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Transformada de Laplace

Def: $\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\,dt = F(s)$

Qdo esta integral converge será a transformada.

Ex: $f(t) = e^{at}$

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} e^{at}\,dt = \int_0^\infty e^{-(s-a)t}\,dt =$$

$$= -\frac{1}{s-a} e^{-(s-a)t}\Big|_0^\infty = \lim_{b\to\infty} \frac{-1}{s-a} e^{-(s-a)b} + \frac{1}{s-a} = \frac{1}{s-a}$$

Obs: $\int_0^\infty f = \lim \int_0^b f$

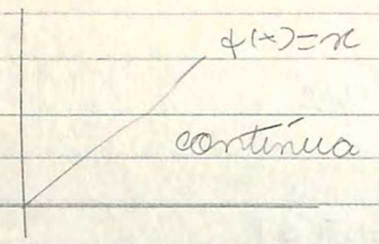
f	$\mathcal{L}[f]$	f	$\mathcal{L}[f]$
e^{at}	$\frac{1}{s-a}$	1	$1/s \quad s>0$
		t	$1/s^2 \quad s>0$
$\cos at$	$\frac{s}{s^2+a^2}$	$\text{sen}\, at$	$a^2/s^2+a^2 \quad s>0$
		t^2	$\frac{2}{s^3} \quad s>0$

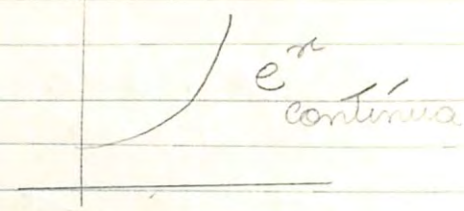
Calcular a transformada de Laplace

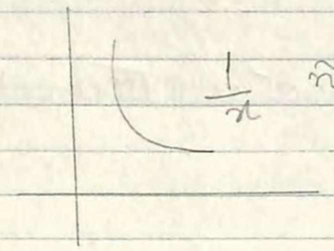
1) $\mathcal{L}(1) = \int_0^\infty e^{-st}\,dt = -\frac{1}{s} e^{-st}\Big|_0^b = \lim_{b\to\infty} -\frac{1}{s} e^{-sb} + \frac{1}{s}$

$= \frac{1}{s}$ //

$f: [0, \infty] \to R$







ñ é contínua
não é seccional/e contínua

Condições para que uma função seja seccional/e contínua.

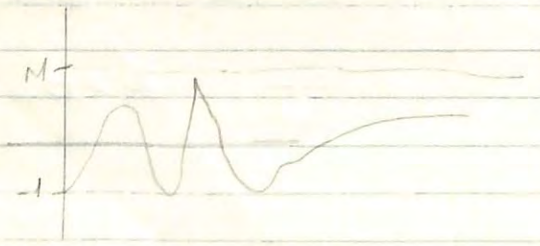
① f tem um nº finito de descontinuidade

② lim laterais nos pontos de descontinuidade são finitos

Função limitada

$f[0, +\infty] \to R$ é limitada $\iff \exists M > 0 \; |f(x)| \le M$

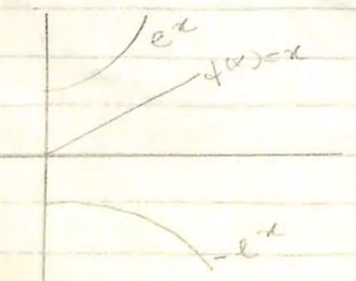
$\forall x \in [0, \infty]$



$-M \le f(x) \le M$

$\forall x \in [0, \infty]$

$f(x) = x$



$|f(x)| \le 1 . e^{1x}$

f é de ordem exponencial, se existe um t_0, $M > 0$, e tais que $|f(t)| \le M e^{ct}$. $\forall t \geq t$

$|\operatorname{sen} x| \le 1 \le 1 e^{0t}$

$|\cos x| \le 1 \le 1 e^{0t}$

e^{t^2} não é de ordem exponencial

$$\lim_{t \to \infty} \frac{e^{t^2}}{M e^{ct}} = +\infty \quad \forall M, c$$

$$\lim_{t \to \infty} \frac{1}{M} \left(e^{t^2 - ct} \right) = +\infty$$

Teorema: se $f[0, +\infty] \to R$ satisfaz as condições de Dirichlet, com abscissa de convergência c, então $\exists \mathcal{L}[f(t)](s) = F(s) \; \forall s > c$

$$f(t) = \operatorname{senh}(t) = \frac{e^{at} - e^{-at}}{2}$$

Transformada de Laplace

① $f: [0, \infty] \to R$

$\mathcal{L}(f) = F(s) = \int_0^\infty e^{-st} f(t) \, at$

a) $\mathcal{L}(e^{at}) = 1/s-a \quad s > a$
b) $\mathcal{L}(\operatorname{sen} at) = a/s^2+a^2 \quad s > 0$
c) $\mathcal{L}(\cos at) = s/s^2+a^2 \quad s > 0$
d) $\mathcal{L}(1) = 1/s$
e) $\mathcal{L}(t) = 1/s^2$
f) $\mathcal{L}(t^2) = \frac{2}{t^3}$

Solução

a) $\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t) dt$

só terá sentido q/do isso for neg.

$$\mathcal{L}(e^{at}) = \int_0^\infty e^{-st} e^{at} dt = \int_0^\infty e^{(-s+a)t} dt = \frac{e^{(-s+a)t}}{(-s+a)} = \frac{e^{(a-s)t}}{a-s} \Big|_0^\infty = 0 - \frac{e^0}{a-s} = \frac{1}{a-s}$$

1) $\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) = e^{2t}$

2) $\mathcal{L}^{-1}\left(\frac{s-2}{s^2-s-6}\right)$

$$\frac{s-2}{s^2-s-6} = \frac{a}{s+2} + \frac{b}{s-3} = \frac{a(s-3)+b(s+2)}{(s-3)(x+2}$$

$$= \frac{(a+b)s + (-3a+2b)}{(s-3)(s+2)}$$

$$\begin{cases} a+b=1 \\ -3a+2b=2 \end{cases} \qquad \begin{matrix} b = 1/5 \\ a = 4/5 \end{matrix}$$

$$\mathcal{L}^{-1}\left(\frac{s-2}{s^2-s-6}\right) = \mathcal{L}^{-1}\left(\frac{4/5}{s+2} + \frac{1/5}{s-3}\right) = \frac{4}{5}e^{-2t} + \frac{1}{5}e^{3t}$$

3) $\mathcal{L}^{-1}\left(\frac{3}{s^2+9}\right) = \text{sen } 3t$

4) $\mathcal{L}^{-1}\left(\frac{s}{s^2+16}\right) = \cos 4t$

5) $\mathcal{L}^{-1}\left(\frac{s+7}{s^2+6s+25}\right)$

$$\frac{s+7}{s^2+6s+25} = \frac{s+3+4}{(s^2+6s+9)+16} = \frac{s+3}{(s+3)^2+16} + \frac{4}{(s+3)^2+16} =$$

$e^{3t}\,\text{sen } 4t +$

Propriedades

1) $\mathcal{L}(f(t)) = F(s) \rightarrow \mathcal{L}(e^{at}f(t)) = F(s-a)$

$$\mathcal{L}(e^{3t}\,\text{sen } 5t) = \frac{5}{(s-3)^2+25}$$

$\mathcal{L}^{-1}[F(s-a) = e^{at}f(t)$

1) $\mathcal{L}(e^{7t}t) = \frac{1}{(s-7)^2}$

3) $\mathcal{L}(e^{7t}\cos 8t) = \frac{s-7}{(s-7)^2+64}$

4) $\mathcal{L}(e^{t}\,\text{sen } 8t) = \frac{8}{(s-1)^2+64}$

5) $\mathcal{L}(e^{3t}\,\text{sen } 7t) = \frac{7}{(s-3)^2+49}$

2ª prop. $\mathcal{L}(f(t)) = F(s)$

$\mathcal{L}(-t f(t)) = F'(s)$

$\mathcal{L}^{-1}[F'(s)] = -t f(t)$

1) $\mathcal{L}(t\,\text{sen } 8t) = \frac{16s}{(s^2+64)^2}$

2) $\mathcal{L}(t\,\underbrace{\text{senh } 3t}_{\frac{3}{s^2-9}}) = \frac{6s}{(s^2-9)^2}$

Se $\mathcal{L}\{f(t)\} = F(s) \rightarrow \mathcal{L}(t^n f(t)) = (-1)^n F^n(s)$

$\therefore \mathcal{L}^{-1}((-1)^n F^n(s)) = t^n f(t)$

3) $\mathcal{L}(f(t)) = F(s) \rightarrow \mathcal{L}(f'(t)) = sF(s) - f(0)$

$$\mathcal{L}(t\,\underbrace{e^{3t}\,\text{sen } 8t}_{\frac{8}{(s-3)^2+64}}) = -\left(-\frac{8 \times 2(s-3)}{[(s-3)^2+64]^2}\right. = \frac{16(s-3)}{[(s-3)^2+64]^2}$$

4ª propriedade

$$\mathcal{L}\left\{\int_0^t f(r)\,dr\right\} = \frac{1}{s}\,\mathcal{L}\{f(t)\}$$

Calcular

a) $\mathcal{L}(e^{3t}\,\text{sen}\,8t) = \frac{8}{(s-3)^2+64}$

b) $\mathcal{L}(t\,\text{senh}\,3t) = \frac{3}{s^2-9} = \frac{6}{(s^2+9)^2}$

c) $\mathcal{L}(t\,e^{3t}\,\text{sen}\,5t) = \frac{10(s-3)}{[-(s-3)^2+25]^2}$

d) $\mathcal{L}^{-1}\left(\frac{5}{s^2+25}\right) = \text{sen}\,5t$

e) $\mathcal{L}^{-1}\left(\frac{s+7}{s^2+6s+25}\right) = \frac{s+7}{s+6s+9+16} = \frac{s+7}{(s+3)^2+16} =$

$= \frac{s+3}{(s+3)^2+16} + \frac{4}{(s+3)^2+16}$

$e^{-3t}\cos 4t + e^{-3t}\,\text{sen}\,4t$

f) $\mathcal{L}^{-1}\left(\frac{-4s}{(s^2+4)^2}\right) = t\,\text{sen}\,2t$

g) $\mathcal{L}^{-1}\left(\frac{8s+2}{s^2+5s+4}\right) = \frac{A}{s+1} + \frac{B}{s+4} = \frac{A(s+4)+B(s+1)}{(s+1)(s+4)}$

$-3B = -30$ $\quad B = 10$

$3A = -6$ $\quad A = -2$

$\mathcal{L}^{-1}\left(\frac{-2}{s+1}\right) + \mathcal{L}^{-1}\left(\frac{10}{s+4}\right) = -2e^{-t} + 10e^{-4t}$

h) $\mathcal{L}^{-1}\left(\frac{1}{s^2+4s+13}\right) = \frac{1}{3}\,\mathcal{L}^{-1}\left(\frac{3}{(s+2)^2+9}\right) = \frac{1}{3}e^{-2t}\,\text{sen}\,3t$

i) $\mathcal{L}^{-1}\left(\frac{2}{s^2(s^2+4s+13)}\right) = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+4s+13}$ - pq o termo é irredutível

$= A(s(s^2+4s+13)) + B(s^2+4s+13) + s^2(Cs+D)$

$A = \frac{-8}{169}$ $\quad B = \frac{2}{13}$ $\quad C = \frac{8}{169}$

$= \frac{-8}{169} + \frac{2t}{13} + \frac{8}{169}e^{-2t}\cos 3t - \frac{10}{507}e^{-2t}\,\text{sen}\,3t$

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③ a) $f(t) = c$

$\mathcal{L}(f(t)) = \mathcal{L}(c)$

$= \int_0^\infty e^{-st}\,c\,dt = c\left[\frac{e^{-st}}{s}\right]_0^\infty = \lim_{b\to\infty} \frac{c\,e^{-sb}}{s} - \frac{c\,e^{-0}}{s} = \frac{-c}{s}$

b) $f(t) = 3t - \frac{1}{2}\cos 5t =$

$\frac{3}{s^2} - \frac{1}{2}\,\frac{s}{s^2+25}$ //

i) $\mathcal{L}(\cosh at) = \mathcal{L}\left(\frac{e^{at}}{2} + \frac{e^{-at}}{2}\right)$

$= \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \left[\frac{s}{s^2+a^2}\right]$

ii) $\mathcal{L}(\operatorname{senh} at) = \mathcal{L}\left(\frac{e^{at} - e^{-at}}{2}\right)$

$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \frac{a}{s^2-a^2}$

iii) $\mathcal{L}(\operatorname{senh}^2 at) = \mathcal{L}\left(\frac{e^{2at} + e^{-2at} - 2}{4}\right)$

$= \frac{1}{4}\left[\frac{1}{s-2a} + \frac{1}{s+2a} - \frac{2}{s}\right]$

$\mathcal{L}^{-1}\left(\frac{-1}{(s-2)^2}\right) = e^{2t} \cdot f(t) = -e^{2t} \cdot t$

$\mathcal{L}(f(t)) = -\frac{1}{s^2} = -t$

— x —

$\mathcal{L}^{-1}\left(\frac{-4s}{(s^2+4)^2}\right) =$

$\frac{-4s}{(s^2+4)^2} = \frac{d}{ds}\,\frac{2}{s^2+4} = \mathcal{L}(\operatorname{sen} 2t)$

$\Rightarrow \mathcal{L}^{-1} = -t \operatorname{sen} 2t$

$\mathcal{L}(t\, e^{5t})$

1) $\mathcal{L}(e^{at} f(t)) = F(s-a)$

$\mathcal{L}(e^{5t}\, t) = \frac{1}{s^2} = \frac{1}{(s-5)^2}$

$\mathcal{L}(e^{-t}\, t \cos 2t) =$

$f(t) = t \cos 2t$

$\mathcal{L}(f(t)) = -\frac{s^2-4}{(s^2+4)^2}$

$+\frac{(s+1)^2-4}{((s+1)^2+4)^2}$

— x —

$F(s) = \frac{s+3}{s^2+2s+10} = \frac{(s+1)+2}{(s+1)^2+9}$

$= e^{-t} f(t)$

$\mathcal{L}(f(t)) = \frac{s+2}{s^2+9} = \frac{s}{s^2+9} + \frac{2}{s^2+9}$

$= \cos 3t + \frac{2}{3}\,\frac{3}{s^2+9}$

$= \left(\cos 3t + \frac{2}{3} \operatorname{sen} 3t\right) e^{-t}$ //

41
25
16

— x —

$\mathcal{L}^{-1}\left(\frac{15}{s^2+10s+41}\right) = \mathcal{L}^{-1}\left(\frac{15}{(s+5)^2+16}\right)$

$= \mathcal{L}^{-1}\, \frac{15}{4} \cdot \left(\frac{4}{(s+5)^2+4^2}\right) = \frac{15}{4}\, e^{-5t} f(t)$

$\mathcal{L}(f(t)) = \frac{4}{s^2+4^2} = \operatorname{sen} 4t \Rightarrow \frac{15}{4}\, e^{-5t} \operatorname{sen} 4t$

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1) $\mathcal{L}(t^2-2) = \mathcal{L}(t^2) - \mathcal{L}(2)$

$= \frac{2}{t^3} - \frac{2}{s}$ //

② $e^{-at}\,\mathrm{sen}\,wt = F(s-a)$

$= \frac{w}{s^2+w^2} = \frac{w}{(s+a)^2+w^2}$

③ $\mathcal{L}(2t^2-3t+4) = \frac{4}{t^3} - \frac{3}{t^2} + \frac{4}{t}$

④ $\mathcal{L}(\mathrm{senh}^2 at) = \mathcal{L}\left(\frac{e^{2at}+e^{-2at}-2}{4}\right)$

$= \frac{1}{4}\left[\frac{1}{s-2a} + \frac{1}{s+2a} - \frac{2}{s}\right]$

5) $t\,e^{5t} = \frac{d}{ds}\,\frac{1}{s-5} = \frac{1}{(s-5)^2}$

$= \frac{-1}{(s-5)^2}$

⑦ $\mathcal{L}^{-1}\left(\frac{3}{s+2}\right) = 3e^{-2t}$

⑥ $\mathcal{L}^{-1}\left(\frac{s+2}{(s+2)^2+w^2}\right) = e^{-2t}\,f(t)$

$\mathcal{L}(f(t)) = \frac{s}{s^2+w^2} = \cosh wt$

$\Rightarrow f(t) = e^{-2t}\cosh wt$

① def: $\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\,dt$

② $\mathcal{L}(e^{at} f(t)) = F(s-a)$

③ $\mathcal{L}(t\,f(t)) = -F'(s)$

④ $\mathcal{L}(f'(t)) = sF(s) - f(0)$

⑤ $\mathcal{L}\{H_a(t)\,f(t-a)\} = e^{-as}F(s)$

⑥ $\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\{f(t)\}\,\mathcal{L}(g(t))$

⑦ $\mathcal{L}^{-1} F(s-a) = e^{at} f(t)$

⑧ $\mathcal{L}^{-1}\{e^{-as}F(s)\} = H_a(t) - f(t-a)$

⑨ $\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f(t) * g(t)$

calcule $\mathcal{L}$

a) $\mathcal{L}(e^{2t}\,\mathrm{sen}\,3t) = \frac{3}{(s-2)^2+9}$

b) $\mathcal{L}(t^3\,\mathrm{sen}\,3t) = (-1)^3\left(\frac{3}{s^2+9}\right)'''$

c) $\mathcal{L}(e^{-3t}\cos(2t+4)\} = \mathcal{L}\{e^{-3t}(\cos 2t\cos 4 - \mathrm{sen}\,2t\,\mathrm{sen}\,4)\}$

$= \cos 4\; e^{-3t}\cos 2t - \mathrm{sen}\,4\,\mathcal{L}\,e^{-3t}\,\mathrm{sen}\,2t =$

$\cos 4\;\frac{s+3}{(s+3)^2+4} - \mathrm{sen}\,4\;\frac{2}{(s+3)+4}$

d) $\mathcal{L}\{t\,e^{2t} f(t)\} = (s-2)\frac{d}{ds}F(s-2) + F(s-2)$

e) $\mathcal{L}\{f(t)\}$ onde

$$f(t)=\begin{cases}0 & t<1/2\\ 1+t & t\geq \frac{1}{2}\end{cases}$$

f) $\mathcal{L}\{f(t)\}$ onde

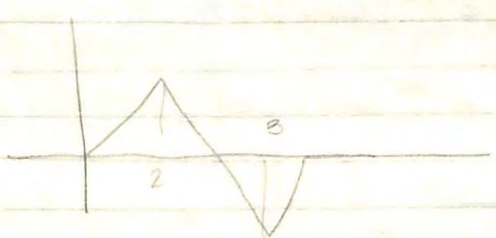
$$f(t)=\begin{cases}\text{sen}\,t & t<2\pi\\ 0 & t>2\pi\end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^{2\pi} e^{-st}\,\text{sen}\,t + \int_{2\pi}^{\infty} e^{-st}\,(0)\,dt =$$

$$F(s)\,(1+s^2) = \frac{s}{1+s^2}\left[-e^{-st}\,\text{sen}\,t - \frac{e^{-st}}{s}\cos t\right]_0^{2\pi}$$

$$= \frac{s}{s^2+1}\left[-e^{-2\pi}+1\right] = \frac{1-e^{-2\pi s}}{s^2+1}$$

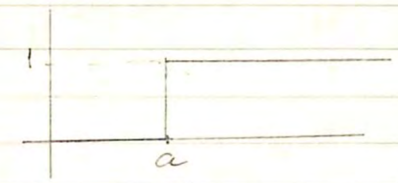
g) $$f(t)=\begin{cases}t & t<2\\ 8-3t, & 2\leq t\leq 3\\ t-4 & 3<t\leq 4\\ 0 & t>4\end{cases}$$



$$= \int_0^2 e^{-st}\,t\,dt + \int_2^3 e^{-st}(8-3t)\,dt$$

$$+ \int_3^4 e^{-st}(t-4)\,dt + \int_4^{\infty} e^{-st}\,(0)\,dt$$

função de degrau unitário

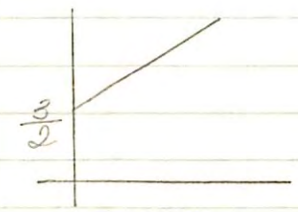


$$H_a(t)\begin{cases}0 & t\leq a\\ 1 & t>a\end{cases}$$

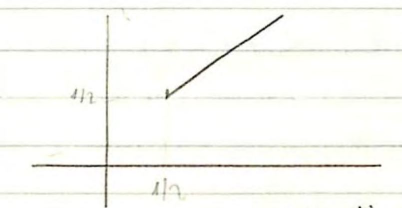
$$\mathcal{L}\{H_a(t)\} = \frac{e^{-as}}{s}$$

Exemplo

$$f(t)=\begin{cases}0 & t<1/2\\ 1+t & t>1/2\end{cases}$$



$f = t + \frac{3}{2}$

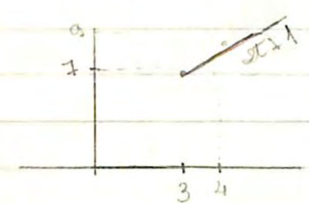


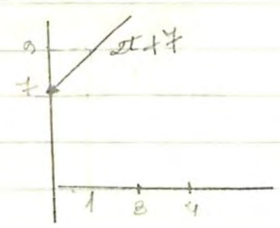
$f(t+1)$

$$\mathcal{L}\{f(t)\} = e^{-s/2}\,\mathcal{L}\{t+\tfrac{3}{2}\} = e^{-s/2}\left[\frac{1}{s^2}+\frac{3}{2s}\right] = e^{-s/2}\,\frac{2+3s}{2s^2}$$

—x—

$$f(t)=\begin{cases}2t+1 & t\geq 3\\ 0 & t<3\end{cases}$$





$$\mathcal{L}\{H_a(t-a)\} = e^{-as}\,\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{\underbrace{H_3(t-3)}_{g(t)}\} = e^{-3s}\,\mathcal{L}\{2t+7\} = e^{-3s}\left[\frac{2}{s^2}+\frac{7}{s}\right]$$

funções periódicas

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \int_0^p + \int_p^{2p} + \int_{2p}^{3p} + \int_{mp}^{(m+1)p} + \dots$$

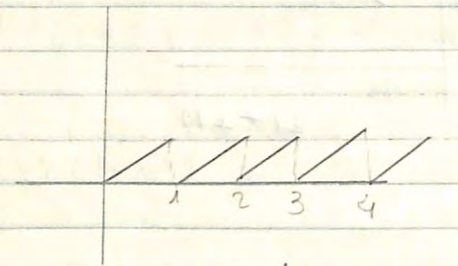
$$f(t) = f(t+p) = f(t+mp)$$

$$\mathcal{L}(f(t)) = \frac{\int_0^p e^{-st} f(t)\,dt}{1 - e^{-ps}}$$

— x —

Exemplos

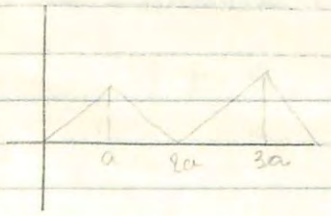
a) calcule $\mathcal{L}\{f(t)\}$



$$\mathcal{L}(f(t)) = \frac{\int_0^1 e^{-sx} x\,dx}{1 - e^{-s}} = \frac{1}{1-e^{-s}} \int_0^1 x e^{-sx} dx =$$

$$= \frac{1}{1-e^{-s}} - e^{-s} - s^2 e^{-s}$$

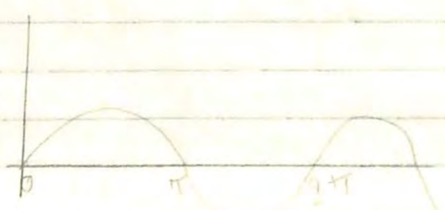
b)



$$\mathcal{L}(f(t)) = \frac{\int_0^{2a} e^{-st} f(t)\,dt}{1 - e^{-2as}}$$

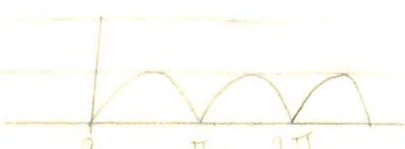
c) $\mathcal{L}\,|\operatorname{sen} t|$

$f(x) = \operatorname{sen} t$



$$\mathcal{L}\{|\operatorname{sen} t|\} = \frac{1}{s^2+1}\left(\frac{1+e^{-\dots}}{1-e^{-\dots}}\right.$$

$f(x) = |\operatorname{sen} t|$



periodo π

$$\int e^{-st} \operatorname{sen} t = \frac{1}{1+s^2}\left[- e^{-st} \operatorname{sen} t - \frac{e^{-st}}{s} \cos t\right]$$

$$\mathcal{L}\left\{\int_0^t f(r)\,dr\right\} = \frac{1}{s}\mathcal{L}\{f(t)\}$$

c) $\mathcal{L}\left(\int_0^t t e^{-2t} \operatorname{sen} t\right)$

$$\mathcal{L}(t e^{-2t} \operatorname{sen} t) = \left(-\frac{1}{(s+2)^2+1}\right)' = \frac{2(s+2)}{((s+2)^2+1)^2} \Rightarrow$$

$$\mathcal{L}\left(\int_0^t t e^{-2t} \operatorname{sen} t\right) = \frac{2(s+2)}{s[(s+2)^2+1]^2}$$

convolução

$$f(t) * g(t) = \int_0^t f(\varepsilon)\, g(t-\varepsilon)\, d\varepsilon$$

$$\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\{f(t)\} \cdot \mathcal{L}\{g(t)\}$$

ou

$$\mathcal{L}^{-1}\{F(s) \cdot G(s)\} = f(t) * g(t)$$

$\int x \operatorname{sen} x = x \cos x - \operatorname{sen} x$

Exemplo

$f(t) = t$

$g(t) = \operatorname{sen} t$

$$f * g = \int_0^t (t-\varepsilon) \operatorname{sen} \varepsilon\, d\varepsilon = \int_0^t (t \operatorname{sen} \varepsilon - \varepsilon \operatorname{sen} \varepsilon)\, d\varepsilon =$$

$$\left[-t\cos\varepsilon - \varepsilon\cos\varepsilon - \operatorname{sen}\varepsilon\right]_0^t = -t\cos t + t\cos t + \operatorname{sen} t - t$$

$$= \operatorname{sen} t - t$$

2) $f(t) = 1$

$g(t) = t$

$$f * g = \int_0^t \varepsilon \, d\varepsilon = \frac{t^2}{2}$$

— x —

$$f(t) * g(t) = \int_0^t f(t-\varepsilon)\, g(\varepsilon)\, d\varepsilon$$

$t - \varepsilon = u$

$d\varepsilon = -du$

$$= \int_t^0 f(u)\, g(t-u)\, du = -\int_t^0 f(u)\, g(t-u)\, du$$

— x —

$$\mathcal{L}\left(\int_0^t (t-\varepsilon)\, \text{sen}\, \varepsilon\, d\varepsilon\right) = \mathcal{L}(t * \text{sen}\, t) = \mathcal{L}(t) \cdot \mathcal{L}(\text{sen}\, t)$$

$$= \frac{1}{s^2} \cdot \frac{1}{1+s^2}$$

Exemplo

$$\mathcal{L}^{-1}\left(\frac{1}{s^2(1+s^2)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s^2} \cdot \frac{1}{1+s^2}\right) = t * \text{sen}\, t =$$

$$= \int_0^t \overset{u}{(t-\varepsilon)}\, \overset{dv}{\text{sen}\, \varepsilon\, d\varepsilon} = -(t-\varepsilon)\cos\varepsilon \Big]_0^t - \int_0^t \cos\varepsilon\, d\varepsilon = -(t-\varepsilon)\cos\varepsilon$$

$$- \text{sen}\, \varepsilon \Big]_0^t = -\text{sen}\, t - (-t) = t - \text{sen}\, t$$

— x —

$$\mathcal{L}^{-1}\left(\frac{1}{(s-2)(s-3)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s-2} * \frac{1}{s-3}\right) = e^{2t} * e^{3t} = \int_0^t e^{2(t-u)} e^{3\varepsilon} d\varepsilon$$

$$= \int_0^t e^{2t-2\varepsilon} \cdot e^{3\varepsilon}\, d\varepsilon = \int_0^t e^{2t+\varepsilon}\, d\varepsilon = e^{2t} \int_0^t e^{\varepsilon}\, d\varepsilon =$$

$$e^{2t}\left[e^{-\varepsilon}\right]_0^t = e^{3t} - e^{2t}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2(s^2+2s+4)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s^2} * \frac{1}{s^2+2s+4}\right) = \frac{1}{\sqrt{3}}\left(\frac{1}{s^2} * \frac{\sqrt{3}}{(s+1)^2+3}\right)$$

$$= \frac{1}{\sqrt{3}}\, t * e^{-t}\, \text{sen}\, \sqrt{3}\, t$$

— x —

Sistema de eq. diferenciais

$$\begin{cases} z'' + y' = \cos t \\ y'' - z = \text{sen}\, t \end{cases} \qquad \begin{cases} \mathcal{L}(z'') + \mathcal{L}(y') = \mathcal{L}(\cos t) \\ \mathcal{L}(y'') - \mathcal{L}(z) = \mathcal{L}(\text{sen}\, t) \end{cases}$$

$$\begin{cases} s^2\mathcal{L}(z) - s\,z(0) - z'(0) + s\mathcal{L}(y) - y(0) = \dfrac{s}{s^2+1} \\ s^2\mathcal{L}(y) - s\,y(0) - y'(0) - \mathcal{L}(z) = \dfrac{1}{s^2+1} \end{cases}$$

$$\begin{cases} s^2\mathcal{L}(z) + s + 1 + s\mathcal{L}(y) - 1 = \dfrac{s}{s^2+1} \\ s^2\mathcal{L}(y) - s - \mathcal{L}(z) = 1/s^2+1 \end{cases}$$

$$\begin{cases} s^2\mathcal{L}(z) + s\mathcal{L}(y) = \dfrac{s}{s^2+1} - s = \dfrac{s^3}{s^2+1} \\ s^2 \times\ -\mathcal{L}(z) + s^2\mathcal{L}(y) = \dfrac{1+s^3+s}{s^2+1} \end{cases}$$

$$\begin{cases} s^2\mathcal{L}(z) + s\mathcal{L}(y) = s^3\ /s^2+1 \\ -s^2\mathcal{L}(z) + s^4\mathcal{L}(y) = s^2 + s^5 + s^3\ /s^2+1 \end{cases}$$

$$s(1+s^3)\,\mathcal{L}(y) = s^2(1+s^3)\ /s^2+1$$

$$\mathcal{L}(y) = \frac{s}{s^2+1} \Rightarrow y = \cos t$$

$y = \cos t \qquad y' = -\operatorname{sen} t \qquad y'' = -\cos t$

$-\cos t - z = \operatorname{sen} t$

$z = -\cos t - \operatorname{sen} t$ //

Obs:

$$\frac{s^2+10s+84}{s^2(s+2)} = \frac{a}{s} + \frac{b}{s^2} + \frac{c}{s+2}$$

— —

calcule $\mathcal{L}^{-1}$

a) $\frac{1}{s(s+1)}$

e) $\mathcal{L}^{-1}\left(\frac{1}{s^2+4s+29}\right)$

b) $\left(\frac{1}{s(s+2)}\right)$

d) $\mathcal{L}^{-1}\left(\frac{2s}{(s^2+1)^2}\right)$

c) $\frac{3e^{-2s}}{3s^2+1}$

f) $\frac{1+e^{-s}}{s}$

a) $1-e^{-t}$

b) $\frac{1}{4} - \frac{1}{4}e^{2t} - \frac{1}{2}te^{t}$

c) $\frac{1}{5}e^{-2t}\operatorname{sen} 5t$

d) $t \operatorname{sen} t$

e) $\sqrt{3}\, u_2 t \operatorname{sen} \frac{t-2}{\sqrt{3}}$

f) $1 + u_1(t) = \begin{cases} 1 & \text{se } t \leq 1 \\ 2 & \text{se } t > 1 \end{cases}$

14) $s^2\mathcal{L}(y) - s y(0) - y'(0) - 2s\mathcal{L}(y) + y(0) - 3\mathcal{L}(y) = 0$

$(s^2-2s-3)\mathcal{L}(y) = +s + 7 - 1$

$(s^2-2s-3)\mathcal{L}(y) = s + 6$

$$\mathcal{L}(y) = \frac{s+6}{(s-1)^2+1} = \frac{(s-1)}{(s-1)^2+1} + \frac{7}{(s-1)^2+1} \qquad \frac{1}{s^2+1}$$

$e^t \cdot \cos t + 7(e^t \operatorname{sen} t)$ //

11) $F(s) = \frac{2s+3}{(s^2-4s+20)} = \frac{2s+3}{(s-2)^2+4^2} = \frac{2(s-2)+7}{(s-2)^2+4^2}$

$= \mathcal{L}\left(\frac{2(s-2)}{(s-2)^2+4^2}\right) + \mathcal{L}\left(\frac{7}{(s-2)^2+4^2}\right) = 2e^{2t}\cos 4t + e^{2t}\frac{7}{s^2+4^2} =$

$2e^{2t}\cos 4t + \frac{7}{4}e^{2t}\frac{4}{s^2+4^2} = e^{2t}\left(2\cos 4t + \frac{7}{4}\operatorname{sen} 2t\right)$ //

16) $s^2\mathcal{L}(y) - s y(0)^{0} - y'(0) - 4s\mathcal{L}(y) + 4y(0)^{0} + 4\mathcal{L}(y) = 0$

$\mathcal{L}(y)(s^2-4s+4) = +2$

$\mathcal{L}(y) = \frac{2}{s^2-4s+4} = \frac{2}{(s-2)^2} = e^{2t}\cdot\mathcal{L}\left(\frac{2}{s^2}\right) = 2e^{2t}\cdot t$ //

17) $s^2\mathcal{L}(y) - s y(0)^{1} - y'(0)^{3} - 4s\mathcal{L}(y) + 4y(0)^{1} + 5\mathcal{L}(y) = 0$

$s^2\mathcal{L}(y) - s - 3 - 4s\mathcal{L}(y) + 4 + 5\mathcal{L}(y) = 0$

$\mathcal{L}(y)(s^2-4s+5) = s - 1$

$$\mathcal{L}(y) = \frac{s-1}{s^2-4s+5} = \frac{s-1}{(s-2)^2+1} = \frac{s-2}{(s-2)^2+1} + \frac{1}{(s-2)^2+1}$$

$e^{2t}(\cos t + \operatorname{sen} t)$ //

8) $\frac{5(s+1)}{s^2-4} = \mathcal{L}\left(\frac{5s}{(s^2-4)}\right) + \mathcal{L}\left(\frac{5}{(s^2-4)}\right)$

$= 5\cosh 2t + \frac{5}{2}\ \frac{2}{s^2-4}$

$= 5\cosh 2t + \frac{5}{2}\operatorname{senh} 2t$

25) $\mathcal{L}\left(\frac{1}{s^2+4s+29}\right) = \frac{1}{(s+2)^2+5^2} = e^{2t}\mathcal{L}\left(\frac{1}{s^2+5^2}\right)$

$\frac{1}{5}e^{2t}\operatorname{sen} 5t$ //

20) $s^2\mathcal{L}(y) - sy(0) - y'(0) + \mathcal{L}(y) = 1/s^2$

$\mathcal{L}(y)(s^2+1) = \frac{1}{s^2} + s - 2$

$\mathcal{L}(y) = \frac{1+s^3-2s^2}{s^2} \times \frac{1}{s^2+1}$

$\mathcal{L}(y) = \frac{1+s^3-2s^2}{s^2(s^2+1)} = \frac{a}{s} + \frac{b}{s^2} + \frac{c}{s^2+1}$

$a(s^3+s) + b(s^2+1) + c(s^2)$

$a\cancel{s^4} + a\cancel{s} + b\cancel{s^2} + b + c\cancel{s^2} =$

$a = 0$

$b+c = -2$

$b = 1$

$c = -3$

$\frac{1}{s^2} + \frac{-3}{s^2+1} = t + 3.\operatorname{sen} t$ //

21) $s^2\mathcal{L}(y) - s\cancel{y(0)}^{0} - y'(0)^{2} + 4\mathcal{L}(y) = -2/s$

$s^2\mathcal{L}(y) - 2 + 4\mathcal{L}(y) = -2/s$

$\mathcal{L}(y)(s^2+4) = -\frac{2}{s} + 2$

$\mathcal{L}(y) = \frac{-2}{s(s^2+4)} + \frac{2}{s^2+4} = \frac{-2}{s} * \frac{1}{s^2+2^2} + \operatorname{sen} 2t$

22) $s^2\mathcal{L}(y) - s\cancel{y(0)} - y'(0) + 2s\mathcal{L}(y) - 2\cancel{y(0)} + 3\mathcal{L}(y) = \frac{3}{s^2}$

$s^2\mathcal{L}(y) - 1 + 2s\mathcal{L}(y) + 3\mathcal{L}(y) = 3/s^2$

$\mathcal{L}(y)(s^2+2s+3) = \frac{3}{s^2} + 1$

$\mathcal{L}(y) = \frac{3+s^2}{s^2}\ \frac{1}{s^2+2s+3}$

$\mathcal{L}(y) = \frac{3+s^2}{s^2(s^2+2s+3)} = \frac{3}{s^2(s^2+2s+3)} + \frac{\cancel{s^2}}{\cancel{s^2}(s^2+2s+3)} = \frac{1}{(s+1)^2+1} = e^{-t}\cos t$

$\frac{A}{s^2} + \frac{B}{s} + \frac{Cs+D}{s^2+2s+3}$

$A\cancel{s^2} + 2A\cancel{s} + 3A + \underline{Bs^3} + \underline{2Bs^2} + 3B\cancel{s} + \underline{Cs^3} + Ds^2 = 3$

$B + C = 0$

$A + 2B + D = 0$

$2A + 3B = 0 \quad 2 = -3B \Rightarrow B = -2/3$

$3A = 3 \Rightarrow A = 1 \quad C = 2/3$

$\frac{1}{s^2} + \frac{-2}{3s} +$

26) $\frac{4}{(s-1)^2} = \frac{4}{(s-1)} \cdot \frac{1}{(s-1)} = 4 \cdot e^t * e^t.$

$\int_0^t 4\, e^{(t-u)} \cdot e^u\, du = 4e^t \int_0^t e^{-u} e^u\, du = 4e^t \int_0^t du$

$4e^t [u]_0^t = 4e^t t$ //

30) $F(s) = \dfrac{1}{s^3(s+1)} = \dfrac{t^2}{2} * e^{-t}$

$\int_0^t e^{-(t-u)} \dfrac{u^2}{2} du = \dfrac{1}{2}\int_0^t e^{-t+u} u^2 du = \dfrac{1}{2} e^{-t}\int_0^t e^{u} u^2 du$

31) $\dfrac{1}{s^2+1} * \dfrac{s}{s^2+1}$ $\cos t * \operatorname{sen} t$

$\int_0^t \cos(t-u) . \operatorname{sen} u \, du = \int_0^t (\cos t \cos u - \operatorname{sen} t \operatorname{sen} u) \operatorname{sen} u$

$\int_0^t \cos t \cos u \operatorname{sen} u - \operatorname{sen} t \operatorname{sen} u^2$

32) $\dfrac{3s^2}{(s^2+1)^2} = \dfrac{3}{s^2+1} * \dfrac{s^2}{s^2+1}$

Rio de Janeiro, 11 de janeiro de 1979

Séries de potências

Def: série do tipo $\sum_0^\infty a_n (x-a)^n$

Obs. toda série de potências

a) Toda série de potências

b)

1) Ex: $\sum_0^\infty n!\,(x-1)^n = 1 + (x-1) + 2(x-1)^2 + 6(x-1)^3 + \dots + R$

onde

$\sum a_n x_n$

a) $\dfrac{|(n+1)!\,(x-1)^{n+1}|}{n!\,(x-1)^n} = \lim_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} = \infty$, $x \neq 1$

É sempre divergente para $x \neq 1$ $l = 1$

② $\sum_1^\infty \dfrac{x^n}{n} = x + \dfrac{x^2}{2} + \dfrac{x^3}{3}$

b) $\dfrac{\left|\frac{x^{n+1}}{n+1}\right|}{\frac{x^n}{n}} = |x| \left(\dfrac{n}{n+1}\right)^{1}$ $\lim_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = |x| < 1$

Pelo critério da razão $d < 1$ para convergir

$-1 < x < 1$ $l = 0$

-1 1 Colocando $x = 1$

$x = 1 \Rightarrow 1 + \frac{1}{2} + \frac{1}{3} + \dots$

série harmônica $\rightarrow$ diver

ge $x > 1$

$x = -1 \Rightarrow -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4}$ converge

$x \geq -1$

3) $\sum_0^\infty \frac{(x+2)^n}{n!} = 1+(x+2)+\frac{(x+2)^2}{2!}+\frac{(x+2)^3}{3!}$

$$\frac{|(x+2)^{n+1}|}{(n+1)!} \cdot \frac{n!}{|(x+2)^n|} = \frac{|x+2|}{n+1}$$

$$\lim_{n\to\infty} \frac{|x+2|}{n+1} = 0 \quad \forall x$$

centro de converg $= -2$

$\forall x$

2) $\sum n^4 (2x)^n$

$$\frac{(n+1)^4 (2x)^{n+1}}{n^4 \, |2x|^n} = \left(\frac{n+1}{n}\right)^4 |2x|$$

$$\lim_{n\to\infty} = |2x| < 1 \Rightarrow |x| < \frac{1}{2}$$

$c = 0$

$|x| < 1/2$

Funções definidas por séries de potências

a) f é contínua em todos os ptos em I

b) f tem derivada em todos os ptos e f' se obtém derivando termo a termo

$$e^x = 1 + x + \frac{x^2}{2!}$$

$$\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

c) f tem integral em todos os ptos

d) os coef C_n são os coeficientes de Ta...

$f(x) = \sum c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots c_n(x-a)^n$

$f'(x) = (x-a) + 2c_2(x-a) + 3c_3(x-a)^2 + \dots n c_n (x-a)^{n-1}$

$f''(x) = 2c_2 + 6c_3(x-a) + \dots + c_n n(n-1)(x-a)^{n-2}$

$f(a) = c_0 \quad f'(a) = c_1 \quad f''(a) = 2c_2$

determinar

a) $\sum_0^\infty x^n = \frac{1}{1-x} = f$

b) $\sum_{n=1}^\infty \frac{x^n}{n} \Rightarrow$

$f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$

$f'(x) = 1 + x + x^2 + x^3 + x^4 = \frac{1}{1-x}$

se $f'(x) = \frac{1}{1-x}$

$\Rightarrow f(x) = -\log(1-x) + c$

c) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots = f$

d) $= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$

e) $\frac{x^2}{2} - \frac{x^3}{3\times 2} + \frac{x^4}{4\times 3} - \frac{x^5}{5\times 4}$

d) $\sum \frac{x^n}{n!}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - x^{10} + \dots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

$e^{x^2} = 1 + x^2 + \frac{x^4}{2!}$

$e^{-x} = 1 + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!}$

c) $f = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$

$f' = 1 - x^2 + x^4 - x^6 + x^8 = \frac{a_1}{1-q}$

$f'(x) = \frac{1}{1+x^2} \therefore f(x) = \text{arc tg } x$

d) $e^{-x} =$

e) $\frac{x^2}{2} - \frac{x^3}{3\times 2} + \frac{x^4}{4\times 3} - \frac{x^5}{5\times 4}$

$f'(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} =$

$f''(x) = 1 - x + x^2 - x^3 = \frac{1}{1+x}$

$f(x) = \int \log(1+x)\,dx = (x+1)\log(1+x) - (x+1) + c$

$f(0) = -1 + c \Rightarrow c = +1$

$\Rightarrow$

$f(x) = (x+1)\log(1+x) - x$

função analítica

uso de série de potências para a resolução de eq diferenciais

$f(x) = \frac{1}{2+3x}$

$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!}$

não precisa achar é só dividir

$f(x) = \frac{1}{2+3x}$

$1 \quad \underline{\lfloor 2+3x}$

$-\frac{1}{2} - \frac{3}{2}x \qquad \frac{1}{2} - \frac{3}{4}x + \frac{9}{8}x^2$

$-\frac{3}{2}x$

$-\frac{3}{4} + \frac{9}{4}$

Ex: $y' + y = x \qquad y(0) = 0$

Ex: $x^2y'' + xy' + x^2y = 0$

$y = a_0 + a_1x + a_2x^2 + a_3x^3 - - -$

$y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \dots n a_n x^{n-1}$

$y'' = 2a_2 + 2.3a_3x + 3.4a_4x^2 + \dots n(n-1)a_n x^{n-2}$

$x^2y'' = 2a_2x^2 + 2.3a_3x^3 + 3.4a_4x^4 + \dots n(n-1)a_nx^n$

$xy' = a_1x + 2a_2x^2 + 3a_3x^3 + 4a_4x^4 + \dots + na_nx^n$

$x^2y = a_0x^2 + a_1x^3 + a_2x^4 + a_3x^5 + \dots a_nx^{n+2}$

$a_1x + x^2(2^2a_2 + a_0) + x^3(3^2.a_3 + a_1) + x^4(4^2a_4 + a_2) + x^5(5^2a_5 + a_3)$

$= 0$

$a_1 = 0$

Calcule $f(t)$

$$\mathcal{L}^{-1}\left\{ \frac{-1}{(s-2)^2} \right\} = e^{2t}\mathcal{L}(f(t))$$

$$\mathcal{L}(f(t)) = \frac{-1}{s^2} \Rightarrow f(t) = -t$$

$$f(t) = -e^{2t}t$$

2) $\mathcal{L}^{-1}\left\{ \frac{1}{(s+1)^3} \right\} = e^{-t}\mathcal{L}\{f(t)\}$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^3} \Rightarrow f(t) = \frac{t^2}{2}$$

$$f(t) = e^{-t}\frac{t^2}{2}$$

3) $\mathcal{L}^{-1}\left\{ \frac{1}{s^2-4s+9} \right\}$

$$\frac{1}{(s-2)^2+5} = e^{2t}.f(t)$$

$$f(t) = \frac{1}{s^2+5} \Rightarrow \frac{\sqrt{5}}{\sqrt{5}}\,\frac{1}{s^2+5}$$

$\sqrt{5}$ sen $\sqrt{5}t$

$$\Rightarrow f(t) = e^{2t}\frac{1}{\sqrt{5}}\,\text{sen}\sqrt{5}t$$

$$\mathcal{L}^{-1}\left\{ \frac{s+5}{s^2-2s+5} \right\} = \frac{s+5}{(s-1)^2+4} = \frac{(s-1)+6}{(s-1)^2+4}$$

$$= e^{t}\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{f(t)\} = \frac{s+6}{s^2+4} = \frac{s}{s^2+4} + \frac{6}{s^2+4}$$

$\cos 2t + 3\,\text{sen}\, 2t$

$$f(t) = e^{t}[\cos 2t + 3\,\text{sen}\, 2t]$$

Resolva

$$\begin{cases} y'' - 2y' + y = te^{t} \\ y(0) = 0, \quad y'(0) = 0 \end{cases}$$

$$\mathcal{L}(y'') - 2\mathcal{L}(y') + \mathcal{L}(y) = \mathcal{L}(te^{t})$$

$$s^2\mathcal{L}(y) - s\overset{0}{\cancel{y(0)}} - \overset{0}{\cancel{y'(0)}} - 2s\mathcal{L}(y) + 2\overset{0}{y(0)} + \mathcal{L}(y) = 1/(s-1)^2$$

$$(s^2-2s+1)\,\mathcal{L}(y) = \frac{1}{(s-1)^2}$$

$$\mathcal{L}(y) = \frac{1}{(s-1)^2} \times \frac{1}{(s^2-2s+1)} = \frac{1}{(s-1)^4} = e^{t}.\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^4} = \frac{t^3}{6}$$

$$y(t) = e^{t}.\frac{t^3}{6} \;//$$

2) $\begin{cases} y'' - y' - 6y = 3t^2 + t - 1 \\ \end{cases}$

$$s^2\mathcal{L}(y) - s\overset{-1}{\cancel{y(0)}} - \overset{6}{y'(0)} + s\mathcal{L}(y) - \overset{-1}{\cancel{y(0)}} - 6\mathcal{L}(y) = \overbrace{\frac{6}{s^3} + \frac{1}{s^2} - \frac{1}{s}}^{c}$$

$$s^2\mathcal{L}(y) + 1 - 6 + s\mathcal{L}(y) + 1 - 6\mathcal{L}(y) = c$$

$$\mathcal{L}(y)(s^2 - s - 6) = \frac{-s^4 + 7s^3 - s^2 + s + 6}{s^3(s+2)(s-3)}$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{(s+2)} + \frac{E}{(s-3)}$$

pag 27

6) $f(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & 1 \le t < 2 \\ 0 & t \geq 2 \end{cases}$

$-tH_1(t) + H_1 t$

$f(t) = t + (-t+1)H_1(t) - H_2(t)$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2} + \frac{e^{-s}}{s} - \left(\frac{se^{-s}+e^{-s}}{s^2}\right) - \frac{e^{-2s}}{s}$$

obs: $tH_1(t) = \frac{d}{ds}\left(\frac{e^{-s}}{s}\right)$

11) $$\frac{2s+3}{s^2-4s+20} = \frac{2(s-2)+7}{(s-2)^2+4^2}$$

$= e^{2t} \cdot \mathcal{L}\{f(t)$

$$\mathcal{L}\, f(t) = \frac{2s+7}{s^2+4^2} = \frac{2s}{s^2+4^2} + \frac{7}{s^2+4^2}$$

$= 2\cos 2t$

$7-4+3$

Rio, 15/1/79

1) Resolva

$(1+x^2)\, y'' + x y' - y = 0$

$$(1+x^2) \sum_2^\infty k(k-1)a_k x^{k-2} + x \sum_1^\infty k a_k x^{k-1} - \sum_0^\infty a_k x^k = 0$$

$$\sum_2^\infty k(k-1)a_k x^{k-2} + \sum_2^\infty k(k-1)a_k x^k + \sum_1^\infty k a_k x^k - \sum_0^\infty a_k x^k = 0$$

1º) colocar todos no mesmo grau

$= 2a_2 + 3\cdot 2a_3 x + \sum_4^\infty k(k-1)a_k x^{k-2} \to$ grau 2

$+ \sum_2^\infty k(k-1)a_k x^k +$

$+ a_1 x + \sum_2^\infty k a_k x^k$

$- (a_0 + a_1 x + \sum_2^\infty a_k x^k = 0$

$$= 2a_2 + 6a_3 x + a_1 x - a_0 - a_1 x + \Big[\overbrace{\sum_4^\infty k(k-1)a_k x^{k-2}}^{\sum_2^\infty (k+2)(k+1)a_{k+2} x^k} + \sum_2^\infty k a_k x^k - \sum_2^\infty a_k x^k +$$

$$+ \sum_2^\infty k(k-1)a_k x^k \Big] = 0$$

$$(2a_2 - a_0) + 6a_3 x + \sum_2^\infty x^k \left[(k+2)(k+1)a_{k+2} + k a_k - a_k + k(k-1)a_k\right] = 0$$

$2a_2 - a_0 = 0$

$6a_3 = 0$

$(k+2)(k+1)a_{k+2} + a_k(k^2-1) = 0$

$$a_{k+2} = \frac{-(k^2-1)a_k}{(k+2)(k+1)} = \frac{-\cancel{(k+1)}(k-1)a_k}{(k+2)\cancel{(k+1)}} = -\frac{(k-1)a_k}{(k+2)}$$

$a_0 = a$

$a_1 = b$

$\boxed{a_2 = \frac{a_0}{2} = \frac{a}{2}}$

$a_3 = 0$

$a_4 = -\frac{1}{4} a_2 = -\frac{a}{2\cdot 4}$

$a_5 = \cancel{-\frac{2}{5}} a_3$

$a_6 = \frac{-3}{6} a_4 = \frac{3}{2\cdot 4\cdot 6} a$

$a_7 = \cancel{-\frac{4}{7}} a_5$

$a_8 = -\frac{5}{8} a_6 = -\frac{3\cdot 5\, a}{2\cdot 4\cdot 6\cdot 8}$

$$y = a + bx + \frac{ax^2}{2} - \frac{ax^4}{2.4} + \frac{3a}{2.4.6}x^6 - \frac{3.5a}{2.4.6.8}x^8 + \frac{3.5.7a}{2.4.6.8.10}x^{10}$$

$$\frac{(-1)}{2(1.2.3.4.5)} = \frac{(2k-2)!}{2^{2k}(k!)^2}x^{2k}$$

$$y = a + bx + a\sum_1^\infty \frac{(2k-2)!}{2^{2k}(k!)^2}x^{2k}(-1)^{k+1}$$

—/—

$$x^2y'' + xy' + x^2y = 0$$

$$\overbrace{\sum_2^\infty k(k-1)a_kx^k}^{2º} + \overbrace{\sum_1^\infty ka_kx^k}^{1º} + \overbrace{\sum_0^\infty a_kx^{k+2}}^{2º} = 0$$

(a) (b) (c)

$$(a) + a_1x + \sum_2^\infty ka_kx^k$$

$$a_1x + \sum_2^\infty k(k-1)a_kx^k + \sum_2^\infty ka_kx^k + \sum_0^\infty a_kx^{k+2}$$

$$a_1x + \sum_0^\infty x^{k+2}[(k+2)(k+1)a_{k+2} + (k+2)a_{k+2} + a_k] = 0$$

$$a_1x + \sum_0^\infty x^{k+2}[(k+2)a_{k+2}(k+1+1) + a_k]$$

$a_0 = a_0$

$a_1x = 0$

$a_{k+2} = \frac{-a_k}{(k+2)^2}$

$a_0 = a_0$

$a_1 = 0$

$a_2 = \frac{-a_0}{2^2}$

$a_3 = \frac{-a_1}{3^2} = 0$

$a_4 = \frac{-a_2}{4^2} = \frac{a_0}{2^2 4^2}$

$a_5 = \frac{-a_3}{5^2} = 0$

$a_6 = \frac{-a_4}{6^2} = \frac{-a_0}{2^2 4^2 6^2}$

$$y = a_0 - \frac{a_0x^2}{2^2} + \frac{a_0x^4}{2^2 4^2} - \frac{a_0}{2^2 4^2 6^2}x^6 + \frac{a_0}{2^2 4^2 6^2}$$

$$y = a_0 + a_0\sum_1^\infty \frac{(-1)^n}{2^{2k}(k!)^2}x^{2k}$$

$$(1+x^2)y'' - 4xy' + 6y = 0$$

$$(1+x^2)\sum_2^\infty k(k-1)a_kx^{k-2} - 4x\sum ka_kx^{k-1} + 6\sum_0^\infty a_kx^k = 0$$

$$\underbrace{\sum_2^\infty k(k-1)a_kx^{k-2}}_{0} + \underbrace{\sum_2^\infty k(k-1)a_kx^k}_{2} - 4\underbrace{\sum_1^\infty ka_kx^k}_{1} + 6\underbrace{\sum_0^\infty a_kx^k}_{0} = 0$$

$$2a_2 + 6a_3x + \sum_4^\infty k(k-1)a_kx^{k-2} + \sum_2^\infty k(k-1)a_kx^k - 4[a_1x + \sum_2^\infty ka_kx^k] +$$

$$+ 6[a_0 + a_1x + \sum_2^\infty a_kx^k] = 0$$

$$\sum_2^\infty (k+2)(k+1)a_{k+2}x^k$$

$$(2a_2 + 6a_0) + x(6a_3 + 2a_1) + \left[\sum_4^\infty k(k-1)a_kx^{k-2} + \sum_2^\infty (k-1)ka_kx^k\right.$$

$$\left. - 4\sum_2^\infty ka_kx^k + 6\sum_2 a_kx^k\right] = 0 \qquad k \geq 2$$

$$(2a_2 + 6a_0) + x(6a_3 + 2a_1) + \sum_2^\infty x^k[k(k-1)a_k - 4ka_k + 6a_k +$$

$$+ (k+2)(k+1)a_{k+2}]$$

$2a_2 + 6a_0 = 0$

$6a_3 + 2a_1 = 0$

$(k-2)(k-3)$

$k^2 - 5k + 6$

$$(k+2)(k+1)a_{k+2} + a_k(k^2 - k - 4k + 6) = 0$$

$$a_{k+2} = -\frac{(k-2)(k-3)}{(k+2)(k+1)}a_k$$

$a_0 = 1$

$a_2 = -6/2 = -3$

$y = 1 + x - \frac{1}{3}x^3$ //

$a_3 = -\frac{2a_1}{6} = -1/3$

$a_4 = 0$ $a_7 = 0$

$a_5 = 0$ $a_8 = 0$

$a_6 = 0$

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8) $\frac{5(s+1)}{s^2-4} = \frac{5(s+1)}{(s+2)(s-2)} = 5(s-2)$

—x—

① Resolver: (det onde converge)

$y' + xy = 0$

Resp: $y = a_0\left(1 - \frac{x^2}{2} + \frac{x^4}{2.4} - \frac{x^6}{2.4.6} + \dots\right) = a_0\left(e^{-\frac{x^2}{2}}\right)$

② $y' = x - 2y$, com cond. iniciais $y(0)=0$

Resp: $y = \sum_2 \frac{(-1)^n (2x)^n}{4n!} = \frac{1}{4}e^{-2x} - \frac{1}{4} + \frac{x}{2}$

③ $y'' = x + y^2$
$y(0)=0$ encontrar os 4 primeiros termos
$y'(0)=1$

Resp: $y = x + \frac{x^3}{3!} + 2\frac{x^4}{4!} + 16\frac{x^6}{6!}$

④ Idem $y'' = (2x-1)y - 1$ $y(0)=0$ $y'(0)=1$

Resp: $y = x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{5}{24}x^4 + \dots$

⑤ $xy' = 1 - x + 2y$ em pot de $x-1$

Resp: $y = a_0[1 + 2(x-1) + (x-2)^2 + \frac{1}{2} + (x-1)$

8) $y'' + xy = 0$

Resp: $a_n = -\frac{1}{n(n-1)} a_{n-3}$ $n \geq 3$

$y = a_0\left(1 - \frac{x^3}{2} + \frac{x^6}{180} + \dots\right) + a_1\left(x - \frac{x^4}{12} + \right.$

10) $\frac{\cos et}{2} + \frac{1}{2}$ //

Det a solução geral em $x=1$ de $y'' + (x-1)y = e^x$

(série de pot em $x-1$)

$y = a_0 + a_1(x-1) + a_2(x-1)^2 + \dots$

$x - 1 = z$

$y'' + zy = e^{z+1}$

$y = \sum_0^\infty a_k z^k$; $y' = \sum_1^\infty k a_k z^{k-1}$ $y'' = \sum_2^\infty k(k-1)a_k z^{k-2}$

$\underbrace{\sum_2^\infty k(k-1)a_k z^{k-2}}_{0} + \underbrace{\sum_0^\infty a_k z^{k+1}}_{1} = e \cdot e^z$

$2a_2 + \sum_3^\infty k(k-1)a_k z^{k-2} + \sum_0^\infty a_k z^{k+1} = e\left[1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots\right]$

$2a_2 + \sum_0^\infty z^{k+1}[(k+3)(k+2)a_{k+3} + a_k] = e + ez + \frac{ez^2}{2!} + \frac{ez^3}{3!}$

$2a_2 = e$ $= e + e\sum_1^\infty \frac{z^n}{n!}$

$2a_2 + \sum_0^\infty z^{k+1}[(k+3)(k+2)a_{k+3} + a_k] = e + e\sum_0^\infty \frac{z^{n+1}}{(n+1)!}$

$2a_2 = e$

$\cancel{z^{k+1}}(k+3)(k+2)a_{k+3} + a_k = \frac{e\cancel{z^{k+1}}}{(k+1)!}$

$(k+3)(k+2)a_{k+3} = -a_k + \frac{e}{(k+1)!}$

$a_{k+3} = \frac{-a_k}{(k+3)(k+2)} + \frac{e}{(k+3)!}$

$a_0 = a_0$ $a_2 = \frac{e}{2}$ $a_4 = -\frac{a_1}{12} + \frac{e}{4!}$

$a_1 = a_1$ $a_3 = \frac{-a_0}{6} + \frac{e}{3!}$

$a_5 = -\frac{e}{2.20} + \frac{e}{5!}$

$a_6 = \frac{a_0}{6.30} - \frac{e}{30.3!} + \frac{e}{6!}$

$y = a_0 + a_1(x-1) + \frac{e}{2}(x-1)^2 + \left(-\frac{a_0}{6} + \frac{e}{3!}\right)(x-1)^3$

8) $\mathcal{L}^{-1}\left(\frac{5s+5}{s^2-4}\right) = \frac{5s}{s^2-4} + \frac{5}{s^2-4}$

$5\left(\frac{s}{s^2-4}\right) + 5\left(\frac{1}{s^2-4}\right)$

$= 5\cosh 2t + \frac{5}{2}\,\text{senh}\,2t$

9) $e^{-2t}\cos wt$

11) $\frac{2s+3}{s^2-4s+4+16} = \frac{2s+3}{(s-2)^2+16} + \frac{7}{(s-2)^2+16}$

$= \frac{2(s-2)}{(s-2)^2+16} + \frac{7}{4}\,\frac{4}{(s-2)^2+16}$

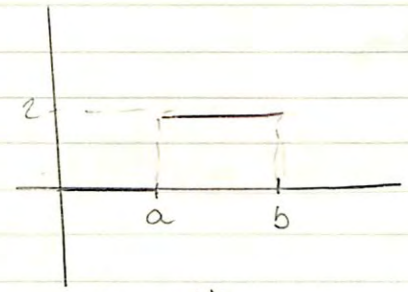
$= 2e^{2t}\cos 4t + \frac{7}{4}e^{2t}\,\text{sen}\,4t$

29) $F(s) = \frac{s^2(2s+3)}{s^2} + 2e^{-3s}$

$= \frac{2e^{-3s} + 2s^3 + 6s^2}{s^2}$

calcule $\mathcal{L}(t)$

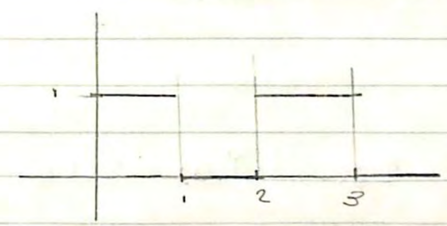
$f(t) = \begin{cases} 0 & 0 \le t \le a \\ 2 & a \le t \le b \\ 0 & t \ge b \end{cases}$



$f(t) = 2H_a - 2H_b$

$\mathcal{L}(f(t)) = 2\frac{e^{-as}}{s} - 2\frac{e^{-bs}}{s}$

2) $f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & 1 \le t < 2 \\ 1 & 2 \le t \le 3 \\ 0 & t \ge 3 \end{cases}$

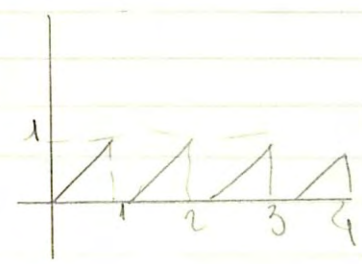


$f(t) = 1 - H_1(t) + H_2(t) - H_3(t)$

$\mathcal{L}f(t) = \frac{1}{s} - \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}$

$y(s) = \frac{e^{-s}}{s^2} \quad \mathcal{L}(H_1(t).f(t-1))$

$f(t) = t \Rightarrow f(t) = H_1(t).(t-1)$//



$f(0)t = \begin{cases} t & 0 \le t \le 1 \\ 0 & t \ge 1 \end{cases}$

$\phi(t) = t - tH_1(t)$

$$y(s) = \frac{e^{-3s}}{s^2-2s+3} = e^{-3s}\frac{1}{(s-1)^2+2} =$$

$H_3(t) \cdot f(t-3)$

$$f(t) = e^t \frac{\operatorname{sen}\sqrt{2}t}{\sqrt{2}}$$

$$f(t) = H_3(t)\, e^{(t-3)} \frac{\operatorname{sen}\sqrt{2}(t-3)}{\sqrt{2}}$$

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① $\begin{cases} z'' + y' = \cos t \\ y'' - z = \operatorname{sen} t \end{cases}$

$$\begin{cases} s^2\mathcal{L}(z) - s z(0) - z'(0) + s\mathcal{L}(y) - y(0) = \frac{s}{s^2+1} \\ s^2\mathcal{L}(y) - s y(0) - y'(0) - \mathcal{L}(z) = \frac{1}{s^2+1} \end{cases}$$

$$\begin{cases} s^2\mathcal{L}(z) + s + \cancel{1} + s\mathcal{L}(y) - \cancel{1} = s/s^2+1 \\ s^2\mathcal{L}(y) - s - \mathcal{L}(z) = 1/s^2+1 \end{cases}$$

$$\begin{cases} s^2\mathcal{L}(z) + s\mathcal{L}(y) = \frac{s}{s^2+1} - s \\ s^2\mathcal{L}(y) - \mathcal{L}(z) = \frac{1}{s^2+1} + s \end{cases}$$

$$\begin{cases} s^2\mathcal{L}(z) + s\mathcal{L}(y) = \frac{s - s(s^2+1)}{s^2+1} \\ s^2\mathcal{L}(y) - \mathcal{L}(z) = \frac{1+(s^2+1)s}{s^2+1} \end{cases}$$

$$\mathcal{L}(z) = -\frac{1+s^3+s}{s^2+1} - s^2\mathcal{L}(y)$$

$$s^2\left(-\frac{1-s^3-s}{s^2+1} - s^2\mathcal{L}(y)\right) + s\mathcal{L}(y) = \frac{s-s^3-s}{s^2+1}$$

$$\frac{-s^2+s^5-s^3}{s^2+1} - s^4\mathcal{L}(y) + s\mathcal{L}(y) = \frac{s-s^3-s}{s^2+1}$$

$$\mathcal{L}(y)(-s^4+s) = \frac{s^2 - \cancel{s^3} - s}{s^2+1} + \frac{s^2 - s^5 \cancel{+s^3}}{s^2+1} = \frac{2s^2-s^5-s}{s^2+1}$$

$$\mathcal{L}(y) = \frac{\cancel{s}(2s-s^4-1)}{s^2+1} \times \frac{1}{\cancel{s}(-s^3+1)} = \frac{-s^4+2s-1}{(s^2+1)(-s^3+1)}$$

— x —

Sequência

$a_n = \frac{1}{n} \to 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$b_n = \frac{(-1)^n}{n^2} \to -1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}$

$g_n = \frac{n}{2n+3} = \frac{1}{5}, \frac{2}{7}, \frac{3}{9}, \frac{4}{11} \ldots$

$$\lim_{n\to\infty} g_n = \frac{n}{2n+3} = \lim_{n\to\infty} \frac{n}{2n+3} = \lim \frac{1}{2+\frac{3}{n}\,^{\to 0}} = \frac{1}{2}$$

$h_n = n^2 \qquad 1, 4, 9$

$\lim_{n\to\infty} \{h_n\} = \infty$

Definição – Diz-se que l é $\lim_{n\to\infty} a_n$ se dado

$$y(s) = \frac{e^{-3s}}{s^2-2s+3} = e^{-3s}\frac{1}{(s-1)^2+2} =$$

$H_3(t) \circ f(t-3)$

$f(t) = e^t \frac{\operatorname{sen}\sqrt{2}t}{\sqrt{2}}$

$f(t) = H_3(t)\, e^{(t-3)} \frac{\operatorname{sen}\sqrt{2}(t-3)}{\sqrt{2}}$

pag 32

① $\begin{cases} z'' + y' = \cos t \\ y'' - z = \operatorname{sen} t \end{cases}$

$\begin{cases} s^2\mathcal{L}(z) - s z(0) - z'(0) + s\mathcal{L}(y) - y(0) = \frac{s}{s^2+1} \\ s^2\mathcal{L}(y) - s y(0) - y'(0) - \mathcal{L}(z) = \frac{1}{s^2+1} \end{cases}$

$\begin{cases} s^2\mathcal{L}(z) + s + \cancel{1} + s\mathcal{L}(y) - \cancel{1} = s/s^2+1 \\ s^2\mathcal{L}(y) - s - \mathcal{L}(z) = 1/s^2+1 \end{cases}$

$\begin{cases} s^2\mathcal{L}(z) + s\mathcal{L}(y) = \frac{s}{s^2+1} - s \\ s^2\mathcal{L}(y) - \mathcal{L}(z) = \frac{1}{s^2+1} + s \end{cases}$

$\begin{cases} s^2\mathcal{L}(z) + s\mathcal{L}(y) = \frac{s - s(s^2+1)}{s^2+1} \\ s^2\mathcal{L}(y) - \mathcal{L}(z) = \frac{1+(s^2+1)s}{s^2+1} \end{cases}$

$\mathcal{L}(z) = -\frac{1+s^3+s}{s^2+1} - s^2\mathcal{L}(y)$

$s^2\left(-\frac{1-s^3-s}{s^2+1} - s^2\mathcal{L}(y)\right) + s\mathcal{L}(y) = \frac{s-s^3-s}{s^2+1}$

$\frac{-s^2+s^5-s^3}{s^2+1} - s^4\mathcal{L}(y) + s\mathcal{L}(y) = \frac{s-s^3-s}{s^2+1}$

$\mathcal{L}(y)(-s^4+s) = \frac{s^2-\cancel{s^3}-s}{s^2+1} + \frac{s^2-s^5+\cancel{s^3}}{s^2+1} = \frac{2s^2-s^5-s}{s^2+1}$

$\mathcal{L}(y) = \frac{\cancel{s}(2s-s^4-1)}{s^2+1} \times \frac{1}{\cancel{s}(-s^3+1)} = \frac{-s^4+2s-1}{(s^2+1)(-s^3+1)}$

— × —

Sequência

$a_n = \frac{1}{n} \to 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

$b_n = \frac{(-1)^n}{n^2} \to -1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}$

$g_n = \frac{n}{2n+3} = \frac{1}{5}, \frac{2}{7}, \frac{3}{9}, \frac{4}{11} \dots$

$\lim_{n\to\infty} g_n = \frac{n}{2n+3} = \lim_{n\to\infty} \frac{n}{2n+3} = \lim \frac{1}{2+\frac{3}{n}\to 0} = \frac{1}{2}$

$h_n = n^2 \qquad 1, 4, 9$

$\lim_{n\to\infty} \{h_n\} = \infty$

Definição – Diz-se que l é $\lim_{n\to\infty} a_n$ se dado $\varepsilon > 0$

É possível determinar n_0 tq $n > n_0 \Rightarrow$

$|a_n - L| < \varepsilon$

Série

Ex

$1 - \quad 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

$S_n = \frac{a_1 - a_1 q^n}{1 - q}$

$\lim_{n \to \infty} S_n = \frac{a_1}{1-q}$

$S_n = \frac{1}{1 - 1/3} = 3/2$

2) $1 - 1 + 1 - 1 + 1 - 1 + 1 \cdots$

3) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \cdots$ diverge

Séries Positivas
$a_n > 0$

1. critério da integral
2. critério da comparação
3. critério da equivalência

① $\sum_{1}^{\infty} \frac{2n}{3n+1} = \lim_{n \to \infty} \frac{2}{3 + \frac{1}{n}} = \frac{2}{3} \neq 0$

$\Rightarrow$ diverge

② $\sum_{0}^{\infty} \frac{1}{2n+3} \Longleftrightarrow \int_{1}^{\infty} \frac{1}{2x+3}\,dx =$

$\lim_{A \to \infty} \int_{1}^{A} \frac{1}{2x+3}\,dx = \lim_{A \to \infty} \left[\log(2A+3) - \log 3\right] = \infty$

diverge //

③ $\sum_{1}^{\infty} \frac{1}{n^2+3n+2}$

$\int_{1}^{\infty} \frac{1}{x^2+3x+2}\,dx = \lim_{A \to \infty} \int_{1}^{A} \frac{1}{x^2+3x+2}\,dx =$

$\lim \log \frac{x+1}{x+2} \Big|_{1}^{A} \Rightarrow$ converge

teste da comparação

Se $a_n > b_n$ $\sum_{0}^{\infty} a_n$ converge $\Rightarrow \sum b_n$ converge

Se $a_n < b_n$ e $\sum a_n$ diverge $\Rightarrow \sum b_n$ diverge

Série

① Critério geral

se $\lim_{n \to \infty} a_n \neq 0 \Rightarrow \sum a_n$ diverge

Séries alternadas

I) Se $|a_1| > |a_2| > |a_3| > \dots$

II) Se $\lim_{n \to \infty} a_n = 0 \Rightarrow$ a série converge

critério da equivalência

se $\lim_{n \to \infty} \frac{a_n}{b_n} = 1 \Rightarrow$ elas são da mesma natureza

Ex

$\sum a_n = \sum \frac{n}{n^3+7n+20}$

$\sum b_n = \sum \frac{1}{n^2}$

$\Rightarrow \lim_{n \to \infty} \frac{n}{n^3+7n+20} \cdot n^2 = \lim \frac{n^3}{n^3+7n+20} =$

$\lim_{n \to \infty} \frac{1}{1+\frac{7}{n^2}+\frac{20}{n^3}} = 1$

logo $\sum a_n$ converge

Ex:

$\sum \frac{n}{n^2+7n+5}$

$\sum \frac{1}{n}$ diverge

$\lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{n^2}{n^2+7n+5} = \lim \frac{1}{1+\frac{7}{n}+\frac{5}{n^2}} = 1$

$\Rightarrow \sum a_n$ diverge

convergência absoluta

Def: $\sum_{1}^{\infty} a_n$ diz-se absolutamente convergente se $\sum |a_n|$ for convergente

Ex: $\sum_{1}^{\infty} (-1)^{n+1} \frac{1}{2n}$

Def: $\sum a_n$ diz-se cond. convergente se $\sum |a_n|$ for divergente e $\sum a_n$ for convergente

Ex: $\sum (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} \dots$

$\sum \frac{1}{n}$ é divergente e $(-1)^{n+1}\frac{1}{n}$ é convergente

$\Rightarrow$ condicionalmente convergente critério da raiz

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = d \begin{cases} d<1 & \text{converge} \\ d=1 & ? \\ d=1 & \text{diverge} \end{cases}$$

② critério da Razão

$$\lim \frac{|a_{n+1}|}{a_n} = d \begin{cases} d>1 & \text{diverge} \\ d<1 & \text{converge} \\ d=1 & \end{cases}$$

Ex:

$$\sum (-1)^{n+1} \frac{n}{3^n}$$

usando raiz

$$|a_n| = \frac{n}{3^n}$$

$$\sqrt[n]{\frac{n}{3^n}} = \frac{\sqrt[n]{n}}{3} \approx \frac{1}{3} < 1 \text{ converge}$$

Ex:

$$\sum \frac{2^n}{n!} \quad \lim_{n\to\infty} \frac{|a_{n+1}|}{a_n} = \frac{\cancel{2^{n+1}}}{(n+1)!} \cdot \frac{\cancel{n!}}{\cancel{2^n}} = \frac{2}{n+1}$$

$$\lim \frac{2}{n+1} = 0 \text{ converge}$$

Ex: $\sum \frac{1}{n}$

$$\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 0 \ ?$$

Ex: $\sum \frac{1}{n^2}$

$$\lim \frac{1}{(n+1)^2} \cdot n^2 = \left(\frac{n}{n+1}\right)^2 = 1 \quad ?$$

1 – Medida por Comparação

a) $\sum \frac{n}{n^2+10^6}$

$\sum \frac{1}{n}$ diverge e

$$\lim_{n\to\infty} \frac{n/10^6+n^2}{1/n} = \lim \frac{n^2}{10^6+n^2} = \frac{1}{\frac{10^6}{n^2}+1} = 1$$

$\Rightarrow \sum a_n$ diverge

b) $\sum n e^{-n^2}$

$$\int_0^\infty x e^{-x^2} dx = \frac{1}{2} e^{-x^2}\Big|_0^A = \frac{1}{2} e^{-A^2} - e^{-1} = \frac{1}{2}\left[\frac{1}{e^{A^2}} - \frac{1}{e}\right] = \frac{1}{2}$$

converge

c) $\sum n^{-n^2} = \sum \frac{n}{n^2}$ mas $\frac{1}{n^2}$ converge

$\frac{1}{n^2} < \frac{n}{n^2} \Rightarrow \sum n^{-n^2}$ +b converge//

③ aplicando o teste da razão

$$\sum_1^\infty \frac{n}{1+n^2} = \int_1^\infty \frac{x}{1+x^2}\, dx = \frac{1}{2} \log 1+x^2 \Big|_0^\infty =$$

$\lim\limits_{A\to\infty} \frac{1}{2}\log(1+x^2)\Big|_1^A = \infty$

logo diverge

a) $\sum n e^{-n^2}$

$\lim \frac{(n+1)e^{-(n+1)^2}}{n e^{-n^2}} = 0$ diverge

b) $\sum \frac{n!}{n^n}$

$\lim\limits_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \times \frac{n^n}{n!} = \frac{n^n}{(n+1)^n} = \frac{1}{e} < 1$

converge

4) Qual é o teste

a) $\sum \frac{n^3}{(n+1)!}$

$\frac{(n+1)^3}{(n+2)!} \cdot \frac{(n+1)!}{n^3} = \frac{(n+1)^3}{(n+2)n^3}$

? b) $\sum n!\, 2^{-n}$

$\frac{(n+1)!}{2^{n+1}} \times \frac{2^n}{n!} = \frac{n+1}{2}$?

5) Investigue para que valores de x as séries convergem

a) $\sum_1^\infty \frac{\cos^2 xn}{1+n^2}$

b) $\sum_1^\infty \left(\frac{1}{n} - e^{-nx}\right)$

c) $\sum_0^\infty x^n n!\, n^{-n}$

d) $\sum x^n n!\, 2^{-n^2}$

e) $\sum_2^\infty \frac{x^n}{n(\log n)^{2x}}$

c) $\sum_0^\infty x^n n!\, n^{-n}$

$\frac{|a_{n+1}|}{a_n} = \frac{x^{n+1}(n+1)!\,(n+1)^{-(n+1)}}{x^n n!\, n^{-n}} = \frac{x(n+1)n^n}{(n+1)^{n+1}}$

$= \frac{x n^n}{(n+1)^n} \Rightarrow |x| \cdot \frac{1}{|e|} < 1$

$|x| < e$

d) $\frac{x^{n+1}(n+1)!\, 2^{-(n+1)^2}}{x^n n!\, 2^{-n^2}} = |x| \cdot \frac{(n+1)\, 2^{n^2}}{2^{(n+1)^2}} = \frac{|x|(n+1)2^{n^2}}{2^{n^2}\cdot 2^n \cdot 2}$

$= \frac{|x|(n+1)}{2^{2n+1}}$ $\quad \frac{(n+1)}{2^{n+1}}$ $\forall x$

e) $\frac{x^{n+1}}{(n+1)\log(n+1)^{2(x+1)}} \times \frac{(n\log n)^{2x}}{x^n}$

$|x|\left(\frac{n}{n+1}\right) \times \frac{\log n^{2n}}{(\log n+1)^{2n}} \Rightarrow$ converge $|x|<1$

Para que valores de x as séries convergem absoluta/e

1) $\sum (-1)^n \frac{(x/2)^{2n}}{(n!)^2}$

2) $\sum_{1}^{\infty} n! \left(\frac{x}{n}\right)^n$

3) $\sum_{0}^{\infty} \frac{x^n (n!)^2}{(2n)!}$

4) $\sum_{1}^{\infty} n^2 (x+1)^n$

1) $\sum (-1)^n \frac{(x/2)^{2n}}{(n!)^2}$

$\frac{|a_{n+1}|}{|a_n|} = \frac{(x/2)^{2(n+1)}}{[(n+1)!]^2} \times \frac{(n!)^2}{(x/2)^{2n}} = \frac{(x/2)^2}{(n+1)^2}$

lim $= 0$ $\forall x$ logo converge $\forall x$ absoluta/e

2) lim $\frac{|a_{n+1}|}{|a_n|} = \frac{(n+1)\cancel{!}\, x^{n+1}}{(n+1)^{n+1}} \times \frac{n^n}{\cancel{n!}\, \cancel{x^n}} = |x| \frac{n^n}{(n+1)^n} = \frac{|x|}{e}$

logo $\forall |x| < e$

3) $\frac{x^{n+1}((n+1)!)^2}{(2(n+1))!} \cdot \frac{2n!}{x^{n+1}(n)!} =$

$= |x| \frac{n^2+2n+1}{4n^2+6n+2}$ = lim $|x| \frac{(n+1)}{(2x+1)}$

$\frac{|x|}{4}$ => $|x| < 4$

4) $\sum n^2 (x+1)^n$

$\frac{(n+1)^2 (x+1)^{n+1}}{n^2 (x+1)^n} = \frac{(n+1)^2}{(n)^2} \cdot |x+1|$

lim $= |x+1| < 1$ => logo $-2 < x < 0$

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1) $\sum \frac{1}{2^n}$ $\quad S_n = \frac{1}{a-q} = \frac{1}{1-q} = 2$

2) $\sum_{1}^{\infty} \frac{2}{3^n} = \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \ldots$

$S_n = \frac{2/3}{1+\frac{1}{3}} = 1$

3) $\sum_{1}^{\infty} \frac{1}{n(n+1)}$ $\quad$ como $\frac{1}{n} > \frac{1}{n(n+1)}$ e $\sum \frac{1}{n}$ diverge

=> $\frac{1}{n(n+1)}$ tb diverge

4) idem anterior

6) $n=1 \to 1$ $\quad a_n = a_1 + (n-1)$
$n=2 \to 4$
$n=3 \to 4$

$a_n = \frac{1}{(3n-2)(3n+1)(3n+4)}$

7) $\sum \log\left(1+\frac{1}{n}\right) = \ell_2 + \ell_{3/2} + \ell \frac{4}{3} + \ell \frac{5}{4}$

$= \ell\left(2 \cdot \frac{3}{2} \cdot \frac{2}{3} \cdot \frac{5}{4} \cdot \frac{x}{n-1} \; \frac{n+1}{x}\right) = \ell(n+1)$

$(1+\frac{1}{n})^n = \frac{1}{0}$

$\lim_{n\to\infty} \ell(n+1) = \log_{n\to\infty} = \infty$ não converge

II a) $|1| \neq |2|$ diverge

b) $\lim \frac{n}{n+1} = \frac{1}{1+\frac{1}{n}} = 1$ diverge

2) $\sum_1^\infty \frac{n+1}{n^3+n+4} =$

seja $\sum_1^\infty \frac{1}{n^3}$ e $\frac{1}{n^3} \geq \frac{n+1}{n^3+n+4}$ e $\frac{1}{n^3}$ converge

$\Rightarrow \sum \frac{n+1}{n^3+n+4}$ tb converge

4) $\sum_1^\infty \frac{1}{n \log n}$

$\int_0^\infty \frac{dx}{x \log x} = \lim = \infty \Rightarrow$ diverge

5) $\sum_1^\infty \frac{\ln n}{n}$

como $\frac{1}{n \log n} < \frac{\log n}{n}$ e $\sum \frac{1}{n \log n}$ diverge $\Rightarrow$

diverge

6) $\sum_1^\infty \frac{1}{\sqrt[3]{n^2}} = \int_1^\infty n^{-3/2} = \frac{x^{1/3}}{1/3}$

$\lim_{A\to\infty} \frac{A^{1/3}}{1/3} = \infty$ diverge

14) $\frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} = \lim \frac{3}{n+1} = 0 \Rightarrow$ conv

16) $\sum \frac{1}{n^2}$ converge e $\frac{1}{n^2} \geq \left|\frac{\operatorname{sen} n}{n^2}\right|$

$\Rightarrow \sum \frac{\operatorname{sen} n}{n^2}$ tb converge

19) $\sum_2^\infty \frac{1}{n\sqrt{\ln n}}$ $\qquad \frac{1}{n \log n}$ diverge

$\frac{1}{n \log n} < \frac{1}{n\sqrt{\log n}} \Rightarrow \sum \frac{1}{n\sqrt{\ln n}}$ tb converge //

7) $\cancel{\sqrt{2}} - \sqrt{1} + \sqrt{3} - \cancel{\sqrt{2}} + \sqrt{4} - \sqrt{3} \ldots = -1 + \sqrt{n+1}$

$\lim -1 + \sqrt{n+1} = \infty \Rightarrow$ diverge

8) $\sum_1^\infty \frac{2^n+3^n}{4^n+5^n}$ $\qquad \sum \frac{3^n}{5^n}$ converge

$\frac{2^n+3^n}{4^n+5^n} \cdot \frac{5^n}{3^n} = \frac{10^n+15^n}{12^n+15^n} \overset{\lim}{=} (\frac{5}{4})^n + 1 = 1$ converge

10) $\sum \frac{n^3}{3^n}$

$\lim \frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3} = \frac{(n+1)^3}{3n^3}$

$\lim = \frac{1}{3}$ converge

11) $\sum_1^\infty \frac{3^n}{n^3}$ $\qquad \sum \frac{n^3}{3^n}$ converge

$\frac{3^n}{n^3} \cdot \frac{n^3}{3^n} = 1 \Rightarrow$ tb converge

$$\frac{(n+1)^4}{(n+1)!} \cdot \frac{\cancel{n!}}{n^4} = \frac{(n+1)^4}{(\cancel{n+1}) \cdot n^4} = \frac{(n+1)^3}{n^4} = 0$$

converge

13) $\frac{3^n}{n!}$

$$\lim \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} = \frac{3}{n+1} = 0 \text{ converge}$$

15) $\sum \operatorname{sen} \frac{1}{n^2}$ como

$$\frac{1}{n^2} \geq \operatorname{sen} \frac{1}{n^2} \Rightarrow \operatorname{sen} \frac{1}{n^2} \text{ tb converge}$$

16) $\sum \frac{\operatorname{sen} n}{n^2}$ como $\sum \frac{1}{n^2}$ converge e

$$\frac{1}{n^2} \geq \frac{\operatorname{sen} n}{n^2} \Rightarrow \sum \frac{\operatorname{sen} n}{n^2} \text{ tb converge}$$

18) $\sum \frac{1}{38+\pi n}$ como

$$\sum \frac{1}{n} \qquad \frac{1}{38+\pi n}$$

$$\frac{1}{38+\pi n} \cdot n = \frac{n}{38+\pi n} = \frac{1}{\frac{38}{n}+\pi} = \frac{1}{\pi} \neq 1$$

$$\Rightarrow \frac{1}{38+\pi n} \text{ converge}$$

22) $\sum \operatorname{arc\,sen} \frac{1}{n}$

$$\lim \frac{\operatorname{arc\,sen} \frac{1}{n}}{\frac{1}{n}} = 1$$

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① $\frac{(n+1)\, x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n\, x^n} = \frac{(n+1)\,|x|}{2n} = \lim \frac{1}{2}|x|$

$= \frac{1}{2}|x| < 1 \Rightarrow |x| < 2 \Rightarrow$

$-2 < x < 2$

Substituo

$= 0,\ \frac{x}{2},\ \frac{2x^2}{4},\ \frac{3x^3}{9} \ldots$

$-1,\ 2,\ -\frac{2}{3}$ $\Rightarrow$ diverge $-2 < x$

$x=2 \rightarrow \frac{2}{2},\ \frac{8}{4},\ \frac{24}{9}$ — diverge

$x \in (-2, 2)$

② $\frac{(x-1)^{n+1}}{n+1} \cdot \frac{n}{(x-1)^n} = (x-1)\frac{n}{n+1} \Rightarrow$

$(x-1) < 1$

$-1 < x-1 < 1 \Rightarrow 0 < x < 2$

$\sum \frac{(x-1)^n}{n} = x-1,\ \frac{(x-1)^2}{2},\ \frac{(x-1)^3}{3}$

$x=0 \rightarrow -1,\ \frac{1}{2},\ -\frac{1}{3}$ converge

$x=2 \quad 1,\ \frac{1}{2},\ \frac{1}{3}$ diverge

$x \in [0, 2)$

③ $\frac{(x+1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(x+1)^n} = \frac{n^2}{(n+1)^2}(x+1) \Rightarrow |x+1| \lim \frac{n^2}{(n+1)^2} = |x+1|$

$-1 < x+1 < 1 \quad -2 < x < 0$

$(-1, 0)$

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1)

$$f(x) = f(0) + f'(0)x + f''\frac{x^2}{2!} + f'''\frac{x^3}{3!}$$

$f(x) = e^x$ 1
$f' = e^x$ 1
$f'' = e^x$ 1

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots \frac{x^n}{n!}$$

2) $f(x) = e^{-x}$
$f'(x) = -e^{-x}$
$f''(x) = e^{-x}$

$$f(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$$

3) $f(x) = e^{x^2}$

$f'(x) = 2x e^{x^2}$

$f''(x) = 4x^2 e^{x^2}$

$f(x) = 1$

6) $1 \quad \underline{|\ 1 + x^2}$
$-1 - x^2 \quad 1$
$-x^2$

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12) $(1+x^2)\, y'' + 2xy' = 0$

$$\underbrace{\sum_2^\infty (k-1)k a_k x^{k-2}}_{} + \sum_2^\infty (k-1)k a_k x^k + 2\underbrace{\sum_1^\infty k a_k x^k}_{} = 0$$

$$2a_2 + 6a_3 x + \sum_4^\infty (k-1)k a_k x^{k-2} + \sum_2^\infty (k-1)k a_k x^k + 2a_1 x + 2\sum_2^\infty k a_k x^k = 0$$

$$2a_2 + (6a_3 + 2a_1)x + \sum_2^\infty (k+2)(k+1)a_{k+2}x^k + (k-1)k a_k x^k + 2k a_k = 0$$

$2a_2 = 0$
$a_2 = 0$
$a_0 = 0$
$a_1 = 1$

$6a_3 = -2a_1 \Rightarrow$
$a_3 = -\frac{1}{3}$ //

$$(k+2)(\cancel{k+1})a_{k+2} + (k-1)k a_k + 2k a_k = 0$$
$$k^2 a_k - k a_k + 2k a_k = 0$$
$$a_k(k^2 + k) = k a_k (\cancel{k+1})$$

$(k+2)a_{k+2} + k a_k = 0$

$$a_{k+2} = -\frac{k a_k}{(k+2)}$$

$$a_5 = -\frac{3a_3}{5} = \frac{1}{5}$$

$$a_7 = -\frac{5a_5}{7} = -\frac{1}{7}$$

$$y(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \text{ //}$$

11) $(x-3)y'' + x^2y' + y = 0$ $\quad y(0)=0 \quad y'(0)=6$

$$\sum_{2}^{\infty}(k-1)k a_k x^{k-1} - 3\sum_{2}^{\infty} k(k-1)a_k x^{k-2} + \sum_{1}^{\infty} k a_k x^{k+1} + \sum_{0}^{\infty} a_k x^k = 0$$

$k = k+1$

$$2a_2x + \sum_{3}^{\infty}(k-1)k a_k x^{k-1} - 6a_2 - 18a_3x - 3\sum_{4}^{\infty} k(k-1)a_k x^{k-2}$$

$k = k-1$

$$+ \sum_{1}^{\infty} k a_k x^{k+1} + a_0 + a_1x + \sum_{2}^{\infty} a_k x^k = 0$$

$$= 2a_2x + \sum_{2}^{\infty}(k+1)k a_{k+1} x^k - 6a_2 - 18a_3x - 3\sum_{2}^{\infty}(k+2)(k+1)a_{k+2}x^k$$

$$+ \sum_{2}^{\infty}(k-1)a_{k-1}x^k + a_0 + a_1x + \sum_{2}^{\infty} a_k x^k = 0$$

$$2a_2x - 6a_2 - 18a_3x + a_0 + a_1x = 0$$

$$(k+1)k a_{k+1} - 3(k+2)(k+1)a_{k+2} + (k-1)a_{k-1}$$

$$\begin{cases} y' + 2z' = t \\ y'' - z = e^{-t} \end{cases}$$

$$\begin{cases} s\mathcal{L}(y) - y(0) + 2s\mathcal{L}(z) - 2z(0) = 1/s^2 \\ s^2\mathcal{L}(y) - s y(0) - y'(0) - \mathcal{L}(z) = \dfrac{1}{s-1} \end{cases}$$

$$\begin{cases} s\mathcal{L}(y) - 3 + 2s\mathcal{L}(z) = 1/s^2 \\ s^2\mathcal{L}(y) - 3s + 2 - \mathcal{L}(z) = 1/s-1 \end{cases}$$

$$\begin{cases} s\mathcal{L}(y) + 2s\mathcal{L}(z) = \dfrac{1}{s^2} + 3 \\ s^2\mathcal{L}(y) - \mathcal{L}(z) = \dfrac{1}{s-1} + 3s - 2 \end{cases}$$

$$= \begin{cases} s\mathcal{L}(y) + 2s\mathcal{L}(z) = \dfrac{1+3s^2}{s^2} \\ s^2\mathcal{L}(y) - \mathcal{L}(z) = \dfrac{1+3s^2-3s-2s+2}{s-1} = \dfrac{3s^2-5s+3}{s-1} \end{cases}$$

$$\mathcal{L}(y) = \frac{1+3s^2}{s^2} - 2s\mathcal{L}(z) = \frac{1+3s - 2s^3\mathcal{L}(z)}{s^3}$$

$$s^2\,\frac{1+3s^2-2s^3\mathcal{L}(z)}{s^3} - \mathcal{L}(z) = \frac{3s^2-5s+3}{s-1}$$

$$\frac{1+3s^2-2s^3\mathcal{L}(z)}{s} - \mathcal{L}(z) = \frac{3s^2-5s+3}{s-1}$$

$$\frac{1+3s^2-2s^3\mathcal{L}(z) - s\mathcal{L}(z)}{s} = \frac{3s^2-5s+3}{s-1}$$

$$\frac{1+3s^2+\mathcal{L}(z)(-2s^3-s)}{s} = \frac{3s^2-5s+3}{s-1}$$

$$1+3s^2+\mathcal{L}(z)(-2s^3-s) = \frac{3s^3-5s^2+3s}{s-1}$$

$$\mathcal{L}(z)(-2s^3-s) = \frac{3s^3-5s^2+3s}{s-1} - 1 - 3s^2 = \frac{3s^3-5s^2+3s-s+1}{s-1}$$

$-3s^3 + 3s^2$

$$\mathcal{L}(z) = \frac{-2s^2+2s+1}{s-1} \times \frac{1}{-2s^3-s} = \frac{-2(s-1)^2}{}$$

$$\begin{cases} s\mathcal{L}(y) - y(0) - s\mathcal{L}(z) + z(0) - 2\mathcal{L}(y) + 2\mathcal{L}(z) = \frac{1}{s^2+1} \\ s^2\mathcal{L}(y) - sy(0) - y'(0) + 2s\mathcal{L}(z) + 2z(0) + \mathcal{L}(y) = 0 \end{cases}$$

$$-s\ \begin{cases} s\mathcal{L}(y) - s\mathcal{L}(z) - 2\mathcal{L}(y) + 2\mathcal{L}(z) = 1/s^2+1 \\ s^2\mathcal{L}(y) + 2s\mathcal{L}(z) + \mathcal{L}(y) = 0 \end{cases}$$

//

$$\begin{cases} -s^2\mathcal{L}(y) + s^2\mathcal{L}(z) + 2s\mathcal{L}(y) - 2s\mathcal{L}(z) = \frac{s}{s^2+1} \\ s^2\mathcal{L}(y) + 2s\mathcal{L}(z) + \mathcal{L}(y) = 0 \end{cases}$$

$s^2\mathcal{L}(z) + 2s\mathcal{L}(y) -$

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a) e^x

$f'(x) = e^x$

$f''(x) = e^x$

$$f(x) = f(x) + f'(x)x + \frac{f''(x)}{2!}^2 + \frac{f'''(x)\,x^3}{3!}$$

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

b) $f(x) = e^{-x}$

$f'(x) = -e^{-x}$

$f''(x) = e^{-x}$

$$f(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

4) $\operatorname{sen} x$

$f'(x) = \cos x$

$f''(x) = -\operatorname{sen} x$

$f''(x) = -\cos x$

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \dots$$

6)
$$\begin{array}{l|l} 1 & 1+x^2 \\ \hline -1-x^2 & 1-x^2+x^4-x^6 \\ \quad -x^2 & \\ \quad x^2+x^4 & \\ \qquad x^4 & \\ \quad -x^4-x^6 & \\ \qquad -x^6 & \end{array}$$

$$f(x) = 1 - x^2 + x^4 - x^6$$

11)
$$\begin{array}{l|l} 1 & 2+3x \\ \hline -\frac{3}{2}x & \frac{1}{2} + \frac{2}{3}x \end{array} \quad - \dots$$

Rio, 22 de janeiro de 1979

Série de fourier

Obs.

$$\int \operatorname{sen} \frac{n\pi x}{l} \cdot \operatorname{sen} \frac{k\pi x}{l} = \begin{cases} 0 & n \neq k \\ 1 & n = k \end{cases}$$

$$\int \cos \frac{n\pi x}{l} \cos \frac{k\pi x}{l} = \begin{cases} 0 & n = k \\ 1 & n \neq k \end{cases}$$

$$\int \cos \frac{n\pi x}{l} \operatorname{sen} \frac{n\pi x}{l} = 0$$

$$f(x) = \frac{a_0}{2} + \Sigma A_n \cos \frac{n\pi x}{l} + B_n \operatorname{sen} \frac{n\pi x}{l} //$$

onde $a_0 = \frac{1}{l} \int_{-l}^{l} f(x)dx$

$$A_k = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l}$$

$$B_k = \frac{1}{l} \int_{-l}^{l} f(x) \operatorname{sen} \frac{n\pi x}{l}$$

Função par e impar

Par $\Rightarrow f(x) = f(-x)$ simétrica a y

impar $\Rightarrow f(-x) = -f(x)$ simétrica a x

$$\int_{-a}^{a} \text{impar} = 0$$

$$\int_{-a}^{a} \text{par} = 2 \int_{0}^{a} \text{par} //$$

convergência

f converge para $\frac{1}{2}[f(x^-)+f(x^+)]$

1) determine a série de fourier

a) $f(x)=|x|$ $[-\pi,\pi]$

f é par =>

$a_0=\frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\left[\frac{x^2}{2}\right] = \frac{1}{\pi}\cdot\pi^2 = \pi$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} \overset{p}{x}\cdot \overset{i}{\operatorname{sen} nx} \overset{=i}{} = 0$

$a_n=\frac{2}{\pi}\int_0^{\pi} x\cos nx = \left[\frac{1}{n^2}\cos nx + \frac{x}{n}\cancel{\operatorname{sen} nx}^{\,0}\right]_0^{\pi}$

$\frac{2}{\pi}\left[\frac{1}{n^2}\cos \pi n - \frac{1}{n^2}\right] = \frac{2}{\pi n^2}\left((-1)^{2n-1}-1\right)$

$A_{2n+1} = \frac{2}{(2n+1)^2\pi}\left((-1)^{2n-1}-1\right)$

$f(x)=\frac{\pi}{2}+\sum \frac{2}{(2n+1)^2\pi}\left((-1)^{2n-1}-1\right)\cos n\pi x$ //

$=\frac{\pi}{2}+\frac{4}{\pi}\sum \frac{\cos(2n+1)\pi x}{(2n+1)^2}$ //

$f(x)=\begin{cases}0 & -l\le x<0\\ x & 0\le x\le l\end{cases}$

$a_0=\frac{1}{l}\int_0^l x\,f(x) = \frac{1}{l}\cdot\frac{l^2}{2} = \frac{l}{2}$

$a_k=\frac{1}{l}\int_0^l \overset{i}{x}\,\overset{p}{\cos}\,\frac{n\pi x}{l}\overset{=i}{} = \frac{-l^2}{k^2\pi^2}\left[\cos k\pi - 1\right]$

$b_k=\frac{1}{l}\int_0^l x\operatorname{sen}\frac{n\pi x}{l}$

$\frac{l^2}{n^2\pi^2}\cancel{\operatorname{sen}\frac{n\pi x}{l}}^{\,0} - \frac{l}{n\pi}x\cos\frac{n\pi x}{l}\Big|_0^l = \frac{-l^2}{n\pi}\cos n\pi$

$f(x)=\frac{l}{4}+\sum\frac{-l^2}{k^2\pi^2}\left[\cos k\pi-1\right]\cos\frac{n\pi x}{l}+\frac{-l^2}{n\pi}\cos n\pi\cdot\operatorname{sen}\frac{n\pi x}{l}$ //

—x—

$f(x)=x^2$ $-\pi\le x\le\pi$

como x^2 é par

$a_0=\frac{2}{\pi}\int_0^{\pi}x^2\,dx = \frac{2}{\pi}\left[\frac{x^3}{3}\right] = \frac{2}{\pi}\cdot\frac{\pi^3}{3} = \frac{2\pi^2}{3}$ // $\frac{1}{2}$

$B_k=\frac{1}{\pi}\int_{\pi}^{\pi} \overset{p}{x^2}\,\overset{i}{\operatorname{sen} nx} \overset{=i}{} = 0$

$a_k=\frac{2}{\pi}\int_0^{\pi}x^2\cos nx = 4\cos\frac{k\pi}{k^2} < \frac{(-1)^{2n+1}\cdot 4}{(2n+1)^2}$

$f(x)=\frac{\pi^2}{3}+\sum_1^{\infty}\frac{(-1)^{2n+1}\,4}{(2n+1)^2}\cos(2n+1)x$ //

usando f(x) calcule

$\sum\frac{1}{n^2}$

substitui por π

$\pi^2=\frac{\pi^2}{3}+\sum\frac{4}{n^2}$ => $\pi^2=\frac{\pi^2}{3}+4+\sum\frac{1}{n^2}$

$\frac{\pi^2}{3} + 4\sum \frac{1}{n^2} = \pi^2$

$4\sum \frac{1}{n^2} = \pi^2 - \frac{\pi^2}{3}$

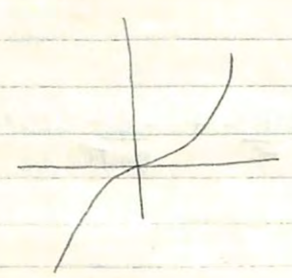
$\sum \frac{1}{n^2} = \frac{\cancel{2}\pi^2}{3} \times \frac{1}{\cancel{4}2} = \frac{\pi^2}{6}$ //

— x —

Extensão par

Ex: Desenvolva $f(x)=x^2$ em

a) série de senos



$\Rightarrow f(x) = \text{impar}$

$a_0 = 0 \quad a_k = 0$

$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \operatorname{sen} nx = \frac{2}{\pi}\int_0^{\pi} x^2 \operatorname{sen} nx$

$= -2\pi \frac{\cos k\pi}{k} + \frac{4}{k^3\pi}(\cos k\pi - 1) =$

$\begin{cases} \frac{2\pi}{k} - \frac{8}{k^3\pi} & k = 1,3,5 \\ -\frac{2\pi}{k} & k = 2,4,6 \end{cases}$

$f(x) = \frac{2\pi}{k} - \frac{8}{k^3\pi}$

Exercícios

1) $f(x) = 2x \quad 0 \le x \le \pi$
$= x \quad -\pi < x < 0$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} = \frac{1}{\pi}\cdot\frac{\pi^2}{2} = -\frac{\pi}{2}$

$+ \frac{2}{\pi}\int_0^{\pi} x = \frac{2}{\pi}\left[\frac{x^2}{2}\right] = \frac{2}{\pi}\frac{\pi^2}{2} = \pi = \frac{\pi}{2}$

$b_n = \frac{1}{\pi}\int_{-\pi}^{0} x \operatorname{sen} nx + \frac{1}{\pi}\int_0^{\pi} 2x \operatorname{sen} nx =$

$= \frac{1}{\pi}\left[\frac{1}{n^2}\cancel{\operatorname{sen} nx} - \frac{x}{n}\cos nx\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{1}{n^2}\cancel{\operatorname{sen} nx} - \frac{x}{n}\cos nx\right]_0^{\pi} =$

$\frac{1}{\pi}\left[\frac{+\pi}{n}\cos -n\pi\right] + \frac{2}{\pi}\left[-\frac{\pi}{n}\cos n\pi\right] =$

$\frac{1}{n}\cos n\pi - \frac{2}{n}\cos n\pi = -\frac{1}{n}\cos n\pi = \frac{(-1)^{2n+1}-1}{n}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{0} x\cos nx + \frac{2}{\pi}\int_0^{\pi} x\cos nx =$

$\frac{1}{\pi}\left[\frac{1}{n^2}\cos nx + \frac{x}{n}\operatorname{sen} nx\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{1}{n^2}\cos nx + \frac{x}{n}\operatorname{sen} nx\right]_0^{\pi}$

$= \frac{1}{\pi}\left[\frac{1}{n^2} - \frac{1}{n^2}\cos n\pi +\right] + \frac{2}{\pi}\left[\frac{1}{n^2}\cos n\pi - \frac{1}{n^2}\right] = \frac{1}{\pi n^2} - \frac{1}{\pi n^2}\cos n\pi$

$+ \frac{2}{\pi n^2}\cos n\pi - \frac{2}{\pi n^2} = -\frac{1}{\pi n^2} + \frac{1}{\pi n^2}\cos n\pi = -\frac{2}{\pi n^2}\cos n\pi$ //

$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left(\frac{\cos x}{1} + \frac{\cos 3x}{3^2} + \cos 5x\right) + 3\left(\operatorname{sen} x - \frac{\operatorname{sen} 2x}{2} + \frac{\operatorname{sen} 3x}{3}\right)$

(2) $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$ $[\pi, \pi]$

$a_0 = \frac{2}{\pi}\int_0^\pi \frac{\pi^2}{12} - \frac{x^2}{4} = \frac{2}{\pi}\left[\frac{\pi^2}{12}x\right]_0^\pi - \frac{2}{\pi}\cdot\frac{1}{4}\left[\frac{x^3}{3}\right]_0^\pi$

$\frac{\pi^2}{6} - \frac{\pi^3}{6\pi} = 0$

$a_n = \frac{2}{\pi}\int_0^\pi \frac{\pi^2}{12}\cos nx - \frac{2}{\pi}\int \frac{x^2}{4}\cos nx$

$= \frac{2\pi^2}{6\cdot 2\pi}\left[\cancel{\frac{\sin nx}{n}}\right]_0^\pi - \frac{1}{2\pi}\int x^2\cos nx$

$-\frac{1}{2\pi}\left[\frac{\pi^2}{n}\cancel{\sin n\pi} - \frac{2}{n^3}\cancel{\sin n\pi} - \frac{2\pi}{n^2}\cos n\pi\right]$

$= \frac{2\pi}{2\pi n^2}\cos n\pi = \frac{1}{n^2}\cos n\pi =$

$f(x) = \sum \frac{1}{n^2}\cos n\pi \cos nx$

12 - 4 | 2
6 - 2 | 2
3 - 1 | 3
2

—x—

b) $x = \pi$

$f(x) = \frac{1}{n^2}(-1)\cdot(-1) = \frac{1}{n^2} = \frac{\pi^2}{12} - \frac{\pi^2}{4} = \frac{\pi^2}{12} - \frac{3\pi^2}{12} = -\frac{\pi^2}{6}$

$\frac{1}{n^2} = \frac{\pi^2}{6}$

em $x = 0$

$\frac{\pi^2}{12} = \sum \frac{(-1)^{n+1}}{n^2}$

(3) $f(x) = -\frac{\pi}{2} - \frac{x}{2}$ $-\pi \le x < 0$

$\frac{\pi}{2} - \frac{x}{2}$ $0 < x < \pi$

f é ímpar

$b_n = \frac{2}{\pi}\int_0^\pi \left(\frac{\pi}{2} - \frac{x}{2}\right)\sin nx$

$u = \frac{\pi}{2} - \frac{x}{2}$
$du = -\frac{dx}{2}$
$dv = \sin nx$
$v = \frac{\cos nx}{-n}$

$\frac{2}{\pi}\left[\left(\frac{\pi}{2} - \frac{x}{2}\right)\frac{\cos nx}{-n}\right] =$

$\frac{2}{\pi}\left[\left(\frac{\pi}{2} - \frac{\pi}{2}\right)\frac{\cos \pi x}{-n} - \left(\frac{\pi}{2} - \frac{0}{2}\right)\frac{\cos 0x}{-n}\right] = \frac{1}{n}$

$\frac{\cancel{2}}{\cancel{\pi}}\frac{\pi}{2n}$

$f(x) = \sum \frac{1}{n}\sin nx$

(4) $f(x) = -x$ $-\pi < x < 0$
$= 0$ $0 < x < \pi$

$a_0 = \frac{1}{\pi}\int_0^\pi x\,dx = -\pi/2$

$a_n = -\frac{1}{\pi}\int_0^\pi x\cos nx = -\frac{1}{\pi}\left[\frac{1}{n^2}\cos nx + \frac{x}{n}\cancel{\sin nx}\right]_0^\pi$

$= -\frac{1}{n^2\pi}\left[\cos nx - 1\right]$ < n par $= 0$; n ímpar $= -\frac{2}{n^2\pi}$

$b_n = -\frac{1}{\pi}\int_0^\pi x\sin nx = -\frac{1}{\pi}\left[\frac{1}{n^2}\cancel{\sin nx} - \frac{x}{n}\cos nx\right]_0^\pi = -\frac{1}{\pi}\left[-\frac{\pi}{n}\cos n\pi\right]$

$= -\frac{1}{n}$

$f(x) = +\frac{\pi}{4} + \sum -\frac{2}{n^2\pi}\cos nx - \frac{1}{n}\sin nx =$

$x = 0$

$-\frac{\pi}{4} = \frac{2}{\pi}\sum\frac{1}{(2n+1)^2} \Rightarrow \sum\frac{1}{(2n+1)^2} = -\frac{\pi^2}{8}$

5) $f(x) = 1 \qquad -\pi < x < 0$

$f(x) = -2 \qquad 0 < x \leq \pi$

$a_0 = -\frac{1}{\pi}\int_0^{\pi} dx + -2\int_0^{\pi} dx =$

$\frac{\pi}{\pi} - \frac{2}{\pi}[\pi] = -1$

$a_n = -\frac{1}{\pi}\int_0^{\pi} \cos nx - \frac{2}{\pi}\int_0^{\pi} \cos nx = \frac{+1}{\pi}\left[\frac{\operatorname{sen} nx}{n}\right]_0^{\pi} + \frac{2}{\pi}\left[\frac{\operatorname{sen} nx}{n}\right]_0^{\pi} \to 0$

$b_n = -\frac{1}{\pi}\int_0^{\pi} \operatorname{sen} nx - \frac{2}{\pi}\int_0^{\pi} \operatorname{sen} nx = -\frac{1}{\pi}\left[\frac{-\cos nx}{n}\right]_0^{\pi} - \frac{2}{\pi}\left[\frac{-\cos nx}{n}\right]_0^{\pi}$

$= \frac{1}{n\pi}[\cos n\pi - 1] + \frac{2}{n\pi}[\cos n\pi - 1] = \frac{\cos n\pi}{n\pi} - \frac{1}{n\pi} + \frac{2\cos n\pi}{n\pi}$

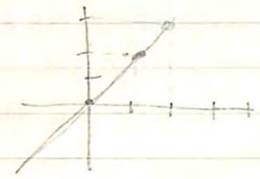
$-\frac{2}{n\pi} = \frac{3}{n\pi}\cos n\pi - \frac{3}{n\pi} \begin{cases} n \text{ par} = 0 \\ n \text{ impar} = -\frac{6}{n\pi} \end{cases}$

$f(x) = -\frac{1}{2} - \frac{6}{\pi}\sum_0^{\infty} \frac{\operatorname{sen}(2n+1)}{(2n+1)}$

b) $x = \frac{\pi}{2}$

$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{5} + \cdots = \frac{\pi}{4}$

6) $f(x) = 2x \qquad f$ é impar



$a_0 = 2\int_0^1 x = 2 \qquad \boxed{a_0 = 2}$

$a_n = 2\int_0^1 2x \cos n\pi x = 4\left[\frac{1}{n^2\pi^2}\cos n\pi x + \frac{1x}{n\pi}\operatorname{sen} n\pi x\right]_0^1$

$= 4\left[\frac{1}{n^2\pi^2}\cos n\pi + \frac{1}{n\pi}\cancel{\operatorname{sen} n\pi} - \frac{1}{n^2\pi^2}\right] =$

$= \frac{4}{n^2\pi^2}\cos n\pi - \frac{4}{n^2\pi^2} \qquad n \text{ impar} = 0$

$b_n = 2\int_0^1 x \operatorname{sen} n\pi x = 2\left[\frac{1}{n^2\pi^2}\operatorname{sen} n\pi x - \frac{1}{n\pi} x \cos n\pi x\right]_0^1$

$= 2\left[\frac{1}{n^2\pi^2}\cancel{\operatorname{sen} n\pi} - \frac{1}{n\pi}\cos n\pi - \frac{1}{n^2\pi^2}\cancel{\operatorname{sen} 0} + 0\right] =$

$= \frac{(-2)^{n+1}}{n\pi} =$

$f(x) = 1 - \frac{2}{\pi}\sum \frac{\operatorname{sen} n\pi x}{n}$

8) $f(x) = x \quad 0 \le x \le 1$
$\quad = 2-x \quad 1 \le x \le 2$

Em senos $a_n = 0$
$a = 0$

$$b_n = \int_0^1 x \,\text{sen}\, \frac{n\pi x}{2} + \int_0^2 2 \,\text{sen}\, \frac{n\pi x}{2} - \int_0^2 x \,\text{sen}\, \frac{n\pi x}{2}$$

$$b_n = \left[\frac{4}{n^2\pi^2} \cancel{\text{sen}\, \frac{n\pi x}{2}} - \frac{2x}{n\pi} \cos \frac{n\pi x}{2}\right]_0^1 + 2\left[-\cos \frac{n\pi x}{2} \cdot \frac{2}{n\pi}\right]_1^2$$

$$+ \left[-\frac{4}{n^2\pi^2} \cancel{\text{sen}\, \frac{n\pi x}{2}} + \frac{2x}{n\pi} \cos \frac{n\pi x}{2}\right]_1^2$$

$$b_n = \frac{4}{n^2\pi^2} \text{sen}\, \frac{n\pi}{2} - \frac{2}{n\pi} \cancel{\cos \frac{n\pi}{2}} - \frac{4}{n\pi} \cancel{\cos n\pi} + \frac{4}{n\pi} \cancel{\cos \frac{n\pi}{2}}$$

$$-\frac{4}{n^2\pi^2} \text{sen}\, n\pi + \frac{4}{n\pi} \cancel{\cos n\pi} - \frac{2}{n\pi} \cancel{\cos \frac{n\pi}{2}} =$$

$$-\frac{4}{n\pi} \cos \frac{n\pi}{2}$$

11) $u_{xx} - \frac{1}{a^2} u_{tt} = 0 \qquad 0 < x < \pi$

$u(0,t) = u(\pi,t) = 0$
$u(x,0) = \text{sen}\, m x$
$u_t(x,0) = 2 \,\text{sen}\, k x$

$$X''T = \frac{1}{a^2} XT''$$

$X'' - \lambda X = 0$
$X(0) = 0$
$X(\pi) = 0$

$X = \text{sen}\, n x$
$y = A_n \cos n x + B_n \,\text{sen}\, n x$

$$u(x,t) = \sum \text{sen}\, n x \,(A_n \cos nat + B_n \,\text{sen}\, nat)$$

$$u(x,0) = \sum B_n \,\text{sen}\, n x = \text{sen}\, m x$$

$$B_n = \frac{2}{\pi} \int_0^\pi \text{sen}\, m x \cdot \text{sen}\, n x = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}$$

$$A_n = \frac{2}{na\pi} \int_0^\pi 2 \,\text{sen}\, k x \,\text{sen}\, n x \quad \begin{vmatrix} 1 = n = n \\ 0 \; m \neq n \end{vmatrix}$$

$$u(x,t) = \text{sen}\, m x \cos mat + \frac{2}{ka} \text{sen}\, k x \,\text{sen}\, kat$$

Separação de variáveis

Resolva

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{a^2}\dfrac{\partial^2 u}{\partial t^2} \\ u(0,t) = (u,l,t) = 0 \\ u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{cases}$$

$u_{xx} = \frac{1}{a^2} u_{tt}$

$X''T = \frac{1}{a^2} XT''$

$X''T = \frac{XT''}{a^2}$

$\frac{X''}{X} = \frac{T''}{a^2 T} \Rightarrow \frac{X''}{X} = \lambda$

$\begin{cases} X'' - \lambda x = 0 \\ T'' - a^2\lambda T = 0 \end{cases}$ $\quad u(x,t) = X(x).T(t)$

$u(0,t) = X(0).T(t)$, $T(t) \neq 0$

$X(0) = 0$

$\begin{cases} X'' - \lambda x = 0 \\ X(0) = X(l) = 0 \end{cases}$ $\quad u(l,t) = X(l).T(t) \Rightarrow X(l) = 0$, $T(t) \neq 0$

$\left.\begin{matrix} \lambda = 0 \\ \lambda > 0 \end{matrix}\right\} =$ Não serve

$\lambda < 0$

$X = \text{sen}\sqrt{-\lambda}\, x$ $\quad \sqrt{-\lambda}\, l = n\pi$

$\sqrt{-\lambda} = \frac{n\pi}{l}$

$X = \text{sen}\, \frac{n\pi}{l} x$

Resolvendo em T p/ $\lambda < 0$

$T_n = A_n \cos \frac{n\pi}{l} at + B_n \text{sen}\, \frac{n\pi}{l} at$ //

$u(x,t) = \sum \text{sen}\, \frac{n\pi}{l} x \left(A_n \cos\frac{n\pi}{l} at + B_n \text{sen}\, \frac{n\pi}{l} at\right)$ //

$u(x,0) = \sum B_n \text{sen}\, \frac{n\pi x}{l} = f(x)$

$\Rightarrow B_n = \frac{2}{l}\int_0^l f(x)\, \text{sen}\, \frac{n\pi x}{l}\, dx$

$A_n = \frac{2}{n a \pi}\int_0^l g(x)\, \text{sen}\, \frac{n\pi x}{l}$ //

— // —

Resolva

$$\begin{cases} u_{xx} = \frac{1}{a^2} u_{tt} \\ u(x,0) = \text{sen}\, 5x \\ u_t(x,0) = 0 \end{cases}$$

$u(x,t) = \sum \text{sen}\, nx\, (A_n \text{sen}\, nat + B_n \cos nat)$

$u(x,0) =$

$B_n = \frac{2}{\pi}\int_0^{\pi} \text{sen}\, 5x\, \text{sen}\, nx = \begin{cases} 1 & m = n \\ 0 \end{cases}$

$B_1 \text{sen}\, x + B_2 \text{sen}\, 2x + B_3 \text{sen}\, 3x + B_4 \text{sen}\, 4x + B_5 \text{sen}\, 5x$

$B_5 = 0$

$f(x) = \text{sen}\, 5x \cos 5at$ //

Equação de la place

$u_{xx} + u_{yy} = 0$

$$\begin{cases} u(0,y) = u(l,y) = 0 \\ u(x,0) = 0 \\ u(x,M) = f(x) \end{cases}$$

$X''Y + Y''X = 0$

$X''Y = -Y''X$

$\frac{X''}{X} = -\frac{Y''}{Y}$

$\begin{cases} X'' + \lambda x \\ Y'' - \lambda y = 0 \end{cases}$

$X = \text{sen}\, \frac{n\pi x}{l}$

$Y = A_n \cosh \frac{n\pi}{l} y + B_n \text{senh}\, \frac{n\pi}{l} y$

$u(x,y) = \sum \text{sen}\, \frac{n\pi x}{l}\left(A_n \cosh \frac{n\pi}{l} y + B_n \text{senh}\, \frac{n\pi}{l} y\right)$ //

$u(x,0) = \sum \operatorname{sen} \frac{n\pi x}{\ell} \; B_n = f(x)$

$B_n = \frac{2}{\ell} \int_0^\ell f(x) \operatorname{sen} \frac{n\pi}{x} \ell$

$u(x,M) = \sum \operatorname{sen} \frac{n\pi x}{\ell} \left[A_n \operatorname{senh} \frac{n\pi M}{\ell} + B_n \cosh \frac{n\pi M}{\ell} \right] = f(x)$

$K_n = \frac{2}{\ell} \int_0^\ell f(x) \operatorname{sen} \frac{n\pi}{x} \ell$

— x —

$u(0,y) = u(\ell,y) = 0$
$u(x,M) = 0$
$u(x,0) = u(x,M) = u(\ell,y) = 0$
$u(0,y) = f(y)$

15) $u(x,t) = \sum \left(A_n \cos \frac{n\pi x}{\ell} + B_n \operatorname{sen} \frac{n\pi x}{\ell} \right) \operatorname{sen} \frac{n\pi x}{\ell}$

$u(x,0) = \sum B_n \operatorname{sen} \frac{n\pi x}{\ell}$

$B_n = \frac{2}{\ell} \int_0^\ell \frac{2\theta_0}{\ell} \, x \operatorname{sen} \frac{n\pi x}{\ell} =$

$\frac{4\theta}{\ell^2} \left[\frac{\ell^2}{n^2\pi^2} \operatorname{sen} \frac{n\pi x}{\ell} - \frac{x\ell}{n\pi} \cos \frac{n\pi x}{\ell} \right]_0^\ell$

$= \frac{4\theta}{\ell^2} \left[\cancel{\frac{\ell^2}{n^2\pi^2} \operatorname{sen} n\pi} + \frac{\ell^2}{n\pi} \cos n\pi \right] =$

$\frac{4}{n\pi} \cos n\pi \implies B_n = \begin{cases} n \text{ par} = \frac{4}{n\pi} \\ n \text{ impar } \frac{4}{n\pi} \end{cases}$

16) $u(x,0) = \sum B_n \operatorname{sen} \frac{n\pi x}{\ell}$

$B_1 \operatorname{sen} \frac{\pi x}{\ell} + B_2 \operatorname{sen} \frac{2\pi x}{\ell} + B_3 \operatorname{sen} \frac{3\pi x}{\ell} + \dots$

$B_1 = 1$
$B_2 = 2$
$B_n = 0 \quad n \neq 1 \neq 2$

$u(x,t) = \operatorname{sen} \pi x \cos \pi x + \operatorname{sen} 2\pi x \cos 2\pi x$ //

9) $u_x(0,t) = u_x(L,t) = 0$

$u(x,0) = x(L - x^2)$

$u_{xx} = u_t$

$X''T = XT'$

$\frac{X''}{X} = \frac{T'}{T} \Rightarrow \begin{cases} X'' - \lambda x = 0 \\ T' - \lambda T = 0 \end{cases}$

1) $X'' - \lambda x = 0$ $\quad u(x,T) = X(x)\,T(t)$

$X(0) = A$ $\quad u_x(x) = X'(x) \cdot t(t)$

$u_x(0,t) = X'(0) \cdot t(t)$

$X = c_1 \cos\sqrt{-\lambda}\, x + c_2 \operatorname{sen}\sqrt{-\lambda}\, x$

$X'(0) = 0$

$X(0) = A$

$u(x,t) = \sum A_n \operatorname{sen} \frac{n\pi x}{l}\, e^{-\frac{n^2\pi^2}{a^2 l^2} t}$

$u(x,0) = \sum A_n \operatorname{sen} \frac{n\pi x}{l} = K \operatorname{sen} \frac{n\pi x}{l}$

$A_n = K$

$u(x,t) = \sum K \operatorname{sen} \frac{n\pi x}{l}\, e^{-\frac{n^2\pi^2}{a^2 l^2} t}$

$u_x(x,t) = \sum \cos \frac{n\pi x}{l} \frac{n\pi}{l} K e^{-\frac{n^2\pi^2 t}{a^2 l^2}}$

1) $u_{xx} = u_t$

$u(x,0) = x$

$u(0,t) = u(l,t) = 0$

$X''T = XT'$

$\frac{X''}{X} = \frac{T'}{T} \Rightarrow \begin{cases} X'' - \lambda x = 0 \\ T' - \lambda t = 0 \end{cases}$ $\quad u(x,t) = X(x)\,T(t)$

$u(0,t) = X(0) \cdot T(t)$

$X(0) = 0$ $\quad \neq 0$

$X(l) = 0$

$X = c_1 \cos\sqrt{-\lambda}\, x + c_2 \operatorname{sen}\sqrt{-\lambda}\, x$

$X(0) = c_1 = 0$

$X = c_2 \operatorname{sen}\sqrt{-\lambda}\, x \Rightarrow \sqrt{-\lambda}\, l = n\pi \Rightarrow \sqrt{-\lambda} = \frac{n\pi}{l}$

$X = \operatorname{sen} \frac{n\pi x}{l}$

em t

$T_n = A_n e^{-\frac{n^2\pi^2}{l^2}}$

$u(x,t) = \sum A_n \operatorname{sen} \frac{n\pi x}{l}\, e^{-\frac{n^2\pi^2}{l^2} t}$

$u(x,0) = \sum A_n \operatorname{sen} \frac{n\pi x}{l} = x$

$A_n = \frac{2}{l}\int_0^l x \operatorname{sen} \frac{n\pi x}{l} = \frac{2}{l}\left[\cancel{\frac{l^2}{n^2\pi^2} \operatorname{sen} \frac{n\pi x}{l}} - \frac{xl}{n\pi} \cos \frac{n\pi x}{l}\right]_0^l$

$A_n = \frac{2}{l}\left[-\frac{ll}{n\pi} \cos n\pi\right] =$

$-\frac{2}{l}\frac{l^2}{n\pi} \cos n\pi = \frac{2l}{n\pi}(-1)^{n+1}$

$u(x,t) = \sum \frac{2l}{n\pi}(-1)^{n+1} \operatorname{sen} \frac{n\pi x}{l}\, e^{-\frac{n^2\pi^2 t}{l^2}}$ //

3) $u(x,t) = \sum A_n \operatorname{sen} \frac{n\pi x}{l}\, e^{-\frac{n^2\pi^2 t}{l^2}}$

$A_n = \frac{2}{l}\int_0^l f(x) \operatorname{sen} n x = \frac{2}{l}\int_0^{l/2} x \operatorname{sen} n x + \frac{2}{l}\int_{l/2}^{l} (L - x) \operatorname{sen} \frac{n\pi x}{l}$

12) $A_m = \frac{2}{l}\int_0^l \operatorname{sen} m\frac{\pi x}{l} \cdot \operatorname{sen} n\frac{\pi x}{l} \;\Big|\; \begin{matrix} =1 & m=n \\ 0 & m\neq n \end{matrix}$

$B_n = \frac{2}{n a l}\int_0^l l \operatorname{sen} k\frac{\pi x}{l} =$

$\frac{2}{l}\cdot\frac{l}{na}\int_0^l \operatorname{sen} k\frac{\pi x}{l} \cdot \operatorname{sen} n\frac{\pi x}{l} = \Big|\begin{matrix} \frac{l}{na} & n=k \\ 0 & m\neq k \end{matrix}$

$u(x,t) = \operatorname{sen} x \frac{m\pi}{l} \cos mat + \frac{2}{ka} \operatorname{sen} kx \operatorname{sen} kat$,,

14) $u(x,t) = \sum \operatorname{sen} nx \,(A_n \cos nat + B_n \operatorname{sen} nat)$

$u(x,0) = 1 - \cos 2x$

$u_t(x,0) = \operatorname{sen} nx \,(-\frac{A_n}{a} \operatorname{sen} \overset{0}{nat} + \frac{B_n}{a} \cos nat)$

$B_n = 0$

$A_n = \frac{2}{\pi}\int_0^\pi \operatorname{sen} nx \quad - \frac{2}{\pi}\int_0^\pi \overset{0}{\operatorname{sen} nx \cdot \cos 2x}$

$\frac{2}{\pi}\left[\frac{-\cos nx}{n}\right]_0^\pi = \frac{2}{n\pi}\left[-\cos n\pi + \cos 0\right] =$

$= \frac{2}{n\pi}(-1)^{n+1} + 1$

$u(x,t) =$

18) $u(x,t) = \sum \operatorname{sen} \frac{n\pi x}{l}\left(A_n \cos \frac{n\pi}{l} at + B_n \operatorname{sen} \frac{n\pi}{l} at\right)$

$A_m = \frac{2}{l}\int_0^l \frac{x}{l} - \frac{x^2}{l} \operatorname{sen} \frac{n\pi x}{l} =$

$= \frac{2}{l^2}\left[\frac{l^2}{n\pi^2} \cancel{\operatorname{sen} \frac{n\pi x}{l}}^{\,0} - \frac{lx}{n\pi} \cos \frac{n\pi x}{l}\right]_0^l - \frac{2}{l}\int x^2 \operatorname{sen} \frac{n\pi x}{l}$

$\frac{2}{l^2}\left[\frac{l^2}{n\pi} \cos n\pi + \right] -$

$-\frac{2}{l}\left[\frac{-l^3}{n\pi} \cdot \cos n\pi + \frac{2l}{n^2\pi^2} \cos n\pi + \frac{2l}{n\pi} \overset{0}{\operatorname{sen} n\pi} - \frac{2l}{n^2\pi^2} \cos n\pi\right]$

$= + \frac{2l^2}{n\pi} \cos n\pi - \cancel{\frac{4}{n^2\pi^2} \cos n\pi} + \cancel{\frac{4}{n^2\pi^2} \cos n\pi}\Big] =$

$= \left[\frac{2}{n\pi} \cos n\pi + \frac{2l^2}{n\pi} \cos n\pi\right] = \frac{2}{n\pi}\left[\cos n\pi + l^2 \cos n\pi\right]$

$\begin{cases} n \text{ par} = & \frac{2l^2}{n\pi} + \frac{2}{n\pi} \\ \text{impar} & -\frac{2}{n\pi} - l^2 \end{cases}$

7) $f(x) = x$ em senos

f é ímpar

$a_0 = 0$

$a_n = 0$

$$B_n = \frac{2}{l}\int_0^l x \,\text{sen}\, \frac{n\pi x}{l} =$$

$$= \frac{2}{l}\left[\frac{l^2}{n^2\pi^2}\,\overset{\to 0}{\cancel{\text{sen}}}\,\frac{n\pi x}{l} - \frac{lx}{n\pi}\cos\frac{n\pi x}{l}\right]_0^l$$

$$= \frac{2}{l}\left[-\frac{l^2}{n\pi}\cos n\pi\right] + =$$

$$= -\frac{2l}{n\pi}$$

$$f(x) = \frac{2l}{\pi}\sum (-1)^{n+1}\frac{\text{sen}\,\frac{n\pi x}{l}}{n} \;//$$

O problema do retângulo

Já

$u_{xx} + u_{yy} = 0$

$\Omega = \{(x,y) \mid 0 < x < L,\ 0 < y < M\}$

$$\begin{cases} u(0,y) = u(l,y) = 0 \\ u(x,0) = 0 \\ u(x,M) = f(x) \end{cases}$$

$u(x,y) = X(x).Y(y)$

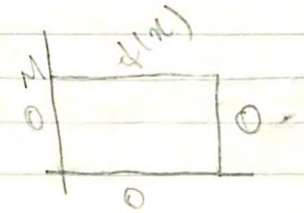
$u(0,y) = X(0).Y(y) \Rightarrow X(0) = 0$

$u(x,0) = X(x).Y(0) \Rightarrow Y(0) = 0$

$u(l,y) = X(l).Y(y) \Rightarrow X(l) = 0$

$\frac{X''}{X} = -\frac{Y''}{Y}$

$$\begin{cases} X'' + \alpha x = 0 \\ Y'' - \alpha y = 0 \end{cases}$$



$$\begin{cases} X'' + \alpha x = 0 \\ X(0) = 0 \\ X(l) = 0 \end{cases}$$

$\alpha = 0$ $X = 0$ } ñ serve

$\alpha < 0$

$\alpha > 0$

$X = c_1 \cos\sqrt{-\alpha}\,x + c_2\,\text{sen}\sqrt{-\alpha}\,y$

$X(0) = c_1 = 0$

$X = c_2\,\text{sen}\sqrt{-\alpha}\,y$

$X(l) = \text{sen}\sqrt{-\alpha}\,l = 0$

$\sqrt{-\alpha}\,l = n\pi \Rightarrow \sqrt{-\alpha} = n\pi/l$

$X = \text{sen}\,\frac{n\pi}{l}x$

$Y'' - \frac{n^2\pi^2}{l^2}Y = 0$

$Y = B_n\left(e^{-\frac{n\pi}{l}} - e^{+\frac{n\pi}{l}}\right) =$

$Y = A_n \cosh\frac{n\pi}{l} + B_n\,\text{senh}\,\frac{n\pi}{l}$

$Y(0) = A_n = 0$

$Y = B_n\,\text{senh}\,\frac{n\pi}{l}y$

$u(x,t) = \sum \operatorname{sen} \frac{\pi n}{l} x \; B_n \operatorname{senh} \frac{n\pi y}{l}$

—x—

ya

$u(x,M) = \sum \operatorname{sen} \frac{n\pi x}{l} \; B_n \operatorname{sen} \frac{\pi n M}{l}$

$B_n \operatorname{sen} \frac{\pi n M}{l} = \frac{2}{l} \int_0^l f(x) \operatorname{sen} \frac{n\pi x}{l}$

$u(x,y) = \frac{2}{l} \sum \frac{\int_0^l f(s) \operatorname{sen} \frac{n\pi s}{l}}{\quad} \operatorname{sen} \frac{n\pi x}{l} \operatorname{senh} \frac{n\pi y}{l}$

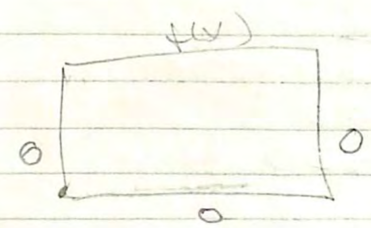
—x—

$u_{xx} + u_{yy} = 0$

$u(x,0) = 0$

$u(x,M) = \frac{3}{5} \operatorname{sen} \frac{2\pi}{l} x$

$u(0,y) = u(l,y) = 0$



$\frac{X''}{X} = -\frac{Y''}{Y}$

$X'' + \lambda x = 0$ $\qquad u: X(x)Y(0)$

$Y'' - \lambda Y = 0$ $\qquad X$

$Y(0) = 0$

$X(0) = 0$

$X(l) = 0$

$\lambda > 0$

$X = c_1 \cos \frac{n\pi x}{l} + c_2 \operatorname{sen} \frac{n\pi x}{l}$

$X = \operatorname{sen} \frac{n\pi x}{l}$

$Y'' - \lambda Y = 0$

$\lambda = 0$

$\lambda > 0$

$Y = A_n \cosh \frac{n\pi}{l} y + B_n \operatorname{senh} \frac{n\pi y}{l}$

$Y(0) = A_n = 0$

$Y = B_n \operatorname{senh} \frac{n\pi y}{l}$

mar

Eu sei que vou te amar

Eu sei que vou te amar
Por toda a minha vida
Eu vou te amar
a cada despidida
Eu vou te amar
Desesperadamente eu sei que vou te amar
Em cada verso meu
será pra te dizer
Que eu sei que vou te amar
Por toda a minha vida
Eu sei que vou chorar
Em cada ausencia tua, eu vou chorar
mas cada volta tua, há de apagar
O que essa tua ausência me causou
Eu sei que vou sofrer
a eterna desventura de viver
a espera de viver ao lado teu
Por toda a minha vida.

Denise
22/11/79

De
Denise
Denise Denise Denise
Denise Denise
Denise Denise Denise Denise
Denise
Denise Denise Denise
Denise

$u(x,t) = \sum A_n \operatorname{sen} \frac{n\pi x}{\ell} \operatorname{senh} \frac{n\pi y}{\ell}$ já

$y = M$

$u(x,M) = \sum A_n \operatorname{sen} \frac{n\pi x}{\ell} \operatorname{senh} \frac{n\pi M}{\ell}$

$A_n = \frac{2}{\ell} \int_0^{\ell} \frac{3}{5} \operatorname{sen} \frac{2\pi}{\ell} x \operatorname{sen} \frac{n\pi x}{\ell}$

$A_n \operatorname{senh} \frac{n\pi M}{\ell} = \frac{6}{5\ell} \int_0^{\ell} \operatorname{sen} \frac{2\pi}{\ell} \operatorname{sen} \frac{n\pi x}{\ell} = \begin{cases} \frac{6}{5\ell} & \text{se } n = 2 \\ 0 & n \neq m \end{cases}$

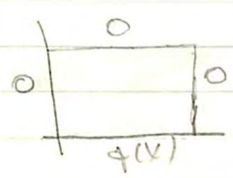
$A_n \operatorname{senh} \frac{n\pi M}{\ell} = \frac{6}{5\ell}$

$A_n = \frac{6}{5\ell \operatorname{senh} n\pi M/\ell}$

— x —

$u(0,y) = u(\ell,y) = 0$

$u(x,M) = 0, \quad u(x,0) = \phi(x)$



$X(0) = 0$

$X(\ell) = 0$

$Y(M) = 0$

$\frac{X''}{X} = -\frac{Y''}{Y} = \lambda \qquad -Y'' = \lambda Y \implies -Y'' - \lambda Y = 0 \quad Y'' + \lambda Y = 0$

$\frac{X''}{X} = -\lambda$

$\begin{cases} X'' + \lambda x = 0 \\ Y'' + \lambda Y = 0 \end{cases}$

$\begin{cases} X'' + \lambda x = 0 \\ X(0) = 0 \\ X(\ell) = 0 \end{cases}$

$\lambda > 0$

$X = c_1 \cos\sqrt{-\lambda}\, x + c_2 \operatorname{sen}\sqrt{-\lambda}\, x$

$X \quad \operatorname{sen} \frac{\pi n x}{\ell}$

$y'' - \lambda x = 0$
$y(0) = 0$
y'

$y_n = A_n \cosh \frac{n\pi y}{l} + B_n \operatorname{senh} \frac{n\pi y}{l}$

$y(M) =$

$u(x,y) = \sum \operatorname{sen} \frac{n\pi y}{M} \; A_n \operatorname{senh} \frac{n\pi}{M} x$

$u(l,y) = \sum \operatorname{sen} \frac{n\pi y}{M} \; A_n \operatorname{senh} \frac{n\pi}{M} l$

$A_n = \frac{2}{M} \operatorname{senh} \frac{n\pi l}{M} = \int_0^l f(y) \operatorname{sen} \frac{n\pi x}{l}$

$u(x,y) = \sum A_n \operatorname{sen} \frac{n\pi x}{l} \operatorname{senh} n\pi \frac{(M-y)}{l}$

—/—

$A_n = \frac{2}{l \operatorname{senh} \frac{n\pi M}{l}} \int_0^l f(x) \operatorname{sen}$

—/—

$\begin{cases} u(0,y) = u(x,0) = u(x,M) = 0 \\ u(l,y) = f(y) \end{cases}$

$u(x,y) = X(x)\,Y(y)$
$u(0,y) = X(0)\,Y(y) \Rightarrow X(0) = 0$
$u(x,0) = X(x)\,Y(0) \Rightarrow Y(0) = 0$
$u(x,M)$ $\quad Y(M) = 0$

$X''Y + XY'' = 0$
$X''Y = -XY''$
$\frac{X''}{X} = -\frac{Y''}{Y}$

$\begin{cases} X'' + \lambda X = 0 \\ Y'' \cancel{+} \lambda Y = 0 \end{cases}$

$X'' + \lambda X = 0$
$\lambda < 0$
$X = c_1 \cos \sqrt{-\lambda}\, x + c_2 \operatorname{sen} \sqrt{-\lambda}\, x$
$X(0) = c_1 = 0$
$X = \operatorname{sen} \sqrt{-\lambda}\, x$

$X(M) = \operatorname{sen} \sqrt{\lambda}\, M = 0$

$\lambda = \frac{n^2\pi^2}{l^2}$

$X = \operatorname{sen} \frac{n\pi}{l} x$

$\lambda \geq 0$
$\{ \; y'' + \lambda x = 0$

$y = A_n \cosh \frac{n\pi}{l} y + B_n \operatorname{senh} \frac{n\pi}{l} y$

$y(0) = B_n = 0$

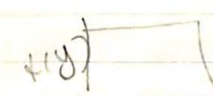
$y = A_n \operatorname{senh} \frac{n\pi y}{l}$

$u(x,y) = \sum \operatorname{sen} \frac{n\pi x}{l} \; A_n \operatorname{senh} \frac{n\pi y}{l}$

$u(l,y) = \sum \operatorname{sen} \frac{n\pi l}{M} \; A_n \operatorname{senh} \frac{n\pi y}{M}$

$A_n \operatorname{senh} \frac{n\pi l}{M} = \frac{2}{l} \int_0^M f(y) \operatorname{sen} \frac{n\pi}{M} y \, dy$ //

—/—



$u(0,y) = f(y)$ $\quad Y(M) = 0$
$u(x,M) = 0 = u(x,0)$ $\quad Y(0) = 0$
$u(l,M) = 0$ $\quad X(l) = 0$

$\begin{cases} X'' + \lambda X = 0 \\ Y'' - \lambda Y = 0 \end{cases} \quad \lambda > 0$

$X = c_1 \cos \sqrt{-\lambda}\, x + c_2 \operatorname{sen} \sqrt{\lambda}\, x$
$X(0) = c_1 = 0$
$X = \operatorname{sen} \sqrt{\lambda}\, x \Rightarrow X(l) = \operatorname{sen} \sqrt{-\lambda}\, l = 0$
$\sqrt{-\lambda}\, l = n\pi$
$\sqrt{-\lambda} = \frac{n\pi}{l}$

$X = \operatorname{sen} \frac{n\pi}{M} x$

$y = A_n \cosh \frac{n\pi y}{M} + B_n \cosh \frac{n\pi y}{M}$

$A = -B\cosh \frac{n\pi l}{M} / \operatorname{senh} \frac{n\pi x}{M}$

$u(x) = -B\cosh \frac{n\pi l}{M} \operatorname{sen} \frac{n\pi x}{M} + B\cosh \frac{n\pi x}{M} \operatorname{senh} \frac{n\pi l}{M}$

$\operatorname{senh} \left(\frac{n\pi}{M}\right)(l-x)$

$u(x,y) = \sum \operatorname{sen} \frac{n\pi}{M} y \, K_n \operatorname{senh} \frac{n\pi}{M}(l-x)$

$K_n \operatorname{senh}\left(\frac{n\pi}{M}\right) l = \frac{2}{M} \int_0^M f(y) \operatorname{sen} \frac{n\pi}{M} y$

— x —

Resumo

$f(x)$

$u(x,y) = \sum A_n \operatorname{sen} \frac{n\pi x}{l} \operatorname{senh} \frac{n\pi y}{l}$

$A_n = \frac{2}{l \operatorname{senh} \frac{n\pi M}{l}} \int_0^l f(x) \operatorname{sen} \frac{n\pi x}{l}$

$f(x)$

$u(x,y) = \sum A_n \operatorname{sen} \frac{n\pi x}{l} \operatorname{senh} n\pi \frac{(M-y)}{l}$

$A_n = \frac{2}{l \operatorname{senh} \frac{n\pi M}{l}} \int_0^l f(x) \operatorname{sen} \frac{n\pi x}{l}$

$f(x)$

$u(x,t) = \sum A_n \operatorname{sen} \frac{n\pi y}{M} \operatorname{senh} \frac{n\pi x}{M}$

$A_n = \frac{2}{M \operatorname{senh} \frac{n\pi}{M} l} \int_0^M f(y) \operatorname{sen} \frac{n\pi}{M} y \, dy$

$f(x)$

$u(x,t) = \sum A_n \operatorname{sen} \frac{n\pi}{M} y / \operatorname{senh} \frac{n\pi}{M}(l-x)$

$A_n = \frac{2}{M \operatorname{senh} \frac{n\pi L}{M}} \int_0^M f(y) \operatorname{sen} \frac{n\pi y}{M} dy$

8) $\begin{cases} u(0,y)=0 \quad u(l,y)=0 \\ u(x,0)=100 \end{cases}$

$\frac{X''}{X} = -\frac{Y''}{Y}$

$\begin{cases} X'' + \lambda x = 0 \\ Y'' - \lambda y = 0 \end{cases}$

$\lambda > 0$

$X = c_1 \cos \sqrt{\lambda} x + c_2 \operatorname{sen} \sqrt{\lambda} x$

$X = \operatorname{sen} \frac{n\pi x}{l}$

$Y = A_n \cosh \frac{n\pi}{l} y + B_n \operatorname{senh} \frac{n\pi y}{l}$

$u(x,y) = \sum \operatorname{sen} \frac{n\pi x}{l} \left(A_n \cosh \frac{n\pi y}{l} + B_n \operatorname{senh} \frac{n\pi y}{l}\right)$

$u(x,0) = \sum \operatorname{sen} \frac{n\pi x}{l} A_n = 100$

$b_n = \frac{2}{l} \int_0^l 100 \operatorname{sen} \frac{n\pi x}{l} dx$

$-\frac{2}{l} \cdot 100 \left[\frac{l}{n\pi} \cos \frac{n\pi x}{l}\right]_0^l = \frac{400}{n\pi}$

$u(x,M) = \sum \operatorname{sen} \frac{n\pi x}{l} \left(A_n \operatorname{senh} \frac{n\pi}{l} M + B_n \cosh \frac{n\pi M}{l}\right) = 0$

$A_n \operatorname{senh} \frac{n\pi M}{l} = 0 = -B_n \cosh \frac{n\pi M}{l}$

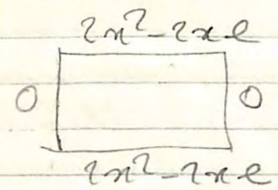
$A_n = 0 - B_n \Rightarrow A_{2n+1} = B_{2n+1}$

$u(x,y) = \sum l_n \operatorname{sen} \frac{n\pi}{l} x \left(-\operatorname{senh} \frac{(2n+1)\pi y}{l}\right.$

$\left. + \cosh \frac{(2n+1)\pi y}{l}\right)$

ya

① $u(0,y) = u(l,y) = 0$
$u(x,M) = u(x,0) = 2x(x-l)$



$u(x,t) = \sum \operatorname{sen} \frac{n\pi x}{l} \left(B_n \operatorname{senh} \frac{n\pi y}{l}\right)$

$+ \sum \operatorname{sen} \frac{n\pi x}{l} \left(B_n \operatorname{senh} \frac{n\pi (M-y)}{l}\right.$

$u(x,t) = \sum A_n \operatorname{sen} \frac{n\pi x}{l} \left(\operatorname{senh} \frac{n\pi y}{l} + \operatorname{sen} \frac{n\pi (M-y)}{l}\right)$

$u(x,0) = \sum A_n \operatorname{sen} \frac{n\pi x}{l}$

$A\, u(x,M) = \sum A_n \operatorname{sen} \frac{n\pi x}{l} \left(\operatorname{senh} \frac{n\pi M}{l} + \operatorname{sen} \frac{n\pi (0)}{l}\right)$

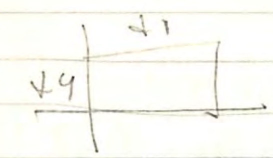
$A_n \operatorname{senh} \frac{n\pi M}{l} = \frac{2}{l} \int_0^l 2x^2 - 2xl \operatorname{sen} \frac{n\pi x}{l}$

$\longrightarrow$

$A_n = \frac{l^2}{[(2n+1)\pi]^3}$

② $u(L,y) = u(x,0) = 0$

$u(x,M) = \begin{cases} x & 0 < x \leq L/2 \\ L-x & L/2 \leq x < L \end{cases}$

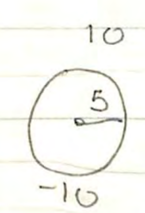


$u(x,y) = \sum \operatorname{sen} \frac{n\pi x}{l} \operatorname{senh} \frac{n\pi y}{l} + \operatorname{sen} \frac{n\pi x}{M} \operatorname{senh} \frac{n\pi (L-x)}{M}$

$A_n = \frac{2}{L \operatorname{senh} \frac{n\pi}{L} M} \left[\int_0^{l/2} x \operatorname{sen} \frac{n\pi x}{l} + \int_{l/2}^{l} (L-x) \operatorname{sen} \frac{n\pi x}{l}\, dx\right] =$

$B_n = \frac{2}{M \operatorname{sen} \frac{n\pi}{M}} \left[\int_0^{M/2} \operatorname{sen} \frac{n\pi y}{M} + \int_{M/2}^{M} (M-y) \operatorname{sen} \frac{n\pi y}{M}\right.$

$\longrightarrow$



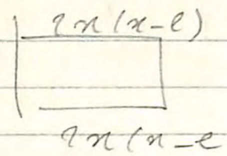
$u(r,\theta) = \frac{a_0}{2} + \sum r^n (a_n \cos n\theta + B_n \operatorname{sen} n\theta)$

$u(r,\theta) = \frac{a_0}{2} + \sum R^n a_n \cos n\theta + R^n b_n \operatorname{sen} n\theta$

$R^n a_n = \frac{1}{l} \int_{\pi}^{\pi}$

$$\begin{cases} u(0,y) = u(2,y) = 0 \\ u(x,M) = u(x,0) = 2x(x-2) \end{cases}$$

x^{-1}



$$u(x,y) = \sum \operatorname{sen} \frac{n\pi x}{l}\left(A_n \cosh \frac{n\pi y}{l} + B_n \cosh \frac{n\pi y}{l}\right)$$

$$u(x,0) = \sum A_n \operatorname{sen} \frac{n\pi x}{l} = 2x(x-l$$

$$B_n = \frac{2}{l}\int_0^l 2x^2 - 2xl \operatorname{sen} \frac{n\pi x}{l}\, dx$$

$$B_n = \frac{-8l^2}{(2n+1)^3\pi^3}$$

$$u(x,M) = \sum \operatorname{sen} \frac{n\pi x}{l}\left(B_n \cosh \frac{n\pi M}{l} + \overset{K_n}{A_n} \operatorname{sen} \frac{n\pi M}{l}\right)$$

$$A_n = \frac{2}{l}\int_0^l 2x(x-l) \operatorname{sen} \frac{n\pi x}{l}$$

$$K_n = \frac{-8l^2}{(2n+1)^3\pi^3}$$

$$A_n \operatorname{senh} \frac{n\pi M}{l} + \frac{-8l^2}{(2n+1)^3\pi^3} \cosh \frac{n\pi M}{l} = \frac{-8l^2}{(2n+1)^3\pi^3}$$

$$a_n = \frac{\frac{8l^2}{(2n+1)^3\pi^3}\left[\cosh \frac{n\pi}{l} - 1\right]}{\operatorname{sen} h \frac{n\pi M}{l}} = e_n\left(\cosh \frac{n\pi}{l} M - 1\right)$$

① $\displaystyle\sum_1^\infty \frac{n+1}{2n+7}$

$$\lim_{n\to\infty} \frac{n+1}{2n+7} = \frac{1+\frac{1}{n}}{2+\frac{7}{n}} = \frac{1}{2} \neq 0 \text{ diverge}$$

② $\displaystyle\sum \operatorname{sen} \frac{n\pi}{2} = 1, 0, -1, 0, -1 \ldots$ diverge

não tem limite

③ $\displaystyle\sum_{n=1}^\infty \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \ldots$

$|a_0| > |a_1| > |a_2| > \ldots$

$\displaystyle\lim_{n\to 0} \frac{(-1)^{n+1}}{n}$ tende a zero $\Rightarrow$ converge

④ $\displaystyle\sum_1^\infty \frac{1}{n}$

$$\int_1^\infty \frac{1}{x}\,dx = \log x\Big|_1^\infty = \lim_{A\to\infty} \log A = \infty \Rightarrow$$

diverge

⑤ $\dfrac{1}{n \log n} \Rightarrow$ diverge

⑥ $\dfrac{1}{n^3+n} < \dfrac{1}{n^3}$ como $\dfrac{1}{n^3}$ converge $\Rightarrow$

$\displaystyle\sum \frac{1}{n^3+n}$ tb converge

7) $\frac{1}{\log n}$ como $\frac{1}{n} < \frac{1}{\log n}$ e $\frac{1}{n}$ diverge

$\Rightarrow \frac{1}{\log n}$ diverge

8) $\frac{1}{n+5}$ diverge

100

3/5

$u(r,\theta) = \frac{a_0}{2} + \sum r^n (a_n \cos n\theta + B_n \operatorname{sen} n\theta)$

$u(R,\theta) = \frac{a_0}{2} + \sum R^n a_n \cos n\theta + R^n b_n \operatorname{sen} n\theta) = f(\theta)$

$R^n a_n = \frac{1}{\pi} \int_{\pi}^{\pi} f(\theta) \cos n\theta \, d\theta = \frac{1}{\pi} \left[\int_{-\pi}^{0} 10 \operatorname{sen} n\theta \, d\theta \right.$

$+ \int_0^{\pi} 10 \operatorname{sen} n\theta \, d\theta$

21) $u(x,y) = \sum B_n \operatorname{sen} \frac{n\pi x}{l} \operatorname{senh} \frac{n\pi y}{l}$

$B_n = \frac{2}{l \operatorname{senh} \frac{n\pi M}{l}} \int_0^l \operatorname{sen} \frac{n\pi x}{l} \, dx$

$= \frac{\frac{2l}{n\pi} \left[\cos \frac{n\pi x}{l}\right]_c^0}{l \operatorname{senh} \frac{n\pi M}{l}} = \frac{2}{n\pi} \operatorname{senh} \frac{n\pi M}{l} (1 - \cos n\pi)$

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10)

$r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

nom $\frac{\partial^2 u}{\partial \theta^2} = 0$

$r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} = 0$

$r z' + z = 0$

27) $a_n R^n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\cos n\theta \, d\theta$

$b_n R^n = \frac{1}{\pi}\left[f(\theta)\, \text{sen}\, n\theta \, d\theta \right]_{\pi}^{\pi}$

$a_n R^n = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0 \cos n\theta + \int_{0}^{\pi} 1 \cos n\theta \right]$

$b_n R^n = \frac{1}{\pi}\int_{-\pi}^{0} 0\, \text{sen}\, n\theta + \int \text{sen}\, n\theta$

$= \frac{1}{n\pi}\left[-\cos n\pi + 1\right]$

$b_{2n+1} R^{2n+1} = \frac{2}{(2n+1)\pi}$

— x —

$u(L,y) = u(x,0) = 0$

$u(x,M) = \begin{cases} x \\ L-x \end{cases}$

$u(x,y) = \sum A_n \, \text{sen} \frac{n\pi x}{L} \, \text{senh} \frac{n\pi}{L} y + B_n \, \text{sen} \frac{n\pi y}{M}$

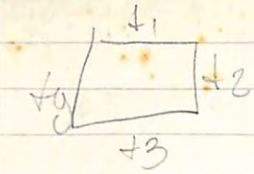
$\text{senh} \frac{n\pi}{M}(L-x)$

$A_n = \frac{2}{L\, \text{senh} \frac{n\pi}{L} M}\left[\int_0^{L/2} x\, \text{sen} \frac{n\pi}{L} x\, dx \right.$

$\left. + \int_{L/2}^{L} (L-x)\, \text{sen} \frac{n\pi}{L} x\, dx\right]$

$B_n = \frac{2}{M\, \text{senh} \frac{n\pi}{M} L}\left[\int_0^{M/2} y\, \text{sen} \frac{n\pi}{M} y\, dy + \int_{M/2}^{M} (M-y)\right.$

$\left. \text{sen} \frac{n\pi}{M} y\, dy\right]$



$f_1 + f_4 = \sum \text{sen} \frac{n\pi x}{L} \, \text{senh} \frac{n\pi}{L} y$

Encontre a solução de Laplace no retângulo $0<x<L$ $0<y<M$

$u(0,y) = u(L,y) = 0$
$u(x,M) = u(x,0) = 2x(x-L)$

$u_{xx} = -u_{yy}$

$X'' Y = -Y'' X$

$\frac{X''}{X} = -\frac{Y''}{Y}$ $\quad \left.\begin{matrix} \lambda = 0 \\ \lambda < 0 \end{matrix}\right\}$ não serve

$\lambda > 0$

$\begin{cases} X'' + \lambda X = 0 \\ Y'' - \lambda Y = 0 \end{cases}$ $\quad X(0) = 0$ $\quad X(L) = 0$

$X = c_1 \cos\sqrt{\lambda}\, x + c_2\, \text{sen} \sqrt{\lambda}\, x$
$X(0) = c_1 = 0$
$X = c_2\, \text{sen} \sqrt{\lambda}\, x$
$X(L) = \text{sen} \sqrt{\lambda}\, L$ $\quad \sqrt{\lambda} L = n\pi$ $\quad \sqrt{\lambda} = \frac{n\pi}{L}$

$\boxed{X = \text{sen} \frac{n\pi x}{L}}$

$Y'' - \lambda Y = 0$

$Y = A_n\, \text{senh} \frac{n\pi y}{L} + B_n \cosh \frac{n\pi y}{L}$ //

$u(x,y) = \sum \text{sen} \frac{n\pi x}{L}\left(A_n \cosh \frac{n\pi}{L} y + B_n\, \text{senh} \frac{n\pi y}{L}\right)$

$u(x,0) = \sum A_n\, \text{sen} \frac{n\pi x}{L}$

0,08794

0,09217

$-4 \quad 3$

Denise

$-1 \quad 2$

$$\rightarrow \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} 1 & \frac{3}{4} & \frac{1}{4} & 0 \\ 1 & 2 & 0 & 1 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & \frac{5}{4} & -\frac{1}{4} & 1 \end{vmatrix}$$

$\frac{4}{5} \cdot \frac{1}{4}$

$8 - 3 = \frac{5}{4}$

$\frac{8}{4} - \frac{3}{4}$

$0{,}03 \left(\frac{3}{100}\right)$ $\frac{81}{10^7}$

$100 \times 100 \times 100 \times 100$ $\frac{1}{10^7}$

$$-\frac{3}{4} \times \begin{vmatrix} 1 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 1 & -\frac{1}{5} & \frac{4}{5} \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$= \frac{2}{20} - \frac{1}{4}$

$\ell+1$

An

$180 \cdot 2 \cdot 30$

$180 \cdot 50 \cdot 2$

$3740{,}00$

$$Y(y) \; \operatorname{sen} \frac{n\pi y}{M} \operatorname{senh} n\pi \frac{\ell}{0} (L - x)$$

$$Y(x) = \sum A_n \operatorname{sen} \frac{n\pi x}{\ell} \operatorname{senh} \frac{n\pi}{\ell} (M - y)$$

$$Y(2) = \sum A_n \operatorname{sen} \frac{n\pi x}{M} \operatorname{senh} \frac{n\pi x}{M}$$

3800 / 250

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$$\psi = \sum \frac{n\pi x}{\ell} A_n \operatorname{senh} \frac{n\pi y}{\ell}$$

$$A_n = \frac{2}{\ell} \int_0^\ell 2x(x-\ell) \operatorname{sen} \frac{n\pi}{\ell} = \frac{-8L^2}{(2n+1)^3\pi^3}$$

$$u(x,y) = \sum \operatorname{sen} \frac{n\pi x}{\ell} \left[\underbrace{a_n \operatorname{senh} \frac{n\pi y}{\ell}}_{K_n} + B_n \cosh \frac{n\pi y}{\ell} \right]$$

$$K_n = \frac{2}{\ell} \int_0^\ell 2x(x-\ell) \operatorname{sen} \frac{n\pi x}{\ell} = \frac{-8L^2}{(2n+1)^3\pi^3}$$

Denise

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