Modern Mathematics in Brazil:
The promise of democratic and effective teaching

Elisabete Zardo Búrigo

Universidade Federal do Rio Grande do Sul

burigo@mat.ufrgs.br

Abstract

The modern mathematics movement, which originated in Europe and in the United States, reverberated in Brazil, more so than in other Latin American countries. In order to understand the local extension of this movement, elements of the reality of secondary education in Brazil are considered as well as the local interpretations and appropriations of the new curricular proposals, in particular by the Grupo de Estudos em Ensino de Matemática (Study Group on Mathematics Teaching – GEEM), created in the city of São Paulo in 1961. The reach of a movement that promised the "modernisation" of teaching, and that identified modernisation with democratisation, must also be understood within the context of optimism and a belief in the benefits of technical progress.

Modern Mathematics in Brazil:
The promise of democratic and effective teaching

The modern mathematics movement, which originated in Europe and the United States, found significant resonance in Brazil, more so than in other Latin American countries. Among the impacts of modern mathematics in this country, the following should be mentioned: the creation of several study groups dedicated to the teaching of mathematics, the systematic realisation of courses for teachers over a fifteen year period, the reformulation of textbooks and the inclusion of new topics in the curriculum programs within primary and secondary education.

As in other countries in this continent, the origins of the adherence to a set of proposals identified as "modern mathematics" can be found externally. The invitation for the participation of teachers from these countries, in training programs at universities in North America, and the promotion of new textbooks are some of the main ways in which such influence was expressed. According to Ruiz and Barrantes (1998), the staging of the first Inter American Conference of Mathematics Education, in December 1961, at the initiative of the International Commission on Mathematical Instruction (ICMI), and the establishment of an Inter American Committee of Mathematics Education, had a core purpose of extending modern mathematics to Latin American countries. The sponsorships by the Organization of American States (OAS), the North American Ford Foundation, the Rockefeller Foundation and National Science Foundation and the resolutions adopted at the Conference would be indicative of the interests involved in the instigation of a series of initiatives aimed at reforming the teaching of mathematics in Latin America.

However, incentives or external pressure are not sufficient to justify the initiative itself, the intensity and the extent of the involvement of Brazilian teachers in the renewal of the teaching of mathematics in this country nor the extent of the changes that were triggered by that

movement. To understand these processes, we must consider elements of Brazilian urban reality in the beginning of the 1960s, alongside the interpretations and local appropriations of the curricular proposals constructed in the United States and European countries (Búrigo, 1989, 1990).

Secondary education in the early 1960s

Since the 1930s, Brazil has lived through a process of intense industrialisation orientated on the internal market, without equal in Latin America (Fonseca, 1987). In the second half of the 1950s this process was accelerated: industrial growth averaged 11% a year, strengthening so-called "heavy industry" - steel, transport and energy production, amongst others - as well as the production of capital goods, and the implementation of a local automotive industry (Nunes; Xausa, 1987).

Urban growth was also strong in the period between 1940 and 1960, whereby the ratio of urban population increased from 31% to 45% of the total population (IBGE, 2007). The diversification of employment, an increased contingent of urban workers and the proliferation of the so-called "middle sectors" is reflected in an increased demand for secondary education[1]. Until the 1920s, secondary education in Brazil was poorly structured: "preparatory exams", which were divided according to the disciplines, were sufficient for entrance into universities (Valente, 2004; Nunes, 2000) [2]. Only in 1929, as a condition of access to higher education, and as a result of the Rocha Vaz reform, was the attendance level requirement instituted in secondary education, which was then organised into six series. In 1960, access to secondary education was still restricted: the enrolment in post-primary education [3] only accounted for 11.5% of the population of 12 to 18 year olds (Gouveia; Havighurst, 1969). Entrance to the first cycle of secondary education, called *ginásio*, was covered by an "examination for admission," in which the mathematics' test served as an important selective function.

Catering to the minority, secondary education, however, was an object of aspiration for the increasing urban sectors. Between 1933 and 1958, the number of enrolments in the first cycle of secondary education increased more than 10 times, from approximately 65 thousand to 659 thousand (IBGE, 2007), and grew rapidly during the 1960s.

A large part of that expansion was concentrated in São Paulo, the most urban and industrial centre in this country. The number of *ginásios* in the city of São Paulo increased from three in 1940 to sixty-five in 1958 (Bontempi Jr., cited in Valente, 2008). The participation of public education in the provision of school places in São Paulo was also above the national average (IBGE, 2007).

This expansion entailed profound changes in the make-up of secondary education.

Firstly, there was a significant demand for teachers. With the creation of Faculties of Philosophy, from the early 30s, the figure of the licensed teacher was instituted as a referent for teacher training and performance in secondary education (Valente, 2005). In 1960, however, "lay" teachers largely predominated, recruited through emergency procedures and without licensing courses [4]. These "lay" teachers were given an opportunity to formalise their appointment through participation in a "sufficiency test", preceded by courses organised by the Secretariat or the Ministry of Education.

The production of textbooks also accompanied the growth of the *ginásios*. Teacher training courses, promoted throughout the countryside of São Paulo, in a joint initiative of the Secretary of Education and publishers, were efficient mechanisms to disseminate these books.

At the institutional level, the traditional Colégio Pedro II [5], established in 1837, was still taken as a referent for secondary education throughout the entire country. The 1951 Ministry of Education and Health Decrees 966 and 1045 established the programs and methodological instructions, approved by the Colégio for the different disciplines, as being mandatory.

The standardisation of education derived from Colégio Pedro II, however, would be

challenged in the context of a search for identity and professional recognition by a wider group of licensed teachers. The first National Congresses on Mathematics Teaching, held in 1955 and 1957 on the initiative, respectively, of the Faculty of Philosophy at the University of Bahia and the Faculty of Philosophy at the University of Rio Grande Do Sul, had the questioning of this as one of its motivations.

In the 1950s, innovative pedagogical experiences were networked through secondary education. The majority of these, as in the case of the experimental classes, as well as the Schools of Application and Faculties of Philosophy, were focusing on methodological innovation, seeking greater student participation and reflecting the influences of "escolanovismo" [6]. In the Ginásios Vocacionais [7], established in São Paulo from 1962, there was still a preoccupation with reflecting on social reality and the general sense of overcoming an exclusively propaedeutic focus. The teaching of mathematics was also the subject of innovative experiments with the adoption of new materials and approaches, many of them reported in the Education Congresses.

The modern mathematics movement in São Paulo

The modern mathematics movement emerges in this context - not as the effect of the aspirations, of the debates and experiments, whereby secondary education was the object, but finding there a favourable environment for the dissemination of the renewal proposals. The teaching of mathematics was already the subject of debate between university professors and secondary school teachers, who saw themselves as responsible for the improvement of programs, and methods of teaching and learning. The government action, focused on the training of "lay" teachers, contributed to the expansion of the scope of this discussion, widening its audience.

The foundation of the Study Group on Mathematics Teaching (GEEM) in Sao Paulo in 1961 was one of the major landmarks of the movement for modernisation. The group originated in

a development course for teachers, organised in a way similar to a seminar offered by the University of Kansas at the time and attended by Professor Osvaldo Sangiorgi, then professor at the Universidade Mackenzie and the traditional Colégio Santa Cruz. The course included the topics: Sets Theory, Lineal Algebra, Mathematical Logic and "modern mathematics practices".

The structure of the course itself, in this first local initiative, already pointed to the reproduction of two elements guiding the movement of modern mathematics in central countries: the sense of adaptation of school mathematics to university mathematics, with the adoption of new subjects and the incorporation of concerns with the accuracy and precision of language and the unification of mathematics through sets and algebraic structures.

The proposals for modernisation through dissemination, however, also dialogued with the schooling demands, frustrated by the scarcity of school places and by the failure rate in admission tests. GEEM's discourse promised to overcome this elitist and ineffective teaching, promoting interest, curiosity and learning:

> Fortunately, for the previous two years, our students in secondary courses and now also in primary courses are receiving a new mathematics message: a mathematics that is within the reach of innate intelligence, through which they learn the subject without getting angry at it for the rest of their lives (Sangiorgi, 1965).

The promise of more effective teaching had one of its main pillars, the correspondence between the structures of intelligence and the "mother structures" of the mathematic building designed by the Bourbaki group, pointed out by Piaget (1955):

> The modern mathematical operation is not easy to teach, but it is much easier, because it reaches the child's mental structures (Sangiorgi, 1963).

GEEM also revealed influences from "escolanovismo", when opposing the understanding of concepts with the mechanisation of algorithms:

> The fundamental difference between the old and the modern method of the teaching of mathematics resides in the fact that the student is no longer obliged to accept fixed recipes that impeded the use of their creative spirit and, they now have permission to use several types of reasoning, thus having the possibility to contribute to improvement of modern techniques (Sangiorgi, 1966).

One oft-cited example was the greatest common divisor of two numbers, where the understanding of the concept "divisor", as a result of the intersection of two sets, came to substitute the blind application of the traditional Euclidian method. The definition of a fraction as an "ordered pair of natural numbers, the second being different to zero" (Sangiorgi, 1971, p. 208) presents a formalist bias which coexisted with the manifested concern in appealing to intuition and contextualisation. As examples attesting to this, the following, among others, can be mentioned: the use of a "special clock" for obtaining real numbers, with only a pointer and ten marks corresponding to the numeric digits (Sangiorgi, 1967) and a film of a container being emptied, projected in reverse, to illustrate multiplication by a negative factor (Babá, 1965).

The reconciliation intended between a more advanced, correct and, at the same time, more accessible mathematics was supported by the different groups composing GEEM. The Group relied on the participation of mathematicians who lent the academic authority of the renowned University of São Paulo, to modern mathematics. Their adhesion to the movement can be partly credited to French influence, and especially, to structuralism, in the origins of the Philosophy Faculty. Connections between the accuracy and effectiveness of mathematics teaching were already components of an academic culture, prevalent between mathematicians, that was understood as being distinctive to that of the engineering course.

At the same time, GEEM relied on the participation of licensed teachers who worked in primary and secondary education; some of them engaged in innovative experiences such as the Colégio de Aplicação of the University of Sao Paulo, Ginásio Vocacional do Brooklin or the Escola Experimental de Lapa. The teachers' participation in the activities of the Group contributed to the presentation of modern mathematics as being, on the one hand, presented as a viable proposal for implementation in schools and, on the other, as a movement for curricular renewal linked to other processes within the renewal of secondary education.

It is interesting to note that the president, and main leader of GEEM, Professor Osvaldo Sangiorgi, was not perceived as one of the most prestigious Brazilians by the organisers of the Inter American Conference of Mathematics Education. At the 1961 Conference, held in Bogota, the mathematician Omar Catunda was a speaker and Alfredo Pereira Gómez was nominated to join a temporary Inter American Committee of Mathematics Education, representing Brazil. At the conference in 1966, Osvaldo Sangiorgi gave a lecture on "Progress of the teaching of Mathematics in Brazil," but the mathematician Leopold Nachbin, participated in the Events Organising Committee and was elected to the Inter American Committee of Mathematics Education representing Brazil (Ruiz and Barrantes, 1998). Thus, the role played by Professor Osvaldo Sangiorgi should be credited, in particular, for his recognition as a renowned professor of secondary education, the construction of an eloquent discourse and his ability to articulate the various actors, especially in Sao Paulo.

The proposals for renewal of the teaching of mathematics considered by GEEM and its members were from diverse origins. The books produced by the School Mathematics Study Group (SMSG) [8] were amongst the main references utilised. But GEEM, in fact, sought interlocution with a broader set of mathematicians and educators such as George Springer, Marshall Stone, George Papy, Caleb Cattegno and Tamas Varga, who, amongst others, gave lectures at seminars sponsored by GEEM.

In August of 1962, Lucienne Felix and French colleagues, at the invitation of GEEM, conducted a three-week course entitled Principles and Methods of the New Pedagogy (Medina, 2008). From 1971, Zoltan Dienes also taught courses in São Paulo and Porto Alegre, at a time when members of GEEM were more concerned with primary education and the discussion of teaching methods.

D'Ambrosio (1987) points out this multiplicity of influences as having resulted in the drafting of an eclectic and inconsistent curriculum. In a more optimistic perspective, this eclecticism

can be interpreted as openness to different perspectives and as an indication that the group was not simply adhering to the proposal of a European or North American group.

The dissemination of modern mathematics by GEEM

GEEM carried out the dissemination of the renewal of education proposals, especially by conducting in-services for teachers and the production of new textbooks for primary and secondary education.

According to Lima and Passos (2008), from 1962 to 1968, GEEM held in-services during the school holidays for teachers in the city of São Paulo every six months. The courses were directed at teachers of state secondary education and relied on support from Government organs. The records from the course, held in February 1965, can offer us an idea of the dynamics of these courses, as analysed by these authors. The course had 400 students and was offered in three different stages, each with a duration of thirteen days and the lessons from 08:00 to 18:00. In addition to the disciplines of mathematics, such as Set Theory, Mathematical Logic, Modern Algebra and Analytic Geometry, the first stage of the courses also included the Modern Practices. In this, teachers were shown the content that they should teach in schools and the approaches for such content. The Modern Practices were organised by themes such as "Multiple and Divisors" or" Rational Numbers" and addressed by different teachers. In the in-services of 1965, twelve teachers conducted the disciplines on Mathematics and seven alternated the presentation of the Modern Practices. At the end of each course, the student-teachers sat a test before obtaining a certificate, which could then be used to further their career.

GEEM also offered in-services for teachers of primary education. According to Medina (2008), the first such in-service took place in February 1963, with the participation of 300 teachers. The offering of these in-services intensified from 1970.

Whilst the in-services offered by GEEM involved some hundreds of teachers every year, the

textbooks produced reached the teachers in the schools in far greater numbers.

Osvaldo Sangiorgi was already a successful author of textbooks in the 1950s. In 1963, he launched a new collection of textbooks for *ginásio* called "Mathematics: Modern Course". Its promotion was accompanied by the conducting of lectures and in-services for teachers in different regions of the country. The collection was Companhia Editora Nacional's best seller throughout the 1960s (Valente, 2008) and, for many teachers, it was the first contact they had with the proposals to modernise the teaching of mathematics.

But the production of new textbooks didn't end with the collection by Sangiorgi. Other members of GEEM also participated in the production of "modern" collections including: Benedict Castrucci, Jacy Monteiro and Renate Watanabe, professors at the University of São Paulo; Ruy Barbosa Madsen, professor of the Faculty of Philosophy of Araraquara, Alcides Boscolo and Scipione di Pierro Netto, secondary education teachers (Miorim, 2005). Anna Franchi, Lucilia Bechara and Manhucia Liberman produced the “Curso Moderno de Matemática para a Escola Elementar” (Modern Mathematic Course for Elementary School) collection, for primary education, of which more than 2 and a half million copies were published between 1967 and 1974 (Villela, 2008).

Other groups of modernisation protagonists

GEEM was not the only group to develop initiatives to modernise the teaching of mathematics in Brazil.

NEDEM - Núcleo de Estudos e Difusão do Ensino da Matemática (Centre for Research and Dissemination of the Teaching of Mathematics) - was created in Curitiba in 1962. The Centre produced two textbook collections which were circulated through the State School System from 1967. The collection for *ginásio* was based on the experiences at the Colégio Estadual do Paraná (Bertoni, 2008).

GEEMPA - Grupo de Estudos em Ensino de Matemática de Porto Alegre (Group for Research on the Teaching of Mathematics in Porto Alegre) - was created in Porto Alegre in 1970, bringing together teachers who were already involved in education reform. GEEMPA participated in an international research project directed by Zoltan Dienes. The concerns of the group were more focused on primary education and methodological issues.

CECIBA – Centro de Estudos e Ciências da Bahia (Centre for Studies and Science of Bahia) - was an organ which promoted courses and debates on issues related to modern mathematics in Salvador (Duarte, 1997).

In Niterói, Arago Backx coordinated experiences for the renewal of the teaching of mathematics in the Centro Educacional (Educational Center) [9] from 1970, influenced by the project headed by George and Frédérique Papy [10] in Belgium (Soares, 2001).

The journey of each of these groups followed its own peculiar dynamics, only partly influenced by GEEM. However, the reach of the action of these groups was largely regional, whilst the effects of the actions of GEEM expanded nationally, spreading the idea of an urgent and necessary reform of the teaching of mathematics.

Autonomy and governmental support

The action of GEEM was encouraged by the support of official bodies and the privileged access to the press that Professor Osvaldo Sangiorgi enjoyed. Following the path of North America and Europe, the Ministry of Education and the Education Secretaries invested in programs to improve the teaching of mathematics and sciences in the 1960s. That investment can be understood on the one hand, as an attempt to overcome a humanist and literary tradition that was, until then, predominant in secondary education and additionally, as part of an effort for scientific training for future technicians in a country in the process of industrialisation. The modern mathematics movement applied for official support with the promise of more "scientific" training and had the endorsement not only of foreign

mathematicians and eminent educators, but of North American bodies and the Organisation of American States itself.

But the appeal of GEEM's discourse, to a large extent, was attributed to the autonomous condition of the teachers' group, in relation to the government. It was not simply one more teaching program drawn up in cabinet or imposed by one school to the others.

The incorporation of elements from modern mathematics in the official curricular proposal was preceded by many debates and even the adoption of a new education program, proposed by GEEM in the IV Brazilian Congress on Mathematics Teaching, held in Belem do Pará in 1962. In 1966, GEEM organised the V Brazilian Congress on Mathematics Teaching in São José dos Campos. It is interesting to note that in this Congress, unlike any previously, courses of mathematics topics predominated, similar to the courses promoted by GEEM.

GEEM, in its first years of existence, promoted weekly Saturday meetings, where the experiences of teachers in the classroom were reported and discussed before being presented as courses in Modern Practice (Madsen, cited in Lima, 2008). Thus, if on the one hand it is true that "reform was definitely not done systematically and rarely was formal evaluation conducted" (D'Ambrosio, 1987, p. 196), it must be recognised that education in the classroom was subjected to debate by GEEM.

According to statements from several members of GEEM [11], neither the participation in meetings or the conducting of classes in the courses were paid, which were construed as militant actions by some of the teachers who were engaged in, and believed in, the process of change in education.

The modern mathematics movement also presented itself as an inclusive movement, which welcomed innovative experiences such as those from Ginásio Vocacional do Brooklin[12]. In this school, the teaching of mathematics was guided by the general objectives of the school, such as "community research" (Bechara, 1965). According to Lucília Bechara, GEEM

welcomed the experiences until the moment "it began to create a commitment to one pedagogy" that, according to Ribeiro (1980), was concerned with criticism of the social context. As well, the Colégio de Aplicação da Universidade in São Paulo networked with the various disciplines through interest centres[13]. The two experiences were interrupted in 1969, within the context of the toughening of the military regime, established in 1964, and the repression of educational debate. Therefore, although the military dictatorship did not completely block the dissemination of modern mathematics ideas, it made the development of some of its currents of thought unfeasible.

Occasionally, members of GEEM referred to modern mathematics as a "libertarian pedagogy", with no pretence of any "conscientisation" or social criticism. The "liberation" referred to the mathematics traditionally taught at the schools; it is apparent that there was also some influence of escolanovismo, with the valorisation of the initiative and creativity of students.

A modern mathematics in a country that wanted to be modern

Modern mathematics mobilised thousands of teachers in courses, provoked changes in programs and textbooks and throughout the 1960s often occupied the front pages of newspapers. It would not have had that reach without the action of militant groups such as GEEM, NEDEM, GEEMPA and other groups who constructed the local appropriation of the proposals to renew the teaching of mathematics.

The feeling of "modernisation", imprinted on the movement for education renewal, favoured its dissemination in various ways.

Amongst the teachers of secondary education, modern mathematics imposed itself as an alternative renewal, presenting itself as articulated and far reaching. The movement was facing a school system considered ineffective and inadequate for the expectations of the democratisation of access to secondary education. It was also facing localised experiences that proposed innovations in the teaching of isolated topics or minor changes in the programs.

Modern mathematics championed a new "attitude" towards mathematics, that is, a change in the school culture itself. The idea that secondary mathematics should approximate higher mathematics was favoured by the presence and authority attributed to licensed teachers. They had their mathematics training "updated" in terms of language and the study of algebraic structures.

Moreover, the reach of a movement that promised "modernisation" of education and which identified modernisation with democratisation, should be understood in a context of optimism and belief in the benefits derived from technical progress. This was Brazil in the early 1960s: a society permeated by major social conflicts, but at the same time seeing itself as a "developing nation". The debate between the people's "nationalist" path and the development path, "associated" with foreign capital, resulted in the installation of a military dictatorship in 1964 which lasted for twenty years. But the optimism would only be exhausted much later; in the 1960s, the dominant social imaginary in the country was still that of a nation that would follow the path of developed countries, which was able to produce steel and automobiles, to generate employment and train technicians. In a country where fundamental freedom had been limited, and in some cases eliminated, the opportunities for consumption grew, especially amongst the urban middle layers.

Modern mathematics gained, in the end, unusual publicity for the movement for curricular renewal. The audience that modern mathematics obtained, with a wide public, can be attributed, in part, to the fetish surrounding school mathematics, on the one hand, but also to the enthusiasm that pierced the whole national imaginary. The automobiles being manufactured in São Paulo were modern, the architecture of the new capital [14], inaugurated in 1960, was modern and the rock that began to be produced in Brazil was modern. The teaching of mathematics had to be as modern as the country wanted and expected it to be.

Acknowledgements

I wish to acknowledge to the chairs and members of the Topic Study Group on The history of the teaching and learning of mathematics for the opportunity of presenting a former version of this paper at the 11th International Congress on Mathematics Education in Monterrey, Mexico. I particularly thank Prof. Gert Schubring for his comments. I thank also Wagner Valente and GHEMAT members for the opportunity of debating research on mathematics modern movement in Brazil.

Notes

[1] In the period referred to here, primary education had a duration of four or five years. Secondary education was organised into two cycles: the first cycle, *ginásio* had a duration of four years, and the second cycle, called *colegial* (high school), had a duration of three years.

[2] In the few institutions it was offered, secondary education was organised according to the exam calendar, whose programs were established by the Colégio Pedro II, created in 1837.

[3] In addition to secondary education dedicated to general training, and with no professionalisation purpose, two-cycle post-primary education also included the following modalities: commercial, industrial, agricultural and normal (teachers' education).

[4] According to the statement of the then Minister, Clóvis Salgado, in 1958 only 16% of teachers currently in secondary education were licensed by the Philosophy Faculties (cited in the Anais do III Congresso Brasileiro de Ensino de Matemática (Annals of the III Brazilian Congress of Mathematics Education), held in 1959).

[5] In the period referred to here, *Colégios* offered secondary education, including its second cycle.

[6] Educational movement appeared in the 1920's, concerned with the democratisation of access to education and defended active methods of teaching, with the influences of John Dewey's ideas.

[7] Ginásios Vocacionais were created as experiences in community education. Their creation was favoured by the Lei de Diretrizes e Bases da Eduação (Guidelines and Bases of Education Law) of 1961, which allowed the creation of experimental schools (Nunes, 2000).

[8] SMSG was created at Yale University in 1958, according to D'Ambrosio (1987).

[9] The Educational Center of Niterói was created in 1960 as an experimental school.

[10] Papy's influence was also present in the experiments developed at the traditional Colégio São Bento and at the public school Colégio Estadual André Maurois, in Rio de Janeiro.

[11] According to Búrigo (1989). Manhucia Liberman manifested the same meaning in an oral statement at the Colóquio Osvaldo Sngiorgi (Symposium Osvaldo Sangiorgi), held in São Paulo in October 2008.

[12] Ginásio Vocacional do Brooklyn, in São Paulo City was created in 1962.

[13] According to Búrigo (1989).

[14] The city of Brasilia, its main monuments and government buildings were planned by the architects Lúcio Costa and Oscar Niemeyer.

References

Babá, Elza (1965). Introdução do conceito de número e numeral de um número. In GEEM, *Matemática moderna para o ensino secundário*. São Paulo: IBECC.

Bechara, Lucília (1965). Alguns dados sobre o desenvolvimento de um moderno planejamento de matemática, iniciado em 1962, na primeira série do Ginásio Vocacional do Brooklin – São Paulo. In GEEM, *Matemática moderna para o ensino secundário*. São Paulo: IBECC.

Bertoni, Neuza B. (2008). A legitimidade curricular da matemática moderna no Paraná: comparando o incomparável. In Búrigo, E.Z., Fischer, M.C.B., Santos, M.B. (Orgs.), *A matemática moderna nas escolas do Brasil e de Portugal: novos estudos*. Porto Alegre: Redes.

Búrigo, Elisabete Z. (1989). *Movimento da matemática moderna no Brasil: estudo da ação e do pensamento de educadores matemáticos nos anos 60*. Porto Alegre: UFRGS.

Búrigo, Elisabete Z. (1990). Matemática moderna: progresso e democracia na visão de educadores brasileiros nos anos 60. *Teoria & Educação,* 2, 255-265.

D'Ambrosio, Beatriz S. (1987). *The dynamics and consequences of the modern mathematics reform movement for Brazilian mathematics education*. Indiana University.

Duarte, Aparecida R. S. (2007). A participação do matemático Omar Catunda no MMM da Bahia. In J. M. Matos, Valente, W. R. (Eds.), *A matemática moderna nas escolas do Brasil e de Portugal: primeiros estudos*. São Paulo: Da Vinci/Capes-Grices.

Fonseca, Pedro C. D. (1987). Estado e industrialização consciente: 1930-1955. *Questões de economia política*, 4, 20-32.

Gouveia, Aparecida J., Havighurst, R. J. (1969). *Ensino médio e desenvolvimento*. São Paulo: Melhoramentos/USP.

Instituto Brasileiro de Geografia e Estatística (IBGE) (2007). *Estatísticas do século XX*. Brasília.

Lima, Flainer R., Passos, L. F. (2008). GEEM – Grupo de Estudos do Ensino da Matemática e o Movimento da Matemática Moderna no Brasil. In Valente, W. R. (Org.), *Osvaldo Sangiorgi: um professor moderno.* São Paulo: Annablume, Brasília: CNPq, Osasco: GHEMAT.

Medina, Denise. História da educação matemática nas séries iniciais: uma cronologia em construção. In Búrigo, E.Z., Fischer, M.C.B., Santos, M.B. (Orgs.), *A matemática moderna nas escolas do Brasil e de Portugal: novos estudos*. Porto Alegre: Redes.

Miorim, Maria Ângela (2005). Livros didáticos de Matemática do período de implantação do Movimento da Matemática Moderna no Brasil. In V Congresso Ibero-Americano de Educação Matemática (V CIBEM), *Annals*.

Nunes, Clarice (2000). O "velho" e "bom" ensino secundário: momentos decisivos. *Revista Brasileira de Educação*, 14, 35-60.

Nunes, H. M. S., Xausa, T. M. (1987). O desenvolvimentismo e o governo Kubitschek. *Questões de economia política*, 4, 33-39.

Piaget, Jean (1955). Les structures mathématiques et les structures opératoires de l'intelligence. In Comission International pour l'Étude et l'Amèlioration de l'Enseignement des Mathématiques, (CIEAEM), *L'enseignement des mathématiques*. Neuchâtel: Delachaux et Niestlé.

Ribeiro, Maria Luísa S. (1980). O Colégio Vocacional “Oswaldo Aranha” de São Paulo. In Garcia, W. E. (Org.), *Inovação educacional no Brasil*. São Paulo: Cortez.

Ruiz, Ángel, Barrantes, Hugo (1998). *La Historia del Comité Interamericano de Educación Matemática*. CIAEM. < http://www.furb.br/ciaem/>

Sangiorgi, O. (1963). Entrevista. Cursos de férias no Mackenzie vão revolucionar ensino da matemática. *A Nação*, 13 de dezembro.

Sangiorgi, O. (1965). Entrevista. Matemática de hoje é de ensinar sem assustar. *Diário Popular*, 3 de fevereiro.

Sangiorgi, O. (1966). Entrevista. Ensino de matemática dá maior liberdade ao aluno. *A Gazeta*, 12 de maio.

Sangiorgi, O. (1967). *Matemática: curso moderno*, vol. 3. São Paulo: Nacional.

Sangiorgi, O. (1971), *Matemática: curso moderno*, vol. 1. São Paulo: Nacional.

Soares, Flávia (2001). *Movimento da Matemática Moderna no Brasil: avanço ou retrocesso?* Rio de Janeiro: PUC-RJ.

Valente, Wagner R. (Org.) (2004). *O nascimento da matemática do ginásio*. São Paulo: Annablume, FAPESP.

Valente, Wagner R. (2005). Do engenheiro ao licenciado: subsídios para a história da profissionalização do professor de Matemática no Brasil. *Diálogo Educacional*, 5(16), 75-94.

Valente, Wagner R. (2008). Osvaldo Sangiorgi, um best-seller. In Valente, W. R. (Org.), *Osvaldo Sangiorgi: um professor moderno.* São Paulo: Annablume, Brasília: CNPq, Osasco: GHEMAT.

Villela, Lucia M. A. Os livros didáticos de maior vendagem, na Companhia Editora Nacional, no período de 1964 a 1980. In Búrigo, E.Z., Fischer, M.C.B., Santos, M.B. (Orgs.), *A matemática moderna nas escolas do Brasil e de Portugal: novos estudos*. Porto Alegre: Redes.