Alexandre Rezende Ferreira

On the Performance of Portfolio Selection Under Increasing Transaction Costs: An analysis based on S&P100 stocks

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Alexandre Rezende Ferreira

On the Performance of Portfolio Selection Under Increasing Transaction Costs:
An analysis based on S&P100 stocks

Esta dissertação foi julgada adequada para obtenção do Título de "Mestre em Economia", e aprovada em sua forma final pelo Programa de Pós-graduação em Economia da Universidade Federal de Santa Catarina.

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"Once you've got a task to do, it's better to do it than live with the fear of it."

(Joe Abercrombie, The Blade Itself)

Sobre a Performance da Seleção de Portfólio Sob Custos de Transação Crescentes:
Uma análise baseada no S&P100

Fevereiro 2016

Abstract:

Two crucial aspects to the problem of investment portfolio selection are the specification of the model for expected returns and its covariances, as well as the choice of the investment policy to be adopted. This dissertation empirically shows that these two aspects are intrinsically attached to the impact due to transaction costs. In order to do that, we implemented 11 different models of covariances to generate a set of 17 portfolio selection policies in a sample composed by the 50 most traded stocks of the S&P100 index from 01/2004 to 01/2014. The performance of those portfolios was evaluated based on different methods and considering the impact of alternative levels of proportional transaction costs. The results indicated that GARCH-type conditional covariances show superior results when compared to the ones obtained with static models only when the level of transaction cost is lower than 10 basis points. Besides, portfolio policies that ignore the covariance structure such as the ones proposed in Kirby & Ostdiek (2012) are more robust specially in scenarios with higher transaction costs. When instead we select the best performing policy each period through a dynamic model selection, we manage to increase the risk adjusted returns to transaction costs as high as 30 basis points.

Keywords: Portfolio optimization, Mean-variance, Volatility timing, Shrinking matrix, Exponentially weighted moving average, Optimal Rolling Estimator, Scalar VECH, Orthogonal GARCH, Conditional correlation, Dynamic model selection, Dynamic Model Average.

Sobre a Performance da Seleção de Portfólio Sob Custos de Transação Crescentes: Uma análise baseada no S&P100

Fevereiro 2016

Resumo:

Dois aspectos cruciais do problema de seleção de portfólio para investimento são a especificação do modelo para os retornos esperados e suas covariâncias, assim como a escolha da política de investimento a ser adotada. Esta dissertação empiricamente mostra que esses dois aspectos estão intrínsicamente associados ao impacto dos custos de transação. Para tanto, nós implementamos 11 modelos diferentes de covariancias para gerar um conjunto de 17 políticas de seleção de portfólio em uma amostra composta pelas 50 ações mais negociadas do índice S&P100 entre 01/2004 e 01/2014. A performance destes portfólios foi avaliada com base em diferentes métodos e considerando o impacto de níveis alternativos de custos de transação proporcionais. Os resultados indicaram que modelos do tipo GARCH de covariâncias condicionais exibiram resultados superiores quando comparados com os obtidos com modelos estáticos apenas quando o nível do custo de transação era inferior a 10 pontos base. Além disso, politicas de seleção de portfólio que ignoram a estrutura das covariâncias como as propostas por Kirby & Ostdiek (2012) são mais robustas especialmente em cenários com custos de transação mais altos. Quando selecionamos a política com melhor performance a cada período através de uma stratégia de seleção dinâmica de modelos, nós conseguimos aumentar os retornos ajustados ao risco para custos de transação tão altos quanto 30 pontos base.

Palavras-chave: Otimização de portfólios, Média-variancia, Volatility timing, Matriz de Encolhimento, Média Móvel Exponencialmente Ponderada, Optimal Rolling Estimator, VECH Escalar, GARCH Ortogonal, Correlação Condicional, Seleção dinâmica de Modelos, Ponderação Dinâmica de Modelos.

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A_i	Asset i
R_i	Return of an asset i
$\mathbf{R}$	Asset return vector
μ_i	Expected return of asset i
$\boldsymbol{\mu}$	Expected return vector
σ_i	R_i standard deviation
ρ_{ij}	Correlation between the returns of assets A_i and A_j $(i \neq j)$.
Σ	Covariance matrix of asset returns
$\mathbf{w}$	Vector of portfolio asset weights
w_i	Portfolio share of total wealth invested in asset A_i
R_p	Portfolio return
μ_p	Expected portfolio return
σ_p	Portfolio return standard deviation
γ	Investor's level of relative risk aversion
β	Asset market factor

κ	Porportional transaction cost

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Chapter 1

Introduction

The stock returns covariance matrix modeling is a key ingredient to the portfolio selection problem. Markowitz (1952) created the basis for the modern portfolio theory when he showed the way which the variances and covariances influence portfolio risk and risk-adjusted portfolio return. Since long ago this aspect drove researchers and market players to enhance the stock return covariance modeling, over the expected return modeling. In fact, empirical evidence suggests that portfolio selection policies that use only the covariance modeling generate superior risk adjusted returns when compared to selection policies that depend on the expected returns; see, for example, Jagannathan & Ma (2003) and DeMiguel et al. (2009).

The literature points to an ample range of possible ways of modeling covariances. The most immediate choice is usually between static models (unconditional) or dynamic models (conditional) as multivariate GARCH models or stochastic volatility (BAUWENS; LAURENT; ROMBOUTS, 2006; SILVENNOINEN; TERÄSVIRTA, 2009). To ground the decision, usually are taken into account pros and cons of each approach as, for example, ease of implementation, processing cost, and the model ability to capture stylized facts such as time variation in the covariance profile among the assets.

The static models are those in which the covariance estimation depends on the sample covariance or on a factor model, without imposing any auto-regressive specification. The most important static models are the ones that shrink the sample covariance matrix to the targets proposed in Ledoit & Wolf (2003a), Ledoit & Wolf (2004) e Ledoit & Wolf (2003b). The dynamic models, in turn, base themselves in the hypothesis that the current covariance depends on the covariance of previous periods, being updated every time according to different econometric specifications based on auto-regressive structures. Among the most important dynamic models are the multivariate GARCH models and the stochastic volatility models cited in Bauwens, Laurent & Rombouts (2006) and in Silvennoinen & Teräsvirta (2009), respectively.[1]

A common point between all these models is the econometric specification differentiation. The dynamic models in particular, posses the biggest implementation requirements when compared to the static models due to their greater parameterization. This greater parameterization can also increase the estimation error and negatively impact the portfolio performance, due to demanding a bigger rebalancing, and, consequently, increased transaction costs. On the other hand, these models can deal with the heteroskedastic behavior of financial series. Hence, the literature regarding the subject still has not found consensus regarding which is the most appropriate form to model covariances.

When we apply these covariances to the problem of portfolio optimization, they present us with a trade off that they provide better risk-adjusted returns, at the cost of instability of the portfolio weights. This instability, in turn, means that the portfolio

1 Bauwens, Laurent & Rombouts (2006) also mentions factor models that use the idea that co-movements of the stock returns are driven by a small number of common underlying variables, which are called factors. As their emphasis is on 'data-driven' models, they refrain from discussing the vast literature on these models. We follow suit and do not include them in our analysis.

requires increased rebalancing that, in presence of proportional transaction costs, will reduce the risk-adjusted net returns obtained. We then ask ourselves: are there better risk-adjusted returns high enough to compensate for the transaction costs this rebalancing incur?

It is worth mentioning that, despite the fact that the estimation of expected returns plays its part in portfolio selection, there is already evidence that this kind of modeling creates policies with high turnovers that might compromise their usage in real situations; see, for example, DeMiguel et al. (2009) and DeMiguel, Nogales & Uppal (2014). So the present work focus on the modeling of covariances using the sample average for expected returns.

The main goal of this work is to analyze in which way the different covariance specifications, used in different strategies of portfolio selection, are impacted by the presence of transaction costs. Hence, we try to understand how the increase in costs decrease the strategies' performance measured as the risk-adjusted return net of transaction costs. This way, the work contributes with this discussion by analyzing the impacts that transaction costs have on the covariance model selection, as well as on the choice of portfolio selection policy. The transaction costs reduce the gross return and this reduction is increasing with the turnover. In this way, a high return, in presence of transaction costs, may be eliminated when faced with a high turnover.

We move one step further by considering a portfolio policy that dynamically selects out of the available portfolios those with the better performance, based on the portfolio return, net of transaction costs. We do this by utilizing the strategy proposed by Raftery, Kárnỳ & Ettler (2010) for model selection when there is uncertainty about the specification. In their paper, at every period of time, they choose the model with the higher probability of being the correct specification, measured by the predictive density of the model. We adapt the strategy as in Koop & Korobilis (2013) and we use the past net returns of a portfolio as its probability of future returns. We use this probability to choose which portfolio selection model to apply each period. With this technique we manage to sucessfully beat the performance of individual policies measured as the risk-adjusted return net of transaction costs.

This work aims to answer a series of fundamental questions: what are the transaction costs that leads different covariance specifications to portfolios with zero average return? How do the better returns obtained with better covariance estimation are negated by increased turnover in the presence of transaction costs? Can we choose winners in these covariances in different periods of time to revert this trade-off?

To answer these questions, we implemented 11 different covariance models to create a set of portfolios from different asset selection strategies in a sample composed by the 50 most traded stocks from the S&P100 index. Out of the 11 covariance models, 3 were the most important static models proposed by Ledoit & Wolf (2004), Ledoit & Wolf (2003a) and Ledoit & Wolf (2003b). In short, these models shrink the sample covariance matrix towards a target covariance. In this case, the target matrices are the identity matrix, constant correlation matrix and market factors matrix, respectively.

The other 8 models belong to the class of dynamic models and were some of the ones listed in Bauwens, Laurent & Rombouts (2006) and Silvennoinen & Teräsvirta (2009): Exponential weighted moving average (EWMA); Optimal Rolling Estimator (ORE) from Foster & Nelson (1994); Scalar VECH from Bollerslev, Engle & Wooldridge (1988); Orthogonal GARCH (O-GARCH) from Alexander (2001); Conditional correlation models with several alternative specifications like the constant conditional correlation (CCC) proposed by Bollerslev (1990) and the dynamic conditional correlation model (DCC) proposed by Engle (2002). The models include auto-regressive components in their specifications, being the EWMA specification a weighted average of the previous observed covariances. The models ORE, VECH e O-GARCH use an auto-regressive specification for the covariance matrix, while the conditional correlation models decompose the covariance matrix in an univariate volatility matrix and a correlation matrix. Each of these specifications are detailed in section 2.5.

The portfolio selection policies used were: equally weighted portfolio ($1/N$) where each asset has the same weight in the portfolio studied by DeMiguel, Garlappi & Uppal (2009); the mean-variance policy initially proposed by Markowitz (1952), in short-selling constrained and unconstrained form, using risk aversion tending to infinity (named the minimum-variance policy) and with norm-restriction as in DeMiguel, Nogales & Uppal (2014). We also used policies that ignore the covariances proposed by Kirby & Ostdiek (2012), using different parameters for risk aversion and distance from the norm. With this variety of policies and specifications for the covariance matrix, we aim to evaluate not only a certain specification but instead to compare how the different specifications most used in the literature react in presence of transaction costs.

The most important result is that small transaction costs, sometimes lower that 20 basis points, are enough to nullify the portfolios' average net returns. When we compare the specifications among themselves, very low transaction costs, lower than 10 basis points, are enough to make their Sharpe coefficients not reject the hypothesis that the static specifications were different from the dynamic ones. Moreover, we are able to create better risk-adjusted returns by selecting different policies each period instead of relying on a single policy the whole period considered.

The remainder of the document is organized this way: chapter 2 details the portfolio selection policies used and the methodology for their evaluation. Chapter 3 describes the steps to implement the analysis, including the data sample used and how performance was measured. Chapter 4 analyzes the results found in the study. Lastly, Chapter 5 summarizes and concludes.

CHAPTER 2

Portfolio Selection and Optimization

Consider an investment universe with N assets $A_1, A_2, \ldots, A_N$ with uncertain future returns $R_1, R_2, \ldots, R_N$. Let $\mathbf{R}$ be the return vector:

$$\mathbf{R} = [R_1, R_2, \ldots, R_n]^\intercal .$$

The expected return vector $\boldsymbol{\mu} = E\left(\mathrm{R}\right)$ contains as its elements $\mu_i = E\left(R_i\right)$, $i = 1, \ldots, n$, such that:

$$\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_n]^\intercal .$$

The covariance matrix of the returns, $\Sigma = Var(R)$, contains as its elements $\sigma_{ii} = \sigma_i^2$ and $\sigma_{ij} = \sigma_{ji} = \rho_{ij}\sigma_i\sigma_j$ (for $i \neq j$), where σ_i is R_i standard deviation and ρ_{ij} is the correlation between the returns of assets A_i and A_j (for $i \neq j$). The covariance matrix Σ is symmetric and written as:

$$\Sigma = (\sigma_{ij})_{i,j=1,\ldots,N} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix}.$$

All valid covariance matrices are positive semidefinite, or equivalently, all eigenvalues are non-negative. A portfolio is represented by the N-dimensional vector $\mathbf{w}$, such that:

$$\mathbf{w} = [w_1, w_2, \ldots, w_n]^\intercal,$$

and w_i is the share of total wealth invested in asset A_i. The portfolio return R_p is linearly dependent of these weights and is the weighted average of the returns of each asset involved, where the weight of each asset is the portfolio share invested in the asset (TSAY, 2010).

$$R_p = w_1 R_1 + \cdots + w_n R_n = \sum_{i=1}^{N} w_i R_i = \mathbf{w}^\intercal \mathbf{R}.$$

Therefore, the expected portfolio return, μ_p, is the weighted average of each asset expected return and the portfolio variance σ_p^2 is a quadratic function of the weight vector. We can denote these variables by:

$$\mu_p = E(R_p) = E(\mathbf{w}^\intercal \mathbf{R}) = \mathbf{w}^\intercal \boldsymbol{\mu} \tag{2.1}$$

$$\sigma_p^2 = Var(R_p) = Var(\mathbf{w}^\intercal \mathbf{R}) = \mathbf{w}^\intercal \Sigma \mathbf{w}. \tag{2.2}$$

2.1 Portfolio selection policies

Assume that there are N risky assets with expected returns μ_t and covariance matrix Σ_t. Suppose that there is no risk free asset and that the investor need to allocate all his wealth among the N risky assets.

With 23.202 citations counted on Google Scholar at this articles' writing date, the mean-variance portfolio described by Markowitz (1952) continues to be the reference in any study about portfolio selection. To solve the mean-variance trade-off between risk and return the investor needs to find the weight vector that satisfies:

$$\begin{aligned} &\min_{w_t} w_t^\intercal \Sigma_t w_t - \frac{1}{\gamma} w_t^\intercal \mu_t, \\ &\text{s.t. } w_t^\intercal e = 1 \end{aligned} \tag{2.3}$$

where $\gamma > 0$ represents the investor's level of relative risk aversion and e is a vector of ones. It can be seen that the component $w_t^\intercal \Sigma_t w_t$ represents the portfolio risk. The component $w_t^\intercal \mu_t$ represents the portfolio return and the γ parameter determines the trade-off between expected return and portfolio risk. Since this formulation has no restrictions to it, we refer to it as the Unrestricted Mean-Variance Portfolio (MeVU).

When considering the case where the investor risk aversion tends to the infinity ($\gamma \to \infty$) the problem can be represented as,

$$\begin{aligned} &\min_{w_t} w_t^\intercal \Sigma_t w_t, \\ &\text{s.t. } w_t^\intercal e = 1. \end{aligned} \tag{2.4}$$

In this case the investor only cares about reducing the portfolio risk, without considering the expected return. This is an important portfolio because the expected returns is subjected to bigger estimation errors when compared to the estimation of covariances (MERTON; MORTON, 1980). We refer to it as the Unrestricted Minimum-Variance Portfolio (MiVU).

Frequently though, portfolio managers are subject to constrains when using the policies. We analyze the case where the portfolio is subject to a short-sale constrain. That gives us the following formulation for the mean-variance problem,

$$\begin{aligned} \min_{w_t} \; & w_t^\intercal \Sigma_t w_t - \frac{1}{\gamma} w_t^\intercal \mu_t, \\ \text{s.t. } & w_t^\intercal e = 1 \\ & w_t \geq 0 \end{aligned} \tag{2.5}$$

where the restriction $w_t \geq 0$ represents the short-sale restriction. We refer to it as the Restricted Mean-Variance Portfolio (MeVC). In the same way, the Restricted Mean-Variance Portfolio is given by:

$$\begin{aligned} \min_{w_t} \; & w_t^\intercal \Sigma_t w_t, \\ \text{s.t. } & w_t^\intercal e = 1 \\ & w_t \geq 0. \end{aligned} \tag{2.6}$$

As in DeMiguel, Nogales & Uppal (2014) we also consider the Mean-Variance Portfolio with Norm Restriction (NCMV2.5) given by:

$$\min_{w_t} w_t^\intercal \Sigma_t w_t - \frac{1}{\gamma} w_t^\intercal \mu_t,$$
$$\text{s.t. } w_t^\intercal e = 1 \tag{2.7}$$
$$\|w_t - w_{0t}\|_1 = \sum_{i=1}^{N} |\,(w_t)_i - (w_{0t})_i\,| \leq \delta$$

where w_{0t} is the weight vector of the Restricted Mean-variance (MeVC) portfolio and δ is the maximum deviation from the norm. With this restriction, we require that the weights of this portfolio remain close to the weights of the MeVC portfolio, as the aggregate absolute distance between the weights cannot be greater than the norm. We use the norm-1 in relation to the restricted mean-variance because of the stability of its weights. We considered δ equals to 2.5%, 5.0% e 10.0% according to DeMiguel, Nogales & Uppal (2014). But we report the results just for δ equals to 2.5% to simplify the reading as the different values had little impact in the result.

Besides those, we implemented the policies proposed by Kirby & Ostdiek (2012) which ignore the asset returns covariances and, because of that, aid in evaluating the advantages and disadvantages of estimating the covariances. These policies are:

Volatility Timing Portfolio (VT1) where the weights are given by:

$$\hat{w}_{it} = \frac{\left(\frac{1}{\hat{\sigma}^2_{it}}\right)^\eta}{\sum\limits_{i=1}^{N}\left(\frac{1}{\hat{\sigma}^2_{it}}\right)^\eta} \tag{2.8}$$

where $\hat{\sigma}_{it}$ is the i^{th} asset returns conditional volatility and η is a parameter that measures the investor's risk aversion. The greater the value of η the greater the weight of the less risky asset in the portfolio.

Reward-to-risk with Expected Returns Portfolio (RwRS1) :

$$\hat{w}_{it} = \frac{\left(\frac{\hat{\mu}^{+}_{it}}{\hat{\sigma}^{2}_{it}}\right)^{\eta}}{\sum_{i=1}^{N}\left(\frac{\hat{\mu}^{+}_{it}}{\hat{\sigma}^{2}_{it}}\right)^{\eta}} \tag{2.9}$$

where $\hat{\sigma}_{it}$ is the i^{th} asset returns conditional volatility, $\hat{\mu}^{+}_{it}$ is the i^{th} asset expected return, assuming that the investor drops assets with expected return lower than zero ($\hat{\mu}_{it} < 0$) and η is a parameter that measures the investor's risk aversion. The greater the value of η the greater the weight of the less risky asset in the portfolio.

Reward-to-risk with 4-factor Model Portfolio (RwR4F1) :

$$\hat{w}_{it} = \frac{\left(\frac{\hat{\beta}^{+}_{it}}{\hat{\sigma}^{2}_{it}}\right)^{\eta}}{\sum_{i=1}^{N}\left(\frac{\hat{\beta}^{+}_{it}}{\hat{\sigma}^{2}_{it}}\right)^{\eta}} \tag{2.10}$$

where $\hat{\sigma}_{it}$ is the i^{th} asset returns conditional volatility, $\hat{\beta}^{+}_{it}$ is the i^{th} asset market beta, assuming that the investor drops assets with beta lower than zero ($\hat{\beta}_{it} < 0$) and η is a parameter that measures the investor's risk aversion. The greater the value of η the greater the weight of the less risky asset in the portfolio. We compute $\hat{\beta}^{+}_{it}$ using the 4-factor model proposed by Carhart (1997) as an extension to the 3-factor model proposed by Fama & French (1992)

We considered η equals to 1, 2 e 4 according to Kirby & Ostdiek (2012). But we report the results just for η equals to 1% to simplify the reading as the different values had little impact in the result.

Following DeMiguel, Garlappi & Uppal (2009) we also consider the $1/N$ portfolio. Considered the "naive" portfolio, is an equally weighed combination of all assets considered in this analysis. It is worth noting that the asset's weights are defined by the percentage of the financial resources it uses. Therefore, this portfolio also needs rebalancing because, with the highs and lows of each asset in the portfolio, at the end of the period after the returns are compounded, the weights in financial terms of each asset will have changed.

Table 1 presents the list of the policies implemented and their respective acronym used in the remainder of the dissertation.

Table 1 – Portfolio Selection Policies: the following table presents the portfolio selection policies considered in this paper and the codes used to refer to them.

Acronym	Portfolio
1/N	Portfolio $1/N$
MiVU	Unrestricted Minimum-variance Portfolio
MeVU	Unrestricted Mean-variance Portfolio
MiVC	Restricted Minimum-variance Portfolio
MeVC	Restricted Mean-variance Portfolio
NCMV2.5	Mean-variance with Norm Restriction (delta=2.5%)
VT1	Volatility timing (eta=1)
RwRS1	Reward-to-risk with Expected Returns (eta=1)
RwR4F1	Reward-to-risk with 4-factor Model (eta=1)

2.2 Expected returns and covariance models

As in Bauwens, Laurent & Rombouts (2006), we define the asset return generation process by the following equation:

$$R_t = \mu_{t|\Omega} + H_{t|\Omega}^{1/2} \varepsilon_t, \tag{2.11}$$

where Ω is the information available up to $t-1$, μ is the expected return at period t, given the available information, $H_{t|\Omega}^{1/2}$ is a N x N positive definite matrix such that $H_{t|\Omega}$ is the covariance matrix, and ε_t is a N x 1 random vector with $E(\varepsilon_t) = 0$ and

$Var(\varepsilon_t) = I_N$. In the unconditional model we assume that $\mu_{t|\Omega}$ and $H_{t|\Omega}$ are independent of previous values every period, while in the conditional model they are updated according the chosen specification. Since the return is given by the expected return and their covariance matrix, the following sections describe the different techniques used in their estimation.

2.3 Expected returns

In this paper we assume that the expected returns are described by the sample average:

$$\mu_{t|\Omega} = \frac{1}{T-1}\sum_{i=1}^{T} R_i. \tag{2.12}$$

We chose not to impose a conditional auto-regressive structure to model the expected returns because this kind of estimation usually contains a large error, damaging the portfolio's performance as in DeMiguel, Nogales & Uppal (2014).

2.4 Static Shrinkage Models

In their work, Ledoit & Wolf (2003b) argue that it is known that a way to obtain a better estimator is to simply take a weighted average between a biased estimator, with little estimation error, and an unbiased, but with a lot of estimation error. This process is called shrinkage of an unbiased estimator towards a target, represented by the biased estimator. This idea can be summarized in the following equation:

$$\sigma_t = \hat{\delta} F_t + (1-\hat{\delta}) S_t, \tag{2.13}$$

where $\delta \in \{\delta \in \mathbb{R} | 0 \leq \delta \leq 1\}\mathbb{R}$ is the shrinkage coefficient, F is the target covariance matrix, highly structured, and S is an unstructured estimator. The sample covariance is most frequently used as this unstructured estimator, so we follow and also use the sample covariance in this work.

As for the target covariance, Ledoit & Wolf (2003a) say that it needs to have a small number of free parameters, meaning it is highly structured. And it also needs to reflect the important characteristics of the variable being estimated. For this paper, we chose as target covariances the ones described in Ledoit & Wolf (2003a), Ledoit & Wolf (2003b) and Ledoit & Wolf (2004). This last one, Ledoit & Wolf (2004) use as target the identity matrix. So, $F_t = \mathrm{I}$.

Ledoit & Wolf (2003a) uses a model of constant correlation between the assets for the shrinkage target F_t. We are able to use this target only because all assets considered belong to the same class: stocks. To define this covariance matrix, let H_t be the sample covariance and h_{ij} be the element of H_t at row i and column j. So the sample correlation is given by:

$$r_{ij} = \frac{h_{ij}}{\sqrt{h_{ii} h_{jj}}},$$

whose average is:

$$\bar{r} = \frac{2}{(N-1)\,N} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} r_{ij}.$$

Therefore the constant correlation matrix F_t can be created using the sample variances and the sample average correlation:

$$\begin{aligned} f_{ii} &= h_{ii}, \\ f_{ij} &= \bar{r}\sqrt{h_{ii} h_{jj}}. \end{aligned} \tag{2.14}$$

Lastly, Ledoit & Wolf (2003b) assume an one factor market model for the return of asset j at time t:

$$r_{j,t} = \alpha_j + \beta_j r_{M,t} + \epsilon_{j,t},$$

where $r_{M,t}$ is the market index return at time t, β_j is the market factor, α_j is the intercept and $\epsilon_{j,t}$ is the error. Assuming that $r_{M,t}$ and $\epsilon_{j,t}$ are uncorrelated and that $\epsilon_{i,t}$ and $\epsilon_{j,t}$ are uncorrelated for $i \neq j$, the target covariance matrix is:

$$F_t = s_{m,t} B B^{\intercal} + D_t, \tag{2.15}$$

where B is the vector of βs, $s_{m,t}$ is the sample variance of $r_{M,t}$ and D_t in the diagonal matrix consisting of the variance of the sample errors.

2.5 Multivariate GARCH models

The conditional variance models used were the ones described in Engle & Sheppard (2008) and Becker et al. (2014). This dissertation focus on the models of multivariate GARCH and 8 different specifications largely used in the literature are implemented:

Exponentially Weighted Moving Average (EWMA): The EWMA model is defined by:

$$H_t = \alpha R_{t-1}{}' R_{t-1} + (1 - \alpha)\, H_{t-1}, \tag{2.16}$$

where $\alpha \geq 0$. When we use the value 0,04 for α, the model EWMA is equivalent to the Riskmetrics^{TM} approach. Zaffaroni (2008) shows that despite allowing gains in processing and supplying a simple way to impose that the resulting matrix be positive semi-definite, the EWMA approach produces inconsistent estimates for the parameters. In this model we use maximum likelihood to estimate the parameter α.

Optimal Rolling Estimator (ORE): As in Foster & Nelson (1994) we can define the model as:

$$H_t = \sum_{t=1}^{\infty} \Omega_{t-k} \odot R_{t-k}^{\intercal} R_{t-k}$$

where Ω_{t-k} is the symmetric weight matrix and $\odot$ symbolizes the Hadamard product (element to element multiplication). The optimal balancing proposed by Foster & Nelson (1994) is given by:

$$\Omega_{t-k} = \alpha \exp\left(-\alpha k\right) \iota^{\intercal} \iota,$$

where α is a non-negative parameter and ι is the vector of ones. The estimator can be rewritten as:

$$H_t = \alpha \exp\left(-\alpha\right) R_{t-k}^{\intercal} R_{t-k} + \exp\left(-\alpha\right) H_{t-1}. \qquad (2.17)$$

Scalar VECH: The scalar VECH specification of Bollerslev, Engle & Wooldridge (1988) is:

$$H_t = C^{\intercal} C + \alpha R_{t-1} {R_{t-1}}^{\intercal} + \beta H_{t-1}.$$

Instead of estimating the $N(N+1)/2$ elements of C, we use the technique suggested by Engle & Mezrich (1996). The idea is to estimate the intercept matrix through an auxiliary estimator given by $\hat{C}'\hat{C} = \bar{S}\,(1-\alpha-\beta)$, where $\bar{S} = \frac{1}{T}\sum_{t=1}^{T} R_t R_t{}^{\intercal}$, so that the Scalar VECH becomes:

$$H_t = \bar{S}\,(1-\alpha-\beta) + \alpha R_{t-1} R_{t-1}{}^{\intercal} + \beta H_{t-1}, \tag{2.18}$$

where the covariance is stationary given that $\alpha + \beta < 1$.

Ortoghonal GARCH (OGARCH): The O-GARCH model of Alexander (2001) belongs to the factor models class and is capable of obtaining great processing gains through dimension reduction. The model is given by:

$$H_t = W\Omega_t W, \tag{2.19}$$

where W is a $N \times k$ matrix whose columns are given by the first k eigenvectors of the $t \times N$ matrix of asset returns, and Ω_t is a $k \times k$ diagonal matrix whose elements are given by $h_{f_{kt}}$ where $h_{f_{kt}}$ is the conditional variance of the k-th principal component and follows a GARCH(1,1) process. The O-GARCH model was implemented using 3 principal components.

Conditional Correlation Models: This class of models is defined as:

$$H_t = D_t \Psi_t D_t, \tag{2.20}$$

where D_t is the $N \times N$ diagonal matrix with the diagonal elements given by $h_{i,t}$, where $h_{i,t}$ is the conditional variance of the i-th asset and follows a process GARCH(1,1), and Ψ_t is a conditional correlation symmetric matrix containing the elements $\rho_{ij,t}$, where $\rho_{ii,t} = 1, i, j = 1, \ldots, N$. We considered 4 alternative specifications for Ψ_t:

- The Constant Conditional Correlation model (CCC) proposed by Bollerslev (1990);
- The Dynamic Conditional Correlation model (DCC) proposed by Engle (2002);
- The Asymmetric DCC model (ASYDCC) proposed by Cappiello, Engle & Sheppard (2006);
- The Dynamic Equicorrelation model (DECO) proposed by Engle & Kelly (2012).

Engle & Colacito (2006) and Engle & Sheppard (2008) study the performance of alternative conditional correlation models in portfolio selection problems.

The multivariate GARCH models are usually estimated through quasi maximum likelihood (QML). However, this estimator is highly biased in high dimensions, as shown in Engle, Shephard & Sheppard (2008) and Hafner & Reznikova (2012). So, the parameters for the EWMA and VECH models are estimated through the composite likelihood method (CL) proposed by Engle, Shephard & Sheppard (2008). As for the conditional correlation models, their estimation can be divided in volatility and correlation. The volatility refers to estimate the conditional univariate variance through QML assuming Gaussian innovations. The correlation matrix parameters in the models DCC and ASYDCC are estimated using the method CL. As shown by Engle, Shephard & Sheppard (2008), the CL estimator generates more precise estimations for the parameters when compared to the two steps procedure proposed by Engle (2002), especially in big problems.

It is important to mention that other multivariate GARCH specifications, besides the ones cited, were proposed in the literature, as discussed in Bauwens, Laurent &

Rombouts (2006) and Silvennoinen & Teräsvirta (2009). Although we can broaden the dissertation to include them, we decided to focus on the 8 specifications most commonly used to reduce the computing effort when evaluating the models.

Table 2 presents the list of the covariance models implemented and their respective acronym used in the remainder of the dissertation.

Table 2 – Covariance Models: the following table presents the covariance models considered in this paper and the codes used to refer to them.

Acronym	Covariance model
CORR	Shrinkage model with constant correlation target
PARA	Shrinkage model with identity matrix target
MARKET	Shrinkage model with market model target
EWMA	Exponentially Weighted Moving Average
ORE	Optimal Rolling Estimator
VECH	Scalar VECH
OGARCH	Ortoghonal GARCH
CCC	Constant Conditional Correlation
DCC	Dynamic Conditional Correlation
ASYDCC	Asymmetric Dynamic Conditional Correlation
DECO	Dynamic Equicorrelation

2.6 Policy combination

It is naive to think that the portfolio manager will follow a single strategy for the whole period. It is likely that when faced with diminishing returns on a certain policy, the manager would switch, choosing a policy that would obtain higher returns, or even combining them in hopes to outperform each of them. Therefore, the policies described need to pass through a process of selection and combination of policies to understand how this enhances the result obtained to the investor against using a single policy continuously.

The work of Raftery, Kárnỳ & Ettler (2010) proposes a methodology to combine econometric specifications in the presence of uncertainty about the model that can be adapted to combine policies. The Dynamic model average (DMA) stems from the hypothesis that there are K models and that $L_t \in \{1, 2, \ldots, K\}$ determines which model is applicable each period. Starting from previsions from all models, the DMA approach computes the probability that the model k is the best model to forecast the observations for period t, given the information available in $t-1$, meaning $Prob\left(L_t = k|y_{t-1}\right) = \pi_{t|t-1,k}$. Once $\pi_{t|t-1,k}$ is computed, these probabilities can be used to weight the model's forecasts. The dynamic model selection (DMS) approach selects the model with the greatest $\pi_{t|t-1,k}$ each period and use it to compute the portfolio weights as in Koop & Korobilis (2013).

Given the starting probability, $\pi_{0|0,k}$, the probability transition is given by:

$$\pi_{t|t-1,k} = \frac{\pi^{\alpha}_{t-1|t-1,k}}{\sum\limits_{l=1}^{K} \pi^{\alpha}_{t-1|t-1,l}}$$

where $0 < \alpha \leq 1$ is a forgetting factor to reduce the computational cost. The probability of each model is given by:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} \cdot p_k\left(y_t|\underline{y}_{t-1}\right)}{\sum\limits_{l=1}^{K} \pi_{t|t-1,l} \cdot p_l\left(y_t|\underline{y}_{t-1}\right)}$$

where $p_k\left(y_t|\underline{y}_{t-1}\right)$ is the predictive density of the model k evaluated in y_t, and that is a measure of the forecast performance. The DMA approach consists in using the weighted average of the individual forecasts using $\pi_{t|t-1,k}$ to compute the weights of each model. The DMS approach instead only selects the models with the highest predictive probability, $\pi_{t|t-1,k}$, in each period, and uses it in the forecasting.

To evaluate investment policies, the algorithm can be adapted to use the policies' returns instead of the predictive likelihood as follows:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} \cdot (1 + R_{net,kt})}{\sum_{l=1}^{K} \pi_{t|t-1,l} \cdot (1 + R_{net,kt})} \tag{2.21}$$

where $R_{net,kt}$ is strategy k return net of transaction cost, at time t, which implies that the probabilities will be updated according to the policy performance in the previous period. To understand the part of $R_{net,kt}$ in the portfolio policy selection, it is possible to rewrite equation 2.21 as:

$$\pi_{t|t-1,k} \propto \left[\pi_{t-1|t-2} \cdot (1 + R_{net,kt})\right]^{\alpha}$$

$$\pi_{t|t-1,k} \propto \prod_{t=1}^{T-1} (1 + R_{net,kt})^{\alpha^{T-t}}. \tag{2.22}$$

Rewriting this way is easy to realize how the probability of each policy is given by its past performance, with recent periods being given greater weights. The greater weights recent periods receive is controlled by the forgetting factor, α and it has an exponential decay. For instance, if $\alpha = 0.99$, the portfolio performance five years ago receives 80% as much weight as the performance last period. If $\alpha = 0.95$, then the portfolio performance five years ago receives only about 35% as much weight. These considerations suggest that we focus on values of α near one and in our analysis we set $\alpha = 0.99$.

2.7 Transaction Costs

When rebalancing his portfolio to the new weight allocation given by the selection policies, the portofolio manager is faced with transaction costs. As Kolm, Tütüncü &

Fabozzi (2014) points out, transaction costs consist of direct costs, such as comissions and taxes, bid-ask spread, and indirect costs, such as slippage. Slipagge is the difference between the price prevailing at the time the trade is antecipated, t_0, and the volume weighted average price over the period, $[t_0, t_0 + T]$, over which it executes. Somewhat simplistically, slippage is due to (a) random price changes in the securities that occur in $[t_0, t_0 + T]$, and (b) "market impact costs," i.e. price chances incurred because of the trade itself. In general, we expect that a trade moves the price against the buyer or seller. That is, the price is pushed upwards when buying and downwards when selling. The market impact portion of the slippage can be substantial when the ratio of the trade size to the average trade volume is high and is often modeled as an increasing function of this quantity.

As in Olivares-Nadal & DeMiguel (), for small trades, which do not impact the market price, the transaction cost is assumed to proportional to the amount traded on each asset. So when we compute the return net of trasaction costs, we get:

$$R_{net,t} = \left(1 - \kappa \sum_{j=1}^{N} \left| w_{j,t} - w_{j,(t-1)^+} \right| \right) (w_t)^\intercal R_{t+1}, \tag{2.23}$$

where, again, $w_{j,t}$ e $w_{j,(t-1)^+}$ are, respectively, the weight of the assets in the portfolio after rebalancing and the weights of the assets before rebalancing, but after the period return have been computed and κ is the proportional transaction cost.

In his work, French (2008) tries to estimate how much americans spend on transaction costs each year for different active investing options, and compare it with the cost of passive investing. In his estimations, americans spend 21 basis points in total trading, as a fraction of the total portfolio.

Chapter 3

Implementation

The expanding window technique was used to evaluate the portfolios, meaning that starting from a data section, we estimated expected returns and covariances and computed the first period, and at each following period we included the data point from the current period and reestimated the returns and covariances to process the period.

The initial estimation window used was 1500 periods. With it we computed the expected returns in period 1501 using the sample expectation as described in chapter 2.3. After we computed the covariance matrices using each of the techniques described in sections 2.4 and 2.5. These two were the basis for each of the portfolios described in section 2. This portfolio remained active in the period and at the end of it the returns were computed as described in 3.2.

Contrary to the rolling window technique, where the oldest observation is discarded, in this technique all observations are kept and the new one is added. That way, we estimate the returns for period $t + 1$, we compute the covariance matrices, and use both to rebalance the portfolios. Meaning that the data up until period t is used to compose the portfolios that will be present in period $t + 1$, out of the sample. This process is repeated until the end of the data set is reached.

We tried different rebalancing frequencies including daily, weekly and monthly rebalancing, but the results found were not significantly different. To simplify the presentation, only the daily rebalancing results are reported.

3.1 Data

We used the 50 most traded stocks with regards to the financial volume from the S&P100 index, between 01/01/2004 and 01/01/2014. With a ten year horizon, the sample goes through periods of growth, recession and economic recovery, including periods of high and low market volatility, so that we do not restrict the results to a determined sample characteristic. The sample contains 2516 observations with daily data of returns, evaluated as the logarithmic difference of the closing prices. The table 14 lists the used assets and their first four moments. It is possible to see in it some stylized facts of financial time series like excess kurtosis.

3.2 Performance evaluation methodology

At each period, each of the strategies described in section 2 were used alongside each of the covariance specifications to rebalance the portfolios. At each period there was an extra data point of the returns incurred in the previous period, that were added to the covariance estimations, changing the weights of each asset in the portfolio.

So, it is possible to say that in each period there were two weights. One before rebalancing - the weights of each asset in the portfolio after the asset price changes. And another, after rebalancing according to the pair portfolio selection policy and covariance specification.

In order to evaluate the result of the described policies, we evaluate its gross return, net return, and the break-even transaction cost for each of the policies using each of the covariances.

The gross return of a portfolio is given by:

$$R_{gross,t} = (w_t)^\intercal R_{t+1}, \tag{3.1}$$

where w_t is the asset weight vector in the portfolio at period t, after rebalancing and R_{t+1} is the return vector at period $t+1$. In evaluating the gross returns we considered its averages and standard deviations with the goal of observing which portfolios incur in higher returns in the absence of transaction costs, and their variability. The portfolio average return is given by:

$$E[R] = \frac{1}{T}\sum_{i=1}^{T} R_i. \tag{3.2}$$

And the return variance is given by:

$$Var[R] = \frac{\sum_{i=1}^{T}\left(R_i - E(\overline{R})\right)^2}{T-L}. \tag{3.3}$$

The average return can be understood as the portfolio performance since more return is preferable to less return. And the variance can be thought of as the risk measure of the portfolio: the bigger the variance, the greater the chance of extreme results. To evaluate the portfolio against the two measures we compute the Sharpe ratio, as proposed in Sharpe (1966), given by:

$$S_a = \frac{E[R]}{\sqrt{Var[R]}}. \tag{3.4}$$

We also measure the portfolio turnover, according to:

$$\text{Turnover} = \frac{1}{T-1} \sum_{t=1}^{T-1} \sum_{j=1}^{N} \left(\left| w_{j,t} - w_{j,(t-1)^+} \right| \right), \tag{3.5}$$

where $w_{j,t}$ and $w_{j,(t-1)^+}$ are, respectively, the weight of the assets in the portfolio after rebalancing and the weights of the assets before rebalancing, but after the period return have been computed. Given that, we can see whether the different portfolio selection policies and covariance models create portfolios more unstable, requiring more rebalancing.

Afterwards, to evaluate the impact of the transaction costs in policies and covariances, these were subjected to increasing transaction costs form 1 to 50 basis points, understood as a transaction cost equals to 0.01% to 0.5% of the rebalanced value of the portfolio. The returns net of transaction costs are then computed:

$$R_{net,t} = \left(1 - \kappa \sum_{j=1}^{N} \left| w_{j,t} - w_{j,(t-1)^+} \right| \right) (w_t)^\intercal R_{t+1}, \tag{3.6}$$

where, again, $w_{j,t}$ e $w_{j,(t-1)^+}$ are, respectively, the weight of the assets in the portfolio after rebalancing and the weights of the assets before rebalancing, but after the period return have been computed and κ is the proportional transaction cost. These net returns also had their Shape ratio computed to see the transaction cost impact.

To compare the static and dynamic covariance specifications, we used the test proposed by Ledoit & Wolf (2008) to check if the Shape ratios of the net returns, resulting from a given dynamic covariance are different from a reference static covariance model.

The test has the following hypothesis:

$$\begin{cases} H0: & \text{The difference between the Sharpe ratios is 0;} \\ H1: & \text{The difference between the Sharpe ratios is different from 0.} \end{cases} \tag{3.7}$$

Lastly, we computed the transaction costs that lead the portfolio to the break-even. Initially we looked for the transaction costs that lead the average net return to zero. To do that we submitted the portfolios to increasing transaction costs, starting at zero basis points and increasing by 1 at every interaction. The procedure was repeated until the average return changed to a negative number and this cost was stored as the break-even transaction costs.

The second break-even evaluated was the transaction cost that would take the portfolio from being superior to a benchmark, to being inferior to it. To that goal, for each portfolio selection policy, we compared the Sharpe ratios between dynamic covariance models and the static ones. We started by computing the time series of net returns, one with a dynamic models and another with a static one. The two net returns series were then submitted to the test proposed by Ledoit & Wolf (2008) to check if they were statistically different from each other. This procedure was repeated for increasing transaction costs, until the test rejected the null hypothesis and the Sharpe ratio of the dynamic model was inferior to the one used as benchmark.

Chapter 4

Results

The presentation of the results is organized in three parts to facilitate their analysis and discussion. The section 4.1 shows the first analysis in terms of portfolio mean, variance, turnover and Sharpe ratio under zero transaction costs. The section 4.2 discusses, for the different covariance specifications, the transaction costs that zeroes the average net return and the one that makes the specifications clearly different. The section 4.3 analyzes the results of using the DMA and DMS strategies. Lastly the section 4.4 analyzes how the results react to different levels of risk aversion and rebalancing frequencies to evaluate how the results change when these parameters change. The tables and graphs referenced throughout this whole section can be found in the appendix.

4.1 Descriptive Statistics

Table 3 shows the average gross return obtained by each portfolio selection policy and the corresponding covariance model used. In practically all the portfolios, the gross return obtained using dynamic specifications is equal or superior to the return obtained with static specifications. As for the portfolio policies that use only variances and ignore the covariances, the returns are closer to each other, but still, the dynamic specifications have equal or superior returns than the static specifications in almost all cases. The dynamic covariance specifications are different among themselves, and when we look at the averages, we can see that the specifications EWMA and VECH display better results than the others.

Table 4 shows the standard deviation of the returns of the portfolio policies. Here the comparison is not so straightforward. We can observe that the results are similar among policies when using dynamic covariances or static covariances. An exception is the DCC specification which delivered portfolios with higher standard deviations. Given that the variance of the returns is the risk measure most commonly used in the literature, it is interesting to notice that the higher portfolio returns observed in the dynamic covariance specifications did not come with a higher level of risk.

The turnover obtained for the portfolios is reported on table 5. Here it is easy to see how in all portfolios the turnover is superior using dynamic specifications for the covariances, than with static covariances. The only exceptions are the portfolios reward-to-risk using sample return (RwRS1), when coupled with the ORE covariance specification. With a turnover of just 0.016 it is equal to the ones obtained with the static covariances.

It is also interesting to associate this table with the results reported in table 3. The higher returns obtained with the dynamic specifications is usually associated with a higher portfolio turnover. The most extreme case is that of the DCC specification that has a reasonably superior return (0.069 on average), when compared to the other portfolios, but it was coupled with the highest turnover among all portfolios observed (0.991 on average). The dynamic specification OGARCH delivered portfolios with a relatively low turnover and on average did not show returns superior to the static competitors.

Therefore, even though specifications with greater structure are linked to higher average returns, they come associated with higher portfolio instability.

To give further support to these observations, table 6 presents the Sharpe ratio in the absence of transaction costs. We can observe a pattern similar to that of the returns, with the specifications EWMA and VECH displaying higher figures than the ones given by static specifications, when compared within the same portfolio selection policy. Their direct competitors when we looked at the returns where the specifications DCC and ASYDCC, were highly penalized by their high standard deviation and now are not as close competitors as when only the returns were considered.

The asterisks in the table 6 mark the instances in which the statistical test rejects the hypothesis that they are equal to those found for the static specification PARA. It is interesting to note that the specifications OGARCH, CCC, DCC and DECO in no case produced figures that rejected the hypothesis of equality of the Sharpe ratios. In other words, these specifications did not have Sharpe ratios statistically superior to the benchmark specification PARA and they displayed higher turnovers. The other dynamic specifications managed to present indexes that were statistically superior in several portfolio selection policies.

It is important now to observe how the Sharpe ratio evolves with the increase of the transaction cost. The graphs 1 and 2 show the evolution of the Sharpe ratio with the increase of the transaction cost, from 0 to 50 basis points, for all policies considered. Each graph evaluates a portfolio selection policy and each line of the graph depicts the evolution of the ratio for each covariance specification. At the transaction costs where the hypothesis that indexes are different is not rejected, the line changes from full line to dotted line.

The analysis of these charts start with the line for the specification PARA, used as benchmark. It is the one colored in black. It is interesting how it has a small slope in policies that ignore the covariances, showing the high stability of this specification and its low necessity for rebalancing. Therefore, it requires a high transaction cost to penalize this specification. In the specifications that ignore the covariances, the slope of the benchmark increases, but it is only high in the portfolios RwRS.

In these graphs, the Sharpe ratio presented at the table 6 is the point where the lines intercept the vertical axis, which corresponds to no transaction costs. Given that the lines are decreasing with the increase in transaction costs, the ratios that didn't have an asterisk at table 6 already start as a dotted line, showing that a statistical test do not indicate statistical difference between the portfolio and the benchmark. Even when there was an asterisk it is easily seen that in almost all specifications few basis points were enough to make the statistical difference disappear. The only exceptions are the portfolios RwRS. In these portfolios, the slope of the benchmark curve is similar to the specifications EWMA and ORE, so that it intercepts the EWMA curve at a high transaction cost and does not intercepts the ORE at all.

An important point to keep in mind when looking at these results given by 6. The first reaction is to look at the turnover to explain all the penalties to a portfolio caused by the transaction cost. But we can't ignore that we need to check the magnitude of the portfolio return. The proportional transaction cost is a function of the turnover, but the transaction costs impact the returns. So a high enough return would compensate a high turnover. The variance is important since we analyze the Sharpe ratio, but the turnover has little impact in it.

Finally, table 7 shows the numbers for the previous analysis for the portfolio $1/N$. Since this portfolio does not use neither the variances nor the covariances, its result is not influenced by the specifications, therefore we need a single number for each statistic. The portfolio displays low turnover, as expected, since its rebalancing is only necessary to keep the ratio of the assets constant between themselves, after the returns where observed. Given its low turnover, its Sharpe ratio is not highly affected by the transaction costs. As the portfolio also fails to generate extraordinary returns, its Sharpe ratio is not superior to those found for the static specifications of the different portfolio policies.

4.2 Break-even Analysis

Table 8 presents the lowest transaction cost that makes the average return of the policy be lower than zero. In the presence of high transaction costs, portfolios with higher break-even values are preferable over portfolios with lower values.

It is interesting to note that the static specifications need transaction costs far higher than the dynamic specifications in order to be neutralized. This difference is even higher in policies that consider covariances than the ones that do not. Only the policy RwRS1 when combined with the ORE covariance specification provides higher break-even value then the static covariances. And that is due not only to a comparable turnover, but also due to a higher average return. This could also be observed in the figure 2 where the curve corresponding to the ORE specification with almost parallel to the benchmark.

The specification EWMA and VECH that had noticeable higher returns but the high turnover rapidly neutralized the average return. So the high return was not high enough to compensate for the high turnover. The opposite happened with the specification ORE: there were no outstanding returns when compared with other dynamic specifications, but its low turnover lead to a lower penalization of its returns, leading to high break-even among those specifications.

The second point of notice is the transaction cost that makes the difference between a dynamic specification and the static benchmark to be statistically different. That is, the statistical test rejects the null hypothesis that the indexes are equal. Portfolio managers that face transaction cost above the ones depicted on the table would have better Sharpe ratios if they had used the static benchmark specification instead of a dynamic covariance specification. Therefore, in the presence of high transaction costs, portfolios with higher break-even are preferable to portfolios with lower break-evens.

In the table 9 we can see that for the specifications CCC, DCC and DECO, a transaction cost lower than 20 basis points (and some times even lower than 10 basis points) was enough to make the Sharpe ratios lower and statistically different from the benchmark.

Again the ORE specification was the one with the better results among the dynamic specifications, because of low turnover and higher returns. In the policy RwRS1 its performance even surpass that of the benchmark, independent of transaction cost. In second place comes the specification OGARCH that did not present high turnover, but failed to generate high returns and did not have a high Sharpe ratio.

4.3 Combination analysis

We start our analysis by looking at the combination of the portfolios using the DMA technique described in section 2.6. Figure 3 shows the overall participation of the portfolios in the composition of the DMA weights. It shows the aggregated participation across all periods analyzed. It is interesting to note here, how on the absence of transaction costs, the portfolios had almost equal participation in the strategy. But as the transaction costs increase, some portfolios start to lag behind and amount for a lesser part of the DMA portfolio, noticeably the MiVU and the MeVU, using the covariances ORE, OGARCH, CCC and DECO. Both portfolios did have noticeable high turnover, as we can see in table 5, and so fail to generate strong net returns under high transaction costs.

Figures 5 and 6 further complement the analysis by comparing the Sharpe ratio of individual portfolios against that of the DMA strategy. Since the strategy is an weighted average of the other portfolios it's resulting Sharpe ratio curve is among the other curves. Therefore, the strategy fails at producing better Sharpe overall than the ones used in its composition. We can see from the participation of each portfolio in the DMA (figure 3) that they had similar participation so we can infer that portfolios that stood above the DMA curve are better than the ones below it. Therefore, at high transaction costs, it is clear that constrained policies (MiVC, MeVC and NCMV2.5) when combined with static covariance specifications (CORR, PARA and MARKET) are clearly better than the others.

Now we turn our attention to select a winner portfolio through the DMS technique described in section 2.6. Figure 4 show the percentage of times each portfolio was selected across all periods. It is interesting to notice a couple patterns that occur when the transaction cost increases: first, the portfolio $1/N$ grows in participation. This is to be expected since as transaction cost grows, the stability of the weights provided by this portfolio becomes important. Being able to pay less to transact is more important than search for better returns. Second, there is more usage of portfolios with dynamic covariances but as the transaction costs grow, so increases the participation of portfolios with static covariances, while the dynamic ones decrease dramatically. This mirrors the findings in figures 1 and 2 as we could see that the performance of portfolios with dynamic covariances degraded rapidly.

Figures 7 and 8 round up this analysis by displaying how the performance of the

DMS strategy compare with the individual portfolios. First, it is interesting to notice how it gives a far improved performance when faced with low and average transaction costs. One thing I find counter intuitive though is how the portfolio manages to degrade to equal and below the other portfolios considered when faced with high transaction costs. One would expect that as an strategy based on selecting the best performing strategy in recent periods, the portfolio DMS would always stay above the others. A possible explanation is that the measure of performance used for strategy is the Sharpe ratio, while the performance measure used for selection is the net returns. But at the time of writing there was still uncertainty about it.

4.4 Robustness checks

To conclude the analysis of the results, it is important to check the impact that different parametrizations have on the results. As discussed, table 8 displays the results when daily rebalancing is considered for the portfolios. The tables 10 and 11 display the same analysis for weekly and monthly rebalancing, respectively.

The relations between dynamic and static specifications remained the same as the one in table 8. But the break-even values increase when we reduce the rebalancing frequency. Table 12 brings only the averages to simplify comparison.

When observing the policies that rely on covariance information, the dynamic specifications have break-evens far superior when the rebalancing frequency is reduced. Meanwhile, the break-even for the static specifications remain unchanged. Still, the break-even for the static specifications is at least an order of magnitude superior. As for the policies that do not rely on covariance information, the increase in the break-even in the dynamic specifications was less pronounced when we reduced the rebalancing frequency. On the static ones, on the other hand, the growth of the break-even was more pronounced.

All in all, a portfolio manager thinking in an aggressive daily balance should focus his efforts in the static specifications, but, the lower the rebalancing frequency it aims towards, the more he can change his focus to the dynamic specifications in order to obtain higher net returns.

Table 13 shows the effect of different levels of the risk aversion. Its impact in the portfolios' break-even is practically neglectable. It becomes a little more pronounced on greater break-evens, but yet, made no difference in the results.

Chapter 5

Concluding Remarks

The literature presents some divergence about the best way to specify the covariances. On one side there are the static specifications (or unconditional) where the covariance estimation depends on the sample covariance and/or a factor model, independent of an auto-regressive structure. On the other side there are the dynamic models based on the hypothesis that the covariance in the current period depends on the covariance of previous periods, being updated at each period. Therefore, these last models use auto-regressive structures either for the whole matrix or for one or more parts of this matrix.

This work adds to this discussion by comparing these specifications in their applicability. In the real world the portfolio manager faces a trade-off where they rebalance the portfolio looking for higher returns but this rebalancing incur in transaction costs. In this work we can see that when the dynamic and static covariance specifications are compared within this scenario, the static specifications were clearly superior given the stability of the portfolio weights. The dynamic specifications, on the other hand, even though at first glance they were able to generate higher returns, this excess returns were converted into losses at low transaction costs. Based on this we suggest that future works on covariances using portfolio selection pay extra attention to this aspect when evaluating their findings.

Looking forward, this work can be enhanced so that the result can be even more general by using markets simulations instead of real market data. We used a sample big enough to make the results cover a broad range of market conditions, but still the work could be further analyzed through several simulations. We raise an alert to the computational cost of this suggestion. It was not promptly included in this work because of the high processing cost. Using a sample of this size, all the steps and tests took three to four days of processing to complete with the available hardware. So in order to run thousands of simulations a great processing power need to be amassed.

Appendix A

Portfolios Descriptive Statistics

Table 3 – Average Portfolio Gross Return: the table shows the average gross return obtained by the portfolios for each period of the sample as described in section 3.2, as well as their average by portfolio selection policy and covariance specification.

	MiVU	MeVU	MiVC	MeVC	NCMV2.5	NCMV5	NCMV10	VT1	VT2	VT4	RwRS1	RwRS2	RwRS4	RwR4F1	RwR4F2	RwR4F4	Average
EWMA	0.071	0.076	0.070	0.071	0.071	0.072	0.074	0.062	0.063	0.064	0.066	0.081	0.108	0.059	0.060	0.062	0.071
ORE	0.064	0.065	0.060	0.061	0.061	0.062	0.064	0.062	0.061	0.057	0.064	0.077	0.102	0.059	0.058	0.059	0.065
VECH	0.077	0.083	0.072	0.074	0.073	0.074	0.075	0.062	0.063	0.062	0.064	0.076	0.099	0.060	0.062	0.066	0.071
OGARCH	0.065	0.064	0.056	0.056	0.057	0.058	0.058	0.061	0.059	0.053	0.060	0.064	0.076	0.057	0.057	0.059	0.060
CCC	0.060	0.060	0.062	0.062	0.063	0.063	0.063	0.062	0.062	0.062	0.061	0.066	0.077	0.059	0.059	0.062	0.063
DCC	0.049	0.105	0.074	0.075	0.075	0.076	0.077	0.062	0.062	0.062	0.061	0.066	0.077	0.059	0.059	0.062	0.069
DECO	0.069	0.069	0.060	0.060	0.060	0.060	0.059	0.062	0.062	0.062	0.061	0.066	0.077	0.059	0.059	0.062	0.063
ASYDCC	0.077	0.078	0.073	0.075	0.074	0.075	0.077	0.061	0.062	0.062	0.062	0.075	0.104	0.057	0.057	0.058	0.070
Average	0.066	0.075	0.066	0.067	0.067	0.067	0.068	0.062	0.061	0.061	0.063	0.072	0.090	0.059	0.059	0.061	0.066
CORR	0.052	0.052	0.055	0.055	0.056	0.056	0.057	0.061	0.058	0.052	0.061	0.066	0.081	0.057	0.055	0.054	0.058
PARA	0.053	0.052	0.055	0.055	0.056	0.057	0.058	0.061	0.059	0.053	0.061	0.067	0.082	0.057	0.056	0.054	0.059
MARKET	0.052	0.052	0.055	0.055	0.056	0.057	0.058	0.061	0.058	0.052	0.061	0.066	0.081	0.057	0.055	0.054	0.058
Average	0.052	0.052	0.055	0.055	0.056	0.057	0.058	0.061	0.058	0.053	0.061	0.067	0.081	0.057	0.055	0.054	0.058

Table 4 – Portfolio Gross Return Standard Deviation: the table shows the average standard deviation obtained by the portfolios for each period of the sample as described in section 3.2, as well as their average by portfolio selection policy and covariance specification.

	MiVU	MeVU	MiVC	MeVC	NCMV2.5	NCMV5	NCMV10	VT1	VT2	VT4	RwRS1	RwRS2	RwRS4	RwR4F1	RwR4F2	RwR4F4	Average
EWMA	0.687	0.708	0.692	0.694	0.683	0.676	0.664	0.899	0.778	0.708	0.904	0.887	1.021	0.966	0.932	0.918	0.801
ORE	0.648	0.654	0.700	0.701	0.692	0.685	0.675	0.899	0.780	0.715	0.910	0.902	1.061	0.969	0.934	0.909	0.802
VECH	0.726	0.751	0.696	0.697	0.687	0.679	0.668	0.901	0.779	0.713	0.902	0.871	0.971	0.969	0.936	0.926	0.804
OGARCH	0.690	0.692	0.723	0.723	0.715	0.707	0.694	0.928	0.810	0.732	0.939	0.898	0.915	1.005	0.988	0.965	0.820
CCC	0.675	0.675	0.711	0.711	0.702	0.695	0.685	0.913	0.793	0.723	0.910	0.869	0.937	0.979	0.949	0.946	0.804
DCC	1.237	2.936	0.741	0.742	0.733	0.726	0.716	0.913	0.793	0.723	0.910	0.869	0.937	0.979	0.949	0.946	0.991
DECO	0.707	0.710	0.713	0.712	0.706	0.699	0.688	0.913	0.793	0.723	0.910	0.869	0.937	0.979	0.949	0.946	0.810
ASYDCC	0.682	0.706	0.725	0.732	0.717	0.710	0.698	1.003	0.913	0.793	0.991	1.023	1.322	1.056	1.064	1.083	0.889
Average	0.757	0.979	0.713	0.714	0.704	0.697	0.686	0.921	0.805	0.729	0.922	0.898	1.013	0.988	0.963	0.955	0.840
CORR	0.666	0.667	0.716	0.716	0.708	0.702	0.691	0.921	0.803	0.732	0.928	0.887	0.931	0.999	0.978	0.953	0.812
PARA	0.669	0.670	0.718	0.718	0.709	0.702	0.692	0.925	0.809	0.733	0.932	0.895	0.950	1.003	0.984	0.962	0.817
MARKET	0.668	0.669	0.717	0.717	0.709	0.702	0.692	0.921	0.803	0.732	0.928	0.887	0.931	0.999	0.978	0.953	0.813
Average	0.668	0.669	0.717	0.717	0.709	0.702	0.692	0.922	0.805	0.732	0.929	0.890	0.938	1.001	0.980	0.956	0.814

Table 5 – Portfolio Turnover: the table shows the average turnover obtained by the portfolios for each period of the sample as described in section 3.2, as well as their average by portfolio selection policy and covariance specification.

	MiVU	MeVU	MiVC	MeVC	NCMV2.5	NCMV5	NCMV10	VT1	VT2	VT4	RwRS1	RwRS2	RwRS4	RwR4F1	RwR4F2	RwR4F4	Average
EWMA	0.296	0.307	0.074	0.074	0.077	0.079	0.083	0.016	0.027	0.046	0.021	0.031	0.042	0.017	0.028	0.046	0.079
ORE	0.089	0.090	0.023	0.023	0.024	0.025	0.027	0.008	0.010	0.014	0.016	0.019	0.021	0.011	0.013	0.018	0.027
VECH	0.473	0.496	0.141	0.139	0.145	0.149	0.158	0.030	0.055	0.095	0.032	0.055	0.082	0.030	0.054	0.093	0.139
OGARCH	0.051	0.052	0.031	0.032	0.032	0.034	0.037	0.011	0.014	0.014	0.018	0.025	0.035	0.013	0.019	0.032	0.028
CCC	0.403	0.393	0.216	0.216	0.222	0.228	0.238	0.055	0.112	0.206	0.056	0.108	0.185	0.053	0.100	0.180	0.186
DCC	1.828	4.156	0.300	0.296	0.307	0.314	0.329	0.055	0.112	0.206	0.056	0.108	0.185	0.053	0.100	0.180	0.537
DECO	0.264	0.261	0.209	0.210	0.215	0.221	0.227	0.055	0.112	0.206	0.056	0.108	0.185	0.053	0.100	0.180	0.166
ASYDCC	0.410	0.404	0.170	0.163	0.174	0.178	0.186	0.029	0.055	0.112	0.031	0.052	0.059	0.028	0.051	0.093	0.137
Average	0.477	0.770	0.146	0.144	0.150	0.154	0.161	0.032	0.062	0.112	0.036	0.063	0.099	0.032	0.058	0.103	0.162
CORR	0.019	0.021	0.005	0.005	0.005	0.005	0.006	0.007	0.006	0.005	0.016	0.019	0.023	0.010	0.011	0.012	0.011
PARA	0.021	0.022	0.005	0.005	0.005	0.006	0.006	0.007	0.006	0.005	0.016	0.019	0.022	0.010	0.011	0.012	0.011
MARKET	0.021	0.022	0.005	0.005	0.005	0.006	0.006	0.007	0.006	0.005	0.016	0.019	0.023	0.010	0.011	0.012	0.011
Average	0.020	0.022	0.005	0.005	0.005	0.006	0.006	0.007	0.006	0.005	0.016	0.019	0.023	0.010	0.011	0.012	0.011

Table 6 – Sharpe ratios based on portfolio gross returns: the table shows the Sharpe ratio for the portfolios, in the absence of transaction costs for the whole sample as described in section 3.2. The asterisks represent the ratios that, when compared to the index of benchmark specification PARA, reject the hypothesis of equality when applied the test proposed by Ledoit & Wolf (2008).

	MiVU	MeVU	MiVC	MeVC	NCMV2.5	NCMV5	NCMV10	VT1	VT2	VT4	RwRS1	RwRS2	RwRS4	RwR4F1	RwR4F2	RwR4F4	Average
EWMA	0.103	0.107	0.101*	0.103*	0.104*	0.107*	0.111*	0.069	0.081*	0.090*	0.073*	0.092*	0.106*	0.061	0.065*	0.068	0.090
ORE	0.099	0.100*	0.086	0.087	0.088	0.091	0.095	0.069*	0.078*	0.080*	0.070*	0.086*	0.096	0.060*	0.062*	0.065	0.082
VECH	0.106	0.111	0.103*	0.106*	0.106*	0.108*	0.113*	0.069	0.081*	0.087*	0.071*	0.087*	0.102	0.062*	0.066*	0.072*	0.091
OGARCH	0.094	0.093	0.078	0.078	0.080	0.081	0.083	0.066	0.073	0.073	0.064	0.071	0.083	0.057	0.058	0.061	0.075
CCC	0.088	0.088	0.087	0.087	0.089	0.091	0.092	0.067	0.078	0.086	0.067	0.076	0.082	0.060	0.062	0.065	0.079
DCC	0.039	0.036	0.100	0.101	0.103	0.105	0.108	0.067	0.078	0.086	0.067	0.076	0.082	0.060	0.062	0.065	0.077
DECO	0.097	0.097	0.084	0.084	0.085	0.086	0.086	0.067	0.078	0.086	0.067	0.076	0.082	0.060	0.062	0.065	0.079
ASYDCC	0.112*	0.111*	0.101*	0.102*	0.103*	0.106*	0.110*	0.061	0.067	0.078	0.063	0.073	0.078	0.054	0.054	0.054	0.083
Average	0.092	0.093	0.093	0.093	0.095	0.097	0.100	0.067	0.077	0.083	0.068	0.080	0.089	0.059	0.061	0.064	0.082
CORR	0.078	0.078	0.076	0.076	0.078	0.080	0.083	0.066	0.073	0.072	0.066	0.075	0.087	0.057	0.057	0.056	0.072
PARA	0.079	0.078	0.077	0.077	0.080	0.081	0.084	0.066	0.072	0.072	0.065	0.075	0.087	0.057	0.056	0.056	0.073
MARKET	0.078	0.078	0.077	0.077	0.079	0.081	0.084	0.066	0.073	0.072	0.066	0.075	0.087	0.057	0.057	0.056	0.073
Average	0.078	0.078	0.077	0.077	0.079	0.081	0.084	0.066	0.072	0.072	0.065	0.075	0.087	0.057	0.057	0.056	0.073

Table 7 – Portfolio $1/N$ Statistics: the table shows the descriptive statistics for the $1/N$ portfolio: average return, standard deviation of the returns, turnover and the Sharpe Index of the net returns when the transaction costs is 0bp, 10bp, 20bp and 50bp.

Average Return	Standard Deviation	Turnover	Sharpe 0bp	Sharpe 10bp	Sharpe 20bp	Sharpe 50bp
0.060	1.112	0.008	0.054	0.054	0.053	0.051

Appendix B

Sharpe Index Evolution

Table 8 – Break-even transaction costs: the table shows the lowest transaction cost, in basis points, so that the average return of the portfolio is less or equal to zero. That is, when subject to transaction costs lower than the ones in the table, the portfolios have positive average net return.

	MiVU	MeVU	MiVC	MeVC	NCMV2.5	NCMV5	NCMV10	VT1	VT2	VT4	RwRS1	RwRS2	RwRS4	RwR4F1	RwR4F2	RwR4F4	Average
EWMA	25	25	95	98	94	93	90	403	241	141	321	267	260	353	223	140	179.3
ORE	72	73	261	267	253	249	238	749	626	408	416	415	498	566	457	337	367.8
VECH	17	17	52	54	51	50	48	214	116	66	202	140	123	202	117	74	96.4
OGARCH	129	125	184	177	179	173	159	565	434	387	334	256	217	459	311	191	267.5
CCC	16	16	30	29	29	28	27	113	56	31	111	63	42	115	61	36	50.2
DCC	3	3	26	26	25	25	24	113	56	31	111	63	42	115	61	36	47.5
DECO	27	27	30	29	29	28	27	113	56	31	111	63	42	115	61	36	51.6
ASYDCC	19	20	44	47	43	43	42	217	113	56	203	148	179	208	115	65	97.6
Average	38.5	38.3	90.3	90.9	87.9	86.1	81.9	310.9	212.3	143.9	226.1	176.9	175.4	266.6	175.8	114.4	144.7
CORR	270	248	1194	1172	1140	1064	926	850	974	1173	392	350	362	597	525	454	730.7
PARA	254	235	1167	1126	1101	1032	923	847	967	1162	392	352	373	596	525	452	719.0
MARKET	254	235	1180	1146	1126	1036	932	850	974	1173	392	350	362	597	525	454	724.1
Average	259.3	239.3	1180.3	1148.0	1122.3	1044.0	927.0	849.0	971.7	1169.3	392.0	350.7	365.7	596.7	525.0	453.3	724.6

Table 9 – Break-even of the Sharpe ratio against the benchmark PARA: the table shows the lowest transaction cost, in basis points, that makes the Sharpe ratio of each portfolio when compared to the benchmark PARA through the test proposed by Ledoit & Wolf (2008) reject the null hypothesis that the ratios are equal. Therefore, when the portfolios are subject to transaction costs higher than the ones in the table, the portfolios have Sharpe ratios lower and different than the benchmark. The 'X' in the table are portfolios that are always superior to the benchmark. It is important to associate this table with table 6. The various ratios presented in table 6 that are not statistically different from the benchmark continue not to be until the value presented in this table. So the majority of the portfolios did not present a Sharpe ratio greater than benchmark for any transaction cost.

	MiVU	MeVU	MiVC	MeVC	NCMV2.5	NCMV5	NCMV10	VT1	VT2	VT4	RwRS1	RwRS2	RwRS4	RwR4F1	RwR4F2	RwR4F4	Average
EWMA	15	17	41	43	41	41	40	79	62	52	200	208	215	106	88	71	82.4
ORE	38	41	71	79	68	68	72	387	190	106	X	X	X	583	387	260	334.4
VECH	11	12	23	25	23	23	22	31	26	22	58	57	62	41	37	36	31.8
OGARCH	96	98	19	19	18	18	17	67	48	34	56	34	36	74	68	72	48.4
CCC	7	8	9	9	9	9	8	14	11	11	16	12	10	18	17	16	11.5
DCC	2	2	12	13	12	12	11	14	11	11	16	12	10	18	16	15	11.7
DECO	16	17	9	9	8	8	7	14	11	11	16	12	10	18	16	15	12.3
ASYDCC	12	12	19	22	19	19	20	10	14	17	23	44	82	13	16	20	22.6
Average	24.6	25.9	25.4	27.4	24.8	24.8	24.6	77.0	46.6	33.0	173.1	172.4	178.1	108.9	80.6	63.1	69.4

Figure 1 – Evolution of Sharpe ratio with the increase of transaction cost for the Minimum Variance and Mean Variance Portfolios: the graph shows how the transaction cost reduces the Sharpe ratio for each portfolio. Each graph compares a covariance specification with the benchmark PARA using the test proposed by Ledoit & Wolf (2008). The dotted line indicates the part where the result of the test do not rejects the null hypothesis that the ratios are different from each other.

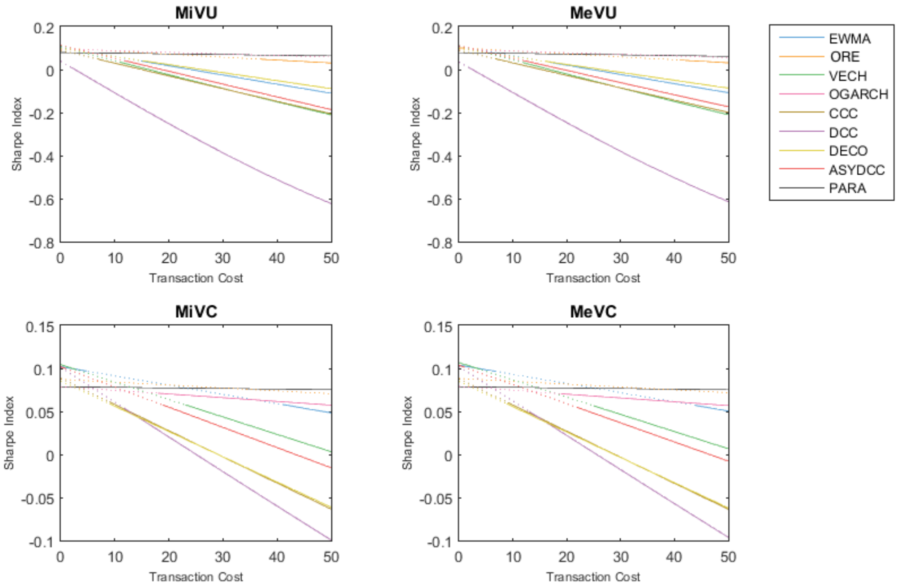
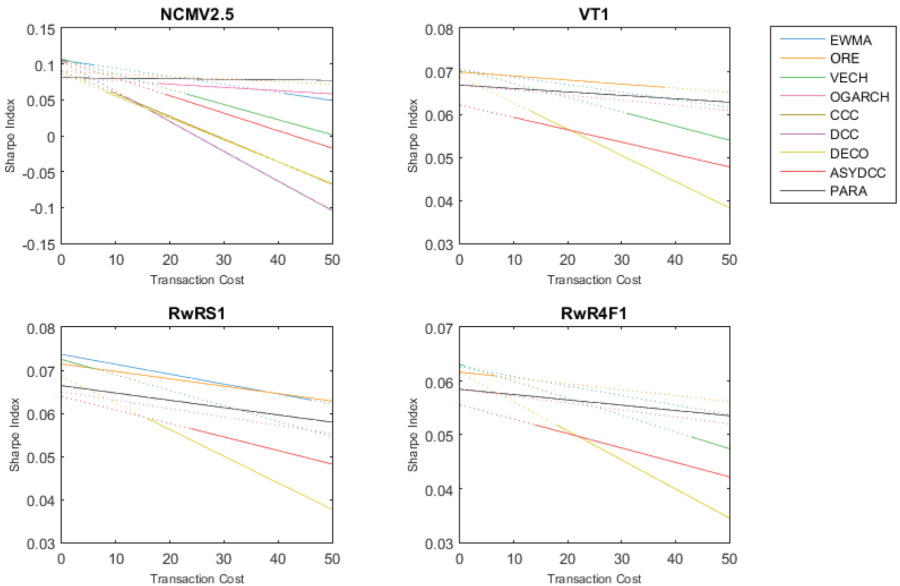


Figure 2 – Evolution of Sharpe ratio with the increase of transaction cost for the portfolios that ignore covariance: the graph shows how the transaction cost reduces the Sharpe ratio for each portfolio. Each graph compares a covariance specification with the benchmark PARA using the test proposed by Ledoit & Wolf (2008). The dotted line indicates the part where the result of the test do not rejects the null hypothesis that the ratios are different from each other.



Appendix C

Portfolio Selection and Combination

Figure 3 – Portfolio contribution in the DMA strategy: the graph shows the contribution of each portfolio in the weights of the combination portfolio DMA at the transaction costs 0bp, 10bp, 20bp and 50bp. Each bar shows, in aggregate, the contribution of each portfolio for the weights of the DMA across all periods. The policies are: 1 - $1/N$; 2 - MiVU; 3 - MeVU; 4 - MiVC; 5 - MeVC; 6 - NCMV2.5; 7 - RwRS1; 8 - VT1; 9 - RwR4F1. The covariances are: 1 - EWMA; 2 - ORE; 3 - VECH; 4 - OGARCH; 5 - CCC; 6 - DCC; 7 - DECO; 8 - ASYDCC; 9 - CORR; 10 - PARA; 11 - MARKET.

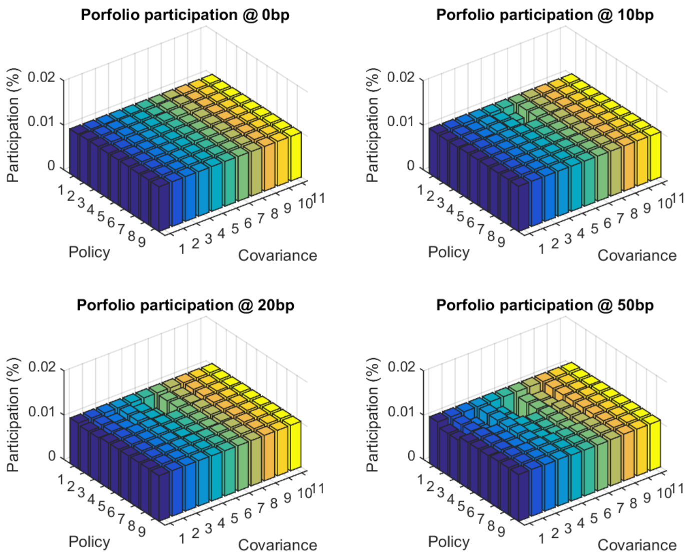


Figure 4 – Portfolio participation in the DMS strategy: the graph shows the participation of each portfolio in the weights of the combination portfolio DMS at the transaction costs 0bp, 10bp, 20bp and 50bp. Each bar shows the percentage of times an specific portfolio was selected as the weights of the DMS portfolio. The policies are: 1 - $1/N$; 2 - MiVU; 3 - MeVU; 4 - MiVC; 5 - MeVC; 6 - NCMV2.5; 7 - RwRS1; 8 - VT1; 9 - RwR4F1. The covariances are: 1 - EWMA; 2 - ORE; 3 - VECH; 4 - OGARCH; 5 - CCC; 6 - DCC; 7 - DECO; 8 - ASYDCC; 9 - CORR; 10 - PARA; 11 - MARKET.

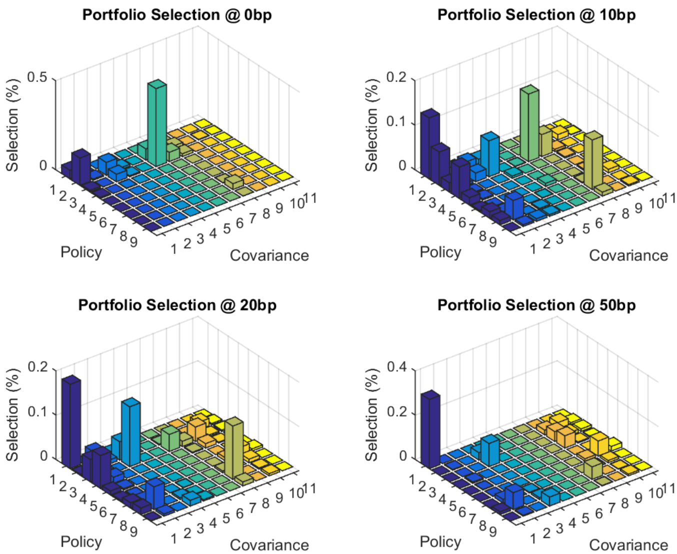


Figure 5 – Evolution of Sharpe ratio with the increase of transaction cost in the DMA strategy - the Minimum Variance and Mean Variance Portfolios: the graph shows how the transaction cost reduces the Sharpe ratio for each portfolio. Each graph compares a covariance specification with the DMA portfolio using the test proposed by Ledoit & Wolf (2008). The dotted line indicates the part where the result of the test do not rejects the null hypothesis that the ratios are different from each other.

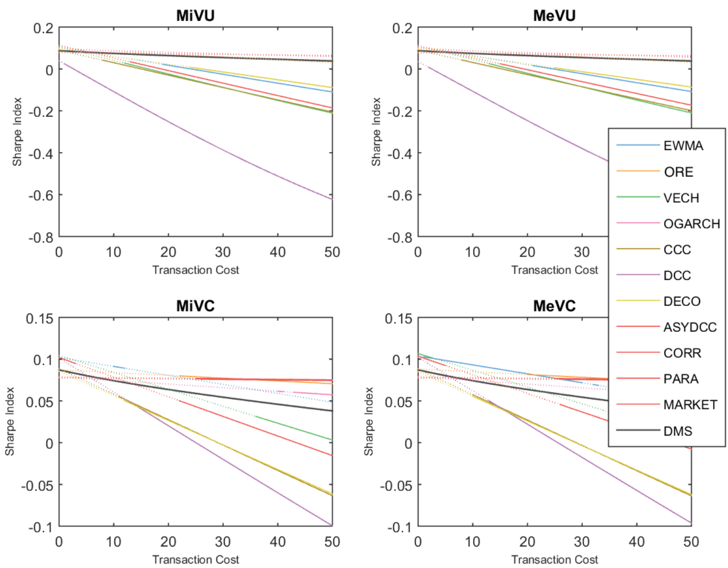


Figure 6 – Evolution of Sharpe ratio with the increase of transaction cost in the DMA strategy - portfolio policies that ignore covariance: the graph shows how the transaction cost reduces the Sharpe ratio for each portfolio. Each graph compares a covariance specification with the DMA portfolio using the test proposed by Ledoit & Wolf (2008). The dotted line indicates the part where the result of the test do not rejects the null hypothesis that the ratios are different from each other.

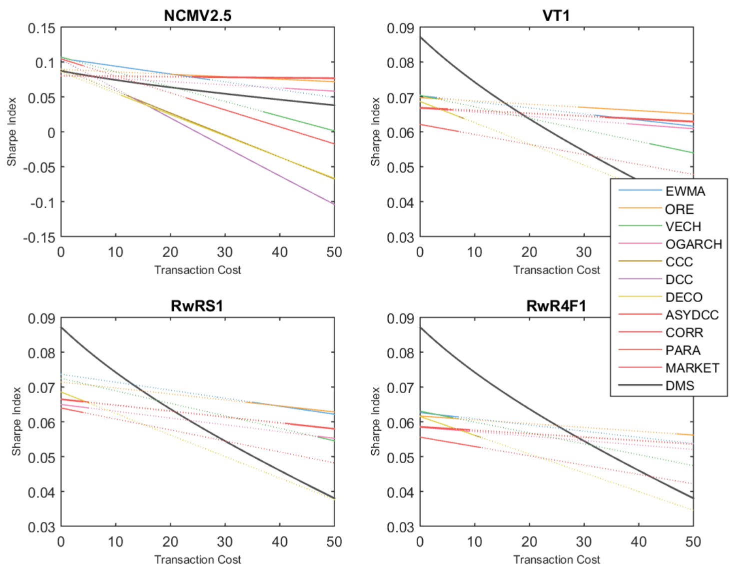


Figure 7 – Evolution of Sharpe ratio with the increase of transaction cost in the DMS strategy - Minimum Variance and Mean Variance Portfolios: the graph shows how the transaction cost reduces the Sharpe ratio for each portfolio. Each graph compares a covariance specification with the DMA portfolio using the test proposed by Ledoit & Wolf (2008). The dotted line indicates the part where the result of the test do not rejects the null hypothesis that the ratios are different from each other.

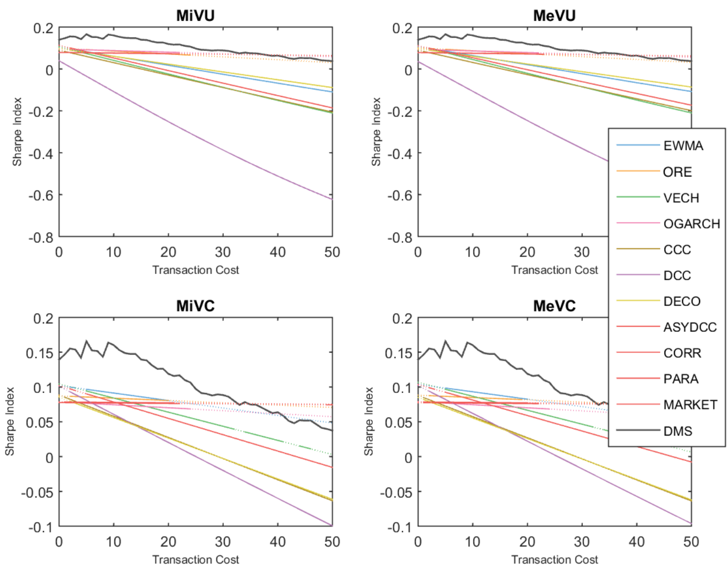
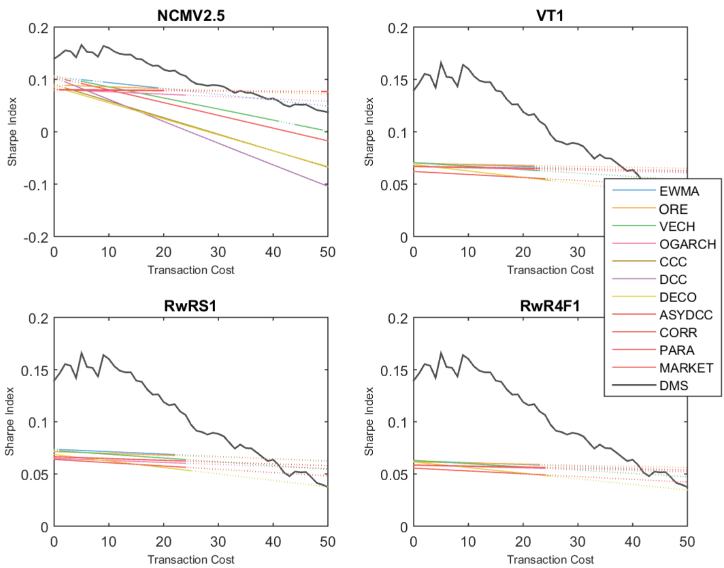


Figure 8 – Evolution of Sharpe ratio with the increase of transaction cost in the DMS strategy - portfolio policies that ignore covariance: the graph shows how the transaction cost reduces the Sharpe ratio for each portfolio. Each graph compares a covariance specification with the DMA portfolio using the test proposed by Ledoit & Wolf (2008). The dotted line indicates the part where the result of the test do not rejects the null hypothesis that the ratios are different from each other.



Appendix D

Robustness Analysis

Table 10 – Break-even transaction cost for null average portfolio return with weekly rebalancing: the table shows the lowest transaction cost, in basis points, so that the average return of the portfolio is less or equal to zero, with weekly rebalancing. Meaning that when subject to transaction costs lower than the ones one the table, the portfolios have positive average net return. And when subject to higher transaction costs, average net return negative.

	MiVU	MeVU	MiVC	MeVC	NCMV2.5	NCMV5	NCMV10	VT1	VT2	VT4	RwRS1	RwRS2	RwRS4	RwR4F1	RwR4F2	RwR4F4	Average
EWMA	40	41	160	163	158	155	150	468	327	216	419	380	388	420	305	216	250.4
ORE	99	99	365	370	353	347	331	747	667	495	531	554	674	607	527	435	450.1
VECH	30	30	91	94	90	88	85	292	182	113	289	225	208	278	183	126	150.3
OGARCH	176	170	279	270	271	262	242	615	527	514	441	372	346	521	404	293	356.4
CCC	27	27	58	58	57	56	53	186	104	60	187	121	87	186	112	70	90.6
DCC	7	4	46	46	46	45	44	186	104	60	187	121	87	186	112	70	84.4
DECO	48	48	57	56	54	53	50	186	104	60	187	121	87	186	112	70	92.4
ASYDCC	35	35	82	88	81	80	78	305	186	104	298	245	311	290	185	118	157.6
Average	57.8	56.8	142.3	143.1	138.8	135.8	129.1	373.1	275.1	202.8	317.4	267.4	273.5	334.3	242.5	174.8	204.0
CORR	273	254	1196	1172	1143	1074	942	848	970	1163	509	490	552	637	584	543	771.9
PARA	258	242	1173	1141	1113	1047	934	845	963	1152	508	492	563	635	584	539	761.8
MARKET	259	241	1184	1154	1132	1050	943	848	970	1163	509	490	552	637	584	543	766.2
Average	263.3	245.7	1184.3	1155.7	1129.3	1057.0	939.7	847.0	967.7	1159.3	508.7	490.7	555.7	636.3	584.0	541.7	766.6

Table 11 – Break-even transaction cost for null average portfolio return with monthly rebalancing: the table shows the lowest transaction cost, in basis points, so that the average return of the portfolio is less or equal to zero, with monthly rebalancing. Meaning that when subject to transaction costs lower than the ones one the table, the portfolios have positive average net return. And when subject to higher transaction costs, average net return negative.

	MiVU	MeVU	MiVC	MeVC	NCMV2.5	NCMV5	NCMV10	VT1	VT2	VT4	RwRS1	RwRS2	RwRS4	RwR4F1	RwR4F2	RwR4F4	Average
EWMA	71	70	292	297	285	278	267	580	476	353	557	562	620	523	430	336	374.8
ORE	138	138	537	544	522	508	480	770	749	639	644	706	872	654	605	542	565.5
VECH	54	54	190	194	185	181	169	433	318	223	440	392	393	401	298	221	259.1
OGARCH	232	221	450	440	436	416	383	680	649	692	551	500	501	594	515	419	479.9
CCC	65	67	159	162	156	152	144	346	232	157	354	285	268	328	222	150	202.9
DCC	12	7	106	106	104	101	96	346	232	157	354	285	268	328	222	150	179.6
DECO	98	100	151	150	145	140	131	346	232	157	354	285	268	328	222	150	203.6
ASYDCC	71	69	190	203	189	186	181	470	346	232	461	451	682	430	314	216	293.2
Average	92.6	90.8	259.4	262.0	252.8	245.3	231.4	496.4	404.3	326.3	464.4	433.3	484.0	448.3	353.5	273.0	319.8
CORR	279	263	1211	1193	1158	1090	966	847	967	1159	621	634	744	674	643	631	817.5
PARA	265	251	1192	1169	1133	1071	956	845	961	1149	620	635	752	673	641	624	808.6
MARKET	265	250	1202	1182	1149	1080	963	847	967	1159	621	634	744	674	643	631	813.2
Average	269.7	254.7	1201.7	1181.3	1146.7	1080.3	961.7	846.3	965.0	1155.7	620.7	634.3	746.7	673.7	642.3	628.7	813.1

Table 12 – Break-even transaction cost for null average portfolio return with different rebalancing frequencies: this table repeats the averages presented in tables 8, 10 and 11 together for easy comparison

	MiVU	MeVU	MiVC	MeVC	NCMV2.5	NCMV5	NCMV10	VT1	VT2	VT4	RwRS1	RwRS2	RwRS4	RwR4F1	RwR4F2	RwR4F4	Average
Daily																	
Dynamic	38.5	38.3	90.3	90.9	87.9	86.1	81.9	310.9	212.3	143.9	226.1	176.9	175.4	266.6	175.8	114.4	144.7
Static	259.3	239.3	1180.3	1148.0	1122.3	1044.0	927.0	849.0	971.7	1169.3	392.0	350.7	365.7	596.7	525.0	453.3	724.6
Weekly																	
Dynamic	57.8	56.8	142.3	143.1	138.8	135.8	129.1	373.1	275.1	202.8	317.4	267.4	273.5	334.3	242.5	174.8	204.0
Static	263.3	245.7	1184.3	1155.7	1129.3	1057.0	939.7	847.0	967.7	1159.3	508.7	490.7	555.7	636.3	584.0	541.7	766.6
Monthly																	
Dynamic	92.6	90.8	259.4	262.0	252.8	245.3	231.4	496.4	404.3	326.3	464.4	433.3	484.0	448.3	353.5	273.0	319.8
Static	269.7	254.7	1201.7	1181.3	1146.7	1080.3	961.7	846.3	965.0	1155.7	620.7	634.3	746.7	673.7	642.3	628.7	813.1

Table 13 – Break-even transaction cost for null average portfolio return with different risk aversions (γ): the table shows the lowest transaction cost, in basis points, so that the average return of the portfolio is less or equal to zero, when the portfolio selection policy consider different risk aversion parameter values. Meaning that when subject to transaction costs lower than the ones one the table, the portfolios have positive average net return. And when subject to higher transaction costs, average net return negative.

	$\lambda = 1$					$\lambda = 2$				
	MeVU	MeVC	NCMV2.5	NCMV5	NCMV10	MeVU	MeVC	NCMV2.5	NCMV5	NCMV10
EWMA	25	98	94	93	90	25	96	94	92	88
ORE	73	267	253	249	238	73	265	255	246	236
VECH	17	54	51	50	48	17	53	51	50	48
OGARCH	125	177	179	173	159	127	181	181	174	159
CCC	16	29	29	28	27	16	29	29	28	27
DCC	3	26	25	25	24	3	26	25	25	24
DECO	27	29	29	28	27	27	29	29	28	27
ASYDCC	20	47	43	43	42	20	45	43	43	41
CORR	248	1172	1140	1064	926	262	1189	1133	1074	949
PARA	235	1126	1101	1032	923	247	1158	1109	1047	924
MARKET	235	1146	1126	1036	932	248	1174	1120	1059	943
Average	93.1	379.2	370.0	347.4	312.4	96.8	385.9	369.9	351.5	315.1

	$\lambda = 5$					$\lambda = 10$				
	MeVU	MeVC	NCMV2.5	NCMV5	NCMV10	MeVU	MeVC	NCMV2.5	NCMV5	NCMV10
EWMA	25	96	93	91	87	25	96	93	91	87
ORE	73	262	255	249	237	73	262	255	250	237
VECH	17	52	51	49	47	17	52	51	49	47
OGARCH	128	183	181	174	159	129	183	181	174	159
CCC	16	29	29	28	27	16	29	29	28	27
DCC	3	26	25	25	24	3	26	25	25	24
DECO	27	30	29	28	27	27	30	29	28	27
ASYDCC	19	44	43	42	40	19	44	43	42	40
CORR	267	1193	1136	1078	958	269	1194	1136	1078	960
PARA	252	1165	1112	1052	934	253	1165	1113	1053	936
MARKET	253	1180	1126	1065	952	254	1180	1127	1061	952
Average	98.2	387.3	370.9	352.8	317.5	98.6	387.4	371.1	352.6	317.8

Appendix E

Assets Considered

Table 14 – Assets Considered and its four first moments: the table shows which assets were utilized to compose the sample used in the study and their first four moments. We used daily data from 01/2004 to 01/2014. We can observe in the table some of the stylized facts of financial time series: the returns series present asymmetry showing that returns are stronger in one direction than in the other; all the assets have high kurtosis when compared to the normal distribution whose kurtosis is 3.

Ticker	Company	Mean	Variance	Asymmetry	Kurtosis
AAPL	Apple Inc.	0.159	5.445	-0.092	7.968
BAC	Bank of America Corp	-0.026	13.051	-0.307	24.099
MSFT	Microsoft Corporation	0.024	2.881	0.082	14.252
INTC	Intel Corporation	0.002	3.654	-0.308	8.000
CSCO	Cisco Systems. Inc.	-0.001	4.025	-0.404	13.272
GE	General Electric Company	0.009	3.783	-0.012	14.653
F	Ford Motor Company	0.004	8.980	-0.075	18.679
PFE	Pfizer Inc.	0.010	2.277	-0.357	11.060
ORCL	Oracle Corporation	0.044	3.583	-0.042	8.336
WFC	Wells Fargo & Co	0.029	8.662	0.878	24.915
JPM	JPMorgan Chase & Co.	0.029	7.398	0.337	18.011
C	Citigroup Inc	-0.080	14.564	-0.480	36.052
EMC	EMC Corporation	0.026	4.166	0.194	7.124
T	AT&T Inc.	0.032	1.995	0.528	14.249
XOM	Exxon Mobil Corporation	0.045	2.532	0.016	17.286
FCX	Freeport-McMoRan Inc	0.038	10.390	-0.497	8.497
CMCSA	Comcast Corporation	0.039	3.827	0.042	16.210
HPQ	Hewlett-Packard Company	0.013	4.413	-0.481	15.320
KO	The Coca-Cola Co	0.031	1.372	0.427	16.323
GILD	Gilead Sciences. Inc.	0.093	4.032	-0.126	9.436
QCOM	QUALCOMM. Inc.	0.046	4.058	0.001	9.491
MS	Morgan Stanley	-0.010	12.801	1.635	53.171
EBAY	eBay Inc	0.022	5.593	-0.010	10.961
VZ	Verizon Communications Inc.	0.037	1.897	0.303	11.446
HAL	Halliburton Company	0.059	6.861	-0.504	10.038
MRK	Merck & Co.. Inc.	0.019	3.291	-2.412	46.201
WMT	Wal-Mart Stores. Inc.	0.024	1.463	0.144	10.106
TXN	Texas Instruments Incorporated	0.022	3.795	-0.257	6.888
ABT	Abbott Laboratories	0.038	1.642	0.089	9.418
HD	Home Depot Inc	0.043	3.036	0.406	7.956
COP	ConocoPhillips	0.054	3.748	-0.457	10.739
MO	Altria Group Inc	0.066	1.627	-0.171	20.595
USB	U.S. Bancorp	0.025	5.727	-0.024	19.092
LOW	Lowe's Companies. Inc.	0.029	3.911	0.287	7.734
JNJ	Johnson & Johnson	0.034	1.020	0.584	16.367
BMY	Bristol-Myers Squibb Co	0.042	2.157	0.083	8.260
DIS	Walt Disney Co	0.052	3.116	0.412	11.689
PG	Procter & Gamble Co	0.030	1.261	-0.187	10.414
SLB	Schlumberger Limited.	0.052	5.713	-0.554	10.360
CVX	Chevron Corporation	0.055	2.901	0.074	17.512
SBUX	Starbucks Corporation	0.064	4.504	0.275	8.587
CVS	CVS Health Corp	0.059	2.929	-1.073	21.903
MDLZ	Mondelez International Inc	0.032	1.606	-0.122	8.323
AXP	American Express Company	0.037	6.019	0.069	14.596
DOW	Dow Chemical Co	0.017	5.462	-0.433	11.118
UNH	UnitedHealth Group Inc.	0.040	4.811	0.417	27.028
GS	Goldman Sachs Group Inc	0.028	6.317	0.304	18.093
AMGN	Amgen. Inc.	0.026	2.789	0.729	12.046
TWX	Time Warner Inc	0.023	3.565	-0.645	20.400
CAT	Caterpillar Inc.	0.041	4.474	-0.198	8.585

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