 64, 125)$
$a_4 = 4^3 = 64$
$a_5 = 5^3 = 125$

4. $a_n = n^2$
$a_1 = 1^2 = 1$
$a_2 = 2^2 = 4$
$a_3 = 3^2 = 9$
$a_4 = 4^2 = 16$
$a_5 = 5^2 = 25$
$a_6 = 6^2 = 36$
$a_7 = 7^2 = 49$
140

5. $a_n = \sqrt[n+1]{n}$

$a_1 = \sqrt[2]{1}$

$a_2 = \sqrt[3]{2}$

$a_3 = \sqrt[4]{3}$

$a_4 = \sqrt[5]{4}$

$a_5 = \sqrt[6]{5}$

6. $a_n = \sqrt{n}$ $\{0, 1, 4, 9, 16, 25\}$

7. $a_n = 2^n$
$a_1 = 2^1 = 2$
$a_2 = 2^2 = 4$
$a_3 = 2^3 = 8$

$a_4 = 2^4 = 16$
$a_5 = 2^5 = 32$
$a_6 = 2^6 = 64$
$a_7 = 2^7 = \boxed{128}$

8- $a_n = \dfrac{(n+1)^2 - (n-1)^2}{2}$

p/ ser Par tenho que ter par X ∀ número

$a_n = \dfrac{n^2+2n+1-(n^2-2n+1)}{2}$

$a_n = \dfrac{\cancel{n^2}+2n\cancel{+1}-\cancel{n^2}+2n\cancel{-1}}{2}$

$a_n = \dfrac{4n}{2}$

$a_n = 2n$

par x ∀ número = par

9- $a_n = 2n-1$

$a_1 = 1$

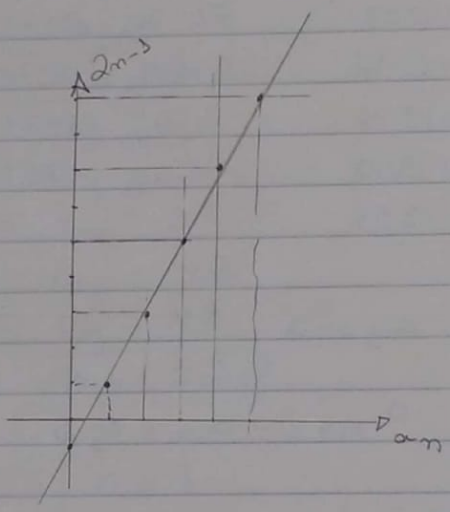
$a_2 = 3$

$a_3 = 5$

$a_4 = 7$

$a_5 = 9$

$a_0 = -1$



10- $a_n = (-1)^n \dfrac{2n+1}{n+2}$

$a_0 = (-1)^0 \dfrac{2.0+1}{0+2} \Rightarrow 1 . \dfrac{1}{2} = 1/2$

$a_1 = (-1)^1 \dfrac{2.1+1}{1+2} = -1 . \dfrac{3}{3} = -1$

$a_2 = (-1)^2 \dfrac{2.2+1}{2+2} = 1 . \dfrac{5}{4} = 5/4$

$a_3 = (-1)^3 \dfrac{2.3+1}{3+2} = -1 . \dfrac{7}{5} = -7/5$

$a_4 = (-1)^4 \dfrac{2.4+1}{4+2} = \dfrac{9}{6} = 3/2$

$$a_5 = \frac{(-1)^5 \cdot 2 \cdot 5 + 1}{5+2} = -1 \cdot \frac{11}{7} = -11/7$$

* Como eu quero 6 sequências, tenho [illegible]

ímpar > 5

dá nº negativo $(-1)^5 = \underline{\underline{-1}}$ então, por analogia é o $\boxed{7}$

11- 1930

1978 → 13º

$13 - 11 = \boxed{2 \text{ copas}}$

$a_1 = 1930$ | $a_8 = 1958$
$a_2 = 1934$ | $a_9 = 1962$
$a_3 = 1938$ | $a_{10} = 1966$
$a_4 = 1942$ | $\boxed{a_{11} = 1970}$ → tinha que ser
$a_5 = 1946$ | $\left.\begin{matrix} a_{12} = 1974 \\ a_{13} = 1978 \end{matrix}\right\} 2$
$a_6 = 1950$
$a_7 = 1954$

12- $a_1 = 1$

$$a_{n+1} = a_n + 0,5$$

$n = 1 \Rightarrow a_2 = 1 + 0,5 = 1,5$
$n = 2 \Rightarrow a_3 = 1,5 + 0,5 = 2,0$
$n = 3 \Rightarrow a_4 = 2,0 + 0,5 = 2,5$
$n = 4 \Rightarrow a_5 = 2,5 + 0,5 = 3,0$
$n = 5 \Rightarrow \boxed{a_6 = 3,5}$
$n = 6 \Rightarrow a_7 = 4,0$
$\boxed{a_8 = 4,5}$

13/06 13- $a_1 = 5$ $\quad n \geq 1$

$$a_{n+1} = a_n + 3$$

$n = 1 \Rightarrow a_2 = a_1 + 3 = 5 + 3 = 8$
$n = 2 \Rightarrow a_3 = a_2 + 3 = 8 + 3 = 11$

$a_4 = 34$
$a_5 = 17$
$a_6 = 20$
$a_7 = 23$
$a_8 = 26$
$a_9 = 23$

R: Quatro números são pares.

14- $a_0 = 2$ $\quad n \in N$
$a_{n+1} = a_n + 5$

$n=0 \Rightarrow a_{0+1} = a_0 + 5 \Rightarrow a_1 = 2+5 = 7$ +5
$n=1 \Rightarrow a_{1+1} = a_1 + 5 \Rightarrow a_2 = 7+5 = 12$
$a_3 = 17$
$a_4 = 22$
$a_5 = 27$
$a_6 = 32$
$a_7 = 37$
$a_8 = 42$
$a_9 = 47$
$\boxed{a_{10} = 52}$ //

15- $a_1 = -2$ $\quad n \geq 1$
$a_{n+1} = -3 . a_n$ $\quad$ 6° e 7° termo?

$n=1 \Rightarrow a_{1+1} = -3 . a_1 \Rightarrow a_2 = -3.(-2) = 6$ x3
$n=2 \Rightarrow a_{2+1} = -3 . a_2 \Rightarrow a_3 = -3.6 = -18$
$a_4 = 54$
$a_5 = -162$
$a_6 = 486$
$a_7 = -1458$

18
x3
54

pensar

16 - $n \in \mathbb{N}$

$f(0) = 2$ $\qquad f(n+1) = f(n) + \frac{2}{3}$ $\qquad f(10) = ?$

$a = \frac{\Delta y}{\Delta x}$

$f(0+1) = f(0) + \frac{2}{3}$

$f(1) = 2 + \frac{2}{3} = \frac{6+2}{3} = \frac{8}{3}$

$f(1+1) = f(1) + \frac{2}{3}$

$f(2) = \frac{8}{3} + \frac{2}{3} = \frac{10}{3}$

$f(2+1) = f(2) + \frac{2}{3}$

$f(3) = \frac{10}{3} + \frac{2}{3} = \frac{12}{3}$

$\frac{10}{3} - \frac{8}{3} = \frac{2}{3}$

f(4) = 14	28	48
f(5) = 16	30	50
f(6) 18	32	52
f(7) 20	34	54
f(8) 22	36	56
f(9) 24	38	58
f(10) 26	40	60
	42	62
	44	64
	46	$\frac{66}{3} = \boxed{22}$

Progressão Aritmética (P.A)

Chama-se progressão aritmética uma sequência em que a partir do 2º elemento, a diferença entre cada elemento e seu antecessor é uma constante. Esta constante é chamada de razão da P.A. e é indicada pela letra r

$(2, 4, 6, 8, 10 \ldots)$ $\quad 2-4 = 6-4 = 8-6 = 10-8 \ldots = 2$

$r = 2$

Scanned by CamScanner

$(a_1, a_2, a_3, a_4 \ldots)$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 \ldots = r$

A P.A pode ser: (~~crescente~~)

a) crescente $r > 0$
(2, 4, 6 ...)

b) decrescente $r < 0$
(8, 6, 4, 2 ...)

c) constante $r = 0$
(2, 2, 2 ...)

13/06

pág 6

18. c) $3 - \sqrt{2} - (3 - 2\sqrt{2})$

$\cancel{3} - \sqrt{2} - \cancel{3} + 2\sqrt{2} = \sqrt{2}$

$3 - (3 - \sqrt{2})$

$3 - 3 + \sqrt{2} = \sqrt{2}$

$3 + \sqrt{2} - (3) = \sqrt{2}$

$3 + 2\sqrt{2} - (3 + \sqrt{2})$

$\cancel{3} + 2\sqrt{2} - \cancel{3} - \sqrt{2} = \sqrt{2}$

19. $\frac{x+5}{2} - \frac{x+3}{2} = \frac{x+5-x-3}{2} = \frac{2}{2} = 1$

$\frac{x+7}{2} - \frac{x+5}{2} = \frac{x+7-x-5}{2} = \frac{2}{2} = 1$

$\frac{x+9}{2} - \frac{x+7}{2} = \frac{x+9-x-7}{2} = \frac{2}{2} = 1$

20. $\frac{a^2+2}{a} - \frac{2a^2+2}{a} = r$

$\frac{a^2 + \cancel{2} - 2a^2 - \cancel{2}}{a} = r$

$\frac{-a^{\cancel{2}}}{\cancel{a}} = r$

$\boxed{r = -a}$

21. $(-2x, 5x-4, 1+3x \ldots)$

a) $a_2 - a_1 = r$

$5x-4-(-2x) = r$

$\boxed{7x-4 = r}$

$a_3 - a_2 = r$

$1+3x-(5x-4) = r$

$1+3x-5x+4 = r$

$\boxed{5-2x = r}$

ou direto

$5x-4-(-2x) = 1+3x-(5x-4)$

$a_2 - a_1 = a_3 - a_2$

$7x-4 = 5-2x$

$7x+2x = 5+4$

$9x = 9$

$\boxed{x = 1}$

outra maneira:

$a_2 = \frac{a_1+a_3}{2}$

$5x-4 = \frac{-2x+1+3x}{2}$

b) $x = 1$

$(-2x, 5x-4, 1+3x \ldots)$

$(-2, 1, 4, \underline{\underline{7}}, \underline{\underline{10}})$

22. $(x^2, 3x, 2x)$ $x = ?$ p/ P.A. decrescente

$3x - x^2 = r$ $\quad$ $2x - 3x = r$

$3x - x^2 = 2x - 3x$

$3x - x^2 = -x$

$-x^2 + 4x = 0$

$x^2 - 4x = 0$ $\quad$ $x' = 0$ P.A. (constante)

$\Delta = 16$ $\quad$ $x'' = 4$ $\quad$ $(16, 12, 8, \ldots)$ $\quad$ $\boxed{r = -4}$

23_ $r = 2x - 3$

$\boxed{r < 0}$

$2x - 3 < 0$

$2x < 3$

$\boxed{x < 3/2}$

p/ valores menores que $3/2$

24_ $(a, ax, 2a)$ $a \neq 0$ P.A

$ax = \frac{2a + a}{2}$

$2ax = 3a$

$2x = \frac{3\cancel{a}}{\cancel{a}}$

$\boxed{x = 3/2}$

ou

$ax - a = 2a - ax$

25_ $(4 + 3\sqrt{5}, \quad , 8 + 5\sqrt{5} \ldots)$

$a_7 = \frac{4 + 3\sqrt{5} + 8 + 5\sqrt{5}}{2}$

$a_7 = \frac{\overset{6}{\cancel{12}} + \overset{4}{\cancel{8}}\sqrt{5}}{\cancel{2}}$

$\boxed{a_7 = 6 + 4\sqrt{5}}$

$6 + 4\sqrt{5} - 4 - 3\sqrt{5} = r$

$\boxed{2 + \sqrt{5} = r}$

26_ $((x+5)^2, (x+3)^2, (x-1)^2)$

$(x+3)^2 - (x+5)^2 = (x-1)^2 - (x+3)^2$

$x^2 + 6x + 9 - (x^2 + 10x + 25) = x^2 - 2x + 1 - (x^2 + 6x + 9)$

$\cancel{x^2} + 6x + 9 - \cancel{x^2} - 10x - 25 = \cancel{x^2} - 2x + 1 - \cancel{x^2} - 6x - 9$

$-4x - 16 = -8x - 8$

$-4x + 8x = -8 + 16$

$4x = 8$

$\boxed{x = 2}$

$$(2+5)^2 = 7^2 = \underline{\underline{49}}$$

$$(49, 25, 1 \ldots)$$

$$r = 25 - 49$$

$$\boxed{r = -24}$$

27.- a) $(x-y, x, x+y)$

$$x - (x-y)$$

$$\cancel{x} - \cancel{x} + y = y$$

$$\cancel{x} + y - \cancel{x} = y$$

¿P.A

$\underline{r = y}$

b) $(x-2y, x-y, x+y, x+2y)$

$$x - y - (x-2y)$$

$$\cancel{x} - y - \cancel{x} + 2y = y \quad \boxed{r = y}$$

$\underline{\underline{\text{no es PA}}}$

$$x+y-(x-y) = \cancel{x} + y - \cancel{x} + y = 2y \quad \boxed{r = 2y}$$

c) $(x-3y, x-y, x+y, x+3y)$

$$x - y - (x-3y) = r \longrightarrow r = 2y$$

$$x + y - (x-y) = r \longrightarrow r = 2y$$

$$x + 3y - (x+y) = r \longrightarrow r = 2y$$

$\underline{\underline{\text{¿P.A}}}$

d) $\dfrac{a+b}{2} - a = r$

$$\frac{a+b-2a}{2} = r$$

$$r = \frac{-a+b}{2}$$

Ex:

28- $(2, 4, 6) \quad r = 2$

$(x-r) \quad x \quad (x+r)$

$$\begin{cases} (x-r) \cdot x \cdot (x+r) = \frac{15}{2} \\ x - \cancel{r} + x + x + \cancel{r} = 6 \Rightarrow 3x = 6 \\ \boxed{x = 2} \end{cases}$$

$(2-r) \cdot 2 \cdot (2+r) = \frac{15}{2}$

$(4-r^2) \cdot 2 = \frac{15}{2}$

$8 - 2r^2 = \frac{15}{2}$

$16 - 4r^2 = 15$

$-4r^2 = -1$

$r^2 = 1/4$

$r = \pm 1/2$

$x = 2 \quad e \quad r = \frac{1}{2} \quad \left(\frac{3}{2}, 2, \frac{5}{2}\right)$

$x = 2 \quad e \quad r = -\frac{1}{2} \quad \left(\frac{5}{2}, 2, \frac{3}{2}\right)$

17/06 29. $(x-r, x, x+r)$

$$\begin{cases} x - \cancel{r} + x + x + \cancel{r} = 15 \rightarrow x = 5 \\ (x-r) \cdot x \cdot (x+r) = 80 \end{cases}$$

$(5-r) \cdot 5 (5+r) = 80$

$(25-r^2) \cdot 5 = 80$

$125 - 5r^2 = 80$

$-5r^2 = -125 + 80$

$-5r^2 = -45$

$r^2 = 9$

$r = \pm 3$

$x = 5 \quad e \quad r = 3 \rightarrow (2, 5, 8)$

$x = 5 \quad e \quad r = -3 \rightarrow (8, 5, 2)$

30- $(x-r, x, x+r)$

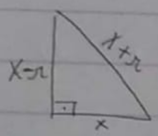
$$x-\cancel{r}+x+x+\cancel{r}=180$$
$$3x=180$$
$$\boxed{x=60}$$

$(30, 60, 90)$

ou

$(90, 60, 30)$

São 30 e 60

31- 

$r=6$

$(x-r, x, x+r)$

$$(x+r)^2 = x^2 + (x-r)^2$$
$$(x+6)^2 = x^2 + (x-6)^2$$
$$x^2+12x+\cancel{36} = x^2 + x^2 - 12x + \cancel{36}$$
$$-x^2+24x=0$$
$$x^2-24x=0$$
$$x'=0$$
$$x''=24$$

~~$x=0$ e $r=6$ ⟶ $(-6, 24, 6)$~~ um lado n pode medir zero

$x=24$ e $r=6$ ⟶ $(18, 24, 30)$

32- $(x-2r, x-r, x, x+r, x+2r)$

$$x-\cancel{2r}+x-\cancel{r}+x+x+\cancel{r}+x+\cancel{2r}=10$$
$$5x=10$$
$$\boxed{x=2}$$
$$(2-2r).(2-r).2.(2+r).(2+2r)=0$$
$$(4-4r^2).(4-r^2).2=0$$

Scanned by CamScanner

$(16-4r^2-16r^2+4r^4) \cdot 2 = 0$

$32-8r^2-32r^2+8r^4=0$

$8r^4-40r^2+32=0$

$r=\pm 2$ ou $r=\pm 1$

$r=2$ e $x=2$ $(-2,0,2,4,6)$

$r=1$ e $x=2$ $(0,1,2,3,4)$

33_ $(x-4r, x-3r, x-2r, x-r, x, x+r, x+2r, x+3r, x+4r)$

$9x=0$

$\boxed{x=0}$

$(x-4r) \cdot (x-3r) \ldots (x) \ldots (x+4r) = ?$

$-4r \cdot -3r \cdot -2r \cdot -r \cdot 0 \ldots = 0$

Descrever por meio de uma fórmula cada uma das sequências abaixo

a) $(3, 6, 9, 12, 15 \ldots)$

$\boxed{a_1 = 3 ; \ a_{n+1} = a_n + 3}$

$a_1 = 3$

$a_2 = a_1 + 3 = 3 + 3 = 6$

$a_3 = a_2 + 3 = 6 + 3 = 9$

$\boxed{a_n = a_{n-1} + 3}$

$a_n = 3n \quad n \in \mathbb{N}$

b) $(1, -1, 1, -1, 1, -1 \ldots)$

$a_1 = 1$

$a_2 = a_1 - 2 = 1 - 2 = -1$

$a_3 = a_2 - 2 =$

1/08 **Fórmula do termo geral (P.A.)**

$(a_1, a_2, a_3, a_4 \ldots)$

$a_2 = a_1 + r$

$a_3 = a_2 + r \Rightarrow a_3 = a_1 + r + r$

$a_3 = a_1 + 2r$

$a_4 = a_3 + r \Rightarrow a_4 = a_1 + 2r + r$

$a_4 = a_1 + 3r$

$a_5 = a_4 + r \Rightarrow a_5 = a_1 + 3r + r$

$a_5 = a_1 + 4r$

$a_n = a_1 + (n-1) \cdot r$

Ex:

Dada a P.A. $(2, 4, 6, 8 \ldots)$

$a_{21} = ?$ $\quad r = 4 - 2 = 2$

$a_{21} = a_1 + (21 - 1) \cdot r$

$a_{21} = 2 + 20 \cdot 2$

$a_{21} = 2 + 40 = \underline{\underline{42}}$

Exercícios página 8.

34. $a_{15} = ?$ $\quad (3, 7, 11, \ldots)$

$7 - 3 = 4$

$a_{15} = 3 + 14 \cdot 4$

$a_{15} = 3 + 56$

$\underline{\underline{a_{15} = 59}}$

35. $a_{20} = ?$ $\quad (-9, -4, 1, 6, \ldots)$

$6 - 1 = \underline{\underline{5}}$

$a_{20} = -9 + 19 \cdot 5$

$a_{20} = -9 + 95$

$\underline{\underline{a_{20} = 86}}$