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Essays on heterogeneous expectations in agent-based macroeconomic models

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Orientador: Prof. Dr. Jaylson Jair da Silveira

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Essays on heterogeneous expectations in agent-based macroeconomic models

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ABSTRACT

This thesis comprises three essays aimed at analyzing the macroeconomic implications of heterogeneous expectations through agent-based macroeconomic models with different closures. In the first essay, we propose a dynamic version of an efficiency wage model augmented by heterogeneous expectations. In an environment where two opposing forces govern the co-evolution of the distribution of expectations about the unemployment rate among workers, the persistence of heterogeneous expectations, as well as the Granger causality of the unemployment rate and expectations, are stylized facts reported in the empirical literature that emerge as properties of the model. In the second essay, we propose an extension of a model with Keynesian closure and heterogeneous agents who use heuristics in their decision-making process, with the possibility of direct and indirect interaction in the goods and labor markets. In this model, we analyzed under which circumstances the introduction of the expectations formation structure guarantees the equilibrium of macroeconomic variables and how exogenous technological shocks can affect this equilibrium. In the second essay, we concluded that, for an empirically relevant combination of the choice intensity parameter of the discrete choice process, a sufficiently high value of the weight of the social component in the choice process is necessary for the model convergence to equilibrium. In addition, we conclude that household expectations can amplify the effect of technological shocks. Finally, the third essay proposes expanding the models presented in the first and second essays to incorporate the effect of networks in forming expectations. For the efficiency wage model, it was found that introducing the network in forming expectations reduces the weight of the observed component of the utility function in the choosing perceptions process. In addition, the model can replicate some stylized facts reported in the empirical literature. In the model with Keynesian closure, we found indications that self-fulfilling movements in expectations are an emergent property of the model and we also found that the model also manages to replicate some of the stylized facts reported in the empirical literature.

Keywords: Agent-based computational model. heterogeneous expectations. Bounded rationality.

RESUMEN

Esta tesis comprende tres ensayos destinados a analizar las implicaciones macroeconómicas de las expectativas heterogéneas a través de modelos macroeconómicos basados en agentes con diferentes cierres. En el primer ensayo, proponemos una versión dinámica de un modelo de salarios de eficiencia aumentado por expectativas heterogéneas. En un entorno en el que dos fuerzas opuestas gobiernan la coevolución de la distribución de las expectativas sobre la tasa de desempleo entre los trabajadores, la persistencia de expectativas heterogéneas, así como la causalidad de Granger de la tasa de desempleo y las expectativas, son hechos estilizados recogidos en la literatura empírica que emergen como propiedades del modelo. En el segundo ensayo, proponemos una extensión de un modelo con cierre keynesiano y agentes heterogéneos que utilizan heurísticas en su proceso de toma de decisiones, con posibilidad de interacción directa e indirecta en los mercados de bienes y de trabajo. En este modelo, analizamos bajo qué circunstancias la introducción de la estructura de formación de expectativas garantiza el equilibrio de las variables macroeconómicas y cómo los shocks tecnológicos exógenos pueden afectar a este equilibrio. En el segundo ensayo, concluimos que, para una combinación empíricamente relevante del parámetro de intensidad de elección del proceso de elección discreta, es necesario un valor suficientemente alto del peso del componente social en el proceso de elección para que el modelo converja al equilibrio. Además, concluimos que las expectativas de los hogares pueden amplificar el efecto de las perturbaciones tecnológicas. Por último, el tercer ensayo propone ampliar los modelos presentados en el primer y segundo ensayos para incorporar el efecto de las redes en la formación de expectativas. Para el modelo de salarios de eficiencia, se ha encontrado que la introducción de la red en la formación de expectativas reduce el peso del componente observado de la función de utilidad en el proceso de elección de percepciones. Además, el modelo puede replicar algunos hechos estilizados recogidos en la literatura empírica. En el modelo con cierre keynesiano, encontramos indicios de que los movimientos autocumplidos en las expectativas son una propiedad emergente del modelo y también encontramos que el modelo también consigue replicar algunos de los hechos estilizados reportados en la literatura empírica.

Palabras clave: Modelo informático basado en agentes. Expectativas heterogéneas. Racionalidad limitada.

RESUMO EXPANDIDO

Introdução

Esta tese é composta por três ensaios que têm como objetivo analisar as implicações macroeconômicas de heterogeneidade de expectativas de trabalhadores e consumidores por meio de modelos macroeconômicos baseados em agentes com diferentes fechamentos. Para isto, foi utilizado como metodologia Modelo Baseado em Agentes, o que torna possível estudarmos a economia como um sistema complexo. Nesse sentido, a presente tese contribui com a literatura de expectativas heterogêneas e sua coevolução com a atividade econômica apresentado dois modelos baseados em agentes macroeconômicos de forma original.

Objetivos

O objetivo geral da presente tese é explorar a dinâmica de heterogeneidade das expectativas e suas implicações macroeconômicas em dois modelos macroeconômicos com diferentes fechamentos. Como objetivos específicos aponta-se a revisão da literatura que apresenta evidência de existência e persistência de heterogeneidade das expectativas, analisar os mecanismos de *feedback* entre as expectativas e as variáveis macroeconômicas, analisar os possíveis desvios do estado estacionário dos modelos que se devem à introdução do mecanismo de formação de expectativas, além de mudanças de respostas em relação a choques de produtividade. Por fim, aponta-se como um último objetivo específico analisar as mudanças nas propriedades emergentes do modelo com a introdução de expectativas interativas.

Metodologia

Para atender aos objetivos da presente tese, foi utilizado como metodologia Modelo Baseado em Agentes. Mais precisamente, no primeiro ensaio, propomos uma versão dinâmica de um modelo de salário eficiência com heterogeneidade de expectativas sobre o desemprego. Em um ambiente em que existem duas forças opostas que regem a coevolução da distribuição das expectativas sobre a taxa de desemprego entre os trabalhadores, a persistência de heterogeneidade das expectativas, assim como a relação de causalidade temporal entre a taxa de desemprego e as expectativas, são fatos estilizados relatados na literatura empírica que surgem como propriedades emergentes do modelo. No segundo ensaio, propõe-se a expansão de um modelo com fechamento Keynesiano e agentes heterogêneos que utilizam heurísticas no seu processo de decisão, com a possibilidade de interação direta e indireta no mercado de bens e no mercado de trabalho. Nesse modelo, foi analisado em que condições a introdução da estrutura de formação de expectativas garante o equilíbrio das variáveis macroeconômicas e como choques tecnológicos exógenos podem afetar esse equilíbrio. Por fim, no terceiro ensaio é proposto uma expansão dos modelos apresentados no primeiro e no segundo ensaio para incorporar o efeito de redes no processo de formação de expectativas. Para isso, foi incorporado uma estrutura de redes em ambos os modelos.

Resultados e discussão

No primeiro ensaio, a análise do modelo de salário eficiência com expectativas heterogêneas apontou que um valor mínimo para o parâmetro relacionado à intensidade da escolha é necessário para que a causalidade temporal entre as expectativas e a taxa de desemprego ocorra como uma propriedade emergente do modelo. Além disso, para valores muito altos do parâmetro relacionado à intensidade da escolha, não há evidência estatística de causalidade entre as expectativas e a taxa de desemprego. Já no segundo ensaio concluímos que, para um valor empiricamente relevante do parâmetro de intensidade de escolha do processo de escolha discreta, é necessário um valor suficientemente alto do peso do componente social no processo de escolha para que o modelo convirja para o equilíbrio. Além disso, concluímos que as expectativas formadas pelos consumidores podem amplificar o efeito de choques tecnológicos. Finalmente, no terceiro ensaio foi encontrado que a introdução da rede no processo de formação das expectativas diminui o peso do componente observado da função utilidade no processo de escolha das percepções. Além disso, o modelo foi capaz de reproduzir alguns fatos estilizados relatados na literatura empírica. No modelo com fechamento keynesiano, foi encontrado indicações de que movimentos autorrealizáveis das expectativas é uma característica emergente do modelo.

Palavras-chave: Modelo computacional baseado em agentes. Expectativas heterogêneas. Racionalidade limitada.

LIST OF FIGURES

Figure 1.1 – Actual and simulated unemployment rate.	39
Figure 1.2 – Actual and simulated BS.	39
Figure 1.3 – Simulated proportion of neutral, optimistic and pessimistic workers. . .	41
Figure 1.4 – Actual proportion of neutral, optimistic and pessimistic workers. . . .	41
Figure 1.5 – Normalized BS and unemployment rate simulated over the last 504 simulation steps.	42
Figure 1.6 – Simulated normalized BS and unemployment rate between simula- tion steps 90 and 140.	43
Figure 1.7 – Observed normalized BS and unemployment rate.	43
Figure 1.8 – Test BS Granger causes the unemployment rate for different combi- nations of parameters ψ and β	45
Figure 1.9 – Test unemployment rate Granger causes BS for different combina- tions of parameters ψ and β	45
Figure 1.10 – Observed normalized BS and yearly change in unemployment rate. .	46
Figure 1.11 – Simulated normalized BS and yearly change in unemployment rate between simulation steps 90 and 140.	47
Figure 2.1 – Seasonally adjusted European Union balance of responses on the general economic situation over the next 12 months, from June 1986 to April 2016.	62
Figure 2.2 – Steady state values of the output gap and unemployment rate for different pairs of the intensity of choice ν and social influence weight ψ in the centralized model.	70
Figure 2.3 – Steady state values of inflation and real wage for different pairs of the intensity of choice ν and social influence weight ψ in the centralized model.	71
Figure 2.4 – Steady state values of balance for different pairs of the intensity of choice ν and social influence weight ψ in the centralized model. . . .	72
Figure 2.5 – Steady state values of the output gap and unemployment rate for different pairs of the intensity of choice ν and social influence weight ψ in the decentralized scenario.	73
Figure 2.6 – Steady state values of inflation and real wage for different pairs of the intensity of choice ν and social influence weight ψ in the decentralized scenario.	74
Figure 2.7 – Steady state values of balance for different pairs of the intensity of choice ν and social influence weight ψ in the decentralized scenario. . . .	75
Figure 2.8 – Emergent macroeconomic dynamics under negative productivity shocks in the centralized scenario considering $\nu = 50$ and $\psi = 10$	77

Figure 2.9–Balance of perceptions under a negative productivity shock in a centralized scenario with $\psi = 10$ and $\nu = 50$	78
Figure 2.10–Micro-level variance in the centralized scenario under negative productivity shocks with $\psi = 10$ and $\nu = 50$	78
Figure 2.11–Emergent macroeconomic dynamics under negative productivity shocks in the centralized scenario considering $\psi = 0$ and $\nu = 0$	79
Figure 2.12–Balance of perceptions under a negative productivity shock in a centralized scenario with $\psi = 0$ and $\nu = 0$	80
Figure 2.13–Micro-level variance in the centralized scenario under negative productivity shocks considering $\psi = 0$ and $\nu = 0$	80
Figure 2.14–Emergent macroeconomic dynamics under negative productivity shocks in the centralized scenario considering $\psi = 1$ and $\nu = 10$	81
Figure 2.15–Balance of perceptions in the centralized scenario under negative productivity shocks considering $\psi = 1$ and $\nu = 10$	81
Figure 2.16–Micro-level variance in the centralized scenario under negative productivity shocks considering $\psi = 1$ and $\nu = 10$	82
Figure 2.17–Emergent macroeconomic dynamics under negative productivity shocks in the decentralized scenario considering $\psi = 10$ and $\nu = 50$	83
Figure 2.18–Balance of perceptions in the decentralized scenario under negative productivity shocks considering $\psi = 10$ and $\nu = 50$	84
Figure 2.19–Micro-level variance in the decentralized scenario under negative supply shocks considering $\psi = 10$ and $\nu = 50$	85
Figure 2.20–Emergent macroeconomic dynamics under negative productivity shocks in the decentralized scenario considering $\psi = 0$ and $\nu = 0$	85
Figure 2.21–Balance of perceptions in the decentralized scenario under negative productivity shocks considering $\psi = 0$ and $\nu = 0$	86
Figure 2.22–Micro-level variance in the decentralized scenario under negative productivity shocks considering $\psi = 0$ and $\nu = 0$	86
Figure 2.23–Emergent macroeconomic dynamics under negative productivity shocks in the decentralized scenario considering $\psi = 1$ and $\nu = 10$	87
Figure 2.24–Balance of perceptions in the decentralized scenario under negative productivity shocks considering $\psi = 1$ and $\nu = 10$	88
Figure 2.25–Micro-level variance in the decentralized scenario under negative productivity shocks considering $\psi = 1$ and $\nu = 10$	88
Figure 3.1 – Illustrations of networks for $n = 4$	94
Figure 3.2–Illustration of the process from a regular ring lattice to a random network.	95
Figure 3.3–Actual and simulated unemployment rate.	100
Figure 3.4–Actual and simulated BS.	101

Figure 3.5 – Simulated proportion of neutral, optimistic and pessimistic workers.	102
Figure 3.6 – Simulated Normalized BS and simulated unemployment rate over the last 504 simulation steps.	102
Figure 3.7 – Simulated Normalized BS and simulated unemployment rate between simulation steps 90 and 140.	103
Figure 3.8 – Simulated Normalized BS and simulated yearly change in the unemployment rate between simulation steps 90 and 140.	104
Figure 3.9 – Observed and simulated balance in the GNR model augmented by interactive expectations.	112
Figure 3.10 – Observed and simulated unemployment by the GNR model augmented by expectations.	113
Figure 3.11 – Observed and simulated inflation in the GNR model augmented by interactive expectations.	113
Figure 3.12 – Simulated co-evolution of the balance and the unemployment rate in the GNR model augmented by interactive expectations.	114
Figure 3.13 – Simulated balance and unemployment rate in the GNR model augmented by interactive expectations in simulation steps 50 and 100.	114
Figure 3.14 – Observed balance and unemployment rate.	115
Figure 3.15 – Observed balance and yearly change in unemployment rate.	116
Figure 3.16 – Simulated balance and yearly change in the unemployment rate.	116
Figure 3.17 – Simulated balance and yearly change in the unemployment rate between steps 50 to 100.	117

LIST OF TABLES

Table 1.1 – Algorithm of unemployment expectation formation for a given period $t \geq 2$ by a worker i that did not have the bias to form pessimistic unemployment expectations.	37
Table 1.2 – Calibrated parameter values.	38
Table 1.3 – Granger causality test for simulated unemployment and BS.	44
Table 1.4 – Granger causality test for BS and yearly change in unemployment. . .	47
Table 2.1 – Mean and standard deviation of the balance of perception about the economic situation for the unemployed group, full-time workers and part-time workers.	63
Table 2.2 – Algorithm to choose a perception about the future economic activity in every period $t \geq 2$	66
Table 2.3 – Range in which the consumption perception bias will be contained for possible perceptions of the future economic situation	66
Table 2.4 – Parameter Values.	68
Table 2.5 – Means and standard deviations of average over simulations that did not converge in the centralized scenario	72
Table 2.6 – Parameter Values.	76
Table 2.7 – Long-run values of output-gap, unemployment, inflation, real wage and balance of perceptions for different combinations of parameters in the centralized scenario. Monte-Carlo standard errors are in parentheses. The values are rounded to 4 decimal places.	83
Table 2.8 – Long-run values of output-gap, unemployment, inflation, real wage and balance of perceptions for different combinations of parameters in the decentralized scenario. Monte-Carlo standard errors are in parentheses. The values are rounded to 4 decimal places.	87
Table 3.1 – Algorithm of unemployment expectation formation for a given period $t \geq 2$ by a worker i that did not have the bias to form pessimistic unemployment expectations.	99
Table 3.2 – Calibrated parameter values.	100
Table 3.3 – Granger causality test for simulated unemployment and BS.	103
Table 3.4 – Granger causality test for BS and yearly change in unemployment. . .	104
Table 3.5 – Algorithm to choose a perception about the future economic activity in every period $t \geq 2$	108
Table 3.6 – Range in which the consumption perception bias will be contained for possible perceptions of the future economic situation	109
Table 3.7 – Parameter Values.	110
Table 3.8 – Calibrated parameter Values.	112

Table 3.9–Granger causality test for simulated unemployment and BS using 3 lags in the VAR model.	118
Table 3.10–VAR estimation of the unemployment equation.	118
Table 3.11–Granger causality test for simulated yearly change in unemployment rate and BS using 3 lags in the VAR model.	119
Table 3.12–Granger causality test for simulated yearly change in unemployment rate and BS using 14 lags in the VAR model.	119
Table A.1–ADF test empirical data.	129
Table A.2–Information criterion empirical data.	129
Table A.3–Granger causality test with 2 lags for the empirical data.	129
Table A.4–Granger causality test with 6 lags for the empirical data.	129
Table A.5–ADF test yearly.	130
Table A.6–Information criterion empirical data.	130
Table A.7–Granger causality test with two lags for the empirical data.	130
Table A.8–Granger causality test with 6 lags for the empirical data.	130
Table B.1–ADF test empirical data in level.	131
Table B.2–Information criterion empirical data.	131
Table B.3–Granger causality test with 2 lags for the empirical data.	131
Table B.4–Granger causality test with 7 lags for the empirical data.	131
Table B.5–ADF test empirical data in level.	132
Table B.6–Information criterion empirical data.	132
Table B.7–Granger causality test with 12 lags for the empirical data.	132
Table B.8–Granger causality test with 24 lags for the empirical data.	132

CONTENTS

	INTRODUCTION	16
1	ENDOGENOUSLY TIME-VARYING HETEROGENEITY IN UNEMPLOYMENT EXPECTATIONS: A DISCRETE CHOICE APPROACH IN AN AGENT-BASED MACRODYNAMIC MODEL	19
1.1	A REVIEW ON HETEROGENEOUS UNEMPLOYMENT EXPECTATIONS AND ITS MACROECONOMIC IMPLICATIONS	21
1.1.1	Evidence of heterogeneity in unemployment expectations	21
1.1.2	Evidence of Correlation between Unemployment Expectations and Observed Unemployment	23
1.2	THE PROPOSED AGENT-BASED MODEL	25
1.2.1	The efficiency-wage model with heterogeneous expectations about the unemployment rate	26
1.2.2	Formation of unemployment expectations as a discrete choice process	29
1.2.2.1	Structure of the discrete choice model associated with the formation of unemployment expectations	30
1.2.2.2	The Agent-Based Model of Unemployment Expectations Formation as a Discrete Choice Process	33
1.2.3	Computational implementation and calibration of the proposed agent-based model	35
1.3	EMERGENT PROPERTIES OF THE ABM	40
1.4	FINAL REMARKS	48
2	A MACROECONOMIC ABM EXTENDED FOR HETEROGENEOUS EXPECTATION	49
2.1	A BRIEF LITERATURE REVIEW	50
2.2	THE REFERENCE AGENT-BASED MODEL	53
2.2.1	Determining consumption, production, prices and wages	55
2.2.2	Search and matching in labor and goods market	57
2.2.2.1	Labor market	57
2.2.2.2	Goods market	59
2.2.3	Financial conditions, exit and entry	60
2.3	THE GNR MODEL AUGMENTED BY EXPECTATIONS	61
2.4	EFFECTS OF THE PERCEPTIONS ABOUT THE FUTURE ECONOMIC ACTIVITY ON THE STEADY-STATE PROPERTIES OF GNR MODEL	67
2.5	EFFECTS OF PRODUCTIVITY SHOCKS IN GNR MODEL AUGMENTED BY EXPECTATIONS	75

2.6	FINAL REMARKS	89
3	SOCIAL NETWORK IN MACROECONOMIC AGENT-BASED MODEL	90
3.1	RELATED LITERATURE AND DEFINITIONS ON NETWORK	91
3.1.1	Related literature	91
3.1.2	Definitions on network	93
3.2	INTERACTIVE EXPECTATION IN THE EFFICIENCY-WAGE MODEL AUGMENTED BY EXPECTATIONS	95
3.2.1	Emergent properties	100
3.3	INTERACTIVE EXPECTATION IN GNR MODEL AUGMENTED BY EXPECTATIONS	104
3.3.1	Computational implementation and calibration strategy	109
3.3.2	Emergent properties	112
3.4	CONCLUDING REMARKS	119
	FINAL REMARKS	121
	REFERENCES	122
	APPENDIX A – ESSAY 1	129
A.1	STATISTICS FOR THE CAUSALITY TEST BETWEEN THE UNEM- PLOYMENT RATE AND BS USING DATA FOR THE US.	129
A.2	STATISTICS FOR THE CAUSALITY TEST BETWEEN BS AND YEARLY CHANGE IN THE UNEMPLOYMENT RATE EMPIRICAL DATA.	129
	APPENDIX B – ESSAY 3	131
B.1	STATISTICS FOR THE CAUSALITY TEST BETWEEN THE UNEM- PLOYMENT RATE AND THE BALANCE OF EXPECTATIONS USING DATA FROM ITALY.	131
B.2	STATISTICS FOR THE CAUSALITY TEST BETWEEN THE BAL- ANCE OF EXPECTATIONS AND YEARLY CHANGE IN THE UNEM- PLOYMENT RATE EMPIRICAL DATA.	131

INTRODUCTION

Economists widely accept that expectations matter and influence economic activity. In this context, they have tried to understand how people interpret the world and form expectations about different macroeconomic variables and how these expectations could affect the dynamic of macroeconomic variables.

Traditional macroeconomic models are based on the rational representative agent assumption since the seminal contribution of Muth (1961). On the other hand, in light of empirical evidence, it is plausible to consider heterogeneous expectations and bounded rational agents whose limited information sets and cognitive abilities are not compatible with the hypothesis of rational expectation. As Arthur (1995) suggests, economic agents make their choices based on their current beliefs or expectations about the economy, which are often individual and formulated on the basis of their actions and the beliefs and actions of other agents. These expectations are constantly reformulated and co-evolved with the dynamics of the economy. Moreover, this vast possibility of beliefs and behaviors forces us to think of the economy as a complex system (ARTHUR, 1995).

For macroeconomic analysis, the complexities of the real world have led to the need to explore macroeconomic models that consider the economy as a complex system. This is exactly the methodological core of Agent-Based Computational Economics (ACE), defined by LeBaron and Tesfatsion (2008) as the computational study of the economic process modeled as a dynamic system of interacting agents. As noted by Tesfatsion (2017), ACE is the specialization of the agent-based modeling approach, and it permits the systematic study of locally constructive decision processes in macroeconomic contexts.

According to the research conducted by Haldane and Turrell (2019), agent-based models (ABMs) were not designed to provide a single model for addressing complex problems. Instead, they offer a flexible toolkit to deal with such problems that involve heterogeneous agents. As this approach allows different agents in an economy to exhibit a variety of behaviors, these models are typically solved numerically at an agent level, one behavior at a time.

In this context, this dissertation aims to contribute to the literature on heterogeneous expectations and their co-evolution with economic activity by presenting two macroeconomic agent-based models with different closures in an original way.

In the first essay, we present a dynamic version of the efficiency wage model augmented by expectations developed by Silveira and Lima (2021) in which workers form their perceptions (expectations) based on a discrete choice protocol following the suggestive framework developed in Brock and Hommes (1997). Based on the structure formulated, we can analyze the co-evolution of expectations and the mechanism of

feedback between expectations and the unemployment rate. In this essay, we found that the persistence of heterogeneity of expectations is a stylized fact reported in the empirical literature that can be generated as an emergent property of the model. In addition, the temporal causality between the unemployment rate and expectations is another emergent property reported in the empirical literature that was generated as an emergent property of the model. Another result found was that a minimum value for the parameter related to the intensity of choice is necessary for temporal causality between expectations and the unemployment rate to occur as an emergent property of the model. Furthermore, for very high values of the parameter related to the intensity of choice, there is no statistical evidence of causality between expectations and the unemployment rate.

In the second essay, we propose an extension of the macroeconomic ABM proposed by Guerini, Napoletano, and Roventini (2018) to incorporate the structure of household perceptions of future economic activity and its macroeconomic implications through the consumption channel. The authors developed an ABM in which households and firms interact in good and labor markets according to either centralized or decentralized search and matching protocols and use heuristics in their rules of behavior. In this structure, we are able to analyze the dynamics of the economy for different combinations of parameters that govern the formation of expectations in the model in both centralized and decentralized search and matching protocols. To incorporate expectation formation in this model, we use the suggestive framework of Brock and Hommes (1997). We found that for low values of the intensity of choice and weight of the social component parameters, the economy persistently deviates from the steady-state level. In addition, it was found that the inclusion of heterogeneous expectations in the model can amplify the effects of a negative technological shock.

In the third essay, we expand on the model presented in the first and second essays to incorporate a network structure in the expectation formation process. For the efficiency wage ABM, we found that the introduction of networks in the formation of expectations reduces the weight of the observed component of the utility function in the process of choosing perceptions (expectations). Also, the model can replicate some stylized facts reported in the empirical literature. In the model with a Keynesian closure, we found that endogenous waves of optimism and pessimism can emerge in the model. Moreover, we find self-fulfilling movements in expectations as an emerging property of the model.

Using these models to analyze the co-evolution of expectations and the level of economic activity, the dissertation aims to expand knowledge about the feedback mechanism between the formation of heterogeneous expectations and the level of economic activity. After this introduction, the next three chapters consist of essays 1, 2, and 3 summarized earlier in this introduction. Following that, the general conclusions

of this dissertation will be presented.

1 ENDOGENOUSLY TIME-VARYING HETEROGENEITY IN UNEMPLOYMENT EXPECTATIONS: A DISCRETE CHOICE APPROACH IN AN AGENT-BASED MACRODYNAMIC MODEL

Macroeconomic theory suggests that people's perceptions of macroeconomic indicators can significantly affect economic fluctuations. According to Curtin (2019a), expectations can influence economic decisions such as saving, incurring debt, investing in bonds or stocks, renting or buying a home, and entering or leaving the labor force. Therefore, policymakers must consider survey-based indicators as an important factor while making policy decisions.

As such, many studies are concerned with analyzing the expectation formation of households and professional forecasters. In particular, expectations about the labor market are of fundamental importance for working-age individuals insofar as the risk of job loss is one of the most significant risks faced by them, and it can lead to drops in consumption and significant welfare losses (HENDREN, 2017). Furthermore, as noted by Orland (2017), workers who overestimate the unemployment rate feel disadvantaged in individual wage negotiations and accept lower wage offers than they need to.

Given the potential importance of the unemployment expectations held by economic agents (especially working households) for the observed fluctuations in key macroeconomic variables, several surveys have been conducted in different countries to collect information about them (see, e.g., the US Michigan Survey and the Joint Harmonised European Union Programme of Business and Consumer Surveys). In these surveys, unemployment expectations are usually reported as ranging from pessimism to optimism, also including neutral expectations that unemployment will remain about the same in some near future. These surveys have yielded robust empirical evidence on the existence and persistence of heterogeneity in unemployment expectations across agents or households (see also Orland (2017), Fullone et al. (2007), Blanchflower and Kelly (2008), Kuchler and Zafar (2019), Malgarini and Margani (2008), Curtin (2003), Carroll (2003)). Many of these studies suggest differences in information sets and cognitive abilities to extract and process such information as possible causes of the observed expectation heterogeneity. Curtin (2019a) argues that consumer expectations data on unemployment in most developed economies provide compelling evidence that they do not suffer from the high costs of collecting, processing, and interpreting information about unemployment rate trends (CURTIN, 2019a, p. 64). In light of this, the author proposes a new paradigm to explain how agents' expectations are formed. Curtin (2019b) contends that, in forming their expectations, most agents are not focused exclusively on national economic statistics but rather on the economic conditions they personally face in the labor market. Given this issue, one of the hypotheses considered in developing the model in this essay is that the heterogeneity of expectations originates from the different economic conditions that workers experience.

Meanwhile, other studies have presented robust econometric evidence of correlation between the state of unemployment expectations and the actual level or rate of unemployment (e.g. Curtin (2003), Curtin (2019a), Leduc and Sill (2013), Girardi (2014), Lehmann and Weyh (2016), Dickerson and Green (2012)). Drawing on US and European surveys, these studies have found that households' unemployment expectations are an important driver of actual unemployment, with an increase in pessimism (optimism) leading to a rise (fall) in actual unemployment. This empirical finding is a second factor considered in developing the model in this essay. In particular, one of the channels through which individual expectations affect the observed unemployment rate can be understood through a wage-efficiency model, which will be explored in this research.

In order to explore the dynamics of the heterogeneity of unemployment expectations and its macroeconomic implications, we set forth an agent-based model (ABM) drawing on the heterogeneous expectations-augmented efficiency wage analytical framework proposed by Silveira and Lima (2021). In the latter, the authors show that the significant positive correlation between pessimistic unemployment expectations and the actual unemployment rate observed in the empirical research can arise in their heterogeneous expectations-augmented efficiency wage model of the labor market through a composition effect.

The equilibrium solution derived in Silveira and Lima (2021) is parameterized by a given frequency distribution of unemployment expectations across workers. In the ABM set forth herein, meanwhile, we conceive of such an equilibrium solution as a short-run or temporary one and endogenize the frequency distribution of unemployment expectations across workers in a dynamic version of the heterogeneous expectations-augmented efficiency wage model developed in Silveira and Lima (2021). In our ABM, the revision of unemployment expectations periodically carried out by workers is formally modeled as a discrete choice protocol following the well-known framework developed in Brock and Hommes (1997). As a result, the frequency distribution of unemployment expectations across workers is modeled as endogenously time-varying and coupled to the level of macroeconomic activity, with both experiencing self-sustaining cyclical fluctuations that replicate qualitatively several established stylized facts.

Finally, our aim in this essay is to review the empirical literature that presents evidence of the existence and persistence of heterogeneity of expectations and the feedback between the expectations and the actual unemployment rate, to propose an ABM in which the process of forming expectations can be endogenous in a wage-efficiency model with heterogeneous expectations about the unemployment rate, and to analyze the emergent properties of this model and the implications of heterogeneous expectations on the actual unemployment rate to broaden the understanding of how unemployment expectations influence the observed value of the unemployment rate.

The remainder of this essay is structured as follows: in section 1.1, we review the literature that shows evidence of heterogeneity in unemployment expectations as well as some empirical work that seeks to analyze the feedback between unemployment expectations and the actual unemployment rate. In section 1.2, we present the proposed ABM. In Section 1.3, we present some emergent properties of the ABM. Finally, section 1.4 concludes.

1.1 A REVIEW ON HETEROGENEOUS UNEMPLOYMENT EXPECTATIONS AND ITS MACROECONOMIC IMPLICATIONS

Traditional macroeconomic models assume that agents' expectations regarding macroeconomic variables are rational and homogeneous. This assumption allows for significant simplifications in economic models. As Carroll (2003) asserts, rational expectation models have faced criticisms for failing to replicate several features of macroeconomic data. In response, the literature has explored various alternative assumptions about expectation formation and their implications for macroeconomic dynamics.

In the first subsection, we present some of the empirical literature that presents evidence of the existence and persistence of heterogeneity of unemployment expectations. Many of these studies suggest that differences in information sets and cognitive abilities to extract and process such information are possible causes for the observed heterogeneity of expectations.

In the second subsection, we present econometric evidence of the correlation between pessimism about the labor market and the unemployment rate.

1.1.1 Evidence of heterogeneity in unemployment expectations

The empirical evidence of the existence and persistence of heterogeneity in unemployment expectations presented below consists of analyses from various opinion surveys. In general, these studies show that expectations tend to vary among groups with different socioeconomic characteristics (i.e., concerning gender, education level, and job loss experiences).

Orland (2017) investigated how expectations vary based on sociodemographic characteristics and personality traits. The author conducted his study using opinion survey data from German Ph.D. students across various fields, including engineering, medicine, economics, and humanities. The results were obtained through multinomial logit regression. Among the findings, he discovered that women tend to overestimate the unemployment and inflation rates. Education and exposure to news about consumption can reduce estimation errors. Daily newspaper reading can decrease the likelihood of misinterpreting the unemployment rate by 50%. The author also found that high levels of neuroticism, defined as the degree to which a person experiences the world as

threatening and beyond their control, are associated with curiosity and learning. These levels are related to a higher probability of overestimating unemployment.

In a similar study, Fullone et al. (2007) used Ordinary Least Squares and opinion survey data from the Italian National Institute of Statistics to investigate the influence of various demographic factors on the accuracy level of expectations regarding unemployment, inflation, and GDP, which they call the knowledge score. They found that the respondent's professional category, age, gender, residence area, and education significantly impacted the knowledge score. Additionally, the authors observed that individuals who attach less importance to information tend to have lower knowledge scores. However, the declared desire to be informed did not significantly influence knowledge scores.

Another study that relates differences in expectations to groups with different sociodemographic characteristics is that of Blanchflower and Kelly (2008). The authors conducted their analysis through a logit model using opinion survey data from the United Kingdom, known as the Consumer Confidence Barometer (CCB). They employed a binary dependent variable, indicating whether the individual believes or not that unemployment will increase in the next twelve months. The study found that the probability of believing that the unemployment rate will increase is higher for individuals aged 50 to 64, women, those who left school before the age of 16, and skilled manual workers. This variable was also positively affected by the current unemployment rate.

Kuchler and Zafar (2019), using opinion data from the Survey of Consumer Expectations, which is collected by the Federal Reserve Bank of New York, estimated how personal experiences affect household expectations about aggregate economic outcomes in housing and labor markets. Particularly for the labor market, the authors concluded that the personal experience of unemployment makes respondents significantly more pessimistic about the future unemployment rate in the country. When becoming unemployed, respondents believe that the probability of increasing unemployment in the US in the next twelve months is 4 to 5 percentage points higher than when they were employed. Additionally, Kuchler and Zafar (2019) estimated that, in the expectation formation process, respondents with lower education levels extrapolate their personal experiences with unemployment more than respondents with higher levels of education.

Using consumer opinion survey data from Italy, conducted by the Italian Institute for Studies and Economic Analyses on unemployment expectations, Malgarini and Margani (2008) found significant heterogeneity in the formation of unemployment expectations for different socioeconomic groups, and the hypothesis test of rationality of expectations was rejected for the studied period. The data collected were analyzed based on the consumer's status in the workforce, such as employed, self-employed, and inactive. The researchers pointed out that as collecting information can be expen-

sive, some individuals may rely on low-cost information sources, such as their own subjective perceptions, to make predictions about future unemployment. The authors concluded that these individuals are more likely to base their unemployment expectations on their own specific experiences rather than observing the general dynamics of the labor market while making predictions.

Finally, Carroll (2003) analyzed the relationship between consumer expectation formation and news reporting. The author proposed a model to estimate the evolution of inflation and unemployment expectations using data from the Michigan Survey. The model suggested that agents formed their expectations through media opinions, and the media should report the opinions of professional forecasters taken from the Survey of Professional Forecasters. Carroll (2003) found evidence that expectations are heterogeneous for the two considered opinion surveys. Additionally, the study found that professionals' expectations were more accurate than the general population's predictions. Moreover, the study also found a statistically significant Granger causality, with predictions from professionals influencing the general population's predictions.

1.1.2 Evidence of Correlation between Unemployment Expectations and Observed Unemployment

In this subsection, we present the main studies that provide evidence of the link between expectations of unemployment and the actual unemployment rate. These studies use surveys conducted in the United States and Europe and apply econometric methods to demonstrate that a rise in negative beliefs about job prospects can cause a corresponding increase in unemployment.

Curtin (2003) conducted a comprehensive econometric analysis of the correlation between unemployment expectations and the observed unemployment rate in the United States. The variable used in regressions to represent unemployment expectations was the Balance Score (BS) from the Michigan Survey. This variable and the observed unemployment rate have quarterly periodicity from 1961 to 2002. The author's findings indicated that (i) expectations were positively correlated with future changes in unemployment (1 to 4 quarters ahead); (ii) there is a negative correlation between the past unemployment rate (1 to 4 quarters earlier) and expectations; and (iii) expectations also incorporated contemporary information about the unemployment rate.

Meanwhile, Curtin (2019a) employed the Granger causality test to examine a potential temporal causality between unemployment expectation and actual unemployment rate. The study used data from 15 countries between 1978 and 2016. The author used five lags of the unemployment rate and unemployment expectations to test whether past changes in the unemployment rate can predict changes in expectations and whether past changes in expectations can predict future changes in the unem-

ployment rate. According to the author, it would be natural to expect that both are true, meaning that people form their unemployment expectations based on past events, and their expectations anticipate future changes in the unemployment rate. The analysis showed that in only seven of the fifteen countries analyzed, previous changes in unemployment predicted changes in expectations. On the other hand, in twelve of the fifteen countries, expectations predicted subsequent changes in the unemployment rate. In particular, in the estimated model with U.S. data, using the BS from the Michigan Survey as a measure for unemployment expectations, there was statistical significance that the unemployment rate Granger causes expectations only for the tertiary education group, while the statistical significance that expectations Granger cause the unemployment rate was found for respondents of all education levels. However, Curtin (2019a) emphasizes that a statistically insignificant causality test in the mentioned cases cannot completely rule out the influence that past changes in unemployment have on expectations. In particular, empirical results are sensitive to the number of quarters the variables are lagged in the regressions. That is, most information about recent changes in the official unemployment rate may be incorporated more promptly into expectations rather than having a prolonged or staggered impact on expectations.

Leduc and Sill (2013) used opinion survey data from the Livingston Survey and the Survey of Professional Forecasters to analyze how changes in expectations contribute to fluctuations in aggregate macroeconomic variables. Through a Vector Autoregressive (VAR) model, the authors found that reductions in expectations about unemployment (used as a proxy for future economic activity) are accompanied by an increase in economic activity and a decrease in the observed unemployment rate. In fact, impulse-response analyses showed that, in the impact, a negative innovation in the expected unemployment rate (which means more optimistic expectations) six months ahead leads to a decrease in the current unemployment rate, an increase in inflation, and an increase in the three-month Treasury bond rate. The authors also describe that the findings remain consistent when they use data from the Michigan Survey to measure unemployment expectations.

Using European opinion survey data from the Joint Harmonised EU Programme of Business and Consumer Surveys, Girardi (2014) showed evidence that changes in household unemployment expectations are related to fluctuations in the observed unemployment rate. Through a VAR analysis using four variables (expected and observed unemployment rate, interest rate, and inflation rate), the author found that, on average, a negative shock that reduces household unemployment expectations leads to a decrease in the current unemployment rate.

Lehmann and Weyh (2016) analyzed the forecasting performance of employment expectations on employment growth in 15 European countries from 1998 to 2014, using the Granger causality test and out-of-sample forecasting model. They found that opin-

ion survey expectations three months ahead are an efficient indicator for anticipating employment growth in most countries.

Finally, Dickerson and Green (2012) examined the relationship between labor market expectations and the observed employment level using data on expected job loss and the probability of reemployment after job loss from workers in Germany and Australia. The authors found that in both countries, job loss expectations are strongly correlated with subsequent job loss in the predicted period and that expectations of finding a good replacement after job loss are correlated with subsequent success. On the other hand, it is observed that German and Australian workers show a certain bias toward pessimism in their expectations of job loss.

1.2 THE PROPOSED AGENT-BASED MODEL

The methodology proposed to address the research problems raised in the introduction of this essay involves the development of an Agent-Based Model (ABM) following the framework of Brock and Hommes (1997). ABM gained popularity in the 1990s as computational methods became readily available and proved to be an essential tool for constructing models and simulating the behavior of each agent and their interactions. This approach extends the analysis beyond the representative agent, enabling the investigation of the dynamics of complex economic systems with numerous heterogeneous agents (STEINBACHER et al., 2021).

According to Steinbacher et al. (2021), one can cite the following advantages of ABMs as an analytical methodology: (i) computational power to analytically address social interaction on a large scale; (ii) the ability to use decision rules to model behavior (behavioral heuristics) instead of mathematical optimization; (iii) the growing popularity of behavioral research in economics that aids in the development of ABMs; (iv) the rapid advancement of network theory in social sciences that benefits the formalization of interactions between agents; and (v) improvements in the estimation and calibration of ABMs allow for a better assessment of their fit to empirical data.

As emphasized by Dawid and Gatti (2018), developing an ABM typically involves initially designing the model. This entails determining the types of agents included in the model (e.g., banks, firms, and workers), defining the set of available decisions, their sets of internal states (e.g., wealth, skills, and occupation), and the potential for information exchange with other agents. Following this, the model is encoded, meaning that the designed model is computationally implemented in a programming language, and tests are conducted to ensure its proper implementation. Subsequently, parameters are calibrated, and the emerging properties of the model are analyzed, followed by its validation. Validation, in turn, involves assessing how the model simulations reflect the actual historical behavior of the system being simulated.

In this research, the first step in developing the proposed ABM (model design)

was based on the efficiency-wage model with heterogeneous expectations by Silveira and Lima (2021), which will be presented in subsection 1.2.1. Immediately following, subsection 1.2.2 outlines the design of the proposed ABM. Finally, section 1.2.3 discusses how the ABM was computationally implemented and how its parameters were calibrated.

1.2.1 The efficiency-wage model with heterogeneous expectations about the unemployment rate

Silveira and Lima (2021) proposed a short-term static model for the labor market to demonstrate that the significant positive correlation between pessimistic unemployment expectations and actual unemployment observed in empirical research that uses survey data can arise in an efficiency-wage model where workers exhibit heterogeneity in their unemployment expectations.

In the model proposed by the authors, firms cannot perfectly identify the level of effort of an individual worker, and the worker, in turn, exerts effort at a relatively higher or lower level depending on their expectation about the unemployment rate trend. In this context, it is assumed that a worker with a pessimistic expectation about the unemployment rate has an expected cost of job loss greater than an optimistic worker. Consequently, a pessimistic worker tends to exert more effort than an optimistic worker.

Given the heterogeneity in unemployment expectations and the resulting diversity in the level of effort exerted at work, each firm, unable to perfectly identify the effort level of its employees, sets a uniform wage that maximizes its profits and, consequently, minimizes the cost of labor per unit of average effort.

Silveira and Lima (2021) note that optimistic workers are more costly per unit of effort for the firm than neutral workers and, to a greater extent, pessimistic workers. This is because all hired workers receive the same wage, but neutral workers and, to a greater extent, pessimistic workers provide more effort than optimistic workers.

Regarding expectations about unemployment, workers can, at any given period, be one of three types. A worker can be neutral, optimistic, or pessimistic, denoted as a worker of type n , o , or p , respectively. A worker's effort depends positively on the wage received and negatively on an indicator of wage compensation associated with the expected labor market conditions for the worker. Silveira and Lima (2021) start from the following effort function:

$$\varepsilon_{\tau} = \begin{cases} \left(\frac{w_{\tau} - \mu_{\tau}}{\mu_{\tau}} \right)^{\gamma}, & \text{se } w_{\tau} > \mu_{\tau}, \\ 0 & \text{caso contrário,} \end{cases} \quad (1.1)$$

where ε_{τ} is the level of effort exerted by the worker of type $\tau \in \mathcal{T} = \{n, o, p\}$, $w_{\tau} \in \mathbb{R}_{++}$ is the wage received by the worker of type $\tau \in \mathcal{T}$, and $\mu_{\tau} \in \mathbb{R}_{++}$ is the wage compensation indicator associated with the expected labor market conditions for the worker of type

$\tau \in \mathcal{T}$. The parameter $\gamma \in (0, 1) \subset \mathbb{R}$ measures the effect of increasing effort to pay a worker a higher wage than that associated with the expected labor market conditions for the worker.

The authors assume that the functional form of μ_τ is given by:

$$\mu_\tau = (1 - u_\tau^e) w_{a,t}, \quad (1.2)$$

where $u_\tau^e \in [0, 1] \subset \mathbb{R}$ is the expected unemployment rate by the worker of type τ . Drawing on the empirical literature on heterogeneity in unemployment expectations, the authors assume that the expected unemployment rate is heterogeneous among agents and can be ordered as follows, depending on the type of agent considered:

$$0 < u_o^e < u_n^e = u < u_p^e < 1. \quad (1.3)$$

Each firm, being unable to perfectly identify the type of worker, determines the homogeneous wage w that minimizes the cost of labor per unit of average effort ε . More precisely, each firm determines the quantity of labor L and the wage w that maximizes its profit:

$$\pi = F(\varepsilon L) - wL, \quad (1.4)$$

where $F(\cdot)$ is the production function, with $F(\cdot)' > 0$ and $F(\cdot)'' < 0$ in \mathbb{R}_+ , and the average effort is given by:

$$\varepsilon = \varepsilon_n^\eta \varepsilon_o^\theta \varepsilon_p^\rho, \quad (1.5)$$

where η, θ, ρ denote the proportion of neutrals, optimists, and pessimists, respectively, with $\eta + \theta + \rho = 1$.

Assuming that $w > \mu_\tau$, the first-order conditions for an interior solution to the maximization problem in (1.4) can be expressed as follows:

$$\frac{\partial \pi}{\partial w} = F'(\varepsilon L) L \frac{\partial \varepsilon}{\partial w} - L = 0, \quad (1.6)$$

$$\frac{\partial \pi}{\partial L} = F'(\varepsilon L) \varepsilon - w = 0. \quad (1.7)$$

Substituting (1.6) into (1.7) yields the Solow condition, in which the profit-maximizing pair (w, L) implies the elasticity-wage of the average effort per unit ¹:

$$\frac{\partial \varepsilon}{\partial w} \frac{w}{\varepsilon} = 1. \quad (1.8)$$

Furthermore, given the assumption of the heterogeneous expectations model, the average effort level in (1.8) can be substituted by the definition of ε presented in (1.5). Taking this into account, the condition in (1.8) can be rewritten as follows:

$$\eta \frac{\partial \varepsilon_n}{\partial w} \frac{w}{\varepsilon_n} + \theta \frac{\partial \varepsilon_o}{\partial w} \frac{w}{\varepsilon_o} + \rho \frac{\partial \varepsilon_p}{\partial w} \frac{w}{\varepsilon_p} = 1. \quad (1.9)$$

¹ The first-order conditions are indeed sufficient conditions for a maximum, as it is assumed that the production function is strictly concave.

Equation (1.9) represents the Solow condition weighted by expectations. Utilizing (1.1) and (1.2), the weighted Solow condition can be rewritten as:

$$\left(\frac{\eta}{u_n^e} + \frac{\theta}{u_o^e} + \frac{\rho}{u_p^e} \right) \gamma = 1. \quad (1.10)$$

Silveira and Lima (2021) assume, for analytical simplicity, that expectations about the unemployment rate for pessimists and optimists diverge from neutral expectations by a dispersion factor $\delta \in (0, 1 - \gamma) \subset \mathbb{R}$ as follows:

$$u_\tau^e = \begin{cases} (1 - \delta)u, & \text{if } \tau = o, \\ u, & \text{if } \tau = n, \\ (1 + \delta)u & \text{if } \tau = p. \end{cases} \quad (1.11)$$

Substituting (1.11) into (1.10), the equilibrium unemployment rate of the model will be given by:

$$u^* = \left[1 + \left(\frac{\delta}{1 - \delta} \right) \theta - \left(\frac{\delta}{1 + \delta} \right) \rho \right] \gamma. \quad (1.12)$$

Once the equilibrium unemployment rate is determined, we can use (1.1), (1.2), (1.5), and (1.11) to derive the equilibrium average effort level, which will be given by:

$$\varepsilon^* = \left[\left(\frac{u^*}{1 - u^*} \right)^\gamma \right]^{1 - (\theta + \rho)} \left[\left(\frac{(1 - \delta)u^*}{1 - (1 - \delta)u^*} \right)^\gamma \right]^\theta \left[\left(\frac{(1 + \delta)u^*}{1 - (1 + \delta)u^*} \right)^\gamma \right]^\rho. \quad (1.13)$$

The equilibrium effort of each agent of type $\tau \in \mathcal{T}$ can be deduced as follows. Firstly, substituting (1.11) into (1.2) and considering the symmetric Nash equilibrium condition ($w_{a,\tau} = w, \forall \tau \in \mathcal{T}$), we obtain the labor market compensation indicator at the mentioned equilibrium:

$$\mu_\tau^* = \begin{cases} [1 - (1 - \delta)u^*]w^*, & \text{if } \tau = o, \\ (1 - u^*)w^*, & \text{if } \tau = n, \\ [1 + (1 + \delta)u^*]w^* & \text{if } \tau = p. \end{cases} \quad (1.14)$$

Next, by substituting (1.14) into (1.1), we derive the equilibrium effort for each type of worker:

$$\varepsilon_o^* = \left\{ \frac{w^* - [1 - (1 - \delta)u^*]w^*}{[1 - (1 - \delta)u^*]w^*} \right\}^\gamma = \left[\frac{(1 - \delta)u^*}{1 - (1 - \delta)u^*} \right]^\gamma \quad (1.15)$$

$$\varepsilon_n^* = \left[\frac{w^* - (1 - u^*)w^*}{(1 - u^*)w^*} \right]^\gamma = \left(\frac{u^*}{1 - u^*} \right)^\gamma, \quad (1.16)$$

$$\varepsilon_p^* = \left\{ \frac{w^* - [1 - (1 + \delta)u^*]w^*}{[1 - (1 + \delta)u^*]w^*} \right\}^\gamma = \left[\frac{(1 + \delta)u^*}{1 - (1 + \delta)u^*} \right]^\gamma. \quad (1.17)$$

It is important to note that, as expected, in equilibrium, the worker who holds pessimistic expectations about the unemployment rate provides relatively higher effort due to having a higher expected cost of job loss. On the other hand, the optimistic worker offers relatively lower effort due to having a lower expected cost of job loss. In summary, $\varepsilon_p^* > \varepsilon_n^* > \varepsilon_o^*$.

1.2.2 Formation of unemployment expectations as a discrete choice process

The model explored in this section encompasses the formation of unemployment expectations over the next 12 months as a discrete choice mechanism making use of the "Adaptively Rational Equilibrium Dynamics (ARED)" approach by Brock and Hommes (1997). In the ARED approach, agents base their decisions regarding future predictions of endogenous variables on equilibrium equations. These agents adapt their beliefs over time by choosing from a finite set of different predictors. Each predictor is a function of past observations, and its performance measure is publicly available. It is assumed that agents choose predictors based on their past performance.

An example given by Brock and Hommes (1997) to illustrate this dynamic is a case where agents can choose between predicting through a sophisticated predictor with a high associated cost or a simple predictor with a lower cost. Agents rationally choose among predictors and must make their choices considering the prediction error of the predictor and the associated cost. Suppose that if all agents used the sophisticated predictor, the temporal trajectory of prices would converge to a stable and unique steady state. In contrast, if all agents used the simple predictor, the exact unique steady state would occur, but it would be unstable this time. In an initial state with almost all agents using the simple predictor, prices would diverge from their steady-state value, and the prediction error of the simple predictor would increase. As a result, the number of agents willing to bear a higher cost to obtain the sophisticated predictor would increase. When the intensity of choice to switch between the two beliefs is high, as soon as the benefit of the higher accuracy associated with the sophisticated predictor outweighs its cost, almost all agents would choose the sophisticated predictor. Prices would then be pushed back to their steady-state value and remain there for a while. With prices close to their steady-state value, the prediction error of the simple predictor becomes small again. In contrast, the cost of the sophisticated predictor no longer compensates for the performance differential compared to the simple predictor. When the choice intensity is high, most agents will switch their beliefs to the simple predictor again, and the story repeats. Therefore, an interaction between two opposing forces can lead to very complicated Adaptive Rational Equilibrium Dynamics when the intensity of choice to change belief is high.

Inspired by the ARED approach, the idea of the ABM proposed here is that each agent (worker) can choose, in a given period, to have optimistic, pessimistic, or neutral expectations regarding unemployment. Pessimistic workers should expend more effort than neutral workers, and the latter, in turn, more than optimistic workers. It follows that when the unemployment rate is relatively low, agents generally have better expectations about the labor market and less incentive to exert effort, as the expected cost of job loss is lower (due to a high level of employability). Firms cannot perfectly identify the effort level of their workers. However, considering the equilibrium unemployment

rate, cf. equation in (1.12), obtained by Silveira and Lima (2021), the higher (lower) proportion of optimistic agents compared to pessimistic ones leads to a higher (lower) level of unemployment. In other words, by reinforcing workers' optimism, relatively low unemployment brings a tendency for increased unemployment due to reduced average effort. Symmetrically, in situations with a relatively high unemployment rate, workers become more pessimistic and are incentivized to exert more effort, reducing the equilibrium unemployment rate. This mechanism of interaction between unemployment and workers' expectations about the state of the labor market in an efficiency-wage model seems to generate endogenous fluctuations in unemployment and the composition of expectations about it in the population of workers. Therefore, the expectations of agents (optimistic, pessimistic, or neutral) and, consequently, their effort levels depend on the short-term equilibrium unemployment rate, which, in turn, depends on the proportion of optimistic and pessimistic agents. Thus, as in the example by Brock and Hommes (1997) mentioned earlier, in our model there are also two opposing forces governing the coevolution between the unemployment rate and the distribution of expectations about this rate among workers.

1.2.2.1 Structure of the discrete choice model associated with the formation of unemployment expectations

The structure presented below follows the literature on discrete choice models as presented in Train (2009), which serves as the basis for developing the proposed ABM.

Consider a worker (agent) $i \in \{1, 2, 3, \dots, A\}$, where A is the number of workers. Using the Michigan Survey as a reference, in each period t , a worker i is of type $\tau_i \in \mathcal{T} = \{n, o, p\}$, with the respective subscripts standing for neutral, optimistic, and pessimistic about the unemployment rate in a given period $t + \Delta$ in the future, with $\Delta \in \mathbb{N}_+$ being a given and finite number. In the Michigan Survey, for example, $\Delta = 12$ months. From now on, when we refer to the expected (future) unemployment rate or the unemployment rate in the future, we will be talking about the expectation formed in a given period t about the unemployment rate in the period $t + \Delta$. More precisely, if a worker is of type $\tau_i = o$ (type $\tau_i = p$), she holds an optimistic (pessimistic) unemployment expectation and considers that in the future, the unemployment rate will be lower (greater) than the current one. Meanwhile, a worker of type $\tau_i = n$ considers that in the future, the unemployment rate will be the same as the current one. We consider the next 12 months as the future in the unemployment expectations formation environment studied.

Based on Train (2009), the utility or payoff function of a worker i can be additively decomposed into a deterministic component, denoted by $U^d(\tau_i)$, associated with her observable motivations, and a random component, denoted by $\zeta(\tau_i)$, referring to her

unobservable motivations. Formally:

$$U(\tau_i) = U^d(\tau_i) + \zeta(\tau_i). \quad (1.18)$$

We will make the same assumption usually made in the discrete choice literature that considers that agents of the same type share the same deterministic component in their utility or payoff function. That is, for any two workers i and j , if $\tau_i = \tau_j$, then $U^d(\tau_i) = U^d(\tau_j)$. The unobserved component of the utility or payoff, meanwhile, is possibly and likely heterogeneous across workers: for any two workers i and j , even if $\tau_i = \tau_j$ it may be the case that $\zeta(\tau_i) \neq \zeta(\tau_j)$. In the model set forth herein, this unobserved component reflects idiosyncratic characteristics that affect the formation of unemployment expectations. As emphasized by Curtin (2019a), the heterogeneity of expectations may stem from differences in the sets of information that people consider relevant (or, we would add, are able to collect and process). These sets of information appear to reflect people's economic situations, as individuals share information among their family members, friends, neighbors, co-workers, and many other acquaintances with whom they interact daily Curtin (2019a, p. 47).

At the beginning of each period t , an individual worker forms her unemployment expectation (which we interpret as her choice of type τ_i in that period) by selecting an available alternative that provides her with the highest utility or payoff. Therefore, an individual worker will be of the type $\tau_i \in \mathcal{T}$ whose associated utility satisfies:

$$U(\tau_i) \geq U(\tau'_i), \forall \tau'_i \in \mathcal{T}. \quad (1.19)$$

Considering the utility function in (1.18), the decision-making rule in (1.19) can be re-expressed as follows:

$$U^d(\tau_i) - U^d(\tau'_i) \geq \zeta(\tau'_i) - \zeta(\tau_i), \forall \tau'_i \in \mathcal{T}. \quad (1.20)$$

Note that the inequality in (1.20) implies that even if the deterministic component of the utility associated with the unemployment expectation of type τ_i is strictly greater than the deterministic component of the utility associated with the alternative unemployment expectation of type τ'_i , that is, even if $U^d(\tau_i) - U^d(\tau'_i) > 0$, the unemployment expectation of type τ_i will not necessarily be the one held by an individual worker i in a given period t .

Given the presence of a random component influencing the choice or type of unemployment expectation by the i -th worker, we can only specify her statistical propensity to hold each type of unemployment expectation in \mathcal{T} . Using (1.19) and (1.20), we can formally express the probability with which the i -th individual worker will choose the

unemployment expectation of type τ_i as follows:

$$\begin{aligned}
 Prob(\tau_i) &= Prob(U(\tau_i) \geq U(\tau'_i), \forall \tau'_i \in \mathcal{T}) \\
 &= Prob(\zeta(\tau'_i) - \zeta(\tau_i) \leq U^d(\tau_i) - U^d(\tau'_i), \forall \tau'_i \in \mathcal{T}) \\
 &= \int_{-\infty}^{\infty} I[\zeta(\tau'_i) - \zeta(\tau_i) \leq U^d(\tau_i) - U^d(\tau'_i), \forall \tau'_i \in \mathcal{T}] f(\vec{\zeta}_i)(d\vec{\zeta}_i),
 \end{aligned} \tag{1.21}$$

where $f(\vec{\zeta}_i)$ is the joint probability density function of the vector of random variables $\vec{\zeta}_i$, which is composed of the random variables $\zeta(\tau_i)$, with $\tau_i \in \mathcal{T}$, and $I[\cdot]$ is the indicator function that takes the value of 1 if the expression within brackets is true and zero otherwise. It is important to note that expression in (1.21) indicates that the propensity to choose the strategy τ_i by worker i increases as the differential of observable incentives in favor of this alternative increases.

As pointed out by Train (2009), several discrete choice models can be generated from different specifications of $f(\vec{\zeta}_i)$. In particular, a specification that is considered convenient due to it leading to a closed form for the integral in (1.21) is the logit specification. Such a version is derived by assuming that the random components of the utility functions in (1.18) are independent random variables with the same extreme value probability distribution, which has a Gumbel probability density function (or type I extreme value). Formally:

$$f(\zeta(\tau_i)) = \beta e^{-\beta\zeta(\tau_i)} e^{-e^{-\beta\zeta(\tau_i)}}, \tag{1.22}$$

where $\beta \in \mathbb{R}_+$ is a parametric constant dubbed intensity of choice in Brock and Hommes (1997). The cumulative distribution function associated with the function in (1.22) is given by:

$$F(\zeta(\tau_i)) = e^{-e^{-\beta\zeta(\tau_i)}}. \tag{1.23}$$

Using the functions in (1.22) and (1.23), we can solve the integral in (1.21) to find the choice probability of the i -th worker as the logistic cumulative distribution function:

$$Prob(\tau_i) = \frac{e^{\beta U^d(\tau_i)}}{\sum_{\tau'_i \in \mathcal{T}} e^{\beta U^d(\tau'_i)}} = \frac{1}{1 + \sum_{\tau'_i \in \mathcal{T}, \tau'_i \neq \tau_i} e^{-\beta[U^d(\tau_i) - U^d(\tau'_i)]}}. \tag{1.24}$$

It is important to note that the higher the value taken by the intensity of choice β , *ceteris paribus*, the greater the relative weight of the deterministic component (and hence of the observable motivations) concerning the random component (and hence the idiosyncratic motivations) in determining the propensity of a worker i to hold the unemployment expectation of type τ_i .

The formal structure of the discrete choice model outlined in this section will be used in the next section to analyze the process of the formation of unemployment expectations.

1.2.2.2 The Agent-Based Model of Unemployment Expectations Formation as a Discrete Choice Process

In this subsection, we present the ABM proposed to analyze the coevolution of the formation of unemployment expectations and the observed unemployment rate.

In the environment of unemployment expectation formation considered herein, each worker i , in each period t , becomes optimistic, pessimistic, or neutral depending on her private incentive to obtain the highest wage per unit of effort and her social incentive to somewhat conform with the predominant type of unemployment expectation in the population of workers. The latter incentive reflects the occurrence of a popularity (or diffusion) effect, according to which in selecting what unemployment expectation to hold a worker i is socially influenced as well.

Following Brock and Durlauf (2001), we will decompose the total deterministic utility into two components: the private deterministic utility (associated with private incentives) and the social deterministic utility (associated with social incentives). More precisely, the total deterministic utility in (1.18) will be represented by the sum of the private deterministic utility, denoted by $V(\cdot)$, and the social deterministic utility, denoted by $S(\cdot)$, weighted by a parametric constant $\psi \in \mathbb{R}_{++}$, as follows:

$$U^d(\tau_{i,t}) = V(\tau_{i,t}) + \psi S(\tau_{i,t}). \quad (1.25)$$

It is assumed that workers tend to have higher private utility if they earn higher wages per unit of effort. Meanwhile, the higher the value of the unemployment rate, the less likely a given worker is to have received that wage per unit effort. More precisely, the private utility of the i -th worker, with $i \in \{1, 2, 3, \dots, A\}$, is represented by her wage per unit of effort weighted by the level of employment, as follows:

$$V(\tau_{i,t}) = (1 - u_t^*) \frac{w_{i,t}^*}{\varepsilon_{\tau_{i,t}}^*}, \quad (1.26)$$

where $\varepsilon_{\tau_{i,t}}^*$ is the level of effort provided by a worker holding an unemployment expectation of type $\tau_{i,t} \in \mathcal{T}$, whereas u_t^* and $w_{i,t}^*$ are the unemployment and wage rates in a determined *temporary equilibrium*². Note that the values of the variables featured in (1.26) come from the heterogeneous expectations-augmented efficiency wage model outlined in Silveira and Lima (2021).

From now on, let η_t , θ_t , and ρ_t be the proportions of neutral, optimistic, and pessimistic workers in a given period t , respectively, with $(\eta_t, \theta_t, \rho_t) \in \Delta$, where $\Delta \equiv \{(\eta + \theta + \rho) \in \mathbb{R}_+^3 : \eta + \theta + \rho = 1\}$ is the unit simplex in \mathbb{R}_+^3 composed of all possible frequency distributions of unemployment expectations across workers.

² That is, the equilibrium values of the variables $\varepsilon_{\tau_{i,t}}^*$, u_t^* , and $w_{i,t}^*$ as determined for a given frequency distribution of unemployment expectations across workers (and hence parameterized by it) in period t , as derived in Silveira and Lima (2021).

In Silveira and Lima (2021), the unemployment rate in the temporary equilibrium in a given period t , considering (1.12), can be expressed as follows:

$$u_t^* = \left[1 + \left(\frac{\delta}{1-\delta} \right) \theta_t - \left(\frac{\delta}{1+\delta} \right) \rho_t \right] \gamma, \quad (1.27)$$

where $\delta \in (0, 1-\gamma) \subset \mathbb{R}$ is a parametric constant measuring the dispersion among the three types of unemployment expectation and $\gamma \in (0, 1) \subset \mathbb{R}$ is a parametric constant measuring the effort-enhancing effect of paying to a worker a wage compensation which is higher than the wage compensation associated with her expected labor market conditions.

Meanwhile, using (1.1) and (1.7), and assuming a Cobb-Douglas function of the type $F(\cdot) = A(\epsilon L)^\alpha$, with $A = 1$, the wage rate in the temporary equilibrium in a certain period t , which is by assumption homogeneous across workers, is given by:

$$w_{i,t}^* = \alpha (\epsilon_t^*)^\alpha (1-u_t^*)^{\alpha-1} \equiv w_t^*, \quad (1.28)$$

where ϵ_t^* is the average effort level in the temporary equilibrium in a certain period t , which, considering (2.13), is given by:

$$\epsilon_t^* = \left[\left(\frac{u_t^*}{1-u_t^*} \right)^\gamma \right]^{1-(\theta_t+\rho_t)} \left[\left(\frac{(1-\delta)u_t^*}{1-(1-\delta)u_t^*} \right)^\gamma \right]^{\theta_t} \left[\left(\frac{(1+\delta)u_t^*}{1-(1+\delta)u_t^*} \right)^\gamma \right]^{\rho_t}. \quad (1.29)$$

Finally, taking into account (2.15)-(2.17), the level of effort exerted by a worker i holding an unemployment expectation of type $\tau_{i,t} \in \mathcal{T}$ in the temporary equilibrium in a certain period t is given by:

$$\epsilon_{\tau_{i,t}}^* = \begin{cases} \left(\frac{u_t^*}{1-u_t^*} \right)^\gamma, & \text{if } \tau_{i,t} = n, \\ \left[\frac{(1-\delta)u_t^*}{1-(1-\delta)u_t^*} \right]^\gamma, & \text{if } \tau_{i,t} = o, \\ \left[\frac{(1+\delta)u_t^*}{1-(1+\delta)u_t^*} \right]^\gamma, & \text{if } \tau_{i,t} = p. \end{cases} \quad (1.30)$$

Moreover, the social utility in (1.25) of a worker i holding an unemployment expectation of type τ_i is determined by the popularity (diffusion) of such an expectation in the population of agents. More precisely, we will assume that the social utility of a worker's unemployment expectation is higher when a larger proportion of workers held the same expectation in the previous period. Formally:

$$S_{\tau_{i,t}} = \begin{cases} \theta_t, & \text{if } \tau_{i,t} = o, \\ \eta_t, & \text{if } \tau_{i,t} = n, \\ \rho_t, & \text{if } \tau_{i,t} = p. \end{cases} \quad (1.31)$$

Having specified the private and social components of the utility function represented in (1.25), we can rewrite (1.24) as follows:

$$Prob(\tau_{i,t}) = \frac{1}{1 + \sum_{\tau'_{i,t-1} \in \mathcal{T}, \tau'_{i,t-1} \neq \tau_{i,t-1}} e^{-\beta \{ [V(\tau'_{i,t-1}) + \psi(S(\tau'_{i,t-1}))] - [V(\tau_{i,t-1}) + \psi(S(\tau_{i,t-1}))] \}}}. \quad (1.32)$$

1.2.3 Computational implementation and calibration of the proposed agent-based model

The number of agents (workers) used to implement the proposed ABM was chosen based on the household survey that served as a reference in the model calibration. We implemented the calibration using data from the US Michigan Survey so that the model features 501 agents.³ In the initial period, each type of unemployment expectation (neutral, optimistic, and pessimistic) was held by $\frac{1}{3}$ of the population of agents, so 167 agents held each type of unemployment expectation. Recall that at the beginning of each period t , a worker i forms either a neutral ($\tau_i = n$) or an optimistic ($\tau_i = o$) or a pessimistic ($\tau_i = p$) expectation about the unemployment rate in that period. After establishing the initial conditions, we compute the temporary equilibrium values of the unemployment rate, the wage rate, and the average and individual effort level in period 1 using the equation in (1.27) first and then the equations in (1.28), (1.29), and (1.30). After that, based on the temporary equilibrium value of the unemployment rate, the temporary equilibrium values of the wage and the individual effort level in period 1, we compute the private utility in this period through the expression in (1.26). Then we calculate the social utility in this period through the function in (1.31). After computing the private and the social deterministic utility for the period 1, we compute the deterministic utility in this period using the expression in (1.25). Finally, given the temporary equilibrium value of total private utility in period 1 we are able to compute the probabilities of choice specified in (1.32), which plays a key role in the determination of the frequency distribution of unemployment expectations across workers for the next period.

Following the suggestion presented in Curtin (2019a, p. 6) that agents' expectations reflect their personal experiences, it is reasonable to assume that workers who became unemployed in any of the $t-l$ periods, where $l \in [1, L] \subset \mathbb{N}$ and L should be determined endogenously, will have a higher propensity to hold pessimistic unemployment expectation in period t than those that were employed in all $t-l$ periods. Yet, it does not seem plausible to believe that the weight of the most recent unemployment on the propensity to pessimism is the same as the weight of the previous unemployment state. Alternatively, we assume that the probability with which an unemployed worker in period $t-l$ will hold a pessimistic unemployment expectation in t decreases with l .

With that in mind, we created the following indicator function:

$$I_{t-l}^i = \begin{cases} 1, & \text{if the worker } i \text{ was unemployed in period } t-l, \\ 0, & \text{otherwise.} \end{cases} \quad (1.33)$$

³ The US Michigan Survey has around 500 respondents, but here, we considered 501 agents in order to have the population of agents equally distributed among the three alternative types of unemployment expectation in the initial period.

Assuming that the weight of the unemployment on the propensity to hold pessimistic expectations (ι) decays geometrically with l by a ratio $q \in (0, 1)$ we have that:

$$\iota_{t-l} = q^l \iota_{t-1}. \quad (1.34)$$

Considering that $\sum_{l=1}^L \iota_{t-l} = 1$ and (1.34), we can re-write ι_{t-1} as follows:

$$\iota_{t-1} = \frac{q-1}{q^L-1}. \quad (1.35)$$

Let F_t^i be the unemployment factor of worker i in period t , which we will define as follows:

$$F_t^i = \iota_{t-1} l_{t-1}^i + \iota_{t-2} l_{t-2}^i + \dots + \iota_{t-L} l_{t-L}^i. \quad (1.36)$$

Using (1.33) and (1.34) in (1.36), we are able to express this unemployment factor as:

$$F_t^i = \iota_{t-1} \sum_{l=1}^L q^{l-1} l_{t-l}^i = \frac{q-1}{q^L-1} \sum_{l=1}^L q^{l-1} l_{t-l}^i. \quad (1.37)$$

At the end of each period, given that workers share the same probability of obtaining a job, we randomly choose the workers whom no firm has hired. For example, if the unemployment rate for a determined period was 5%, we randomly select 5% of the population of workers (or the nearest integer) to have faced unemployment in that period. For the workers that have been unemployed in at least one of the L periods, we assume that the bias to pessimism in t is:

$$\mathcal{Q} = \min\{(1 + F_t^i) \text{Prob}(\tau_{i,t} = \rho), 1\}, \quad (1.38)$$

where $\text{Prob}(\tau_{i,t} = \rho)$ is calculated using the equation in (1.32).

Given the value of this \mathcal{Q} , we take a random number $r \in [0, 1] \subset \mathbb{R}$ from a uniform distribution. If $r_{i,t} < \mathcal{Q}$, we have that the worker i in period t holds pessimistic unemployment expectations.

For workers who were (and remained) employed in all L periods previous to t , the type of unemployment expectation that they will hold for the next period is determined by using the probability of choice specified in (1.32). Given the values of those probabilities, we take a random number $r \in [0, 1] \subset \mathbb{R}$ from a uniform distribution to define the unemployment expectation of those workers in any period $t \geq 2$ applying the rules as specified in Table 1.1.

The value of the following parameters was obtained through calibration to find the combination of values that provides the best fit of the model to the empirical data: β (the intensity of choice in (1.22)), ψ (the weight of the social component in the total deterministic utility in (1.25)), δ (the measure of dispersion among the three types of unemployment expectations in (1.27)), q (the decay rate of the unemployment weight on

Table 1.1 – Algorithm of unemployment expectation formation for a given period $t \geq 2$ by a worker i that did not have the bias to form pessimistic unemployment expectations.

Possible cases	Worker i 's unemployment expectation in a period $t \geq 2$
$r \leq \text{Prob}(\tau_{i,t} = p)$	Pessimistic
$\text{Prob}(\tau_{i,t} = p) < r \leq \text{Prob}(\tau_{i,t} = p) + \text{Prob}(\tau_{i,t} = n)$	Neutral
$r > \text{Prob}(\tau_{i,t} = p) + \text{Prob}(\tau_{i,t} = n)$	Optimistic

Source: Own elaboration.

the bias to pessimism in (1.34)) and L (the number of lags at which the unemployment experience is still taken into account on the to bias to pessimism).

Our calibration strategy consisted of finding the combination of parameter values that minimizes the sum of squares of the deviations of the simulated data from the respective observed data. Two empirical time series were used for calibration. The first was the Balance Score (BS) monthly time series, which is calculated from the proportion of optimistic and pessimistic respondents of the Survey of Consumers from the University of Michigan in the United States.⁴ As observed by Curtin (2019a), the BS is a standard method used to report survey expectations data and is defined as the percentage of respondents who thought that the unemployment rate would increase minus the percentage who thought that it would fall, plus 100. The second time series was the U.S. monthly unemployment rate, which is provided by the *Federal Reserve Economic Data*.⁵ The time series extracted for our calibration procedure was from January 1978 to December 2019, totaling 504 months. The combination of parameter values that we selected was the one that minimized the following objective function:

$$\sum_{t=1}^T [(u_t^{\text{observed}} - u_t^{\text{simulated}})^2 + (BS_t^{\text{observed}} - BS_t^{\text{simulated}})^2], \quad (1.39)$$

where $T = 504$ is the total number of periods, u^{observed} is the observed unemployment rate, $u^{\text{simulated}}$ is the simulated unemployment rate, BS^{observed} is the observed BS, and $BS^{\text{simulated}}$ is the simulated BS. The simulated BS was calculated in the same way as the observed BS is calculated in the Michigan Survey, given by:

$$BS_t = 100(\rho_t - \theta_t + 1). \quad (1.40)$$

The function *fminsearchbnd* from MATLAB was used to solve such a minimization problem. This function randomly chooses different combinations of parameters,

⁴ Available at <https://data.sca.isr.umich.edu/#> In the U.S. Michigan survey, households are asked monthly: "How about people out of work during the coming 12 months — do you think that there will be more unemployment than now, about the same, or less?". In addition to 'less', 'more' and 'same', possible answers include 'don't know' and 'no answer', where the latter two usually comprise a very small percentage of all the answers.

⁵ Available at <https://fred.stlouisfed.org/series/UNRATE>.

with their values having to be found within previously established limits. Using a first random combination of parameters, the algorithm computationally calculates simulated values for the BS and the unemployment rate for each simulation step. By inserting the empirical and simulated values into the objective function in (1.39), the value of this function associated with the vector of parameter values used is obtained. This same calculation is done for another random combination of parameters. If this last combination provides a higher accuracy (smaller distance between the simulated and empirical series), it is stored and the previous combination is discarded by the search algorithm of the function *fminsearchbnd*. The process is repeated for 1,000 different combinations of parameters or until the algorithm finds a set that generates a reduction in the value of (1.39) of less than 0.01.

We defined the following plausible ranges within which the *fminsearchbnd* algorithm searched for the parameter values providing the best fit of the model to the empirical data: $0 \leq \beta \leq 10$, $0 \leq \delta \leq 0.97$, $0 \leq \psi \leq 2$, $0 \leq q \leq 1$ and $1 \leq L \leq 12$. Due to the existence of local minima, we tried to obtain the initial configuration of parameters that would generate, after finishing the algorithm, the smallest value for the expression in (1.39). For such a purpose, 400 combinations of initial values were randomly chosen within the range of admissible values previously specified. Among the 400 different initial parameter combinations, the best configuration we found was: $\beta = 7.72$, $\delta = 0.91$, $\psi = 0.33$, $L = 9.09$ and $q = 0.11$. When we use this best configuration of initial parameter values, the combination of parameters ultimately selected by the *fminsearchbnd* function is reported in Table 1.2.

Table 1.2 – Calibrated parameter values.

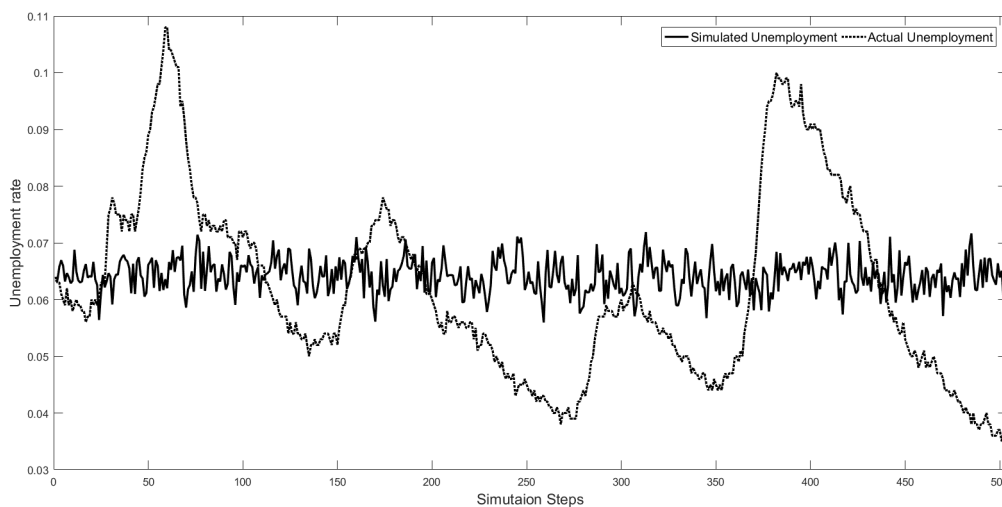
Parameters	Calibrated values
Intensity of choice (β)	5.44
Dispersion among types of unemployment expectations (δ)	0.81
Weight of the social component in the total deterministic utility (ψ)	0.17
The decay rate of the unemployment weight on the pessimism bias (q)	0.26
The number of lags considered on the pessimistic bias (L)	12

Source: Own elaboration.

In addition to the parameters whose calibrated values are reported in Table 1.2, the value of the parameter γ introduced in equation (1.27) is the one suggested by Romer (2012, p. 465), which is $\gamma = 0.03$ (recall that this parameter measures the effort-enhancing effect of paying to a worker a wage compensation which is higher than the wage compensation associated with her expected labor market conditions).

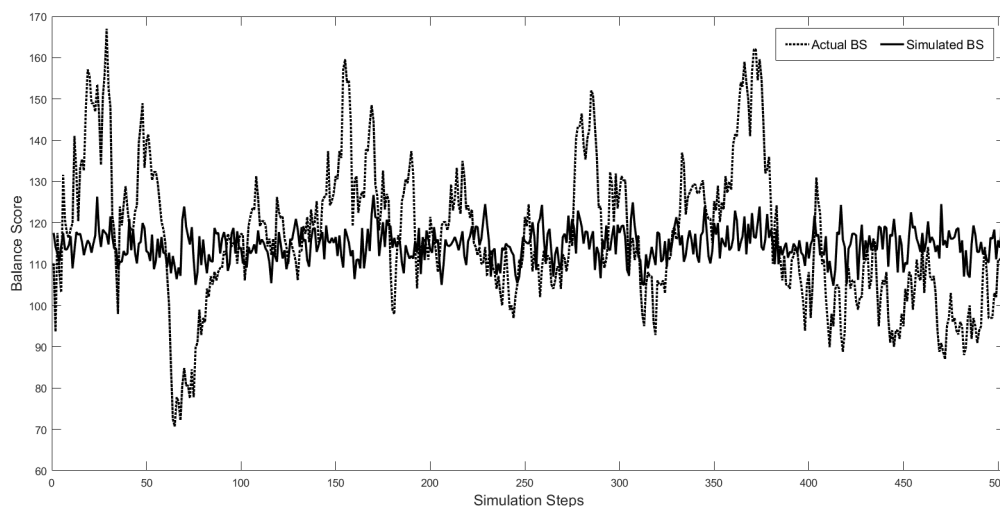
Figures 1.1 and 1.2 display the plots of the actual and simulated unemployment rate and the BS values, respectively. The simulated data were generated with the parameter values reported in Table 1.2 and a uniform frequency distribution of the three

Figure 1.1 – Actual and simulated unemployment rate.



Source: Own elaboration.

Figure 1.2 – Actual and simulated BS.



Source: Own elaboration.

types of unemployment expectations across workers in the initial period, as mentioned earlier in this section.

It can be observed that the simulated time series are relatively close to the actual time series, although the volatility of the latter is considerably higher.

1.3 EMERGENT PROPERTIES OF THE ABM

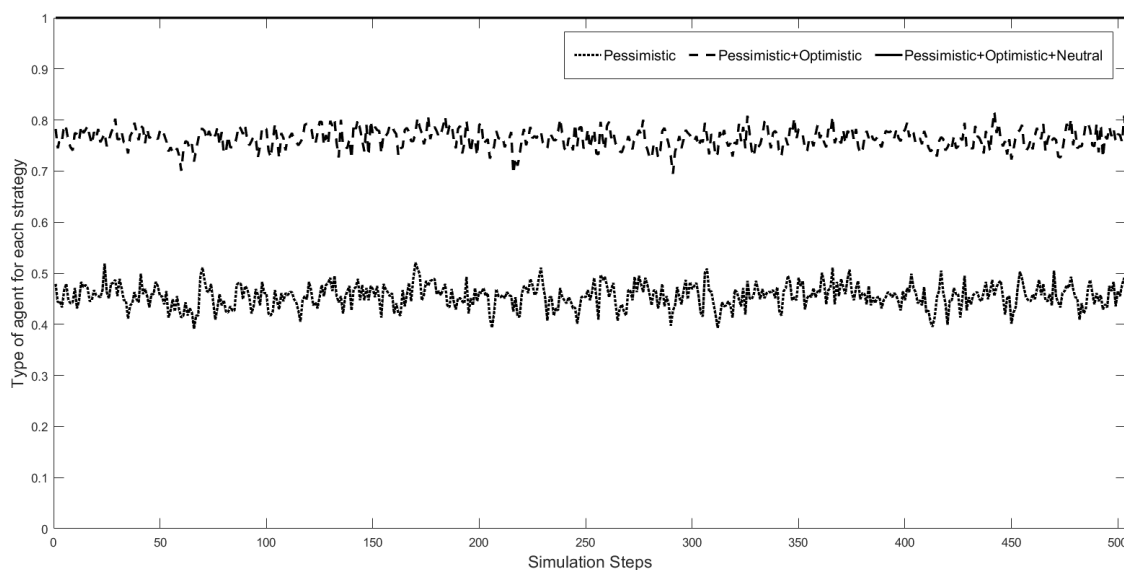
In this section, we present some emergent properties of the ABM set forth in this essay. These simulation results were generated using the set of parameter values found in the calibration of the model and shown in Table 1.2, as well as a uniform frequency distribution of the three types of unemployment expectations across workers in the initial period. We ran each simulation with 566 steps (periods), and all the following emergent properties were generated using simulations with the same random seed. The time interval composed by the first 62 periods was disregarded as the transient interval so only the dynamics in the last 504 periods are shown in the following figures (recall that the time series used in our calibration procedure were from the first month of 1978 to the last month of 2019, thus totaling 504 months).

The first property to be analyzed is the heterogeneity in the choice of expectations regarding unemployment. Figure 1.3 presents the dynamics of the proportion of each type of unemployment expectation in the population of workers over the last 504 steps of the simulation. The small dotted curve represents the proportion of pessimistic workers. The large dotted curve represents the proportion of pessimistic workers plus the proportion of optimistic workers, so the difference between the large dotted and the small dotted curve measures the proportion of optimistic workers. Meanwhile, the proportion of neutral workers is measured by the difference between the bold solid line parallel to the x-axis that cuts the y-axis at 1 and the large dotted curve. Note that the heterogeneity in the frequency distribution of unemployment expectations across workers is persistent, with no single type of expectation ever being held by all workers. Although such heterogeneity is endogenously time-varying, the considered proportions (and hence the resulting heterogeneity) do not seem to present any long-run trend, apparently being subject to mean-reverting dynamics.

A similar pattern can be observed if we consider the actual dynamics of the proportion of each type of unemployment expectation. Just as before, in Figure 1.4 the small dotted curve represents the proportion of pessimistic workers. The large dotted curve represents the proportion of pessimistic workers plus the proportion of optimistic workers, so the difference between the large dotted and the small dotted curve measures the proportion of optimistic workers. Finally, the proportion of neutral workers is measured by the difference between the bold solid line parallel to the x-axis that cuts the y-axis at 1 and the large dotted curve. Similar to what we observed for the simulated series, the heterogeneity in the frequency distribution of unemployment expectations across workers is persistent, and there is not just one type of expectation dominant at any one time. However, we can see that for the real series the volatility is higher. Finally, it can also be observed that the heterogeneity shows no long-term trend and is apparently subject to mean-reversion dynamics.

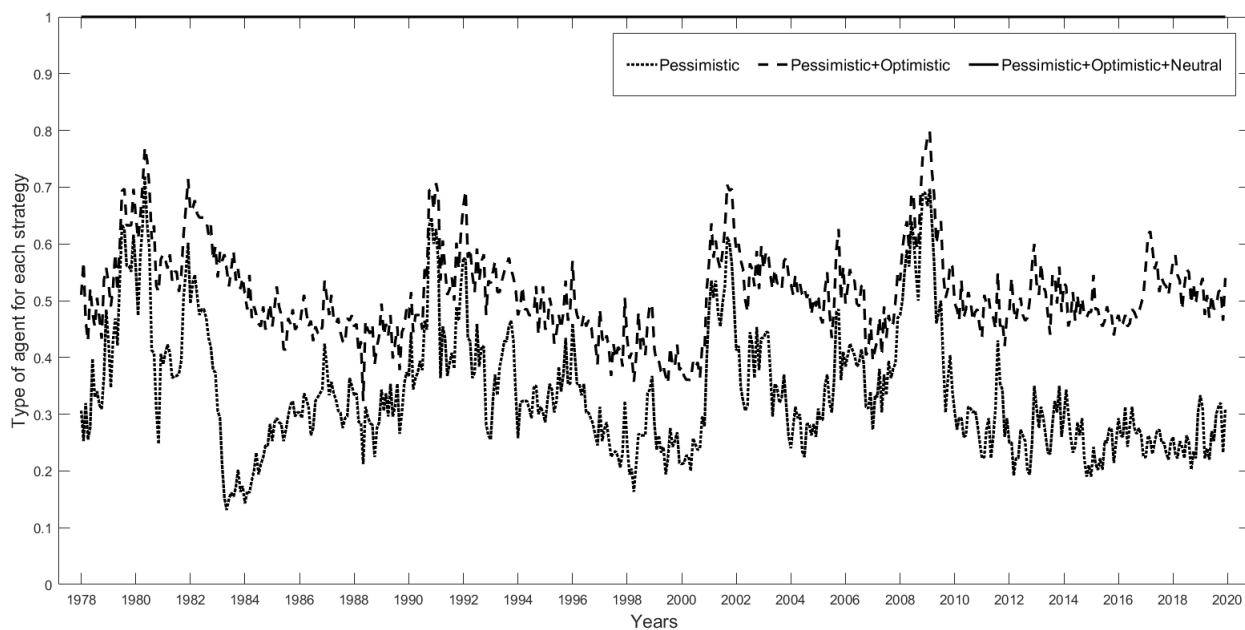
Another interesting emergent property to be reported is the dynamics of the sim-

Figure 1.3 – Simulated proportion of neutral, optimistic and pessimistic workers.



Source: Own elaboration.

Figure 1.4 – Actual proportion of neutral, optimistic and pessimistic workers.

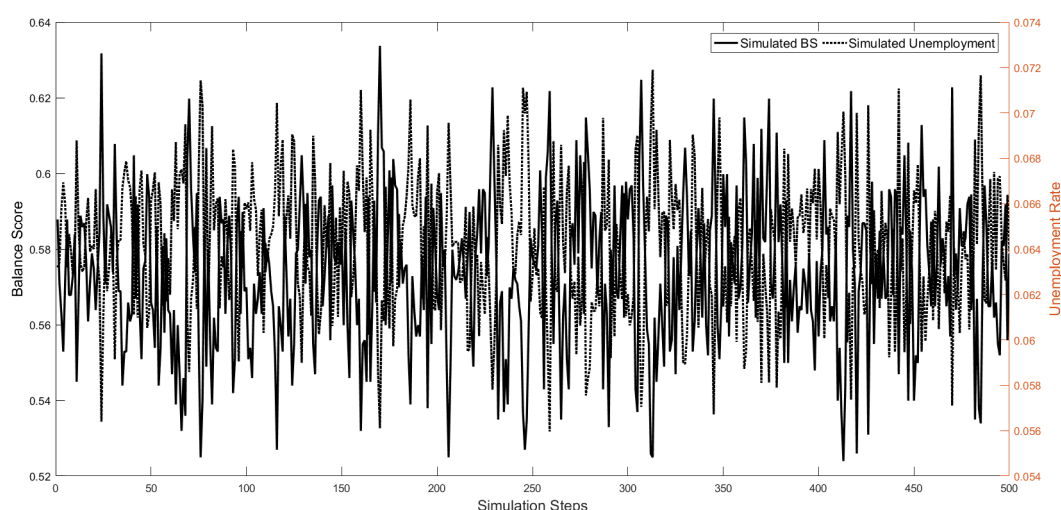


Source: Own elaboration.

ulated values of the unemployment rate and the BS, which are presented in Figures 1.5 and 1.6. In order to have both the BS and the unemployment rate varying in the same interval, we normalize the expression for the BS specified in (1.40) by 200. Consequently, when all workers hold pessimistic (optimistic) unemployment expectations, it follows

that the normalized BS is equal to one (zero) and equal to 0.5 when there is equalization of the pessimistic proportion of workers with the optimistic one. A visual inspection of Figures 1.5 and 1.6 suggests that there is a negative contemporaneous correlation between the BS and unemployment rate. The negative correlation between the BS and the unemployment rate is one of the possible correlations between the considered variables derived in Silveira and Lima (2021). Note from the specification in (1.40) that the value of the BS crucially depends on the proportion of pessimistic workers relative to the proportion of optimistic workers. Moreover, an increase in the proportion of pessimistic workers unambiguously leads to either an increase, a decrease, or a no-change in the BS depending on whether the proportion of optimistic workers either remains constant (or increases less than proportionately or even decreases) or increases more than proportionately or increases proportionately, respectively. In fact, a key result derived in Silveira and Lima (2021) is that a change in the frequency distribution of unemployment expectations across workers impacts non-linearly on actual unemployment. More precisely, whether a higher proportion of workers holding pessimistic unemployment expectations (and hence facing a higher expected cost of job loss) leads to a lower or higher unemployment rate depends on the prevailing configuration of heterogeneity in unemployment expectations across workers. A visual inspection of Figures 1.5 and 1.6 shows moments of high BS followed by periods of high unemployment rates, which is a stylized fact reported in the empirical literature.

Figure 1.5 – Normalized BS and unemployment rate simulated over the last 504 simulation steps.

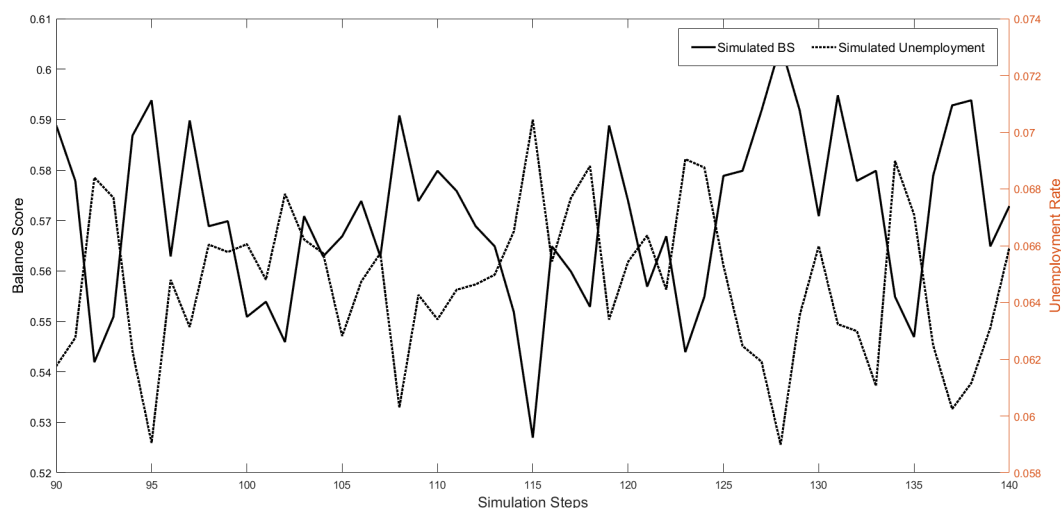


Source: Own elaboration.

To further explore the joint dynamics of the BS and the unemployment rate, Figure 1.6 presents the series of these variables in a window with only 50 simulation

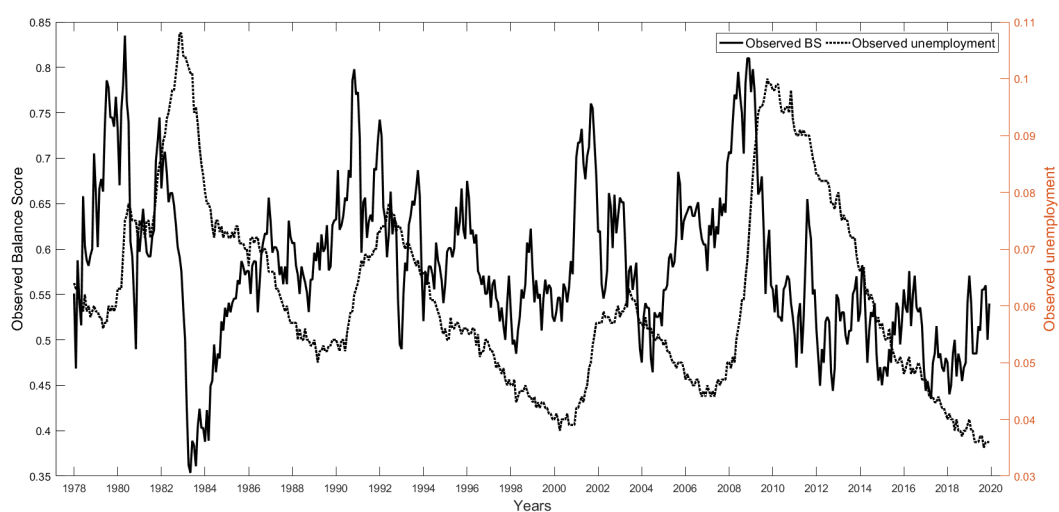
steps, from step 90 to step 140. Note that the dynamics of the trend reversal are irregular. For example, there are situations in which an upward trend is quickly reversed to a downward trend (in just one or two periods) and situations in which the same trend reversal takes longer.

Figure 1.6 – Simulated normalized BS and unemployment rate between simulation steps 90 and 140.



Source: Own elaboration.

Figure 1.7 – Observed normalized BS and unemployment rate.



Source: Own elaboration.

Figure 1.8 shows the observed unemployment rate and BS. It can be seen that, because observed unemployment has longer cycles of growth and decline than

simulated unemployment, the high (low) BS at times of low (high) unemployment is not as clear as in the simulated case. However, it is still possible to see periods in which this fact can be observed more clearly.

As reported in subsection 1.1.2, there is considerable empirical evidence of a correlation between the state of unemployment expectations and the actual unemployment rate. For example, using a Granger causality test, Curtin (2019a) explores the temporal causality between these variables. This test seeks to verify whether a variable can predict future changes in another variable beyond what could be predicted by the latter's own previous changes. Using data ranging from 1978 to 2016, Curtin (2019a) found that the unemployment rate Granger causes the BS and the BS Granger causes the unemployment rate.

In order to check whether the ABM set forth in this paper replicates this empirical evidence, we performed a similar Granger causality test using the series of simulated values of the BS and the unemployment rate. To perform this test, we first checked whether our model could be estimated in level form, for which we applied the Augmented Dickey-Fuller (ADF) unit root test. For both variables of interest, the null hypothesis of a unit root was rejected at a significance level of 5%. We then analyzed three information criteria to select the optimal number of lags for the model to be estimated, namely, the Akaike information criterion (AIC), the Hannan and Quinn information criterion (HQ), and the Schwarz information criterion (SC). All of them recommended that the model should feature only 1 lag.

Table 1.3 presents the statistics of the considered causality test. It can be observed that both the null hypothesis that the unemployment rate does not Granger-cause the BS and the null hypothesis that the BS does not Granger-cause the unemployment rate were rejected at the 5% significance level.

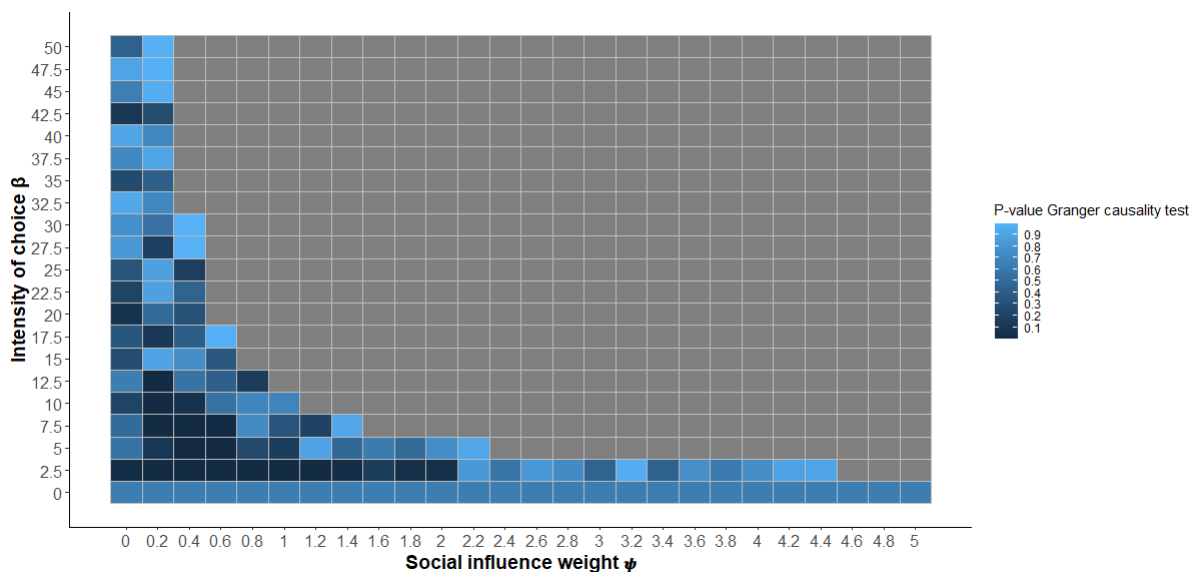
Similar results were obtained when performing the same test with the empirical data described in subsection 1.2.3 (which was used in the calibration). The test statistics are shown in Appendix A. More precisely, we found evidence that the actual unemployment rate causes, in the Granger sense, the actual BS and the actual BS also causes, in the Granger sense, the actual unemployment rate in the period studied. This result is an indication that the simulated model is able to replicate those temporal precedence relationships observed in the empirical data. It should be noted, however, that the information criteria of the empirical data suggested models with 2 or 6 lags.

Table 1.3 – Granger causality test for simulated unemployment and BS.

Null Hypothesis	p-value
Unemployment rate does not Granger-cause BS	0.02
BS does not Granger-cause unemployment rate	0.00

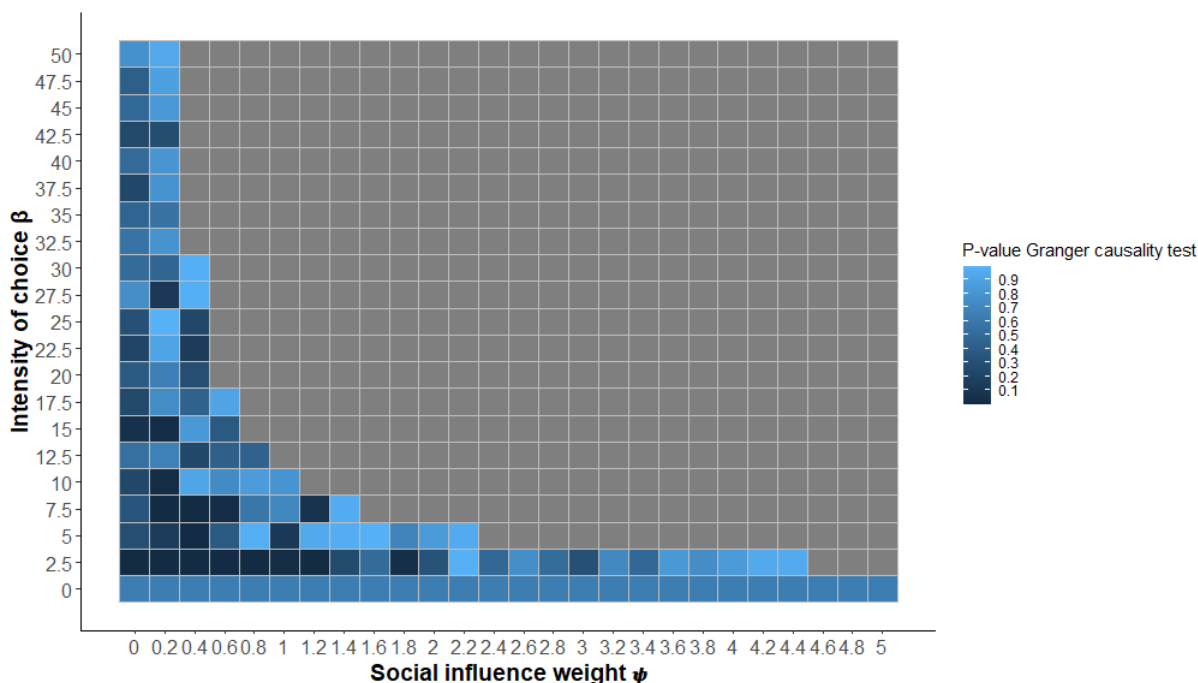
Source: Own elaboration.

Figure 1.8 – Test BS Granger causes the unemployment rate for different combinations of parameters ψ and β .



Source: Own elaboration.

Figure 1.9 – Test unemployment rate Granger causes BS for different combinations of parameters ψ and β .



Source: Own elaboration.

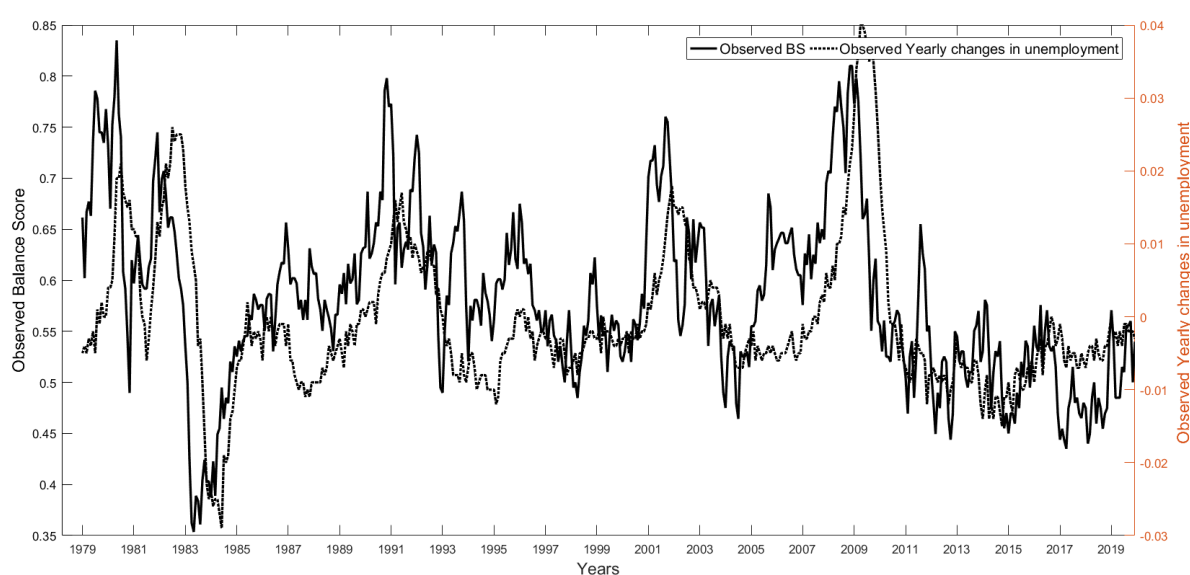
One detail to further investigate is whether the temporal causality still holds for different values of the intensity of choice and for the social influence weight. For this

task, Figures 1.8 and 1.9 show heat maps with the p-value of the Granger causality tests that were applied to the BS and unemployment rate outputs of the model for different combinations of parameters β and ψ . While Figure 1.8 refers to the Granger causality test for the unemployment rate causing the BS, Figure 1.9 refers to the Granger causality test for the unemployment rate causing the BS.

Both figures have cells in gray which represent combinations of parameters that have resulted in the BS and/or unemployment rate remaining constant. For these values, there is no need to test for causality. We will consider causality only if the p-value is less than 0.1. It is interesting to note that when the intensity of choice is zero, there is no significant temporal causality between the BS and the unemployment rate in either direction. Furthermore, for high values of ψ , there is no causality either. More precisely, the maximum value of ψ that generates causality from the BS to the unemployment rate is $\psi = 2$ combined with $\beta = 2.5$. Finally, for very high values of β , there is also no causality. The highest value of β that still generates significant causality from the BS to the unemployment rate is $\beta = 20$ combined with $\psi = 0$.

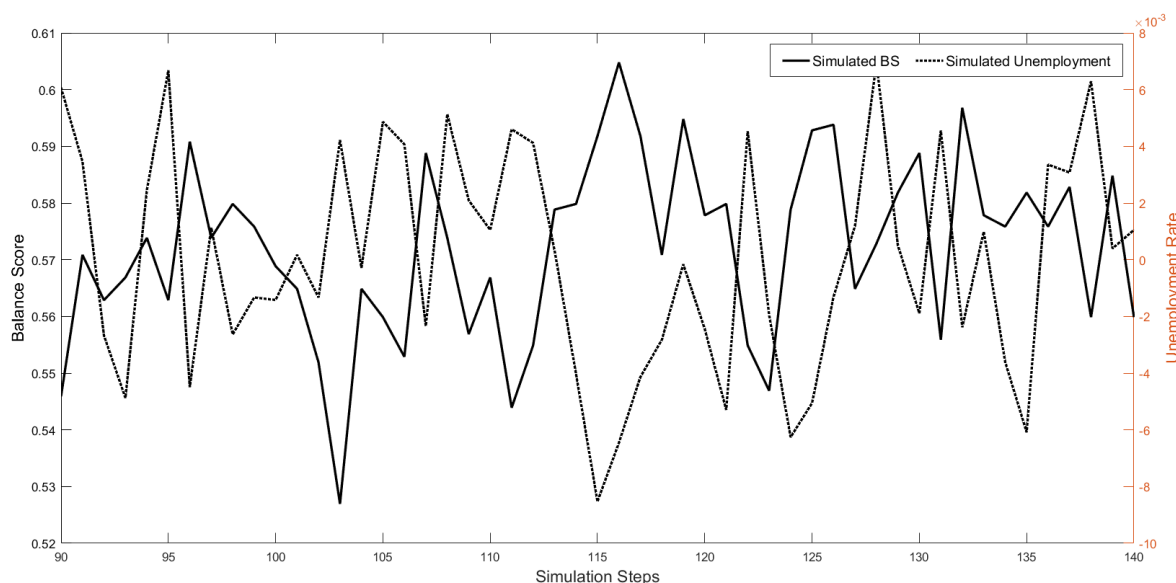
A different approach to analyze the temporal causality between expectations and the unemployment rate is that used by Leduc and Sill (2013) and Girardi (2014). The authors analyze data from Europe on the temporal causality between unemployment expectations, also measured in the form of balances, and the yearly changes in the unemployment rate, measured as the difference between the unemployment rate for a given month and the unemployment rate for the same month of the previous year.

Figure 1.10 – Observed normalized BS and yearly change in unemployment rate.



Source: Own elaboration.

Figure 1.11 – Simulated normalized BS and yearly change in unemployment rate between simulation steps 90 and 140.



Source: Own elaboration.

We reproduced this analysis for the US unemployment rate and expectations data from the Michigan survey. Figure 1.10 shows the graph with the observed variables, while Figure 1.11 shows the co-evolution of the simulated time series between simulation steps 90 and 140. Upon visual inspection, it was found that the same pattern exists in the US as well. To gain more precise insights, we performed the Granger causality test. The test indicates that the BS Granger causes the yearly change in the unemployment rate in both the simulated and observed data, as can be seen in Table 1.4. The only difference was in the best VAR model chosen with the AIC information criterion, which was 13 for the observed data and 12 for the simulated data. Furthermore, while the yearly change in the unemployment rate also causes the BS in the empirical data, we do not have enough statistical evidence for the simulated data at the 1% significance level, as the p-value was 0.22.

Table 1.4 – Granger causality test for BS and yearly change in unemployment.

Null Hypothesis	p-value
BS does not Granger cause a yearly change in unemployment rate simulated data	0.00
BS does not Granger cause a yearly change in unemployment rate observed data	0.00
Yearly change in the unemployment rate does not Granger cause BS observed data	0.00
Yearly change in the unemployment rate does not Granger cause BS simulated data	0.22

Source: Own elaboration.

1.4 FINAL REMARKS

In this essay, we laid down a macroeconomic framework intended to investigate the dynamic of unemployment expectations and its macroeconomic implications using the efficiency wage model augmented by expectation developed in Silveira and Lima (2021). The essay seeks to contribute to the literature that analyzes the formation of expectations in an environment of bounded rationality and heterogeneous agents. To this end, an original framework that follows the suggestive framework proposed by Brock and Hommes (1997) was added to an efficiency wage model, and its macroeconomic implications on the model's dynamics were analyzed.

In Section 2.1, we show some empirical evidence of the persistence of heterogeneous unemployment expectations and their macroeconomic implications. The findings of some of this research were used as the basis for proposing the structure of expectations formation presented in section 2.2, as well as the analysis of the emergent properties and macroeconomic implications presented in section 2.3.

In section 2.3, we found that the dynamic version of the efficiency wage model augmented by expectations manages to qualitatively replicate some of the stylized facts presented in the empirical literature. In particular, the existence and persistence of heterogeneity of unemployment expectations was an emergent property of the model. The Granger causality test showed that there is statistical evidence of temporal causality between the Balance Score (BS) and the unemployment rate, both observed and simulated data. In addition, Granger causality also occurs from the BS to the yearly change in the unemployment rate, both in the observed and simulated data.

Another analysis made in this essay is for which combinations of the intensity of choice and the weight of the social component parameters the causality still occur in the simulation. We found that a minimum level of intensity of choice is required for causality to occur. However, this intensity of choice cannot be too high for causality to still occur. Furthermore, for certain levels of intensity of choice, temporal causality still occurs even without considering the social component.

2 A MACROECONOMIC ABM EXTENDED FOR HETEROGENEOUS EXPECTATION

It is widely accepted that the expectations of agents in an economy play a crucial role in the dynamics of macroeconomic variables. However, there is no agreement on how these expectations are formed. Macroeconomic models use various assumptions about agents' expectations, and these different assumptions may lead to different macroeconomic outcomes.

Behavioral economics suggests that agents have bounded rationality and use heuristics when predicting, which is supported by vast empirical evidence. For instance, Conlisk (1996) provides evidence that people disregard relevant information, use irrelevant information, make mistakes in updating probabilities based on new information, among other behaviors incompatible with rational expectations.

In turn, Agent-Based Models (ABMs) have emerged as a popular methodological tool as they offer a flexible framework for studying bounded rationality in complex systems. According to Fagiolo and Roventini (2016), ABMs consist of models based on more realistic assumptions about how agents behave and interact with each other, compared to the mainstream methodology. This makes these models more interdisciplinary, as the assumptions are based on micro-economic evidence. For instance, instead of assuming perfect rationality, ABMs use the concept of bounded rationality, which is supported by evidence from cognitive psychologists (FAGIOLO; ROVENTINI, 2016).

To analyze macroeconomic dynamics by incorporating the hypothesis of bounded rationality in the formation of expectations and considering the economy as a complex system, we expand on a macroeconomic ABM that enables the analysis of full employment, coordination failures, and involuntary unemployment as emergent properties through the interaction of heterogeneous agents that use heuristics in their decision making. The baseline model used in this essay is the one proposed by Guerini, Napoletano, and Roventini (2018), which is characterized by the presence of a full-employment homogeneous-agents equilibrium. The model has a deterministic skeleton that can be hit by exogenous stochastic shocks. It allows the authors to analyze in which circumstances the economy goes back to the full-employment equilibrium after a shock. In this process, they analyze negative productivity shock under two different scenarios. One is the *centralized matching scenario*, in which a central planner avoids any possible coordination problem in both labor and goods markets. The other is the *decentralized matching scenario*, in which search and matching are local. The authors' main finding is that the economy is always able to return to the full employment equilibrium in the centralized scenario and that the impulse response functions (IRFs) in this scenario are similar to those generated by DSGE models. On the other hand, in the decentralized matching scenario, the economy persistently deviates from full employment equilibrium.

The aim of this essay is to add to the aforementioned model a structure for

forming household perceptions about the future macroeconomic situation, considering, in the vein of ABMs, heterogeneous agents and bounded rationality. We assume that the perception of future economic activity impacts the dynamic of the model through the desired consumption of the households, which is a plausible hypothesis due to the empirical findings in this direction in, e.g., Souleles (2004), Beckmann and Moder (2013), Roth and Wohlfart (2020) and Coibion et al. (2024). The framework developed in this essay makes it possible to analyze the deviations from the steady state that are due to the introduction of the expectations formation structure and also divergences in terms of the economy's responses to exogenous shocks with the introduction of the perception formation mechanism.¹

For this purpose, in the next section, we make a brief literature review on two different fields explored in this essay: (i) macroeconomic models that consider heterogeneous agents and cognitive limitations and (ii) decentralized market economy in macroeconomic models. Section 2.2 presents the baseline model. In section 2.3, we propose an extension of the model presented in section 2.2 to incorporate the formation of perceptions about the future macroeconomic situation. Section 2.4 shows the effects of the structure proposed in the previous section on the steady-state properties of the baseline model. Section 2.5 presents the effects of negative productivity shocks in the dynamics of the model. Finally, section 2.6 presents the final remarks.

2.1 A BRIEF LITERATURE REVIEW

As anticipated in the introduction, the first field of research to be explored in this literature review is heterogeneous agents and cognitive limitations in macroeconomic models, and we will do it by presenting models that use two different methodological approaches, namely, DSGE models and ABMs.

Since economist Robert Lucas raised fundamental criticisms against models based on statistical relationships estimated from historical data, the need for micro-foundations of macroeconomic models has become more prominent. Mainstream macroeconomics considers micro-foundations based on the New Neoclassical Synthesis, which is grounded upon Dynamic Stochastic General Equilibrium (DSGE) models (FAGIOLIO; ROVENTINI, 2016). However, the microfoundation strategy based on a representative agent, typically used in standard DSGE models, may not fully capture the complexities and nonlinearities of real-world economies. Many of these models have strong assumptions which are often criticized in light of empirical and experimental microeconomic evidence (see, e.g., Stiglitz and Gallegati (2011), Christiano, Eichenbaum, and Trabandt (2018), Kirman (2010)). In particular, one of these hypotheses, which is used in this essay, is that heterogeneous agents have bounded rationality, in contrast

¹ from now on we will use perceptions as a synonym for expectations.

to the hypothesis of fully rational and homogeneous agents.

On the other hand, as noted by Violante (2021), academic research has recently shifted toward incorporating heterogeneity in the household sector, as it is done in the Heterogeneous Agent New Keynesian (HANK) models. The HANK model proposed by Kaplan, Moll, and Violante (2018) assumes three groups of households in the economy: "hand-to-mouth" households with high marginal propensities to consume, "middle-class" households with a strong precautionary saving motive, and high net worth individuals with low marginal propensities to consume (VIOLANTE, 2021). The authors also assume an idiosyncratic earnings process that generates a leptokurtic distribution of annual earnings changes. Kaplan, Moll, and Violante (2018) discuss that, in contrast to the Rational Agent New Keynesian (RANK) model, the HANK model shows that the direct effects of interest rate shocks on consumption are small when compared to indirect effects. Direct effects occur when households save less or borrow more after a fall in interest rates, increasing their consumption demand, while indirect effects on consumption arise from the expansion in labor demand and, thus, in labor income, which emanates from the direct effects of the original interest rate cut (KAPLAN; MOLL; VIOLANTE, 2018). Importantly, the authors highlighted that the small role of the direct effect of interest rate changes on aggregate consumption generated by the HANK model is more in line with empirical evidence. Thus, the HANK model proposed by the authors seems to perform better than a typical RANK model.

Besides, after criticism of the fully rational agent hypothesis, a number of behavioral New Keynesian models have been incorporated Brock-Hommes approach into the conventional New Keynesian DSGE model (e.g., De Grauwe (2011), De Grauwe (2012) and Jang and Sacht (2012)).

For instance, De Grauwe (2011) proposed a model in which agents use heuristics to forecast future output and inflation. He assumes that the agents' choice of a particular forecast rule depends on the rule's past performance. As they also consider a random component governing the choice of agents, it implies that choice depends not only on the performance of those forecast rules but also on the parameter that measures the intensity of choice, which they also call "willingness to learn from past performance". To forecast inflation, they use either the central bank's inflation target or extrapolate inflation from the past into the future. As for the output, the author assumes that agents have either an optimism bias or a pessimism bias about the value of the output gap. He found that the behavioral model proposed can produce endogenous and self-fulfilling movements of optimism and pessimism, as well as a strong correlation between the output gap and the percentage of optimists for a given set of parameters. From the sensitivity analysis, he concluded that agents require a certain level of rationality (measured by the intensity of choice parameter), but still face some cognitive limitations in order for endogenous and self-fulfilling movements of optimism

and pessimism to emerge and impact the business cycle.

De Grauwe (2012) used a similar model as the one proposed by De Grauwe (2011). The difference is that in the model proposed by De Grauwe (2012), agents use fundamentalist or extrapolative forecasts for the output gap. Fundamentalists estimate the equilibrium output gap and forecast the output gap to return to a steady state. In the extrapolative forecast, the future output gap will equal the last observed output gap. The main conclusion is the same as that reported in De Grauwe (2011), that is, adding heterogeneous expectations that come from the adaptive learning mechanism results in a model that generates endogenous waves of optimism and pessimism that are self-fulfilling.

De Grauwe and Macchiarelli (2015) add banking credit to the behavioral macroeconomic model proposed by De Grauwe (2012). They found that adding banks to the model intensifies endogenous and self-fulfilling movements of optimism and pessimism.

In turn, Massaro (2013) developed a DSGE model in which a constant proportion over time of agents have rational expectations and the remaining fraction of agents have bounded rational beliefs about the future output and inflation. When performing a simple monetary policy exercise in this model, the author found that, even when considering a small fraction of agents with bounded rationality, the Taylor principle does not necessarily guarantee a determinate equilibrium.

Although there are efforts to incorporate heterogeneity into DSGE models, the literature still reports different perspectives on modeling macroeconomic models and, in particular, the behavior of heterogeneous agents. For instance, Dosi and Roventini (2019) note that due to the flexibility of assumptions of ABMs, those models are an impressive laboratory to analyze macroeconomic dynamics, making it possible to consider the economy as a complex system. Indeed, Riccetti, Russo, and Gallegati (2015) points out that in ABMs, agents may differ in many dimensions, such as income, size, financial fragility, location and information. The same authors highlighted that the direct interaction among heterogeneous agents can hardly be introduced in DSGE models.

As heterogeneity and bounded rationality, along with bottom-up perspective, non-linearity and direct interaction, are the building blocks of ABMs ((FAGIOLO; ROVENTINI, 2012)), a fairly significant amount of ABM research has presented models with this building block that reproduce numerous stylized facts. Fagiolo and Roventini (2012) note that ABMs are often bespoke, adapted to the particular question they are answering. More specifically on research into the formation of expectations, we can cite the K+S ABM developed in Dosi, Napoletano, et al. (2020). The authors analyzed the macroeconomic impact of heterogeneous expectations of firms in an ABM populated by heterogeneous, interacting firms. The model combines Schumpeterian theories of innovation with typical Keynesian features of demand generation. The firms in the model forecast their future demand either by choosing among different heuristics or via recursive least squares

estimations. Their main finding was that the performance of the economy worsens, and the average forecast error does not improve when firms replace simple forecasting rules based on heuristics with a more sophisticated rule that consists of estimating the parameter of their expectation rule via recursive least squares.

Having presented some representative works on heterogeneity and bounded rationality in macroeconomic models, we can now turn to the second field of research to be explored in this essay, which is decentralized search and matching models. Search theory started with the contributions of Mortensen and Pissarides (1999) and has been added to an increasing literature in agent-based models. For instance, Riccetti, Russo, and Gallegati (2015) proposed an ABM in which individuals, firms, and banks interact through a fully decentralized matching process in labor, goods, credit and deposit markets using a protocol common to all markets. The authors show that macroeconomic properties such as nominal GDP growth, unemployment fluctuations and the relationship established by the Phillips curve are emergent properties of this model.

Another representative research in this field, Sepecher and Salle (2015) developed a macroeconomic ABM of a fully decentralized market economy in which unemployed households choose jobs based on the highest wage and select products based on the lowest price. Also, similar to what we are considering in this essay, they proposed a framework for consumer sentiment (pessimistic or optimistic) in which the opinion dynamics model depends on the individual state of each household and also on a component through which the household is influenced by some other households' opinion, but, differently from the structure we are proposing in this essay, households have a probability of $1-p$ of relying only on their own situation, which consists of being pessimistic if they are unemployed or optimistic if they are employed, and they have a probability of p of following the majority opinion of other households. The authors found as an emergent property of the model endogenous waves of pessimism and optimism that feedback into firms' leverage and households' precautionary saving behavior.

Having presented some of the literature that incorporates hypotheses on agent behavior and market interaction used in the ABM proposed in this essay, we present the baseline model used to incorporate the effect of heterogeneous expectations on macroeconomic dynamics in the next section.

2.2 THE REFERENCE AGENT-BASED MODEL

In this section, we will present the ABM proposed by Guerini, Napoletano, and Roventini (2018), which, here on, we will call *GNR model*. In the next section, we will extend this model to incorporate consumers' expectations formation regarding the future macroeconomic situation.

Guerini, Napoletano, and Roventini (2018) developed a model with a deterministic backbone and the possibility of a full-employment equilibrium, in which they could

analyze impulse-response functions (IRF) after a productivity shock in two scenarios. One is the *centralized matching scenario*, in which a fictitious auctioneer is assumed to solve any possible coordination problem among the agents in both labor and goods markets. Conversely, in the *decentralized matching scenario*, agents interact locally in the labor and goods markets, without central coordination. In this structure, matching frictions and heterogeneity among agents can lead to imperfect allocation in labor and goods markets. The authors highlight that the decentralized matching structure is in line with the increasing literature in agent-based models (e.g. Assenza, Gatti, and Grazzini (2015), Ashraf, Gershman, and Howitt (2017), Riccetti, Russo, and Gallegati (2015), Popoyan, Napoletano, and Roventini (2017), Seppecher and Salle (2015)).

In the GNR model, $H \in \mathbb{N}$ households and $F \in \mathbb{N}$ firms interact in goods and labor markets according to the above-mentioned scenarios. Firms produce a consumption good using technology with constant returns to scale that employs only labor, and households supply labor inelastically and consume the final good using the wage paid by firms and their stock of liquid wealth. There is no government or external sector. It is also important to highlight that, as it has been shown by Guerini, Napoletano, and Roventini (2018), the model is stock-flow consistent.

In this ABM, we can find the following sequential order for the microeconomic decisions in any period $t \in \mathbb{N}$: (i) financial state variables are updated, that is, firms update their net-worth and households update their wealth; (ii) firms set their offered wages and sales prices, as well as their expected demand; (iii) households compute their desired consumption levels; (iv) the labor market opens and employers and employees are matched using different protocols (either centralized or decentralized); (v) production takes place and households receive their wages; (vi) the goods market opens and firms and consumers are matched using different protocols; (vii) firms compute their profits and distribute dividends to households; (viii) households calculate their consumption expenditure and their savings; finally, (ix) firms that go bankrupt are removed from the economy and are replaced by new ones at the same rate, while the wealth of defaulted households is reset to a constant value. At the end of each period, aggregate variables such as GDP, investment, and employment are calculated by adding up the respective individual values.

In the following subsection, we explain in detail how microeconomic decisions are made at the firm and household levels and the actual quantity produced by each firm is determined. Subsection 2.2.2 presents the matching protocols between agents in centralized and decentralized scenarios in the labor and goods markets. Finally, the last subsection introduces financial variables and the protocol for entry and exit of firms.

2.2.1 Determining consumption, production, prices and wages

This section outlines the decision-making process of both firms and households. Specifically, it covers the second and third steps described above, which involve the wages offered by firms to workers, the selling prices of their products, the expected demands for their products, and the desired levels of household consumption.

The GNR model involves agents who exhibit adaptive behaviors and employ heuristics. These agents follow linear decision rules that combine two effects: a within-effect that reflects decisions based on the agent's past levels of state variables and a network effect that accounts for the influence an agent receives from their peers.

In each period t , each firm f set its wage offer $W_{f,t}$ as follows:

$$W_{f,t} = W_{f,t-1} + \gamma \Delta P_{f,t-1} + \alpha z_{f,t-1}^{lab} + \beta (\bar{W}_{f,t-1} - W_{f,t-1}), \quad (2.1)$$

where $\gamma \in \mathbb{R}_{++}$, $\alpha \in \mathbb{R}_{++}$ and $\beta \in \mathbb{R}_{++}$ are parameters. The term $\Delta P_{f,t-1}$ represents the impact of change in prices on wages, and it is calculated by subtracting the price in period $t-2$, denoted by $P_{f,t-2}$, from the price in period $t-1$, denoted by $P_{f,t-1}$, that is $\Delta P_{f,t-1} \equiv P_{f,t-1} - P_{f,t-2}$.² The variable $z_{f,t-1}^{lab} = n_{f,t-1}^d - n_{f,t-1}^s$ represents the firm's excess demand for labor and seeks to reflect the firm's attempts to become more competitive in attracting workers. This attempt stems from the fact that when the number of open vacancies exceeds the number of filled vacancies, there will be an increase in the wage offered by the firm. This strategy makes job opportunities more attractive to workers relative to other options available to them, as suggested by Mortensen and Pissarides (1999) and Diamond (1982). Finally, the last term in (2.1) represents the network component by capturing the effect of deviations in the wage of firm f from the average wage of its neighbors in the period $t-1$, denoted by $\bar{W}_{f,t-1}$. As in Guerini, Napolitano, and Roventini (2018), here we also assume that the network is complete so that the neighborhood of a firm f can be defined as $N_f = \{1, 2, \dots, f-1, f+1, \dots, F-1, F\}$. Moreover, in the computation of the average wage, we assume that each firm f randomly assigns heterogeneous weights to its neighbors, which implies that $\bar{W}_{f,t} \equiv \sum_{j \in N_f} \omega_{f,t} W_{j,t-1}$, where $\omega_{f,t} \in \mathbb{R}_{++}$ is a number drawn at random from a uniform distribution with support $(0, 1) \subset \mathbb{R}$ and $\sum_{j \in N_f} \omega_{f,t} = 1$.

In a structure similar to wage decisions, the following decision rule determines the price of each firm:

$$P_{f,t} = P_{f,t-1} + \gamma \Delta W_{f,t-1} + \alpha z_{f,t-1}^{good} + \beta (\bar{P}_{f,t-1} - P_{f,t-1}), \quad (2.2)$$

where γ , α , and β are the same parameters used in (2.1). The second additive component, $\Delta W_{f,t-1} \equiv W_{f,t-1} - W_{f,t-2}$, represents the feedback of wages on price dynamics.³

² This term is inspired by Solow and Stiglitz (1968, p. 544), as noted by Guerini, Napolitano, and Roventini (2018).

³ Guerini, Napolitano, and Roventini (2018) highlights that this term reflects the dynamic correlation between wage adjustments tied to price changes and mark-up pricing as outlined in Solow and Stiglitz (1968).

The third additive component is given by $z_{f,t-1}^{good} = q_{f,t-1}^d - q_{f,t-1}^s$, where $q_{f,t-1}^d$ is the demand of firm f at period $t-1$ and $q_{f,t-1}^s$ is the supply of firm f at the same period. Therefore, the firm increases prices in the presence of excess demand to exploit its market power, following the models presented in Greenwald and Stiglitz (2003) and Diamond (1971). In these models, when demand exceeds supply at the current price, firms have an incentive to raise their prices. Higher prices allow firms to capture some of the surplus generated by the excess demand, leading to increased profits. Finally, the last additive term measures the distance between the firm's price, denoted by $P_{f,t-1}$, and the average price of its neighbors, denoted by $\bar{P}_{f,t-1}$. Following a similar definition to the average wage in (2.1), the average price of the f -th firm's neighbors is defined as $\bar{P}_{f,t-1} \equiv \sum_{j \in N_f} \omega_{f,t} P_{j,t-1}$.

The firms establish their preferred production level using a linear decision rule, which can be defined as follows:

$$\hat{q}_{f,t} = \tilde{q}_f + \alpha z_{f,t-1}^{good} + \beta (\bar{q}_{f,t-1} - q_{f,t-1}), \quad (2.3)$$

where α and β are the same parameters used in (2.1) and the term $\tilde{q}_f \in \mathbb{R}_{++}$ represents the reference production level. According to Guerini, Napoletano, and Roventini (2018), this term is in line with the findings from behavioral economics which suggest that individuals prefer to maintain their current state, also known as status quo biases.⁴ Moreover, people tend to have preferences relative to a reference point or existing situation. This reference point serves as a benchmark against which individuals assess gains or losses.⁵ The deviations from the reference production level are due to the last two components of the equation in (2.3). These components are the excess demand in the goods market from the last period, $z_{f,t-1}^{good}$, and the firm's relative position to its neighbors, given by $\bar{q}_{f,t-1} - q_{f,t-1}$, where $\bar{q}_{f,t-1} \equiv \sum_{j \in N_f} \omega_{f,t} q_{j,t-1}$ is the average production level in the previous period in her neighborhood N_f .

As for the quantity effectively produced and offered by the firms, denoted by $q_{f,t}^s$, it depends on a linear production function that uses only labor, $n_{f,t}$, that is:

$$q_{f,t}^s = a_{f,t} n_{f,t}, \quad (2.4)$$

where $a_{f,t} \in \mathbb{R}_{++}$ is the labor productivity of firm f in period t . Production is assumed perishable and cannot be stored for the next period.

While firms update their decision variables according to the rules in (2.1)-(2.4), households also update their desired consumption level based on the following decision rule:

$$\hat{c}_{h,t} = \tilde{c}_h + \alpha \frac{\Delta A_{h,t}}{P_{t-1}} + \beta (\bar{c}_{h,t-1} - c_{h,t-1}), \quad (2.5)$$

⁴ For more information on status quo biases, see e.g. Kahneman, Knetsch, Thaler, et al. (1991).

⁵ For references on this topic, see e.g. Kőszegi and Rabin (2009) and O'Donoghue and Sprenger (2018)).

where α and β are the same parameters used in (2.1). Analogously to the firms' setting of the desired production level, households also have a consumption reference level $\tilde{c}_h \in \mathbb{R}_{++}$. For each household, deviations from this reference level are due to variations in its real wealth, $\frac{\Delta A_{h,t}}{P_{t-1}}$, reflecting the empirical finding of the wealth effect on consumption.⁶ The last component of the function in (2.5) reflects the network effect. Similar to the firms' decision rules, it is calculated as the difference between a household's consumption and the weighted average of consumption of its neighbors. Here the set of neighbors of household h is defined as $N_h = \{1, 2, \dots, h-1, h+1, \dots, H-1, H\}$. Considering that each household assigns a weight $\omega_{h,t} \in \mathbb{R}_{++}$ that is chosen at random from a uniform distribution with support $(0, 1) \subset \mathbb{R}$ and $\sum_{j \in N_h} \omega_{h,t} = 1$, we can define the weight average consumption in the neighborhood of household h in period t as $\bar{c}_{h,t} \equiv \sum_{j \in N_h} \omega_{h,t} W_{j,t-1}$.

2.2.2 Search and matching in labor and goods market

As anticipated in the introduction of this section, in the GNR model there are two possible mechanisms for the interaction between households and firms in the goods and labor markets: centralized and decentralized. In the centralized scenario, a "fictitious auctioneer" overcomes coordination problems among agents in those markets. In the decentralized scenario, since agents interact locally in both markets, coordination problems can arise in an environment with heterogeneous productivity shocks. In this last scenario, the emphasis is on capturing the dynamics that arise from local interactions and uncoordinated decision-making processes.

2.2.2.1 Labor market

As in Guerini, Napoletano, and Roventini (2018), it is assumed that workers supply labor inelastically and their reservation wage is considered to be zero. Firms demand labor to meet the requirements of their production plans. Additionally, it is assumed that the labor is measured in terms of hours worked.

In the centralized scenario, the "fictitious auctioneer" collects job vacancies from firms and allocates them to workers in proportion to the firms' relative wages. Given the number of firms (F) and the number of households (H), this allocation rule for each firm f is set as follows:

$$n_{f,t}^s = \frac{H}{F} \left(\frac{W_{f,t}}{\bar{W}_t} \right), \quad (2.6)$$

recalling that $W_{f,t}$ is the firm's wage in the period t and $\bar{W}_t \equiv \frac{1}{F} \sum_{f=1}^F W_{f,t}$ is the average market wage at the same period.

⁶ see Sousa (2009) for evidence using Eurozone country data and Jawadi and Sousa (2014) for evidence with data from the United States and the United Kingdom, in addition to the Eurozone.

In turn, each firm's labor demand is determined according to:

$$n_{f,t}^d = \left(\frac{\hat{q}_{f,t}}{a_{f,t-1}} \right) \left(\frac{W_{f,t}}{P_{f,t}} \right)^{-\varphi}, \quad (2.7)$$

remember that $\hat{q}_{f,t}$ is the level of production desired by firm f given by (2.3) and $a_{f,t-1}$ is the specific labor productivity of each firm. According to Guerini, Napoletano, and Roventini (2018), the quotient of these two variables represents genuinely "Keynesian" effects related to expectations regarding the demand for goods. The remaining expression in the function in (2.7) is associated with the impact of real wages on labor demand, and the parameter $\varphi \in \mathbb{R}_{++}$ represents the sensitivity of labor demand to real wage.

Finally, the number of hours effectively worked at the firm level is determined by the short side of the market:

$$n_{f,t} = \min\{n_{f,t}^s, n_{f,t}^d\}. \quad (2.8)$$

This implies that when there is excess demand for labor ($n_{f,t}^d > n_{f,t}^s$), the firm is unable to hire enough workers to reach the desired production level, thus producing $q_{f,t} < \hat{q}_{f,t}$.

In the decentralized scenario, firms post their job offers along with the offered wage. Workers decide whether or not to apply for a job offered by a particular firm with a probability that increases with the offered wage. In this scenario, demand for labor occurs as in (2.7), but here workers actively search for open positions and decide to apply ($\Phi_{h,t} = 1$) or not ($\Phi_{h,t} = 0$) for a particular job according to the following Bernoulli trial:

$$\Phi_{h,t}^{LM} = \begin{cases} 0, & \text{with probability } p_{f,t}^{LM}, \\ 1, & \text{with probability } 1 - p_{f,t}^{LM}. \end{cases} \quad (2.9)$$

In a determined period, a worker can only apply for one position, and one labor unit is offered inelastically by her. The probability of applying ($1 - p_{f,t}^{LM}$) is proportional to the wage offered by the firm relative to the market average, formally:

$$1 - p_{f,t}^{LM} = 1 - \frac{1}{\rho^{LM}} \left[1 - \left(\frac{W_{f,t} - \bar{W}_t}{\bar{W}_t} \right) \right], \quad (2.10)$$

where \bar{W}_t is the average wage in the labor market in the period t and $\rho^{LM} \in (1, \infty) \subset \mathbb{R}$ is a parameter that determines the degree of search frictions (and imperfect information) in the labor market. It follows that the higher (lower) the value of ρ^{LM} , the higher (lower) the probability of workers applying for a position offered in period t by the firm f , for a given difference between the firm's wage and the market average wage. Moreover, higher (lower) values of ρ^{LM} also imply higher (lower) intensity of competition in the recruitment of workers, who become more sensitive to wage differences between firms.

As in the centralized scenario, the hours effectively worked at the firm level are determined as in (2.8). As noted by Guerini, Napoletano, and Roventini (2018), it is

important to observe that, unlike in the centralized scenario, decentralized matching can result in frictional unemployment (or labor rationing) even when the aggregate labor demand and aggregate labor supply are equal.

2.2.2.2 Goods market

In both centralized and decentralized scenarios, the quantity of goods that firms offer remains the same. After the labor market closes and workers are allocated to firms, the linear production function described in function (2.4) determines the quantity of goods produced.

In the centralized scenario, the total amount of desired consumption, given by $\hat{C}_t = \sum_h \hat{c}_{h,t}$, is distributed among firms in a manner that depends on their prices relative to the average market price $\bar{P}_t \equiv \frac{1}{F} \sum_{f=1}^F P_f$. Specifically, a larger portion of the total consumption will be allocated to firms with lower prices than those with higher prices. Formally, the demand for goods of firm f in period t can be expressed as:

$$q_{f,t}^d = \frac{\hat{C}_t}{F} \left[1 - \left(\frac{P_{f,t} - \bar{P}_t}{\bar{P}_t} \right) \right]. \quad (2.11)$$

Analogous to the labor market, the quantity of goods effectively sold is determined by the short-side rule, that is:

$$q_{f,t} = \min\{q_{f,t}^d, q_{f,t}^s\}. \quad (2.12)$$

If demand exceeds supply, consumers are rationed symmetrically. Otherwise, the firm cannot sell its entire production and incur losses.

Similarly to what happens in the labor market, in the decentralized goods market, there is the following Bernoulli trial associated with the decision of potential purchase ($\Phi_{h,t}^{GM} = 1$) or not ($\Phi_{h,t}^{GM} = 0$):

$$\Phi_{h,t}^{GM} = \begin{cases} 0, & \text{with probability } 1 - p_{f,t}^{GM}, \\ 1, & \text{with probability } p_{f,t}^{GM}. \end{cases} \quad (2.13)$$

The probability of success ($p_{f,t}^{GM}$) decreases with the firm's price relative to the market price as follows:

$$p_{f,t}^{GM} = \frac{1}{\rho^{GM}} \left[1 - \left(\frac{P_{f,t} - \bar{P}_t}{\bar{P}_t} \right) \right]. \quad (2.14)$$

In this scenario, each household purchases goods from only one firm and demands a specific number of product units, represented as $\hat{c}_{h,t}$. According to function in (2.14), firms that offer more competitive prices tend to have a higher demand for their products. The parameter $\rho^{GM} \in (1, \infty) \subset \mathbb{R}$ is associated with the level of friction in the goods market. When this parameter is higher (lower), firms are less (more) responsive to differences between their prices and the market average price, which represents a less (more) competitive goods market.

The actual quantity of product sold by a firm, denoted as $q_{f,t}$, is determined by the minimum value between supply and demand, as shown in (2.12). Consumers will be rationed equally if the demand is higher than the supply. On the other hand, if the supply is higher than the demand, the firm will produce a quantity that cannot be sold or stored for the next period.

2.2.3 Financial conditions, exit and entry

As in GNR model, the profit of firm f is computed as the sales revenue minus the labor cost:

$$\Pi_{f,t} = P_{f,t}q_{f,t} - n_{f,t}W_{f,t}. \quad (2.15)$$

When firms generate profits, they distribute a portion of it as dividends to households. Each household receives a fraction of the dividends, which is determined by dividing the total dividends by the number of households. Hence, the dividend received by household h at period t is calculated as:

$$D_{h,t} = \frac{(1-\vartheta)}{H} \sum_{f=1}^F \max\{\Pi_{f,t}, 0\}, \quad (2.16)$$

where $\vartheta \in [0, 1] \subset \mathbb{R}$ is a parameter that determines the fraction of profit retained by all firms.

Moreover, after the matching process between consumers and firms is complete, households effectively consume a quantity of $c_{h,t}$, with $c_{h,t} \leq \hat{c}_{h,t}$. In turn, the household's consumption expenditure is defined by $\sum_{f=1}^F P_{f,t}c_{hf,t}$, where $c_{hf,t}$ is the consumption by household h of the good produced by firm f in period t . The level of savings of household h in period t , denoted by $S_{h,t}$, is determined by the difference between the household income for that period, which is determined by the sum of the wage $W_{h,t}$ and the fraction of firm profits paid in dividends $D_{h,t}$, and the consumption expenditure, that is:

$$S_{h,t} = W_{h,t} + D_{h,t} - \sum_{f=1}^F P_{f,t}c_{hf,t}. \quad (2.17)$$

Assuming that the only asset available in the economy is money, which does not earn interest, the following rule applies to update the wealth of the h -th household:

$$A_{h,t+1} = A_{h,t} + S_{h,t}. \quad (2.18)$$

If a household's current wealth surpasses its initial wealth, the surplus amount is transferred to a bail-out fund for bankrupt households and firms. Conversely, when a family's wealth falls below zero, it is restored to the initial level, utilizing resources from the bail-out fund.

The net worth of firms is updated using the following process:

$$A_{f,t+1} = \begin{cases} A_{f,t} + \vartheta \max\{\Pi_{f,t}, 0\}, \\ A_{f,t} + \min\{\Pi_{f,t}, 0\}. \end{cases} \quad (2.19)$$

When a firm declares bankruptcy due to negative net worth, it is removed from the market and a new firm takes its place. The new company's net worth is determined by the value of the net worth that firms start within the first period of a simulation and is drawn from the rescue fund. The prices, wages, and desired production of the new firm are calculated as averages of the existing firms.

It is important to note that the rescue fund's presence ensures the consistency of the stock flow in the model concerning the entry and exit of households and firms. Based on simulation results of Guerini, Napoletano, and Roventini (2018), as well as the simulation results for the parameterization used in this essay, the fund's resources are always adequate to rescue agents facing bankruptcy.

The next section presents a consumer expectations formation structure, which has been added to their consumption decision rule.

2.3 THE GNR MODEL AUGMENTED BY EXPECTATIONS

In this section, we present the proposed extension of the GNR model to incorporate the perceptions⁷ of households (workers) about the future economic situation. To develop this framework, we took as motivation the *European Commission Business and Consumer Surveys*⁸, hereafter referred to as BCS. This monthly survey, since 1986, asks about 32,000 consumers from European Union countries, "How do you expect the general economic situation in the country to develop over the next 12 months?". The respondents have the following response options: get a lot better, get a little better, stay the same, get a little worse, get a lot worse, and don't know.

As highlighted by Curtin (2019a), differences in expectations reflect differences in economic conditions that people face. In order to have some intuition about this fact, we can analyze the data from BCS. In this survey, answers obtained are aggregated in the form of balance to represent synthetically the overall opinion about future economic activity, as follows:

$$B_t = 100[(\theta_t^+ + 0.5\theta_t) - (0.5\rho_t + \rho_t^-)], \quad (2.20)$$

where $B_t \in [-100, 100] \subset \mathbb{R}$ is the balance in the period t and θ_t^+ , θ_t , ρ_t , and ρ_t^- are

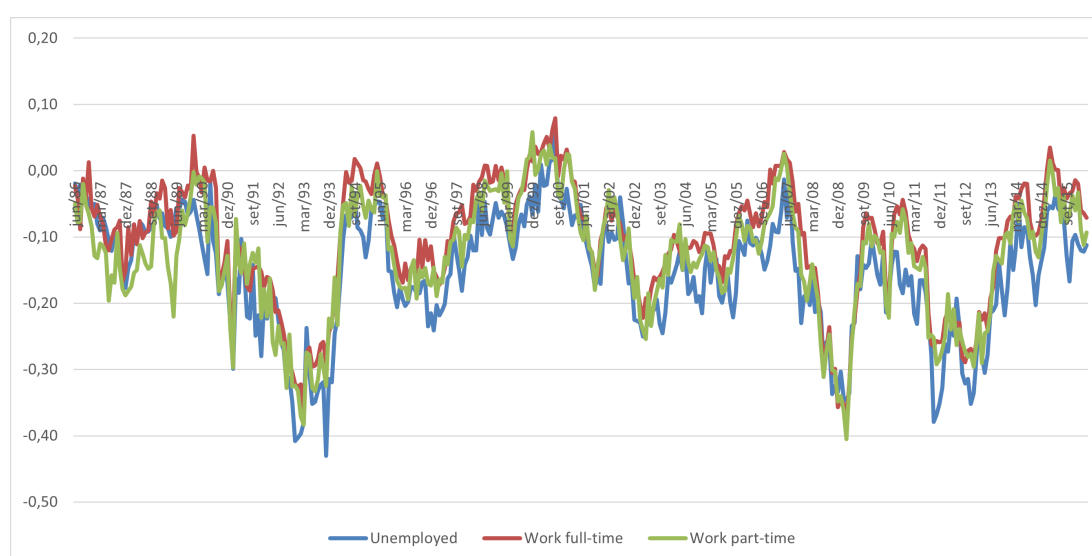
⁷ From now on we will use the term perceptions as a synonym for expectations.

⁸ Available at https://economy-finance.ec.europa.eu/economic-forecast-and-surveys/business-and-consumer-surveys/download-business-and-consumer-survey-data/time-series_en.

the proportions of very optimistic, slightly optimistic, slightly pessimistic, and very pessimistic in the population of respondents for the same period t , respectively.⁹

Figure 2.1 presents the balance of the European Union between June 1986 and April 2016 for three interview groups: unemployed, full-time workers and part-time workers. While Table 2.4 presents the same groups' mean and standard deviation balance. We can realize that the balance is usually lower for those who are unemployed. That is, unemployed people are normally more pessimistic about the future of the economic situation than employed people. Actually, the average balance is higher for full-time workers than for part-time workers and, to a greater extent, for unemployed workers.

Figure 2.1 – Seasonally adjusted European Union balance of responses on the general economic situation over the next 12 months, from June 1986 to April 2016.



Source: Own elaboration using data from the European Commission Business and Consumer Survey.

In the proposed extension of the GNR model, drawing on the BCS, we assume that households can have different levels of optimism or pessimism, or even be neutral, toward the economic situation. More precisely, they can be very pessimistic (vp), slightly pessimistic (sp), neutral (n), slightly optimistic (so), or very optimistic (vo) depending on their belief about whether the economic situation will get a lot worse, a little worse, stay the same, a little better, or a lot better, respectively, over the next 12 months.¹⁰

⁹ The exact equation presented in the survey's User Guide is $B = (PP + \frac{1}{2}P) - (\frac{1}{2}M + MM)$ for balance, where PP is the percentage of very positive responses, P is the percentage of positive responses, M is the percentage of negative responses and MM is the percentage of very negative responses. Furthermore, considering E as the percentage of neutral responses (unchanged) and N as the percentage of respondents with no opinion (don't know), it follows that $PP+P+E+M+MM+N=100$. The percentage of "don't know" answers in the survey is around 2% of all respondents.

¹⁰ As the equation for calculating the balance in the BCS does not explicitly consider the percentage

Table 2.1 – Mean and standard deviation of the balance of perception about the economic situation for the unemployed group, full-time workers and part-time workers.

	Unemployed	Work full-time	Work part-time
Mean	-0.15	-0.11	-0.13
Standard deviation	0.088	0.091	0.089

Source: Own elaboration using data from the European Commission Business and Consumer Survey.

Considering these five types of perceptions, we assume that households (workers) form their perceptions about the future economic situation based on the discrete choice process drawing especially upon Brock and Hommes (1997) and Train (2009), whose general framework has already been presented in subsection 1.2.2.1. More specifically, we assume that agents can choose between the five aforementioned perceptions based on their own experiences in the labor market and the perceptions of other households. Formally, we consider that in each period t , a household (worker) h can assume a type (have a perception) $\tau_h \in \mathcal{T} = \{vp, sp, n, so, vo\}$.

As previously presented in subsection 1.2.2.1, and repeated here for convenience, the utility or payoff function of the household h can be additively decomposed into a deterministic component, denoted by $U^d(\tau_{h,t})$, associated with her observed motivations, and a random component, denoted by $\zeta(\tau_{h,t})$, referring to her unobservable motivations. Formally:

$$U(\tau_{h,t}) = U^d(\tau_{h,t}) + \zeta(\tau_{h,t}). \quad (2.21)$$

Moreover, we consider that each household has two components governing its deterministic utility: a private and a social component. Considering those components, the deterministic utility can be written as follows:

$$U^d(\tau_{h,t}) = V(\tau_{h,t}) + \psi S(\tau_{h,t}), \quad (2.22)$$

where $V(\tau_{h,t})$ is the private utility, $S(\tau_{h,t})$ is the social utility and $\psi \in \mathbb{R}_{++}$ stands for the weight of social utility over deterministic utility. We will refer to this parameter as the *social influence weight*.

We assume that the private utility is associated with the employment history of the h -th household in the last 12 periods, which we will dub *employment indicator*, defined as follows:

$$\mathcal{E}_{h,t-1} = \iota_{t-1} \xi_{h,t-1} + \iota_{t-2} \xi_{h,t-2} + \dots + \iota_{t-12} \xi_{h,t-12}, \quad (2.23)$$

of "don't know" answers, and also considering that this percentage in the BCS is very low (around 2%), by construction we exclude the possibility that the agent in the model may have a "don't know" perception.

where $\iota_{t-\ell} \in \mathbb{R}_+$ is the weight of employment situation in period $t-\ell$ such that $\sum_{\ell=1}^{12} \iota_{t-\ell} = 1$. $\xi_{h,t-\ell} \in [0, 1] \subset \mathbb{R}$ is the number of hours worked by household h in the period $t-\ell$. As in Guerini, Napoletano, and Roventini (2018), here it is also considered that each worker inelastically offers one hour of labor. This implies that, for each household h , the employment indicator $\mathcal{E}_{h,t-1} \in [0, 1] \subset \mathbb{R}$ reaches its highest value when the household has its labor supply employed for all $\ell = 1, 2, \dots, 12$. On the other hand, the households' employment indicator achieves its lowest value, which is equal to zero, when the household is unemployed for all $\ell = 1, 2, \dots, 12$.

Furthermore, as in the first essay of this dissertation, it is assumed here that the weight of past employment, $\iota_{t-\ell}$, on the macroeconomic indicator decays geometrically with time lag ℓ by a factor $q \in (0, 1) \subset \mathbb{R}$, that is:

$$\iota_{t-\ell} = q \iota_{t-\ell-1}. \quad (2.24)$$

Considering that $\sum_{\ell=1}^{12} \iota_{t-\ell} = 1$ and (2.24), it is possible to rewrite ι_{t-1} as follows:

$$\iota_{t-1} = \frac{q-1}{q^{12}-1}. \quad (2.25)$$

We assume the private utility of agents associated with each choice (or perception) depends on whether their employment indicator has improved, worsened, or remained unchanged. In particular, the private utility of pessimism is higher than that of neutral and, to a great extent, of optimism if the h -th household faces a worsening in her employment indicator. On the other hand, if she sees an improvement in her employment indicator, her private utility of optimism is higher than that of neutral and, to a great extent, of pessimism. If there is no change in the indicator, the private utility of the neutral is the highest. Considering it, we can establish the private utility of household h as follows:

$$V(\tau_{h,t}) = \begin{cases} \mathcal{E}_{h,t-1} - \mathcal{E}_{h,t-2}, & \text{if } \tau_{h,t} = vo, so \\ -(\mathcal{E}_{h,t-1} - \mathcal{E}_{h,t-2})^2, & \text{if } \tau_{h,t} = n, \\ -(\mathcal{E}_{h,t-1} - \mathcal{E}_{h,t-2}), & \text{if } \tau_{h,t} = sp, vp. \end{cases} \quad (2.26)$$

Note that $|E_{h,t-1} - E_{h,t-2}| \in [0, 1] \subset \mathbb{R}$, so that $|E_{h,t-1} - E_{h,t-2}| > (E_{h,t-1} - E_{h,t-2})^2$. In other words, if there is a positive change in the employment indicator from $t-2$ to $t-1$, this means that $(E_{h,t-1} - E_{h,t-2}) > 0 > -(E_{h,t-1} - E_{h,t-2})^2 > -(E_{h,t-1} - E_{h,t-1})$. In turn, if there was a negative change in the employment indicator, we would have $-(E_{h,t-1} - E_{h,t-2}) > 0 > -(E_{h,t-1} - E_{h,t-2})^2 > E_{h,t-1} - E_{h,t-1}$.

In turn, the social component of the deterministic utility is associated with the perception of the other households about future economic activity. That is, we consider that the deterministic utility of a given perception, for a given value of the private utility, is higher the higher the proportion of households that held this perception in the previous

period. More precisely, we assume the following functional form for social utility:

$$S(\tau_{h,t}) = \begin{cases} \theta_t^+, & \text{if } \tau_{h,t} = vo, \\ \theta_t, & \text{if } \tau_{h,t} = so, \\ \eta_t, & \text{if } \tau_{h,t} = n, \\ \rho_t, & \text{if } \tau_{h,t} = sp, \\ \rho_t^-, & \text{if } \tau_{h,t} = vp, \end{cases} \quad (2.27)$$

where $\eta_t = 1 - (\theta_t^+ + \theta_t + \rho_t + \rho_t^-)$ is the proportion of neutrals in the population of agents in the period t . It is interesting to note that (2.27) captures how society's views on the future state of the economy can influence the individual's perceptions regarding the same subject. This means that while an individual's perception is influenced by their own past experiences, their decision-making is also influenced by society's forward-looking perspective. Thus, while the private utility has a backward-looking component influencing an individual's perception, the social utility incorporates a forward-looking component into the individual's decision-making.

As described in subsection 1.2.2.1, due to the random component in the utility function, it is only possible to infer the agent's propensity to choose, which is in line with the discrete choice literature. Here we also use the logit specification for the joint probability density function of the vector of random variables $\vec{\zeta}_h$, which is composed of random variables $\zeta(\tau_h)$. Considering the utility function in (2.22), we can establish the probability that a household h in period t has a perception $\tau_{h,t} \in \mathcal{T}$ about the future economic situation of the economy as the following logistic cumulative distribution function:

$$Prob(\tau_{h,t}) = \frac{1}{1 + \sum_{\tau'_{h,t-1} \in \mathcal{T}, \tau'_{h,t-1} \neq \tau_{h,t-1}} e^{-v\{[V(\tau'_{h,t-1}) + \psi(S(\tau'_{h,t-1}))] - [V(\tau_{h,t-1}) + \psi(S(\tau_{h,t-1}))]\}}}, \quad (2.28)$$

where $v \in \mathbb{R}_+$ is the *intensity of choice*. Remembering that the higher the value of v , the greater the relative weight of the deterministic component in comparison with the random component of the utility function in determining the propensity of a household h to hold the perception τ_h about the future economic activity.

Finally, in order to define the perception of a household h in any period $t \geq 2$, we generate a random number $r_{h,t} \in [0, 1] \subset \mathbb{R}$ from a uniform distribution. Considering that $\sum_{\tau \in \mathcal{T}} Prob_{\tau,t} = 1$ for all period $t \in \mathbb{N}$, we apply the rules as specified in Table 2.2.

In this essay, we will extend the GNR model by accounting for the household's perception of the future economic situation of the country, which may affect their current desired consumption. More precisely, We will assume that households with optimistic perceptions of the future economy have a positive bias towards current consumption, while those with pessimistic perceptions have a negative bias. To incorporate this effect,

Table 2.2 – Algorithm to choose a perception about the future economic activity in every period $t \geq 2$

Possible cases	Perception of household h
$r \leq \text{Prob}(\tau_{h,t} = vp)$	Very pessimist
$\text{Prob}(\tau_{h,t} = vp) \leq r \leq \text{Prob}(\tau_{h,t} = vp) + \text{Prob}(\tau_{h,t} = sp)$	Slightly pessimist
$\text{Prob}(\tau_{h,t} = vp) + \text{Prob}(\tau_{h,t} = sp) \leq r \leq \text{Prob}(\tau_{h,t} = vp) + \text{Prob}(\tau_{h,t} = sp) + \text{Prob}(\tau_{h,t} = n)$	Neutral
$\text{Prob}(\tau_{h,t} = vp) + \text{Prob}(\tau_{h,t} = sp) + \text{Prob}(\tau_{h,t} = n) \leq r \leq \text{Prob}(\tau_{h,t} = vp) + \text{Prob}(\tau_{h,t} = sp) + \text{Prob}(\tau_{h,t} = n) + \text{Prob}(\tau_{h,t} = so)$	Slightly optimist
$\text{Prob}(\tau_{h,t} = vp) + \text{Prob}(\tau_{h,t} = sp) + \text{Prob}(\tau_{h,t} = n) + \text{Prob}(\tau_{h,t} = so) \leq r$	Very optimist

Source: Own elaboration.

we include the influence of the household type/perception in period t on its desired consumption defined in equation (2.5). As a result, the desired consumption is now defined as:

$$\hat{c}_{h,t} = \tilde{c}_h + \alpha \frac{\Delta A_{h,t}}{P_{t-1}} + \beta(\bar{c}_{h,t-1} - c_{h,t-1}) + \zeta E(\tau_{h,t}), \quad (2.29)$$

where the variable $E_{h,t} \in (-0.2, 0.2) \subset \mathbb{R}$, which here on we will call *consumption perception bias*, is a bias towards more or less consumption of the household h whose perception (type) is $\tau_{h,t} \in \mathcal{F}$ in period t .

All other variables affecting consumption being equal, household consumption should be lower for pessimistic households than for neutral and, to a greater extent, optimistic households. With this in mind, for each household h , we choose the consumption perception bias as a random scalar drawn from the uniform distribution in the interval $(0, 0.1) \subset \mathbb{R}$, $(0.1, 0.2) \subset \mathbb{R}$, $(-0.1, 0) \subset \mathbb{R}$ and $(-0.2, -0.1) \subset \mathbb{R}$ if $\tau_{h,t} = so$, $\tau_{h,t} = vo$, $\tau_{h,t} = sp$ and $\tau_{h,t} = vp$, respectively. For those households that are neutral, we consider that there is no bias towards more or less consumption, that is, $E(\tau_{h,t} = n) = 0$. Before choosing these upper and lower limits in which $E(\tau_{h,t})$ could fluctuate, we tested for different limits and found the one reported as providing the maximum amplitude that allows the model to continue showing the stock and flow consistency property. Table 2.3 summarizes the possible values for the consumption perception bias.

Table 2.3 – Range in which the consumption perception bias will be contained for possible perceptions of the future economic situation

Ranges for the consumption perception bias
$E(\tau_{h,t} = vp) \in (-0.2, -0.1) \subset \mathbb{R}$
$E(\tau_{h,t} = sp) \in (-0.1, 0) \subset \mathbb{R}$
$E(\tau_{h,t} = n) = 0$
$E(\tau_{h,t} = so) \in (0, 0.1) \subset \mathbb{R}$
$E(\tau_{h,t} = vo) \in (0.1, 0.2) \subset \mathbb{R}$

In the next two sections, we show some of the emergent properties of the GNR

model augmented by expectations.

2.4 EFFECTS OF THE PERCEPTIONS ABOUT THE FUTURE ECONOMIC ACTIVITY ON THE STEADY-STATE PROPERTIES OF GNR MODEL

In this section, we show under what circumstances, and if any, there is a convergence of the GNR model augmented by expectations to the steady-state equilibrium. As shown in Guerini, Napoletano, and Roventini (2018), the GNR model is characterized by the presence of a full-employment homogeneous-agents equilibrium, which can be considered as the benchmark for the dynamics of the economic system.

In order to analyze whether the presence of heterogeneity of perceptions about future economic activity still allows convergence to full employment equilibrium, we carried out several simulations with different sets of values for the two parameters added in the extension of the model, namely, the parameters intensity of choice ν , relate to the stochastic component of the utility function, and the social influence weight ψ , to check if convergence to full employment equilibrium is still a possibility.

For this purpose, each simulation begins with the model's state variables in values consistent with full employment equilibrium and homogeneous agents. Guerini, Napoletano, and Roventini (2018) characterize this equilibrium as follows:

$$\begin{cases} \Delta x_t = 0, \forall x_t \in \Omega, \\ \tilde{y}_t = 0, \tilde{u}_t = 0, \tilde{\pi}_t = 0, \end{cases} \quad (2.30)$$

where x_t is a state vector with all micro and macrostate variables of the model, Ω is all possible state vectors that the economy can assume, \tilde{y}_t is the output gap, while \tilde{u}_t and $\tilde{\pi}_t$ stand for the deviations of unemployment and inflation from their steady-state values, respectively. Also, to initialize the model, we consider that each household demands 1 unit of goods, so each household demands $1/F$ of each firm. As we are considering $H = 200$ households and $F = 20$ firms, we have each firm producing 10 units of goods in the initial period. Furthermore, according to the production function in (2.4) and considering that each household inelastically offers 1 hour of work and that the total labor supply is allocated equally among the firms, we have the labor productivity equal to 1. We can also show that there is only one real wage that satisfies the zero profit condition, that is, considering $\pi_f = p_f q_f^* - w_f n_f^* = 0$ and the production function in (2.4), it follows that $\frac{w_f}{p_f} = a_f$, where q_f^* and n_f^* are the steady-state values of the number of goods sold by firm f and the number of hours worked at the firm level, respectively. It implies that the initial real wage is equal to 1. Besides, for the initialization, we consider $p_f = 10$, so that the nominal wage becomes equal to 10. Finally, we assume that in the first period of each simulation, all households are neutrals concerning future economic activity.

Table 2.4 summarizes the parameter's values used as a reference in all computational simulations to be carried out, unless indicated otherwise. These values are the same as those used in Guerini, Napoletano, and Roventini (2018). In addition to these parameters, we add the parameter q , used in equation (2.24), which represents the decay factor of the weight of past employment. For this parameter, we use the same as in the first essay of this dissertation, that is, $q = 0.26$.

Table 2.4 – Parameter Values.

Parameter	Value	Meaning
MC	30	Number of Monte Carlo realizations
T	1000	Simulation steps
H	200	Number of households
F	20	Number of firms
α	0.4	Sensitivity to economic effects
β	0.4	Sensitivity to social effects
γ	0.4	Sensitivity to price/wage indexation
ϑ	0.5	Percentage of retained profit
φ	5	Sensitivity of labor demand to real wage
ρ^{LM}	2	Matching efficiency in the labor market
ρ^{GM}	2	Matching efficiency in the goods market
q	0.26	Decay rate of the weight of past unemployment

Source: Own elaboration with information provided by Guerini, Napoletano, and Roventini (2018).

We consider that a simulation reached a steady state when both macro and micro variables reached a steady state. We assume that the microstate attained the steady state when the variation in the proportion of agents with each type of perception is less than 0.01 in module for 50 successive periods. In other words, for all $t' = t, t+1, \dots, t+49$, we must have: $|\theta_{t'}^+ - \theta_{t'-1}^+| \leq 0.01$, $|\theta_{t'}^- - \theta_{t'-1}^-| \leq 0.01$, $|\eta_{t'} - \eta_{t'-1}| \leq 0.01$, $|\rho_{t'} - \rho_{t'-1}| \leq 0.01$ and $|\rho_{t'}^- - \rho_{t'-1}^-| \leq 0.01$. In turn, the macrostate variables are considered in a steady state when the deviation from zero of the output gap, inflation, unemployment and excess demand is no more than 0.01, up or down, for 50 successive periods, and the deviation of real wage from 1 is no more than 0.01, up or down, for 50 successive periods.

The experiments reported in the remainder of this section were designed to analyze if the GNR model augmented by expectations set forth in this essay shows the possibility of convergence to a steady state as in the GNR model. In other words, based on several computational simulations, we aim to analyze under which circumstances the co-evolution between perceptions about future economic activity and macroeconomic dynamics is able to lead the economy to a steady state.

To this task, we run simulations with the intensity of choice v from 0 to 50, with increments of 2.5, totaling 21 values, and the social influence weight ψ from 0 to 10, with increments of 0.5, also totaling 21 values. We then run simulations with all possible combinations of these 21 values for each parameter, totaling 441 combinations. For each combination of parameters, we run 30 Monte Carlo simulations, each simulation with a different seed. It is important to note that we repeated the 30 random seeds for each parameter combination, so that we could isolate the stochastic effect when comparing the dynamics of the model from the effect of one parameter combination to another one.

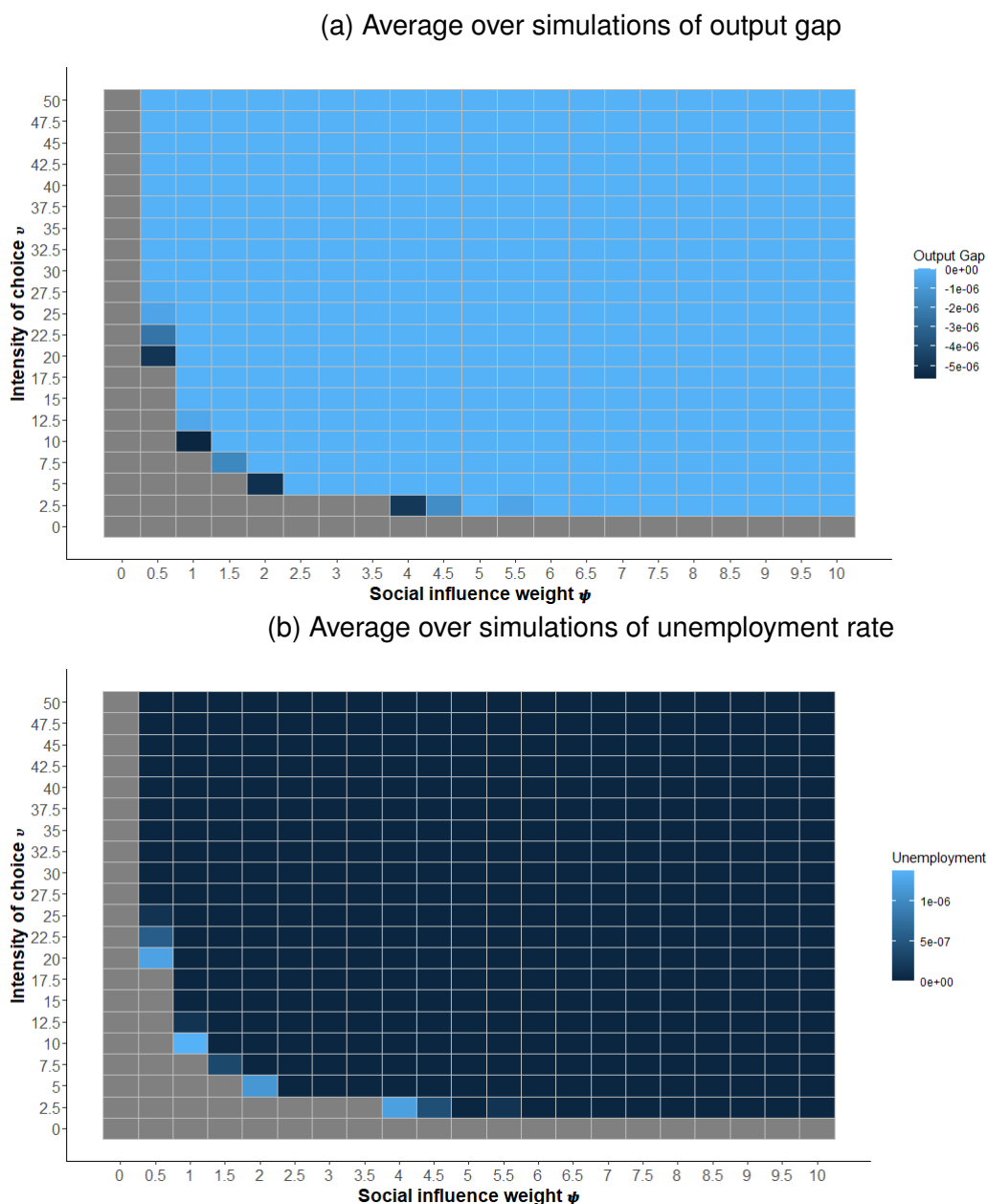
Once these simulations are done, we are able to carry out a robust analysis about with which combination of parameters the model converges to the steady state. To do this, we first took the average value of macro state variables over the range of the respective simulation featuring convergence with the above-mentioned convergence rule. Next, we took only the convergent ones and calculated, for each state variable, the arithmetic mean of their arithmetic means over the range of convergence of such simulations, which from now on we refer to simply as *average over simulations* of each state variable.

It is now possible to analyze potential convergences using heat maps. In Figures 2.2, 2.3 and 2.4, the gray cells indicate combinations of the intensity of choice and the weight of social influence parameters that do not lead to a steady state in the model. In the other cells, convergence occurred. In these cases, the cells display the average values of macroeconomic variables obtained from simulations with the combinations of v and ψ that reached a steady state.

As shown in Figure 2.2 and Figure 2.3, for low values of ψ and v , the model does not converge to a steady state. In particular, regardless of the value of the social influence weight, when the intensity of choice is zero, the model does not converge to the steady state. It follows from (2.28) that the propensity to choose each type of perception about the future economic activity is $1/5$, so the proportion of each perception in the population of households is around $1/5$. In this scenario, considering (2.29), there will be a bias on around $4/5$ of households towards more/less consumption. This deviation of consumption from its steady-state level occurs in every period regardless of the value of the other variables determining consumption in the equation, so the dynamics of the centralized model cannot correct the imbalance caused by the deviation of consumption from neutrality.

Furthermore, regardless of the value of v , the model does not converge to the steady state when the social influence weight is zero. When $\psi = 0$, the utility function is determined only by the private utility, as shown in equation (2.22). As a result, in the period immediately following each simulation's initialization, the employment indicator $\mathcal{E}_{h,t}$ is equal to one for every household, meaning that $U^d(\tau_{h,t+2}) = 0$ for all $\tau_h \in \mathcal{T}$,

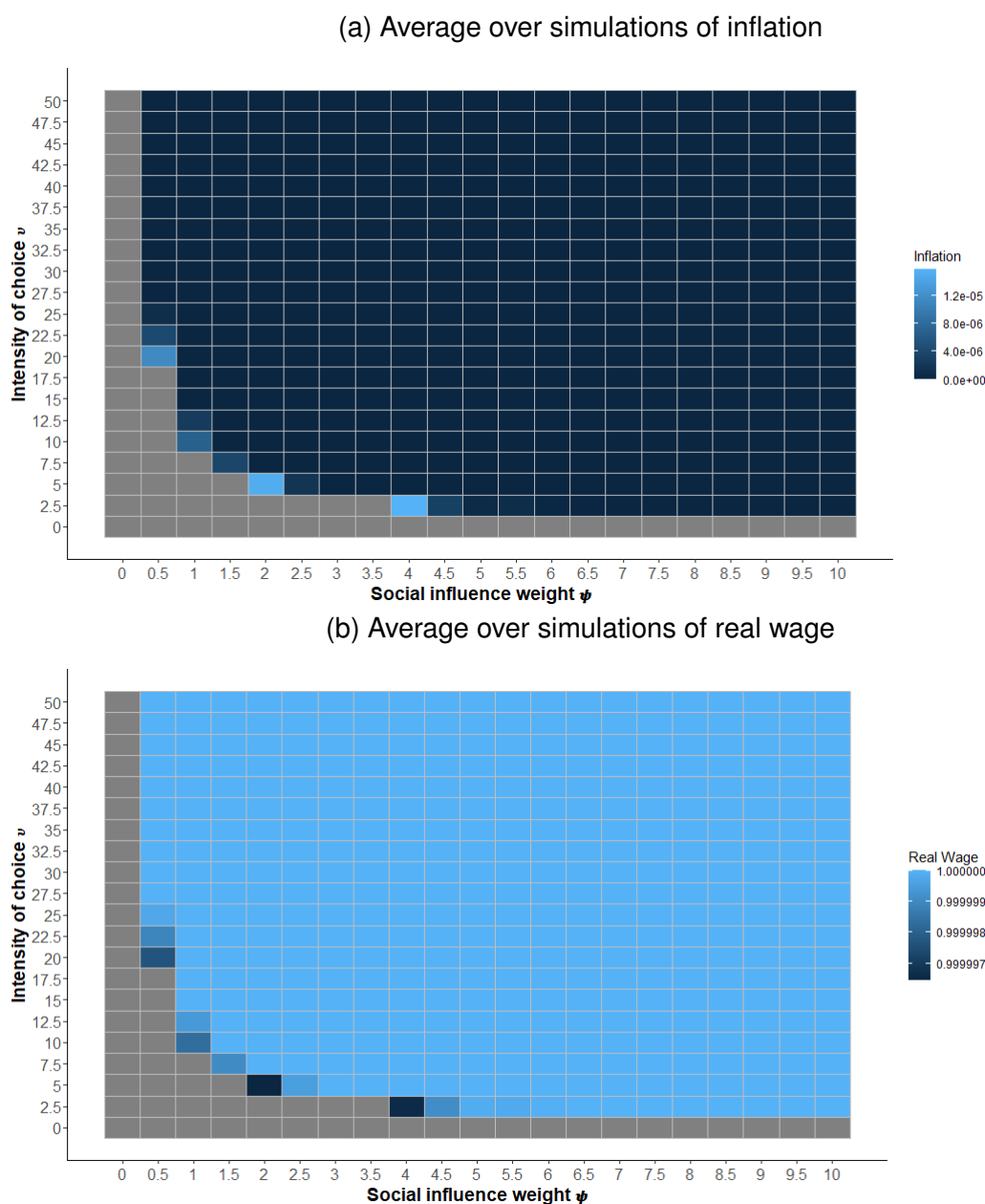
Figure 2.2 – Steady state values of the output gap and unemployment rate for different pairs of the intensity of choice ν and social influence weight ψ in the centralized model.



Source: Own elaboration.

as per equations (2.22) and (2.26). In this case, each agent's perception of the future of economic activity is determined by the random component of the utility function, causing consumption levels to deviate randomly from the steady-state level during each employment indicator's steady-state level. This, in turn, creates another case where the centralized model's dynamics cannot correct the imbalance caused by consumption deviating from neutrality.

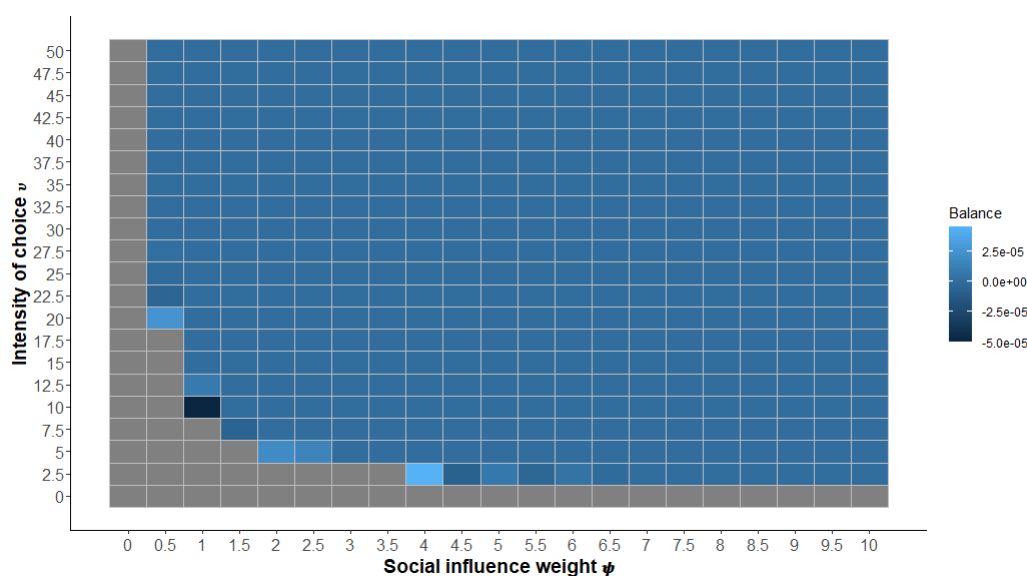
Figure 2.3 – Steady state values of inflation and real wage for different pairs of the intensity of choice ν and social influence weight ψ in the centralized model.



Source: Own elaboration.

It is interesting to point out the boundary that separates the scenarios where convergence occurs from those where it does not, in Figures 2.2, 2.3 and 2.4. This boundary indicates that there is a trade-off between the intensity of choice and the impact of social influence for values of the first parameter that are empirically relevant, which lies between 2 and 10 (see Anufriev, Chernulich, and Tuinstra (2018) and Lux and Zwinkels (2018)). Within this range of values, a decrease in the social influence must be compensated by an increase in the intensity of choice to maintain convergence. This

Figure 2.4 – Steady state values of balance for different pairs of the intensity of choice v and social influence weight ψ in the centralized model.



Source: Own elaboration.

trade-off is no longer present for values of the intensity of choice that are sufficiently high (above 20) or for values of the social influence that are sufficiently high (above 3.5).

Table 2.5 – Means and standard deviations of average over simulations that did not converge in the centralized scenario

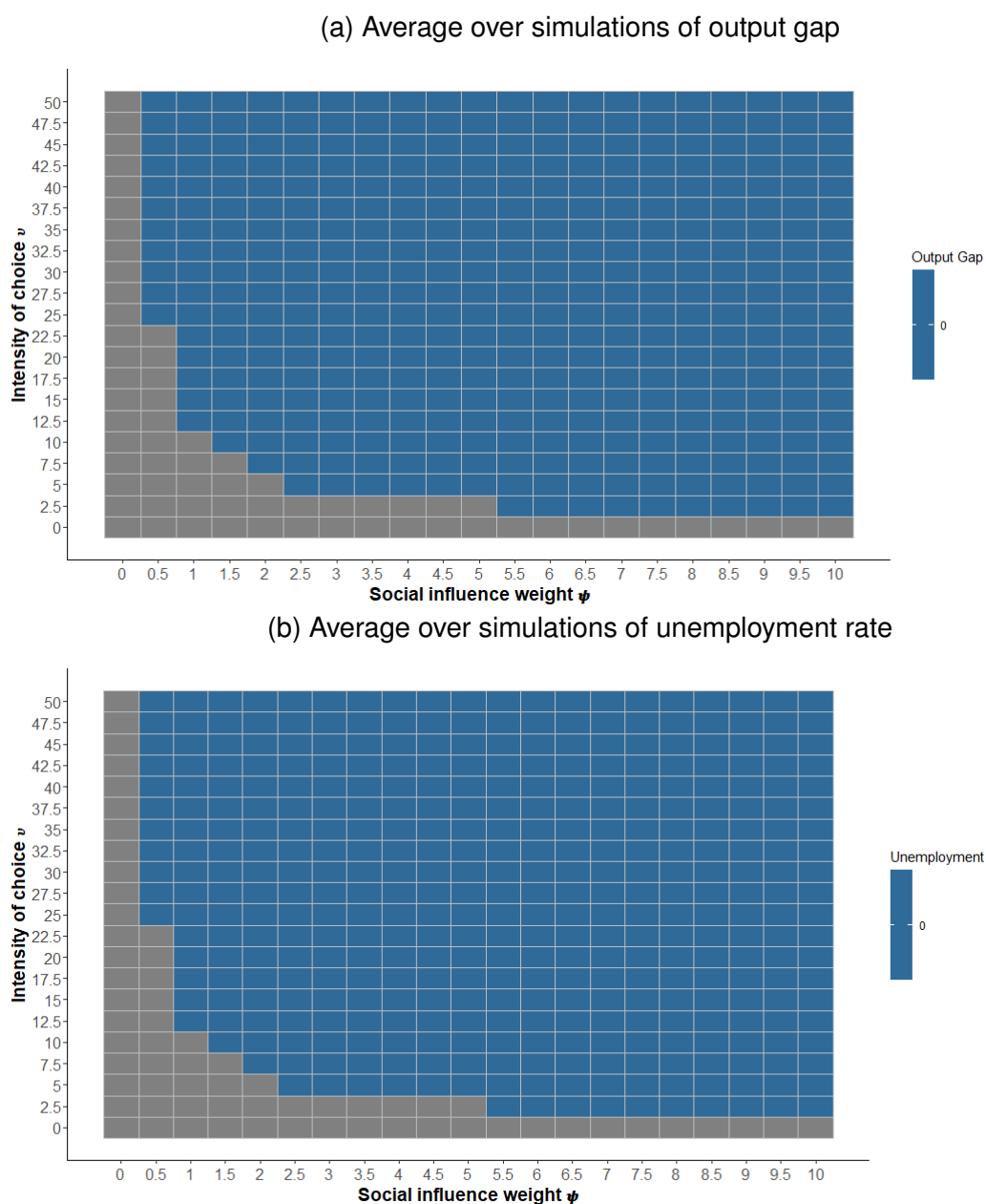
	Output gap	Unemployment	Inflation	Real wage	Balance
Mean	-0.0019	0.0005	0.0004	0.9997	-0.0006
Standard deviation	0.0012	0.0003	0.0002	0.0002	0.001

Finally, Table 2.5 presents the mean and standard deviations of average over simulations that did not converge in the centralized scenario. More precisely, with 63 out of 441 combinations of ψ and v , the model did not converge to the steady state. It can be seen that, on average, there is a bias towards pessimism in those simulations that did not converge to a steady state, which implies lower consumption and, consequently, a lower output gap and real wage, and higher unemployment and inflation than what is found at the steady-state level. Remember that the real wage in the steady state is equal to 1.

In contrast to the centralized scenario, in the decentralized scenario all combinations of parameters that lead to the convergence of the model result in values of the macroeconomic variables that are exactly equal to respective steady-state levels. In

this case, the values of the output gap, unemployment, inflation and balance are all zero, while the real wage is equal to one. This can be observed in Figures 2.5, 2.6 and 2.7.

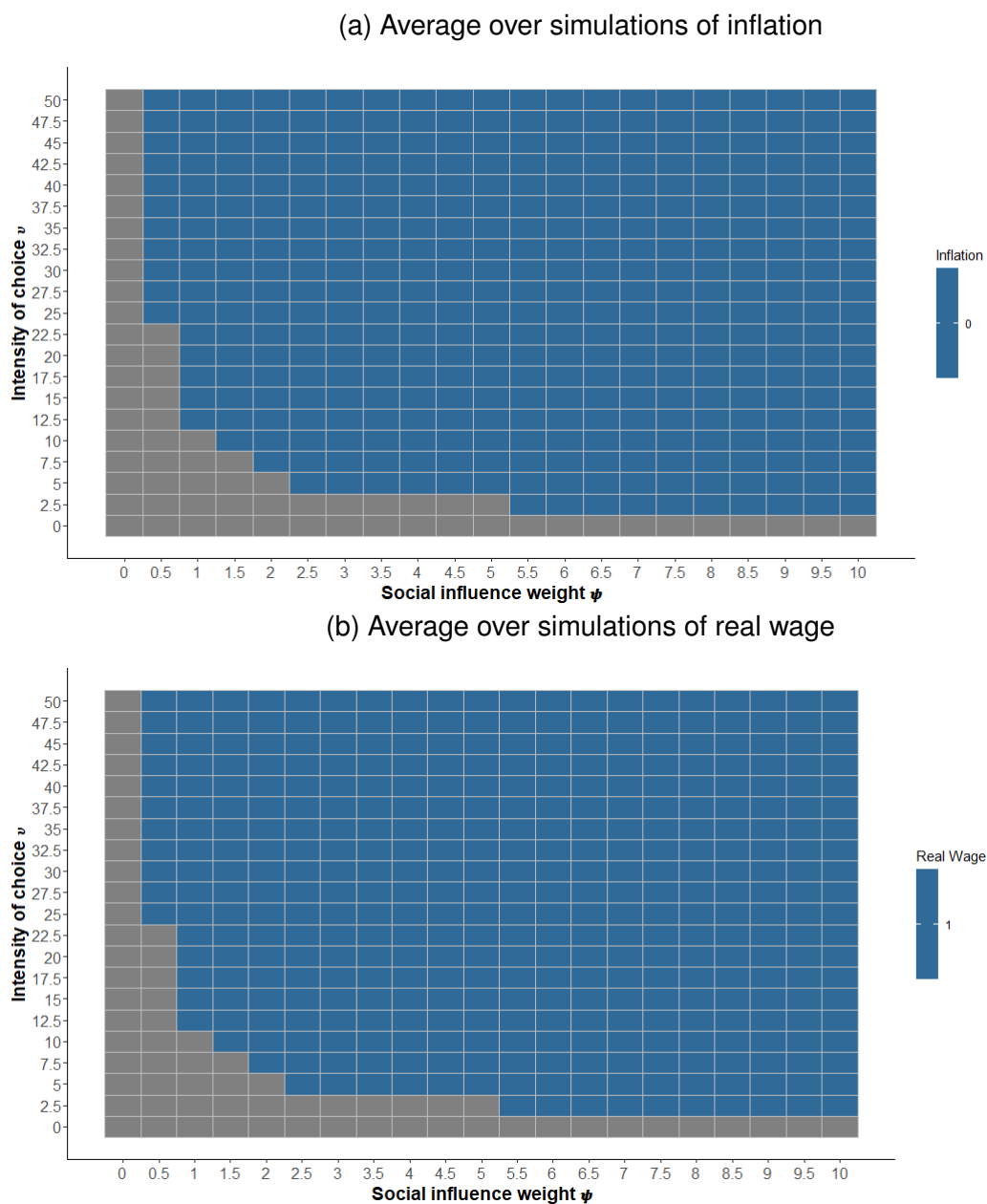
Figure 2.5 – Steady state values of the output gap and unemployment rate for different pairs of the intensity of choice ν and social influence weight ψ in the decentralized scenario.



Source: Own elaboration.

The intuition behind this result is that, for the decentralized case, any small deviation from the steady state leads to larger imbalances at the micro and macroeconomic levels. This sensitivity of the model requires enough higher values for the intensity of

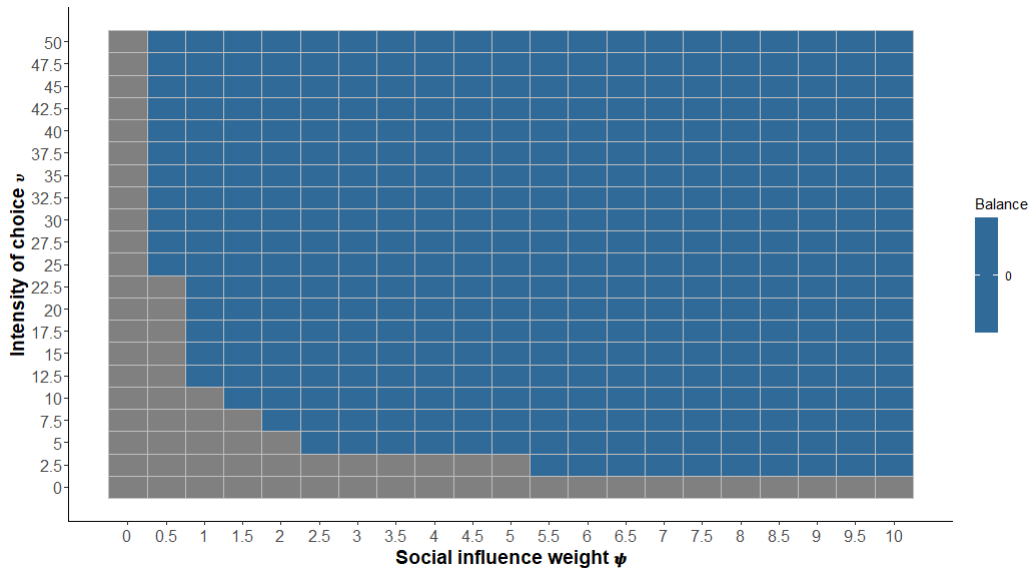
Figure 2.6 – Steady state values of inflation and real wage for different pairs of the intensity of choice ν and social influence weight ψ in the decentralized scenario.



Source: Own elaboration.

choice and social influence parameters so that there is not even the slightest deviation from the steady state for 50 consecutive periods. In fact, for all combinations of parameters that previously took the variables of the centralized model close to the steady state value (with deviations smaller than 0.01) but not exactly equal to the steady state values, in the decentralized model they extrapolate the minimum value required for convergence.

Figure 2.7 – Steady state values of balance for different pairs of the intensity of choice ν and social influence weight ψ in the decentralized scenario.



Source: Own elaboration.

Having analyzed the possibility of convergence to a steady state for different combinations of parameters in both centralized and decentralized scenarios, we can analyze, as it has been done by Guerini, Napolitano, and Roventini (2018), the impacts on macroeconomic variables after a negative productivity shock, which we will do in the next section.

2.5 EFFECTS OF PRODUCTIVITY SHOCKS IN GNR MODEL AUGMENTED BY EXPECTATIONS

In this section, we study the dynamic behavior of the GNR model extended by expectations after a productivity shock. We initialize the simulations in the same way as it has been done in the last section, i.e., setting the initial values of the model's state variables compatible with full employment equilibrium and homogeneous agents with a neutral perception of the future macroeconomic situation. Then we proceed with productivity shocks as proposed by Guerini, Napolitano, and Roventini (2018), that is, in a certain simulation step t^* , we assume that the following technological shock hits the economy at the firm level:

$$a_{f,t} = \tilde{a}(1 + \eta_{f,t}), \text{ with } \begin{cases} \eta_{f,t} = 0, & \text{if } t < t^*, \\ \eta_{f,t} \sim \mathcal{N}(\mu_\eta, \sigma_\eta), & \text{if } t = t^*, \\ \eta_{f,t} = \rho_\eta \eta_{f,t-1}, & \text{if } t > t^*, \end{cases} \quad (2.31)$$

where $\mu_\eta \in \mathbb{R}$, $\sigma_\eta \in \mathbb{R}$, and $\rho_\eta \in \mathbb{R}$ represent, respectively, the mean, standard deviation, and autoregressive persistence of the shock, and $\tilde{a} \in \mathbb{R}$ is the deviation of productivity from its steady state value.

We assume, as in Guerini, Napoletano, and Roventini (2018), that the shock occurs in period $t^* = 50$ after the simulation starts. In addition to the parameters presented in Table 2.4, we used the parameters related to the shock that is summarized in Table 2.6, which are the same as the ones used by Guerini, Napoletano, and Roventini (2018). To mitigate the variability of the results due to the stochastic component, we perform 30 Monte-Carlo simulations, each simulation with a different seed, and all results below refer to the average over the 30 simulations.

Table 2.6 – Parameter Values.

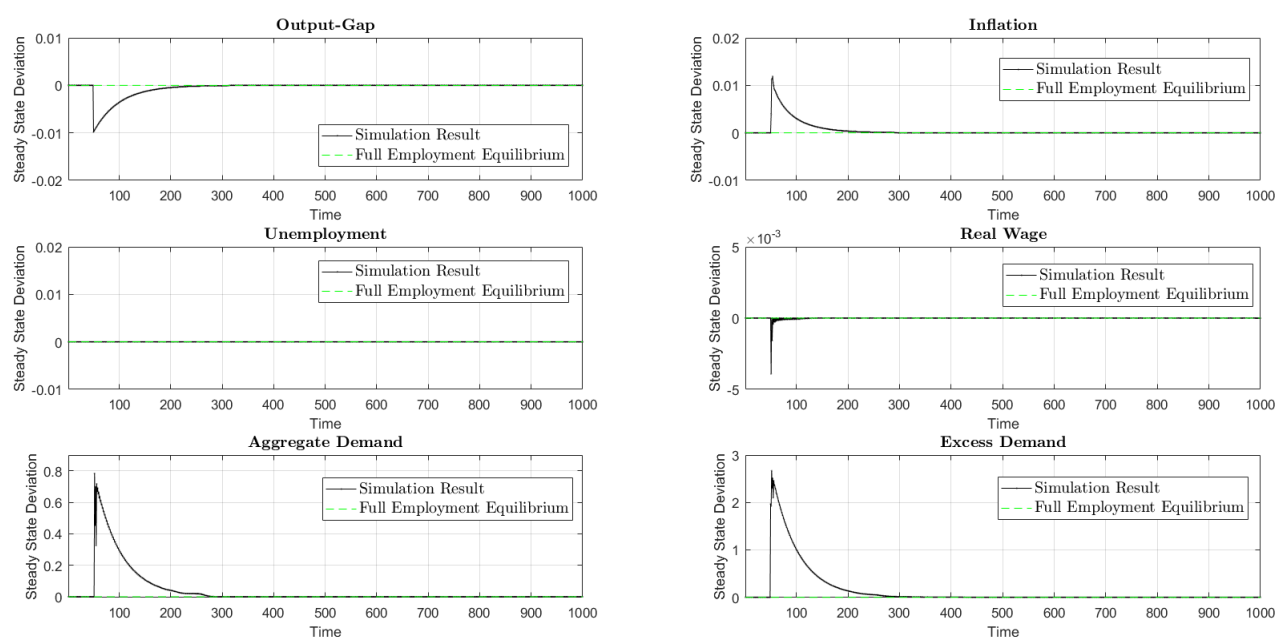
Parameter	Value	Meaning
μ_η	-0.01	Mean of the supply shock
σ_η	0.002	Variance of the supply shock
ρ_η	0.98	Persistence of the supply shock

Source: Own elaboration.

We present impulse-response functions (IRFs) using different combinations of parameters to illustrate the dynamics of the model under a negative productivity shock. We first present the IRFs for a specific combination of parameters in the centralized search and matching scenario. Then, we analyze the IRFs of the decentralized scenario using the same combinations of parameters.

We begin by examining the dynamics of the centralized scenario with the highest combination of parameters used in the heat maps of the previous section. Specifically, we set the intensity of choice parameter ν to 50 and the social influence parameter ψ to 10. Figure 2.8 presents the macroeconomic dynamics extracted from the simulation before the productivity shock until the model returns to the steady-state equilibrium, which happens around the 300th simulation step, while Figure 2.9 shows the balance of perceptions along the same periods. As soon as the negative technological shock occurs at step 50 of the simulation, the production of firms decreases, leading to excess demand in the goods market. This, in turn, forces households to reduce their consumption, resulting in an increase in savings. In such a scenario prices rise, causing firms to hire more workers and generating inflationary pressure on wages. Also, prices increase more than wages as there is excess demand for goods produced by firms, in addition to being indexed to wages. As prices increase more than nominal wages, real wages fall. No frictional unemployment was observed in this scenario. With a high intensity of choice parameter, the perception with the highest deterministic utility will almost

Figure 2.8 – Emergent macroeconomic dynamics under negative productivity shocks in the centralized scenario considering $\nu = 50$ and $\psi = 10$.



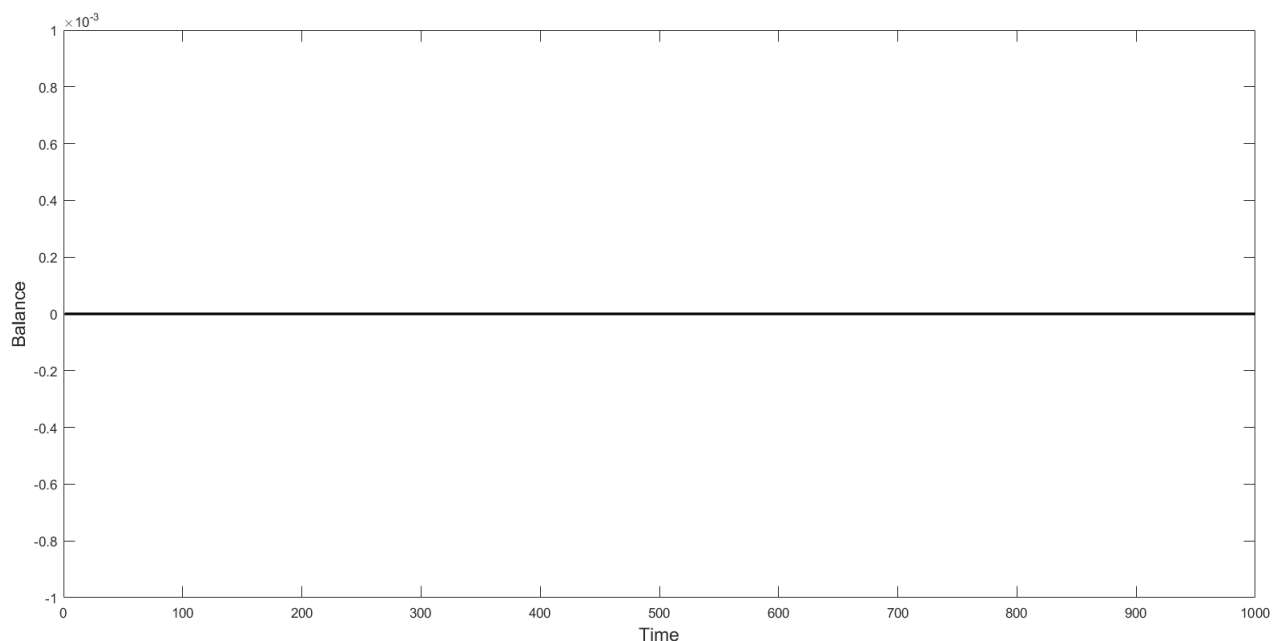
Source: Own elaboration.

certainly be chosen, with the probability increasing according to the difference between the highest deterministic utility and the utility of the other perceptions. Since there is no unemployment even after the shock and the simulation begins with complete neutrality of perceptions, according to equations (2.26), (2.27) and (2.22) the deterministic utility of neutrality is the highest and the difference in relation to the utility of the other perceptions is high due to the high value of ψ chosen for this simulation. Consequently, the probability of choosing the neutral perception is equal to 1 throughout the simulation. No negative bias in consumption as a consequence of perceptions, combined with no frictional unemployment and high savings caused by consumption rationing, are factors that help to keep demand high. Due to the low productivity resulting from the shock, excess demand persists until productivity returns to its equilibrium level, and, consequently, excess demand disappears.

Figure 2.10 shows the evolution of the variance of some microvariables of the model. We can see that microeconomic variables related to households do not change, even after the heterogeneous shock. Additionally, it can be observed that the micro-level variance arising over prices, wages, and firm sales is only temporary and disappears as the shock vanishes.

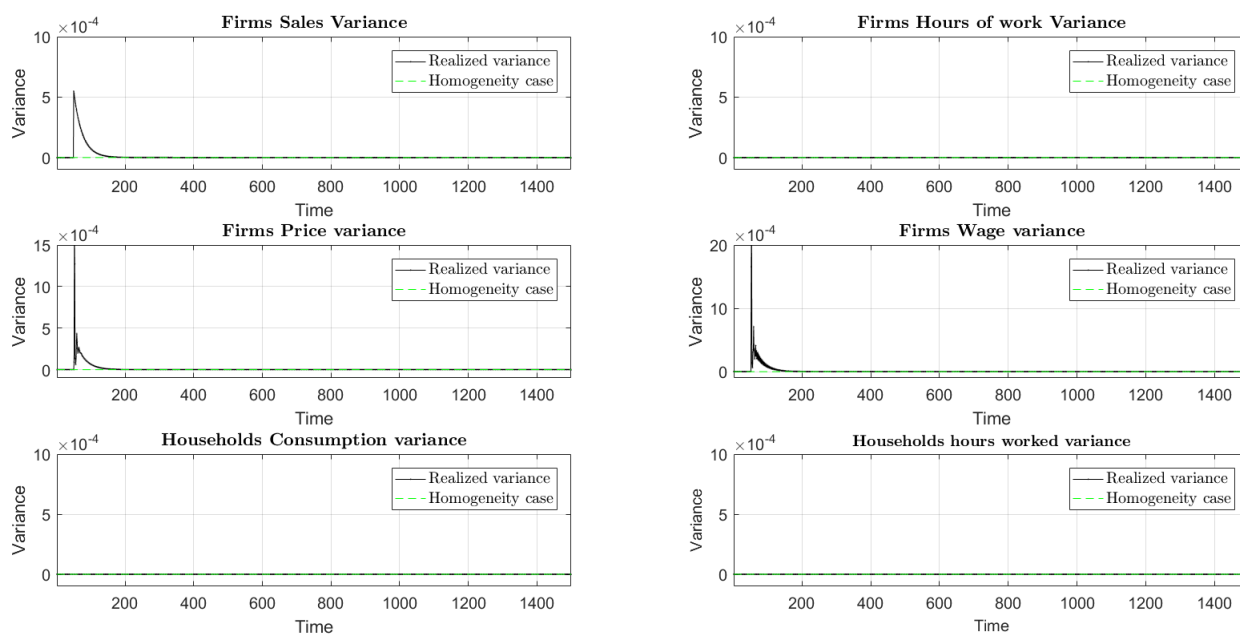
We can now analyze the IRFs under negative technology shock in the centralized scenario with the smallest combination of parameters related to the formation of perceptions, i.e., $\psi = 0$ and $\nu = 0$. Figure 2.12 shows a strong variation in the balance,

Figure 2.9 – Balance of perceptions under a negative productivity shock in a centralized scenario with $\psi = 10$ and $v = 50$.



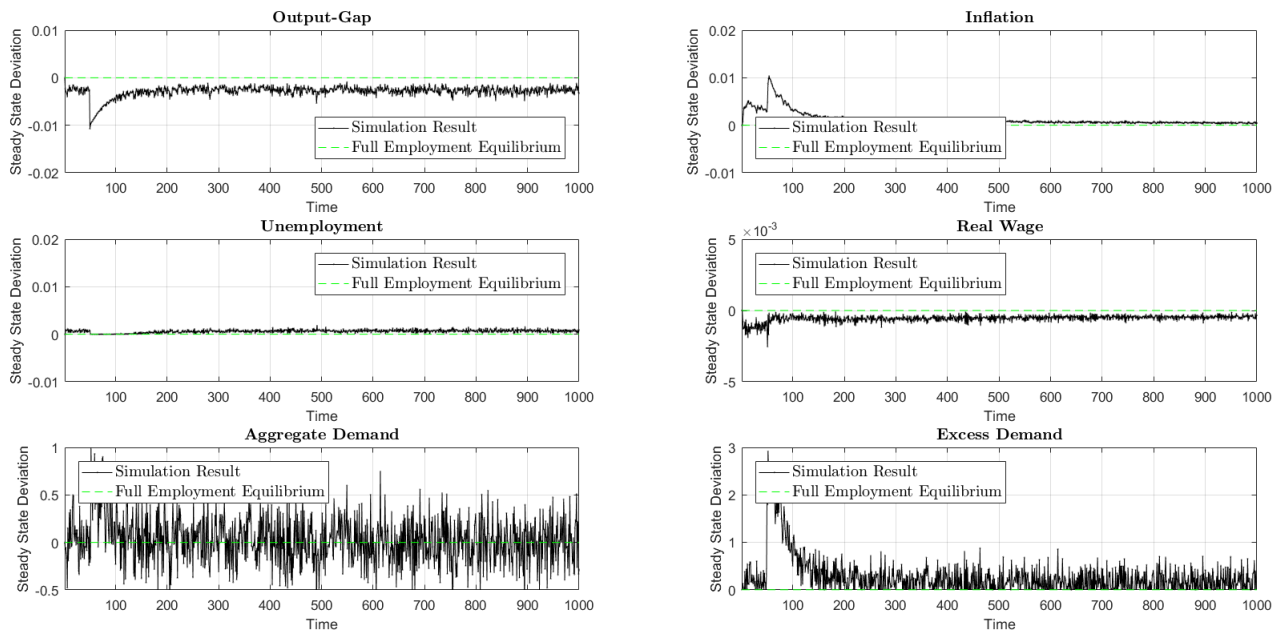
Source: Own elaboration.

Figure 2.10 – Micro-level variance in the centralized scenario under negative productivity shocks with $\psi = 10$ and $v = 50$.



Source: Own elaboration.

Figure 2.11 – Emergent macroeconomic dynamics under negative productivity shocks in the centralized scenario considering $\psi = 0$ and $v = 0$.



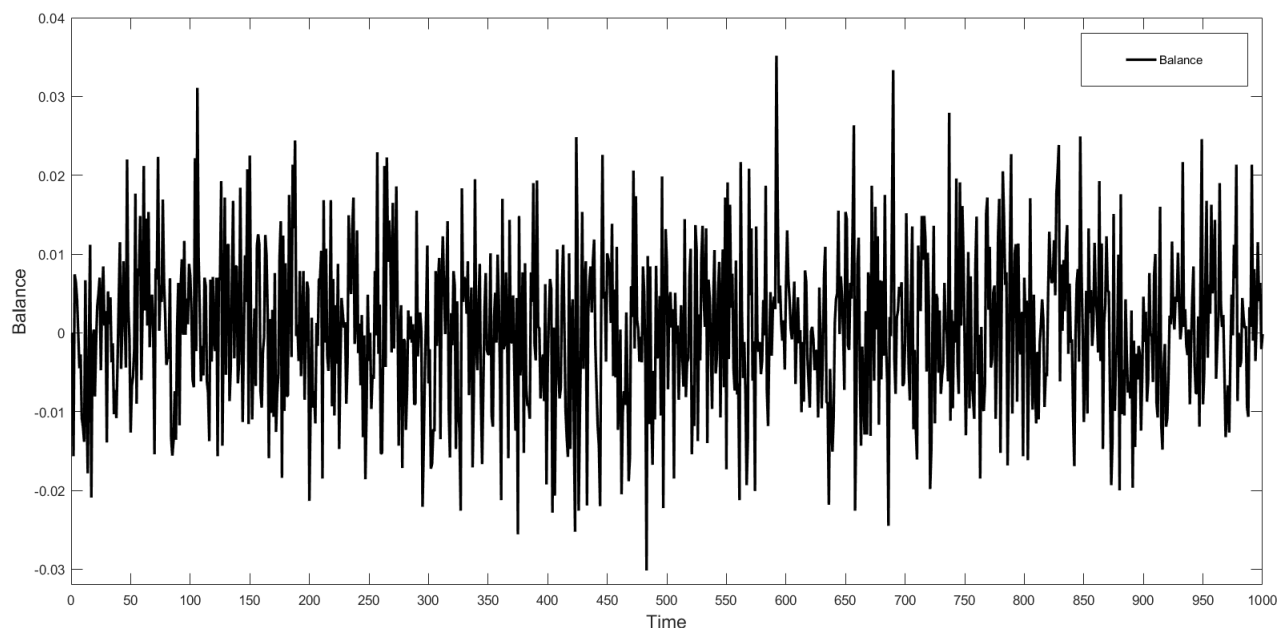
Source: Own elaboration.

which fluctuates around neutrality, but with some periods of great optimism and others of great pessimism. It happens because when $v = 0$, the propensity of choosing each strategy is equal, that is, $1/5$. It leads to a random upward or downward bias in consumption, which is reflected in other macroeconomic variables, as can be seen in Figure 2.11. In particular, it results in aggregate demand alternating around the steady-state level. Additionally, we observe that the absolute value of deviations from the steady state of macroeconomic variables generally increases soon after the shock. However, unemployment decreases immediately after the shock. This is because, in the event of a negative technological shock, firms look for more workers to maintain their production levels, resulting in a short-run reduction in unemployment.

In turn, in Figure 2.13, it can be seen that the variation at the firm level only takes place immediately after the shock and it persists only until productivity returns to its equilibrium level, while the variation in consumption persists throughout the periods.

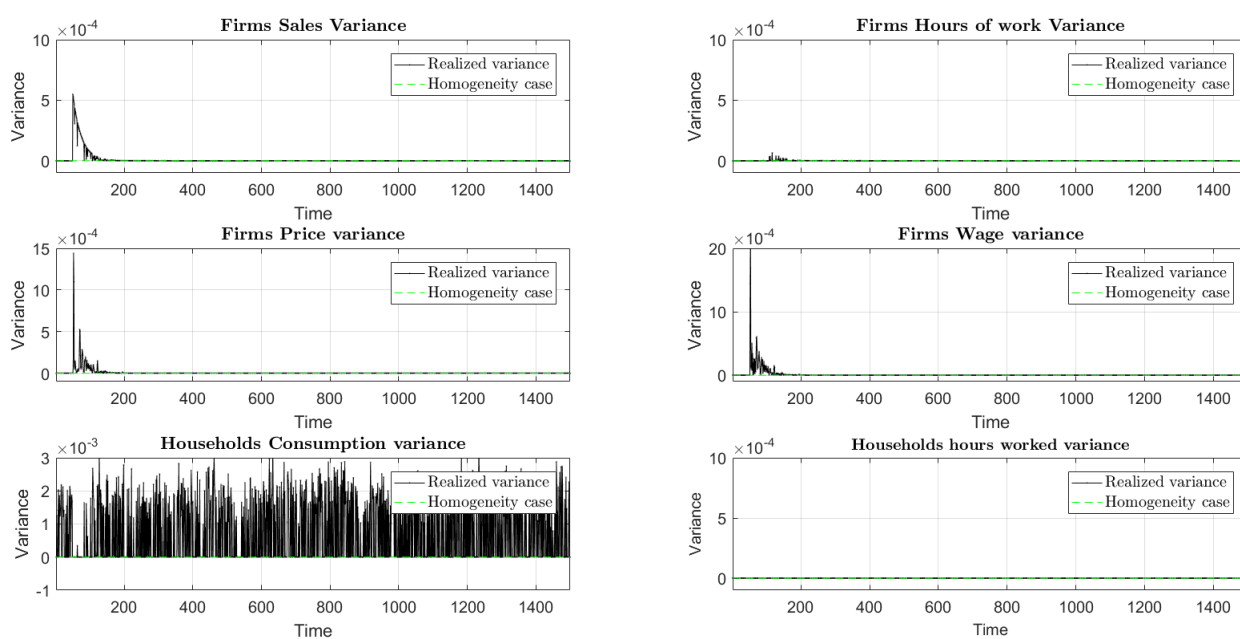
Finally, we analyze the emergent properties with a combination of parameters that also lead to convergence to the steady state of the model in the centralized scenario and are closer to the border between convergence/non-convergence, more precisely, $\psi = 1$ and $v = 10$. In Figure 2.15 we can see that the balance does not vary much and is around zero. Consequently, there is no significant impact on the macroeconomic variables in relation to the model with the maximum ψ and v values, as can be seen by comparing Figure 2.14 with Figure 2.8.

Figure 2.12 – Balance of perceptions under a negative productivity shock in a centralized scenario with $\psi = 0$ and $\nu = 0$



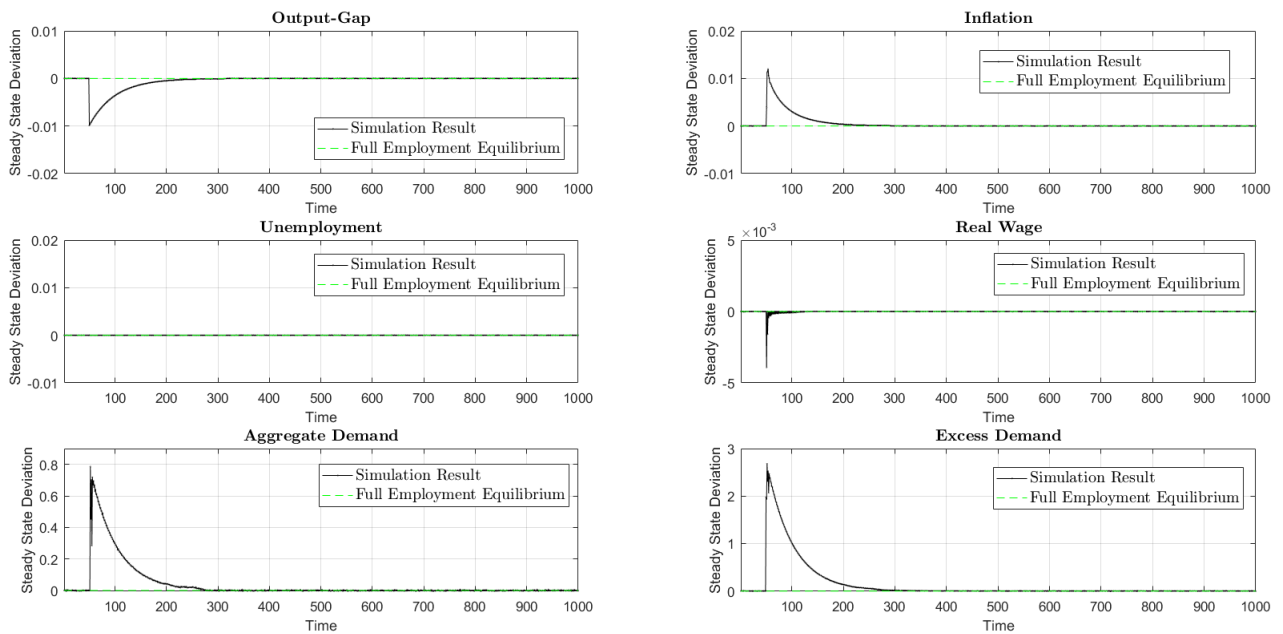
Source: Own elaboration.

Figure 2.13 – Micro-level variance in the centralized scenario under negative productivity shocks considering $\psi = 0$ and $\nu = 0$.



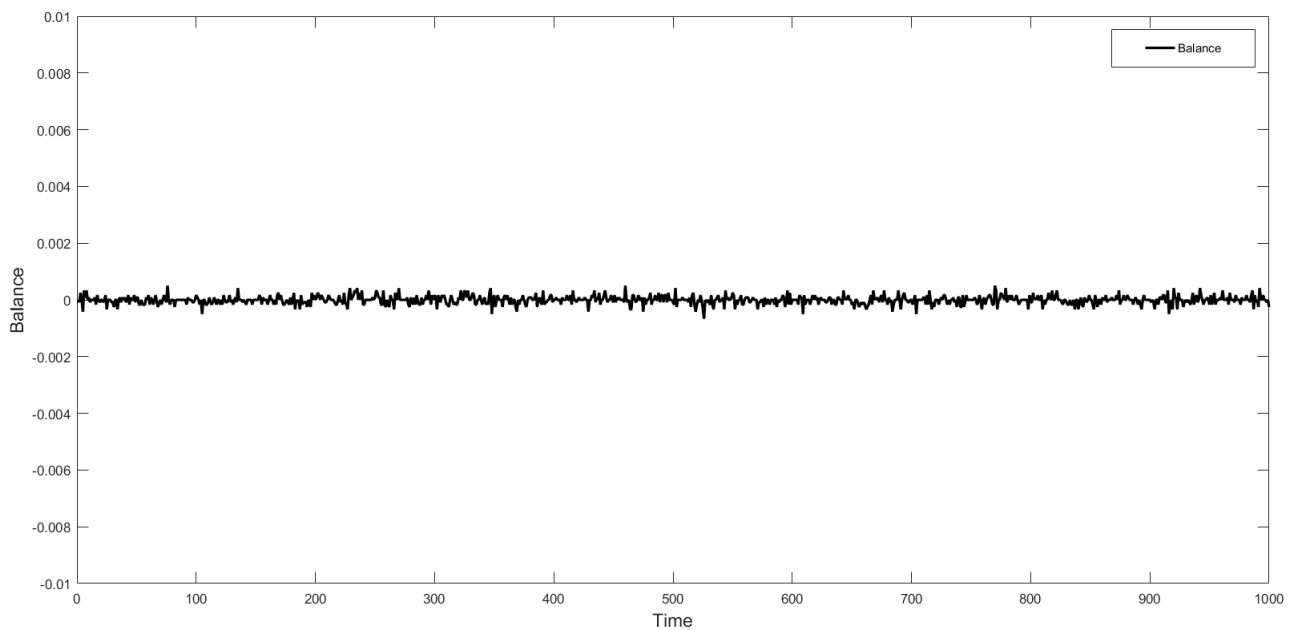
Source: Own elaboration.

Figure 2.14 – Emergent macroeconomic dynamics under negative productivity shocks in the centralized scenario considering $\psi = 1$ and $\nu = 10$.



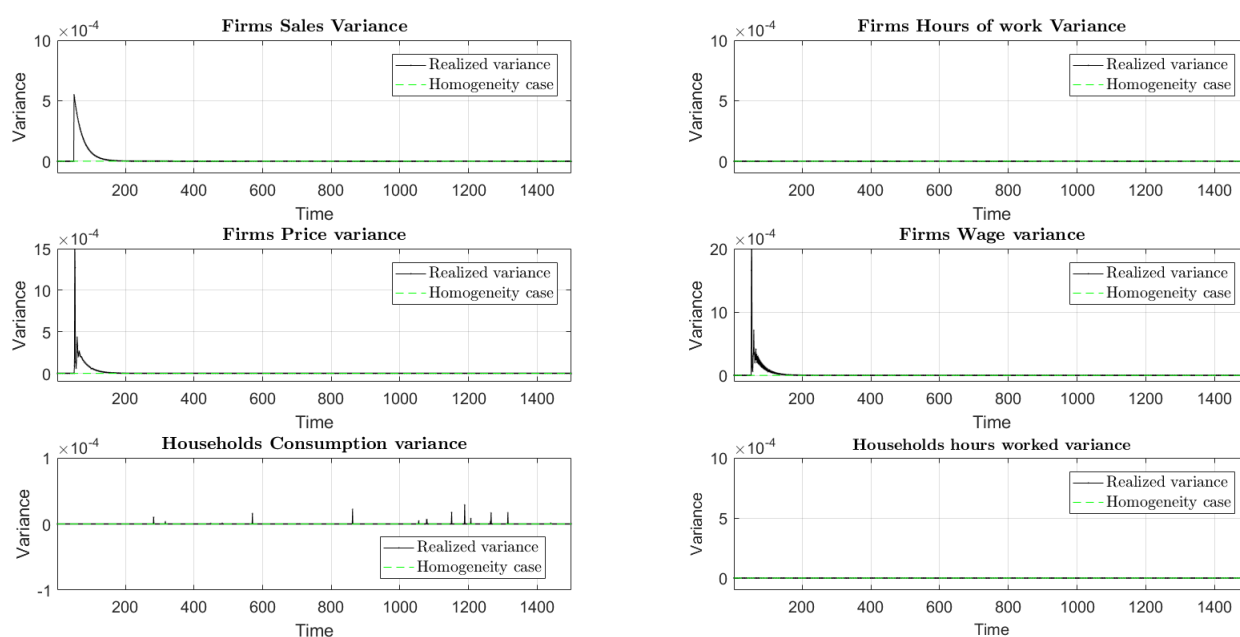
Source: Own elaboration.

Figure 2.15 – Balance of perceptions in the centralized scenario under negative productivity shocks considering $\psi = 1$ and $\nu = 10$



Source: Own elaboration.

Figure 2.16 – Micro-level variance in the centralized scenario under negative productivity shocks considering $\psi = 1$ and $v = 10$.



Source: Own elaboration.

Moreover, when one analyzes the evolution of the variance of some micro variables of the model, shown in Figure 2.16, it can be seen that the heterogeneity at the household level is only on consumption, and it is very mild.

Table 2.7 shows the long-run values of the main macroeconomic variables and of the balance of perceptions. We can note that with the highest combination of parameters ψ and β , macroeconomic variables return to their equilibrium levels of full employment in the long term and agents' expectations remain completely neutral. Moreover, with the smallest combination of parameters ψ and β , macroeconomic variables deviate from their equilibrium levels of full employment in the long term. The average output gap and real wages are below their full employment equilibrium levels, while inflation and unemployment are above their full employment equilibrium levels. With the intermediate combination of parameters ψ and β , there is no significant deviation of the variables from their full employment levels.

Now, using the same combination of parameters as used in the IRFs analysis of the centralized model, we present IRFs of the decentralized model. Starting with the highest combination of parameters, i.e., $v = 50$ and $\psi = 10$, we can see that the macroeconomic dynamics change considerably in relation to the same parameters in the centralized scenario (see Figures 2.8, 2.9 and 2.10). Firstly, as depicted in Figure 2.17, we see that most of the variables, but the inflation, permanently deviate from the equilibrium level of full employment. In particular, the output gap fluctuates around

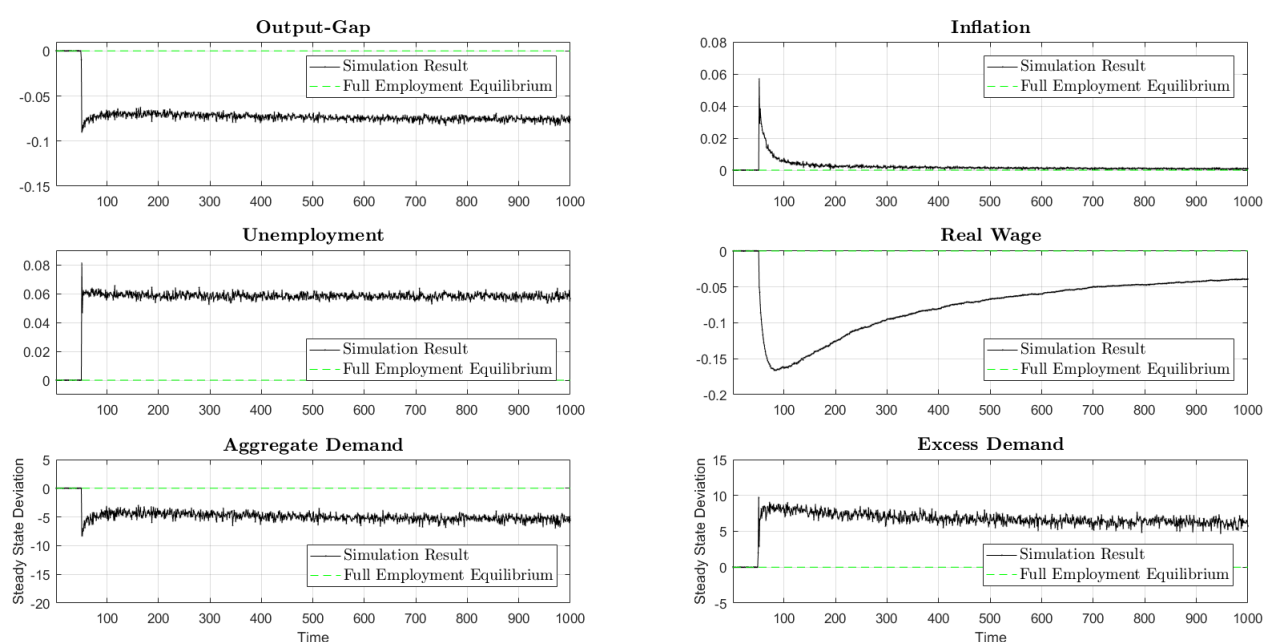
Table 2.7 – Long-run values of output-gap, unemployment, inflation, real wage and balance of perceptions for different combinations of parameters in the centralized scenario. Monte-Carlo standard errors are in parentheses. The values are rounded to 4 decimal places.

Combination of parameters	Output gap	Unemployment	Inflation	Real wage	Balance
$\nu = 50$ and $\psi = 10$	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	1.0000 (0.0000)	0.0000 (0.0000)
$\nu = 0$ and $\psi = 0$	-0.0027 (0.0005)	0.0007 (0.0001)	0.0004 (0.0001)	0.9996 (0.0001)	0.0006 (0.0071)
$\nu = 10$ and $\psi = 1$	-0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	1.0000 (0.0000)	-0.0000 (0.0000)

Source: Own elaboration.

a mean below the full employment level, and unemployment fluctuates above the full employment level. Therefore, even after the shock has vanished, the variables do not return to their full employment levels.

Figure 2.17 – Emergent macroeconomic dynamics under negative productivity shocks in the decentralized scenario considering $\psi = 10$ and $\nu = 50$.

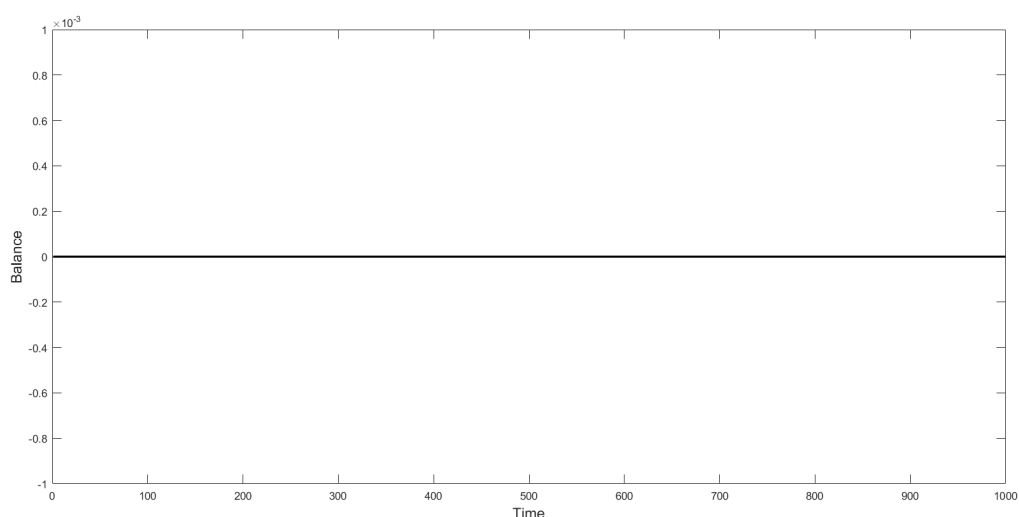


Source: Own elaboration.

However, even with higher unemployment, there is no change in expectations with this combination of parameters, as can be seen in Figure 2.18. With a high intensity of choice parameter, the perception with the highest deterministic utility will almost certainly be chosen. Even in a situation with unemployment, since the simulation begins with complete neutrality of perceptions and the social influence weight is high, the

increase in private utility towards pessimism due to the unemployment rate is not enough to compensate for the greater social utility of neutrality. Consequently, the deterministic utility of neutrality is the highest, and due to the high intensity of choice, the probability of choosing the neutral perception is equal to 1 throughout the simulation.

Figure 2.18 – Balance of perceptions in the decentralized scenario under negative productivity shocks considering $\psi = 10$ and $\nu = 50$.



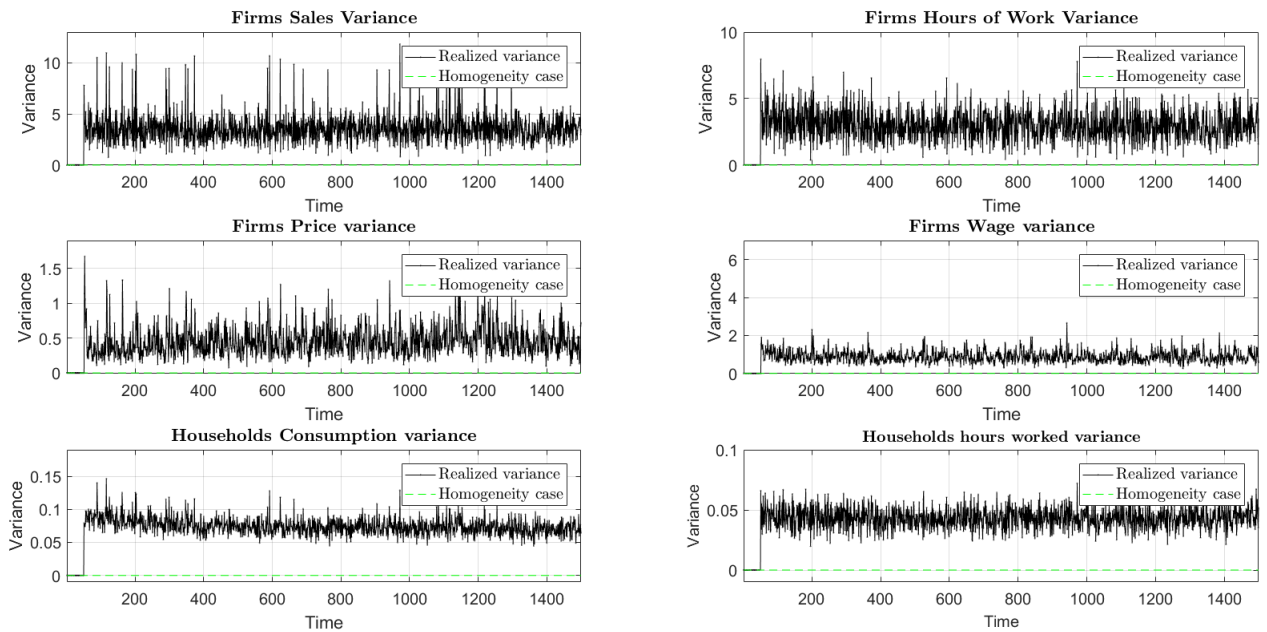
Source: Own elaboration.

Moreover, the technological shock leads to frictional unemployment and mismatches between demand and supply in the goods market in the decentralized scenario. For this reason, micro heterogeneity now occurs not only at the firm level but also at the household level, and it persists even after the shock vanishes, as can be seen in Figure 2.19.

We can now analyze the IRFs under negative technology shock in the decentralized scenario with the smallest combination of parameters related to the formation of perceptions, i.e., $\psi = 0$ and $\nu = 0$. In Figure 2.20, it is noticeable that the macroeconomic variables in the model move away from the steady-state level immediately after initializing the model. In addition, the technological shock seems to have much less impact compared to the case with high ψ and ν values.

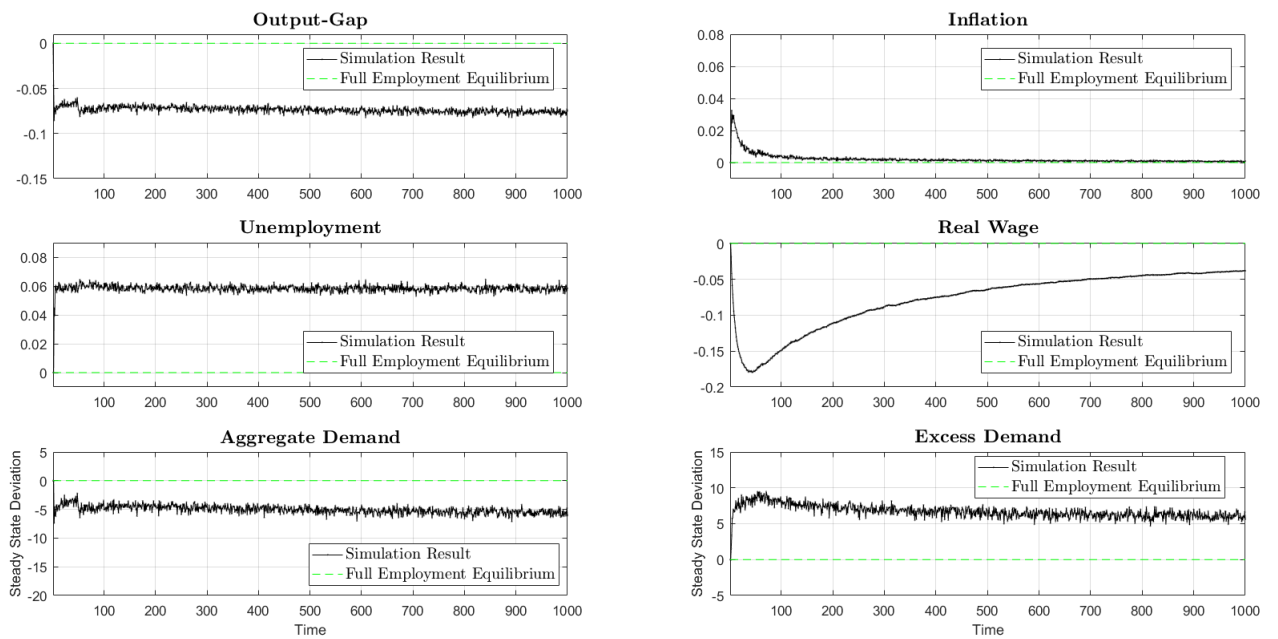
As already mentioned, when $\nu = 0$, each strategy has an equal probability of being chosen, which is $1/5$. Consequently, the balance tends to fluctuate around zero without showing any clear trend, as shown in Figure 2.21. In Figure 2.22, we observe that micro-variance also arises immediately after the model is initialized. This is because perceptions start to vary right after the model is initialized, affecting consumption which, in a decentralized search and matching model, leads to heterogeneity at both household and firm levels.

Figure 2.19 – Micro-level variance in the decentralized scenario under negative supply shocks considering $\psi = 10$ and $v = 50$.



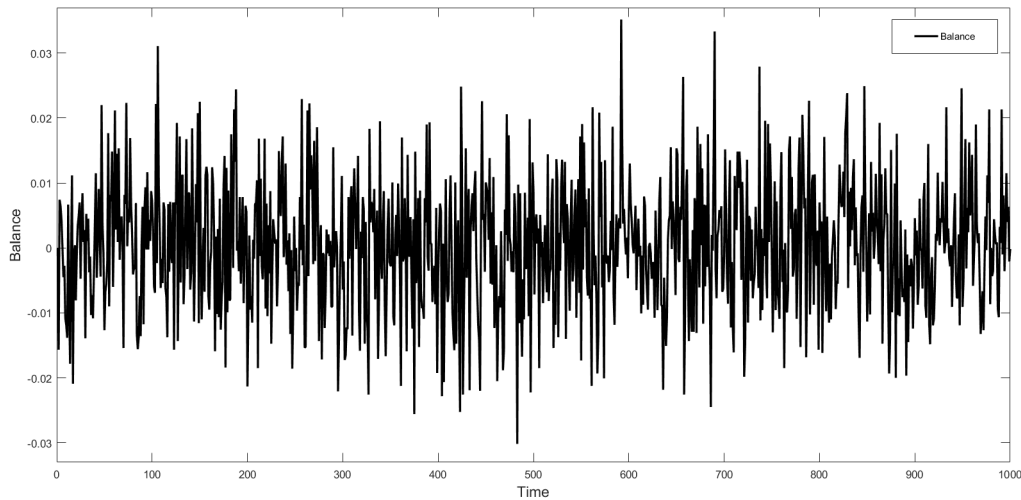
Source: Own elaboration.

Figure 2.20 – Emergent macroeconomic dynamics under negative productivity shocks in the decentralized scenario considering $\psi = 0$ and $v = 0$.



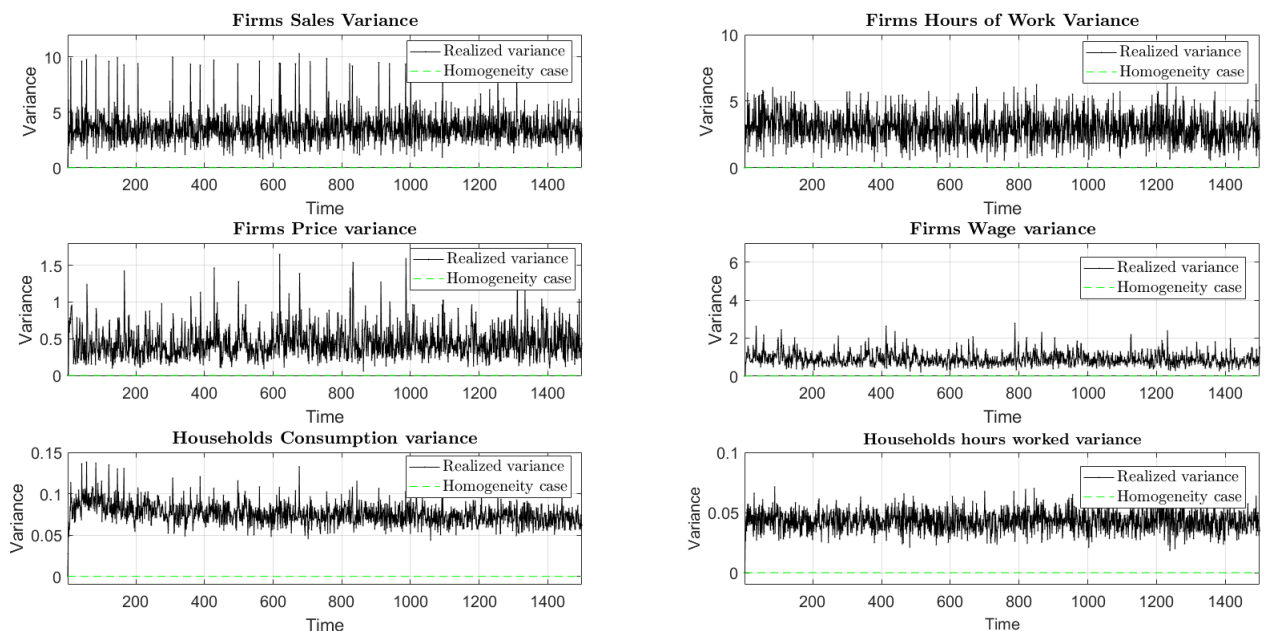
Source: Own elaboration.

Figure 2.21 – Balance of perceptions in the decentralized scenario under negative productivity shocks considering $\psi = 0$ and $\nu = 0$



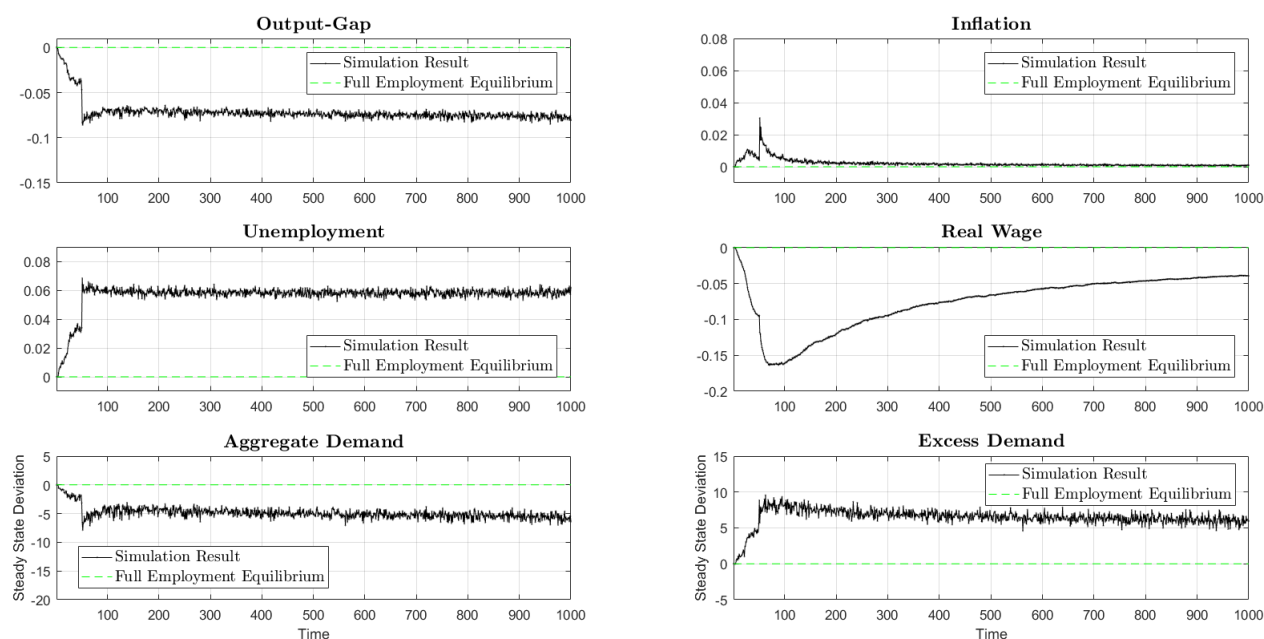
Source: Own elaboration.

Figure 2.22 – Micro-level variance in the decentralized scenario under negative productivity shocks considering $\psi = 0$ and $\nu = 0$.



Source: Own elaboration.

Finally, we analyzed the emergent properties of the decentralized model with the last combination of parameters used in the previous subsection, i.e., $\psi = 1$ and $\nu = 10$. Unlike the centralized model with the same combination of parameters, in the decentralized model, macroeconomic variables deviate from the full employment level

Figure 2.23 – Emergent macroeconomic dynamics under negative productivity shocks in the decentralized scenario considering $\psi = 1$ and $\nu = 10$.

Source: Own elaboration.

even before the shock, which amplifies these deviations. We can analyze this dynamic in Figure 2.23. In Figure 2.24, we can see that the balance is negative right after the model is initialized, and both the level and variation of these variables are amplified after the shock. Figure 2.25 shows that heterogeneity emerges at the firm and household levels shortly after the model is initialized.

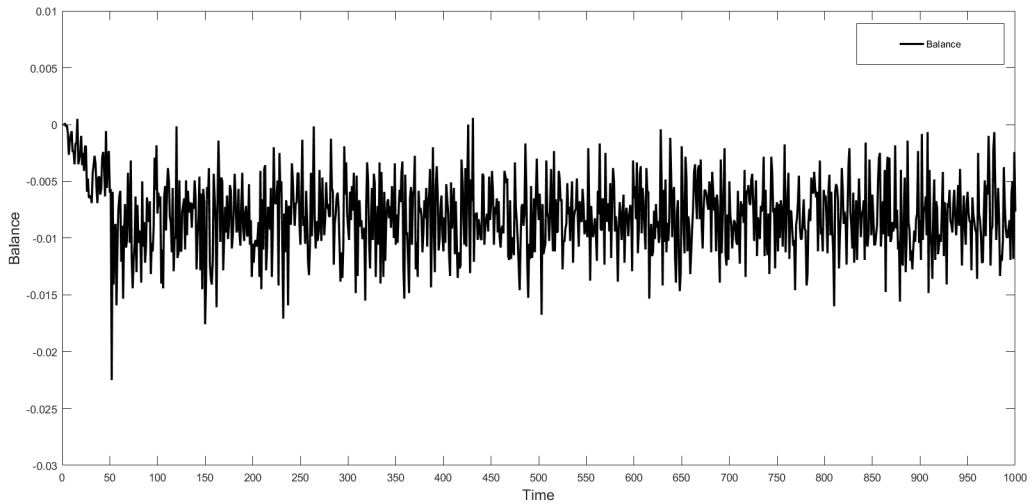
Table 2.8 – Long-run values of output-gap, unemployment, inflation, real wage and balance of perceptions for different combinations of parameters in the decentralized scenario. Monte-Carlo standard errors are in parentheses. The values are rounded to 4 decimal places.

Combination of parameters	Output gap	Unemployment	Inflation	Real wage	Balance
$\nu = 50$ and $\psi = 10$	-0.0761 (0.0013)	0.0579 (0.0014)	0.0008 (0.0001)	0.9600 (0.0022)	0.0000 (0.0000)
$\nu = 0$ and $\psi = 0$	-0.0764 (0.0018)	0.0576 (0.0010)	0.0007 (0.0001)	0.9617 (0.0021)	0.0002 (0.0070)
$\nu = 10$ and $\psi = 1$	-0.0761 (0.0016)	0.0579 (0.0011)	0.0007 (0.0001)	0.9598 (0.0016)	-0.0078 (0.0022)

Source: Own elaboration.

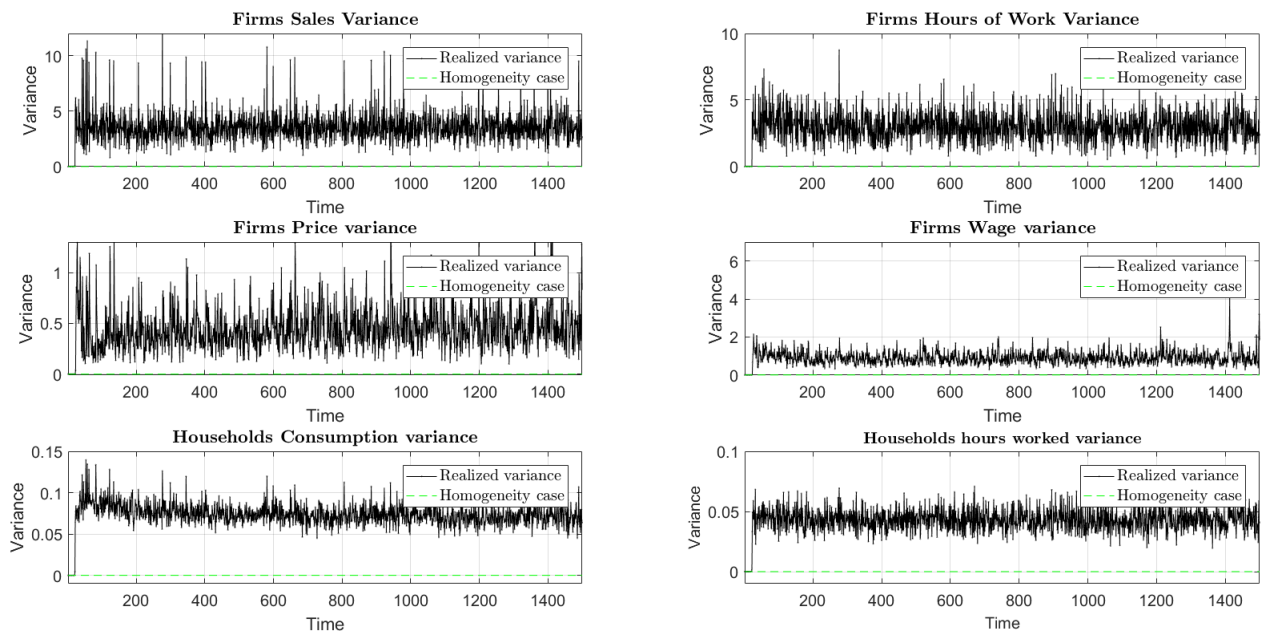
Table 2.8 presents the long-run values of the main macroeconomic variables and of the balance of perceptions. The first thing to note is that, curiously, the deviations of the variables from their equilibrium levels of employment are smaller for the combination

Figure 2.24 – Balance of perceptions in the decentralized scenario under negative productivity shocks considering $\psi = 1$ and $\nu = 10$



Source: Own elaboration.

Figure 2.25 – Micro-level variance in the decentralized scenario under negative productivity shocks considering $\psi = 1$ and $\nu = 10$.



Source: Own elaboration.

of parameters $\beta = 0$ and ψ . For this combination of parameters, the average balance of perceptions is strictly greater than zero, indicating that, on average, agents are more optimistic in the long run. For $\nu = 10$ and $\psi = 1$ agents are, on average, more pessimistic in the long run. For $\nu = 50$ and $\psi = 10$, the balance of perceptions is zero on

average, which indicates that there is no bias towards optimism or pessimism with this combination of parameters.

To summarize, in both centralized and decentralized scenarios, when both the intensity of choice parameter ν and the social influence parameter ψ are high, there will be no variation in expectations. On the other hand, when both parameters are set to a median value of $\psi = 1$ and $\nu = 10$, there is still a variation in expectations, but this is more pronounced in the case of decentralized search and matching. Lastly, when both parameters are set to zero, the variation in expectations is high in both the centralized and decentralized scenarios.

2.6 FINAL REMARKS

In this essay, we aim to analyze the macroeconomic dynamics by considering the economy as a complex system and incorporating the hypothesis of bounded rationality in the formation of expectations. To achieve this, we expand on a macroeconomic ABM that allows us to analyze full employment, coordination failures, and involuntary unemployment as emergent properties through the interaction of heterogeneous agents that use heuristics in their decision-making.

The model used in this essay is based on the one proposed by Guerini, Napoletano, and Roventini (2018), which is characterized by the presence of a full-employment homogeneous-agents equilibrium. This model has a deterministic skeleton that can be affected by exogenous stochastic shocks, which allows us to explore the circumstances under which the economy returns to the full-employment equilibrium after a shock. We expand on this model by using the suggestive framework of Brock and Hommes (1997).

We find that for some combination of parameters related to the expectation formation mechanism the economy persistently deviates from the full employment equilibrium. We also analyze the impact of a negative productivity shock under the centralized matching scenario and the decentralized matching scenario. In the centralized scenario, a central planner avoids any possible coordination problem in both labor and goods markets, and the economy may be able to return to the full employment equilibrium for some combination of parameters related to the expectation formation framework. On the other hand, in the decentralized matching scenario, search and matching are local, and the economy persistently deviates from the full employment equilibrium.

Overall, our findings suggest that the incorporation of bounded rationality in the formation of expectations and the analysis of the economy as a complex system can provide valuable insights into macroeconomic dynamics and the emergence of persistence heterogeneity in expectations.

3 SOCIAL NETWORK IN MACROECONOMIC AGENT-BASED MODEL

Economic agents are often connected in different social networks. Our decisions, beliefs, and behaviors are influenced by people we interact with, even occasionally, as noted by Jackson (2011). Especially in decentralized markets, the terms of trade, prices, and products that emerge can depend on who is connected to whom. For example, in the labor market, both jobs and employees come with many idiosyncrasies. In this context, social networks have the role of mitigating search frictions by spreading information to workers about the specifics of job opportunities and to firms about the potential of workers (JACKSON, 2011).

Another important role of social networks is in influencing learning as well as the diffusion of technology, opinions, and behaviors (JACKSON, 2011). In particular, on the formation of expectations and opinions of economic agents, local interaction appears in various studies (e.g. Brock and Durlauf (2001), Topa (2001), Flieth and Foster (2002), Lux (2009)). Unlike global interaction, which involves interactions across the entire system or population of agents resulting in decisions that are influenced by the aggregate actions of all agents, local interaction refers to the interaction between neighboring agents based on a defined proximity metric (BARGIGLI; TEDESCHI, 2014).

It can be seen that the expectations formation framework coupled with the efficiency wage model, proposed in the first essay, assumes that agents have global interaction in the formation of expectations. The same occurs in the structure of the expectations formation framework coupled with the model with a Keynesian closure. However, as Goyal (2012) note, when we are choosing among alternatives we make use of our past experience as well as the experience of others, especially those who are close to us. Thus, to bring the model closer to reality, it seems plausible to consider that the perceptions about the future unemployment rate could affect the perception of an agent who considers relevant the perceptions of this subset of agents (i.e., the perceptions of her neighborhood), which is in the vein of interactive expectations.

With that in mind, the purpose of this essay is to expand on the ABMs proposed in the first and second essays of this dissertation by including the concept of local interaction and interactive expectations in the formation of perceptions about the future unemployment rate.

In the structure of expectation formation proposed in the first and second essays, agents' choices have a private incentive and a social incentive. As a private incentive, in the efficiency wage model, we consider the agent's efficiency wage weighted by the unemployment rate, while for the model with Keynesian closure, we consider an employment indicator. In this essay, we will continue to consider the same private incentive as before, but now the social component will follow a complex network structure. With this structure, we can analyze what changes in the adjustment of each ABM and in the

procedures for analyzing the feedback of expectations with the macroeconomy when including interactive expectations.

The rest of this essay is structured as follows. In section 3.1 we present a brief review of the literature that incorporates social interaction in the formation of expectations and in macroeconomic models, as well as presenting elements of network theory that will be used as a basis for proposing the network structure which will be used. In section 3.2 we present the ABM of wage efficiency augmented by interactive expectations and its emergent properties. In section 3.3, we present the GNR model augmented by interactive expectations and its emergent properties. Finally, section 3.4 presents the essay's conclusions.

3.1 RELATED LITERATURE AND DEFINITIONS ON NETWORK

3.1.1 Related literature

The study of networks in macroeconomic models has become more popular due to the recognition of the social context in behavioral economics. As Steinbacher et al. (2021) highlight, the 2008 financial crisis led to an increased awareness of the importance of the network properties of economic systems. Schweitzer et al. (2009) argue that the consequences of the 2008 crisis were challenging to predict because it resulted from the dynamic interaction of numerous agents, and the failure of a few main agents cannot explain it.

As Farmer et al. (2012) explain, the economy can be thought of as a complex system consisting of interconnected networks. A noteworthy feature of this approach is that the behavior of the economy as a whole is a result of the several local interactions between individuals within each network and the networks themselves. The authors highlight that this approach is useful in constructing better models for managing financial markets. The micro-level perspective of financial markets, including interactions between different agents and the mechanisms through which assets are traded, makes agent-based models a good methodology for analyzing it. Moreover, as Hommes (2006) points out, heterogeneous agent models have advantages over representative rational agent models because heterogeneity can generate large trading volumes that align with empirical observations and many other stylized facts observed in financial time series. Still on the financial market, Lux (1995) and Lux (1998) presented socio-economic models of the interaction of speculative traders in a financial market. The model is successful in explaining the stylized facts found in empirical time series, such as the distribution of returns like fat tails and high peaks around the mean.

In the macroeconomic theory, there are two branches of study when it comes to social interaction. As noted by Gallegati and Kirman (2012), one branch assumes a representative agent or heterogeneous agents with fixed behavior making decisions in-

dependently of each other. In this approach, information is complete, so agents have no incentive to increase their information through interaction. The other branch considers direct interactions between agents who follow simple behavioral rules. This approach allows for a bottom-up perspective in which the aggregate result differs from what it would be if each agent had acted in isolation.

Macroeconomic research that takes into account the direct interactions between economic agents assumes that these agents interact with a specific group of neighbors, which is defined by a metric of economic or social distance. Furthermore, Topa (2001) emphasizes that this approach involves the impact of other agents' actions on agents' choices and payoffs through various non-market externalities such as imitation, learning, and social pressure. In this context, Topa (2001) presented a model of local interactions in the labor market in which agents exchange information with their neighbors about job opportunities and hiring occurs through informal channels. In the model, employed individuals may become unemployed with some exogenous probability, while unemployed individuals may find a job with a probability that increases with the number of neighbors currently employed. Using data on the spatial distribution of unemployment in Chicago to estimate the parameters of the model, the authors found results that support the hypothesis that local social interaction exists.

Moreover, Brock and Durlauf (2001) analyze the effect of agents' interaction by means of generalized logistic models of discrete choice to analyze the overall behavior of agents. The framework takes into account that the utility of each agent relies on a private component as well as a component that considers social interaction effects. The authors show that when social utility effects on total utility are significant, the existence of multiple equilibria becomes an emergent property of the model.

Flieth and Foster (2002) developed a model of expectation formation of companies in the consumer goods sector and they could identify that there are some periods in which the effects of interaction on expectation formation are higher. The authors also highlighted that it is important to pay attention to neutral answers since this perception is an attractor for decision-makers who change their positive or negative expectations and some individuals, when they find themselves in periods of uncertainty, can remain neutral for long periods.

In another study on interactive expectation formation, Lux (2009) used data from the Business Climate Index for the German economy to estimate parameters of an expectation formation framework with social interaction using the maximum likelihood procedure. Positive values of this index indicate optimism about economic activity, while negative values indicate pessimism about economic activity. The author found evidence that the model has significant explanatory power for the fluctuations of the business climate index

3.1.2 Definitions on network

This subsection sets forth the terminology that comes from standard graph theory. As Jackson (2011) notes, this terminology may have some variation across disciplines. The concepts presented below draw on Jackson et al. (2008), Jackson (2011) and Goyal (2012).

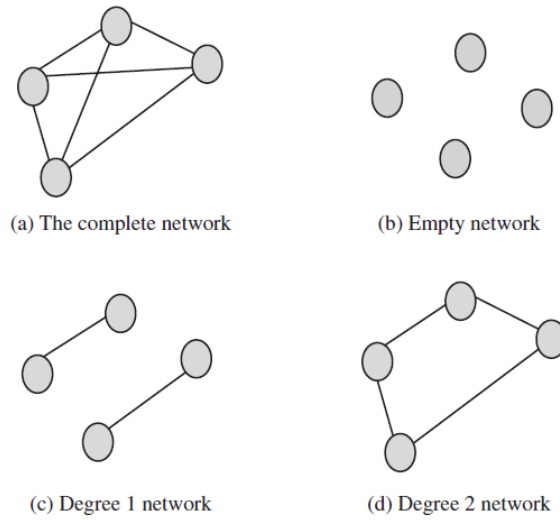
A network has two main components for its formation, namely, nodes and edges. A node might be an individual, a firm, a country, or even a web page, depending on the context. Meanwhile, an edge is a direct connection between two nodes, indicating a direct relation between them. In a social context, a link could be a friendship connection, while in the context of countries, a link could be a free trade agreement or a mutual defense pact (GOYAL, 2012).

Formally, Jackson (2011) note that a network can be represented as a *graph* (N, g) that consists of a set of finite nodes $N = \{1, \dots, n\}$ and a real-valued $n \times n$ matrix g , where $g_{i,j}$ represents the relation between nodes i and j . This square matrix, also known as *adjacency matrix*, lists which nodes are adjacent to each other. In turn, the variable $g_{i,j}$ normally takes on a value of 1 if a link exists between i and j and 0 otherwise. However, creating a weighted adjacency matrix where each edge has an associated weight is also possible. In this case, entries of g record the intensity of the level of relationships.

Moreover, we say that a network is *directed* if it is possible that $g_{i,j} \neq g_{j,i}$ and a network is *undirected* if it is required that $g_{i,j} = g_{j,i}$ for all nodes i and j . One example of a directed network is the follower/following relationships in social media. Goyal (2012) formalizes the *neighbors* of a node i as the set $N_i(g) = \{j \in N | g_{i,j} = 1\}$. Considering $\mathcal{N}_i(g) = |N_i(g)|$ as the number of neighbors of a node i in network g , the network is *regular* if every node has the same number of links, that is, $\mathcal{N}_i(g) = \mathcal{N}$, $\forall i \in N$. In addition, a regular network is said as *complete* if $\mathcal{N} = n - 1$ and a network is called *empty* if $\mathcal{N} = 0$. Finally, the number of nodes that the node i has a direct connection is known as the *degree* of node i , which could also be mathematically represented by $\mathcal{N}_i(g) = |N_i(g)|$. Figure 3.2 illustrates those concepts for a network with 4 nodes.

It is interesting to note that the network used in the ABM proposed by Guerini, Napoletano, and Roventini (2018) is a complete network. In the exposition of the model made in section 2.2, this network is used in the equations for determining the wage, price and desired quantity of production of firms, as well as in the equation for determining the desired consumption of households.

Besides, there are also three more structural properties of a network that are commonly analyzed: the *path* between two nodes, the *distance* between two nodes and the *clustering coefficient*. A path between two nodes in a network is a sequence of nodes connected by edges. In this sequence, each consecutive pair of nodes is directly connected by an edge.

Figure 3.1 – Illustrations of networks for $n = 4$.

Source: Goyal (2012).

The distance between two nodes is the length, or number of nodes, of the shortest path between them. In addition, the average distance represents the average shortest path length between all pairs of nodes in the network.

Finally, the clustering coefficient of a node i is the fraction of all pairs of nodes that are both linked to i that are linked to each other. Formally, the clustering coefficient of a node i in network g is:

$$Cl_i(g) = \frac{\sum_{l \in N_i(g)} \sum_{k \in N_i(g)} g_{l,k}}{\mathcal{N}_i(\mathcal{N}_i - 1)}. \quad (3.1)$$

Besides, the average clustering coefficient of the network g is the arithmetic mean of the coefficients of all the nodes in the network. Formally:

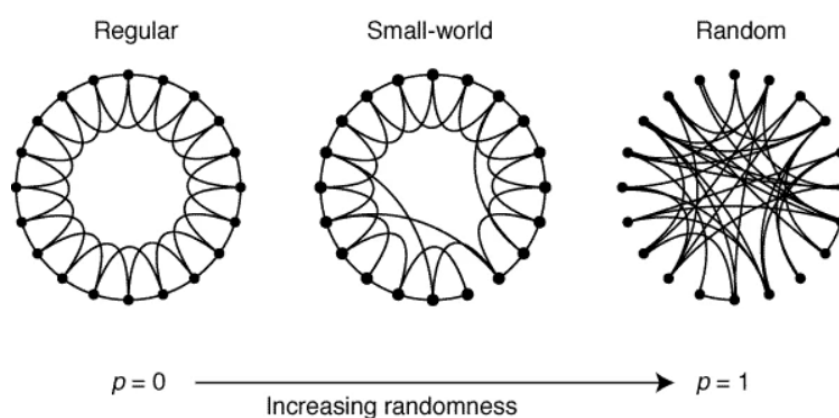
$$Cl^{avg}(g) = \sum_i Cl_i(g)/n. \quad (3.2)$$

Among the stylized facts in the study of social networks, we can highlight the short path length between nodes, high levels of clustering and low average degree relative to the total number of nodes. The *small world* network, introduced by Watts and Strogatz (1998), embodies all these properties. As the authors note, social networks often lie between the regular network, in which each node has the same number of edges and is connected to its nearest neighbor, and the random network, in which edges are placed between nodes randomly. While regular networks exhibit high clustering and long average path lengths, random networks typically have short average path lengths but low clustering coefficients.

To interpolate between regular and random networks, Watts and Strogatz (1998) start by assuming a regular network represented by a ring lattice. In this structure,

nodes are arranged in a circular or ring-like fashion and each node is connected to a fixed number of nearest neighbors. Then, with a probability $p \in [0, 1] \subset \mathbb{R}$, the edge between node i and one of its neighbors is eliminated and is reconnected to a node chosen uniformly at random over all other nodes. This procedure is called *rewrite* and it is carried out for all links of all nodes in the network. Figure 1 illustrates this process for increasing values of p , which we call *rewriting probability*. Watts and Strogatz (1998) propose that for intermediate values of p , the network has a lower path length than the regular network but still has a high cluster coefficient, which are exactly the properties of small-world networks.

Figure 3.2 – Illustration of the process from a regular ring lattice to a random network.



Source: Goyal (2012).

In the next section, we will use the small-world network structure to model the social influence mechanism of the perception of neighbors on each agent's perception of the future unemployment rate in the efficiency-wage model augmented by expectations.

3.2 INTERACTIVE EXPECTATION IN THE EFFICIENCY-WAGE MODEL AUGMENTED BY EXPECTATIONS

In order to analyze the co-evolution of interactive expectations and the unemployment rate in the efficiency wage model, we depart from the ABM proposed in the first essay (see section 1.2). In the aforementioned essay, to structure the ABM, we used as a baseline model the efficiency wage model augmented by heterogeneous expectations proposed by Silveira and Lima (2021), which was presented in subsection 1.2.1. Moreover, the general structure of the discrete choice model that will also serve as a baseline for the expectations formation process was also presented in the first essay, in subsection 1.2.2.1. In order to avoid repetition, we will skip those presentations and go straight to the ABM of unemployment interactive expectations as a discrete choice process. Much of the ABM proposed in this essay is based on the ABM proposed in

subsection 1.2.2.2. Moreover, the contribution of the ABM proposed in this essay is to analyze what changes, if any, occur in the dynamics of the co-evolution of unemployment expectations with the unemployment rate actually observed by assuming that the social component governing the agent's perception depends on local neighborhood influence.

Remember from subsection 1.2.2.2 that, inspired by the Michigan Survey, a worker $i \in \{1, 2, 3, \dots, A\}$, where A is the number of workers, can be of type $\tau_i \in \mathcal{T} = \{n, o, p\}$, with the respective subscripts standing for neutral, optimistic, and pessimistic about the unemployment rate in the immediate future. As it has already been assumed, we consider the deterministic utility, that will govern the probability of choosing perceptions, of the i -th worker as being composed of a private component, denoted by $V(\tau_{i,t})$, and a social component, denoted by $S(\tau_{i,t})$, as follows:

$$U^d(\tau_{i,t}) = V(\tau_{i,t}) + \psi S(\tau_{i,t}), \quad (3.3)$$

where $\psi \in \mathbb{R}_{++}$ is a parameter that represents the weight of the social component in the deterministic utility. The private component is set as:

$$V(\tau_{i,t}) = (1 - u_t^*) \frac{w_{i,t}^*}{\varepsilon_{\tau_{i,t}}^*}, \quad (3.4)$$

where u_t^* , $w_{i,t}^*$ and $\varepsilon_{\tau_{i,t}}^*$ are all determined as in the first essay (see equation (1.27), (1.28) and (1.30), respectively). Just to remind, u_t^* is the temporary equilibrium of the unemployment rate in a given period t , which we also repeat here for convenience:

$$u_t^* = \left[1 + \left(\frac{\delta}{1-\delta} \right) \theta_t - \left(\frac{\delta}{1+\delta} \right) \rho_t \right] \gamma, \quad (3.5)$$

where $\delta \in (0, 1-\gamma) \subset \mathbb{R}$ is a parametric constant measuring the dispersion among the three types of unemployment expectation and $\gamma \in (0, 1) \subset \mathbb{R}$ is a parametric constant measuring the effort-enhancing effect of paying to a worker a wage compensation which is higher than the wage compensation associated with her expected labor market conditions.

Moreover, $w_{i,t}^*$ is the temporary equilibrium wage in a given period t , determined as:

$$w_{i,t}^* = \alpha (\varepsilon_t^*)^\alpha (1 - u_t^*)^{\alpha-1} \equiv w_t^*, \quad (3.6)$$

where ε_t^* is the average effort level in the temporary equilibrium in a certain period t (see equation (1.29) for the definition).

The last component of the private utility in equation (3.4) to be recalled here is the individual effort level of worker i holding the unemployment perception of type $\tau_{i,t}$,

which we also repeat for convenience:

$$\varepsilon_{\tau_{i,t}}^* = \begin{cases} \left(\frac{u_t^*}{1-u_t^*} \right), & \text{if } \tau_{i,t} = n, \\ \left[\frac{(1-\delta)u_t^*}{1-(1-\delta)u_t^*} \right]^\gamma, & \text{if } \tau_{i,t} = o, \\ \left[\frac{(1+\delta)u_t^*}{1-(1+\delta)u_t^*} \right]^\gamma, & \text{if } \tau_{i,t} = p. \end{cases} \quad (3.7)$$

Differently from the first essay, here we change the structure of the social component to consider the formation of expectations from the perspective of interactive expectations. We assume that the agent's perception will be affected by the perception of its neighborhood. More precisely, we consider that an agent's social utility increases when the perception adopted by the neighborhood is similar to her own, and the more different the expectation of the neighborhood is, the lower her social utility. To do this, we assume that A agents are arranged in a regular quadratic network, with each agent connected to the 4 nearest neighbors. Considering a_i the social neighborhood of agent i , we define the impact of the perception of neighbor $j \in a_i$ on the social utility of each perception $\tau_{i,t}$ of worker i in period t as:

$$s_j(\tau_{i,t} = o) = \begin{cases} 1, & \text{if } \tau_{j,t} = o, \\ -1/2, & \text{if } \tau_{j,t} = n, \\ -1, & \text{if } \tau_{j,t} = p, \end{cases} \quad (3.8)$$

$$s_j(\tau_{i,t} = n) = \begin{cases} -1/2, & \text{if } \tau_{j,t} = o, \\ 1, & \text{if } \tau_{j,t} = n, \\ -1/2, & \text{if } \tau_{j,t} = p, \end{cases} \quad (3.9)$$

$$s_j(\tau_{i,t} = p) = \begin{cases} -1, & \text{if } \tau_{j,t} = o, \\ -1/2, & \text{if } \tau_{j,t} = n, \\ 1, & \text{if } \tau_{j,t} = p. \end{cases} \quad (3.10)$$

By taking the arithmetic mean of the impact of each neighbor's perception, we determine the social utility of worker i at t as:

$$S(\tau_{i,t}) = \frac{1}{4} \sum_{j \in a_i} s_j(\tau_{i,t}), \quad (3.11)$$

where $S(\tau_{i,t}) \in [-1, 1] \subset \mathbb{R}$. As a result, if all the neighbors of worker i have the same perception as her, the social utility of worker i associated with this perception reaches its maximum value of 1. On the other hand, if all the neighbors of worker i have a different perception from hers, worker i 's social utility associated with this perception reaches its minimum value of -1.

Having defined the deterministic utility, just as in the first essay, we consider that the probability of choosing perceptions of agent i in period t is given by the logistic

cumulative distribution function derived from the discrete choice process following Train (2009). Formally:

$$Prob(\tau_{i,t}) = \frac{1}{1 + \sum_{\tau'_{i,t-1} \in \mathcal{T}, \tau'_{i,t-1} \neq \tau_{i,t-1}} e^{-\beta\{[V(\tau'_{i,t-1}) + \psi(S(\tau'_{i,t-1}))] - [V(\tau_{i,t-1}) + \psi(S(\tau_{i,t-1}))]\}}}. \quad (3.12)$$

For the computational implementation of the proposed ABM, we chose the same number of agents as in the first essay,¹ i.e. $A = 501$. Moreover, we set the initial configuration of perceptions as 1/3 of the agent population for each type of perception. Recall that at the beginning of each period t , a worker i forms either a neutral ($\tau_i = n$), an optimistic ($\tau_i = o$), or a pessimistic ($\tau_i = p$) expectation about the unemployment rate in that period. Thus, 167 agents hold each type of unemployment expectation. After establishing the initial conditions, we compute the temporary equilibrium values of the unemployment rate, the wage, and the individual effort level in period 1 using the equations in (3.5), (3.6), and (3.7). After that, we compute the private utility in this period through the expression in (3.4).

For the social utility, to allow the possibility of different network structures, we do a rewiring procedure as described in Watts and Strogatz (1998). We start from a regular network structure, with each agent connected to her nearest neighbors, and then with a probability $p \in [0, 1] \subset \mathbb{R}$, which is a parameter to be calibrated, a connection from agent i is eliminated and a new link is created between agent i and any other agent chosen at random. Defined the agents' neighborhood, we calculate the social utility in this period through the function in (3.11).

After computing the private and the social utility for the period 1, we can compute the total deterministic utility using the expression in (3.3). Having defined the deterministic utility of the agents, we are able to compute the probabilities of choice specified in (3.12).

As in the first essay, here we also assume that the agent who was unemployed in one of the previous L periods, where L is a parameter to be calibrated, has a pessimism bias (see equation 1.38). Given the value of this pessimism bias, defined by \mathcal{Q} , we take a random number $r \in [0, 1] \subset \mathbb{R}$ from a uniform distribution. If $r_{i,t} < \mathcal{Q}$, we have that the worker i in period t holds pessimistic unemployment expectations.

For workers who were (and remained) employed in all $t - \ell$ periods, the type of unemployment expectation that they will hold for the next period is determined by using the probability of choice specified in (3.12). Given the values of those probabilities, we take a random number $r \in [0, 1] \subset \mathbb{R}$ from a uniform distribution to define the unemployment expectation of those workers in any period $t \geq 2$ applying the rules as specified in Table 3.1.

¹ Remember that the US Michigan Survey has around 500 respondents, but we considered 501 agents in order to have the population of agents equally distributed among the three alternative types of unemployment expectation in the initial period.

Table 3.1 – Algorithm of unemployment expectation formation for a given period $t \geq 2$ by a worker i that did not have the bias to form pessimistic unemployment expectations.

Possible cases	Worker i 's unemployment expectation in a period $t \geq 2$
$r \leq \text{Prob}(\tau_{i,t} = p)$	Pessimistic
$\text{Prob}(\tau_{i,t} = p) < r \leq \text{Prob}(\tau_{i,t} = p) + \text{Prob}(\tau_{i,t} = n)$	Neutral
$r > \text{Prob}(\tau_{i,t} = p) + \text{Prob}(\tau_{i,t} = n)$	Optimistic

Source: Own elaboration.

In order to make the model fit the empirical data better, we calibrate it by finding the optimal combination of parameters. The parameters that need to be calibrated are the intensity of choice, denoted by β in equation (3.12), the weight of the social component in the total deterministic utility, denoted by ψ in equation (3.3), the measure of dispersion among the three types of unemployment expectations, denoted by δ in equation (3.5), the parameters related to the bias towards pessimism which include the decay rate of the unemployment weight q , the number of lags L at which the unemployment experience is taken into account, and the rewriting probability p . It is worth noting that the rewriting probability parameter is the only additional parameter compared to the ABM of the first essay.

Our calibration strategy follows the same approach as described in section 1.2.3. This involves identifying the combination of parameter values that minimizes the sum of squares of the deviations of the simulated data from the respective observed data. To calibrate our model, we use the same empirical data as in the first essay. This includes the monthly time series for the Balance Score (BS), defined as the percentage of respondents who thought that the unemployment rate would increase minus the percentage who thought that it would fall, plus 100, as provided by the Michigan Survey. Additionally, we use the U.S. monthly unemployment rate, which is available from the *Federal Reserve Economic Data*. For our calibration process, we extracted a time series covering the period from January 1978 to December 2019, which amounts to a total of 504 months.

The function *fminsearchbnd* from MATLAB, which was described in section 1.2.3, was also used here to solve the minimization problem and we defined the following plausible ranges within which the *fminsearchbnd* algorithm searched for the parameter values providing the best fit of the model to the empirical data: $0 \leq \beta \leq 10$, $0 \leq \delta \leq 0.97$, $0 \leq \psi \leq 2$, $0 \leq q \leq 1$, $1 \leq L \leq 12$ and $0 \leq p \leq 1$. As a starting point to begin looking for the minimum value of the objective function, we considered the values of β , ψ , δ , q and L found in the first essay and p equal to 0.1. As a reminder, the values of the respective parameters found in the first essay were $\beta = 5.44$, $\psi = 0.17$, $\delta = 0.81$, $q = 0.26$, and $L = 11.57$. Moreover, we choose 0.1 as a starting point for the rewriting probability

because the network literature of social science normally reports values close to this. When we calibrated this combination of initial parameter values, the combination of parameters ultimately selected by the *fminsearchbnd* function is reported in Table 3.2.

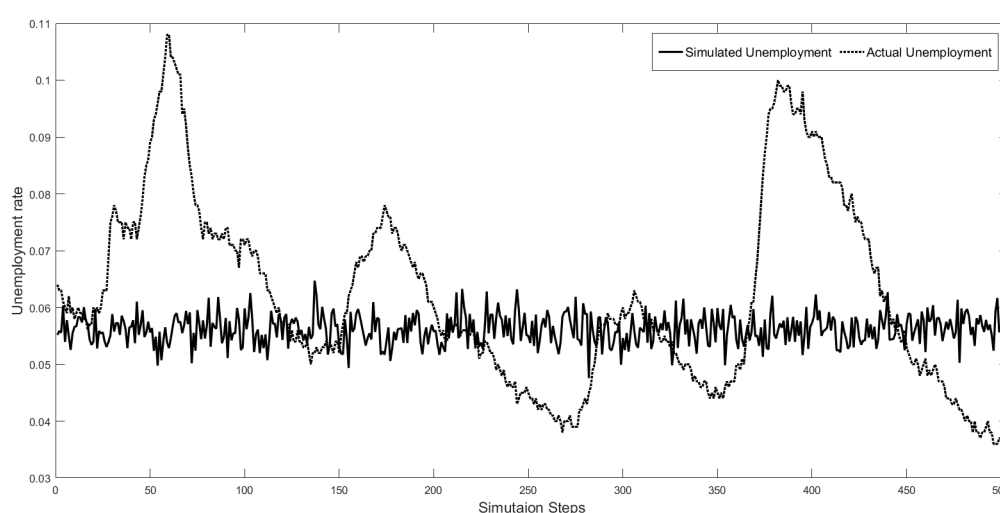
Table 3.2 – Calibrated parameter values.

Parameters	Calibrated values
Intensity of choice (β)	0.45
Dispersion among types of unemployment expectations (δ)	0.8
Weight of the social component in the total deterministic utility (ψ)	0.11
The decay rate of the unemployment weight on the pessimism bias (q)	0.32
The number of lags considered on the pessimistic bias (L)	12
Rewriting probability (p)	0.14

Source: Own elaboration.

The actual and simulated unemployment rate and the BS values can be seen in Figures 3.3 and 3.4 respectively. The simulated data were generated with the parameter values reported in Table 3.2 and a uniform frequency distribution of the three types of unemployment expectations across workers in the initial period, as mentioned earlier in this section. In a similar way to what happened in the first essay, it can be observed that the simulated time series are relatively close to the actual time series on average, although the volatility of the latter is considerably higher.

Figure 3.3 – Actual and simulated unemployment rate.

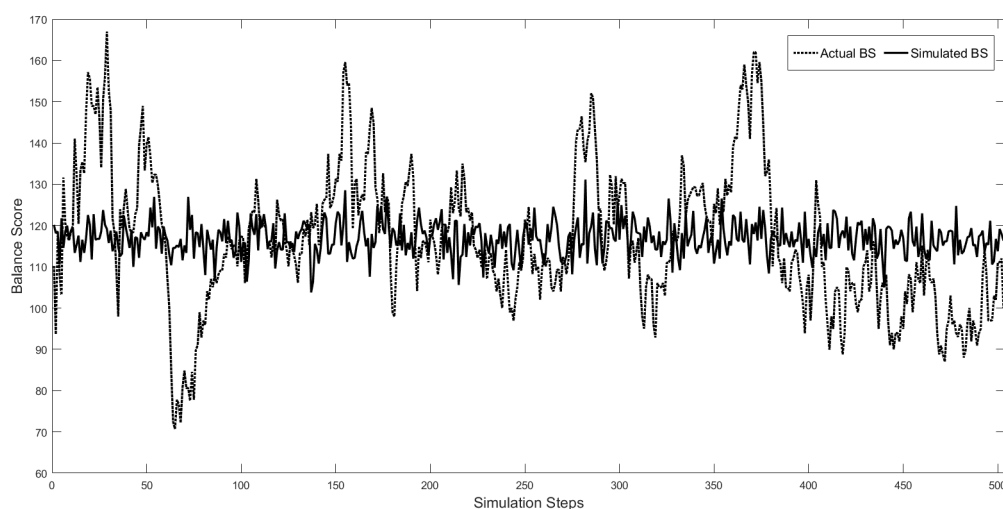


Source: Own elaboration.

3.2.1 Emergent properties

In this subsection, we present the emerging properties of the proposed computational model. These simulation results were generated with a uniform frequency

Figure 3.4 – Actual and simulated BS.



Source: Own elaboration.

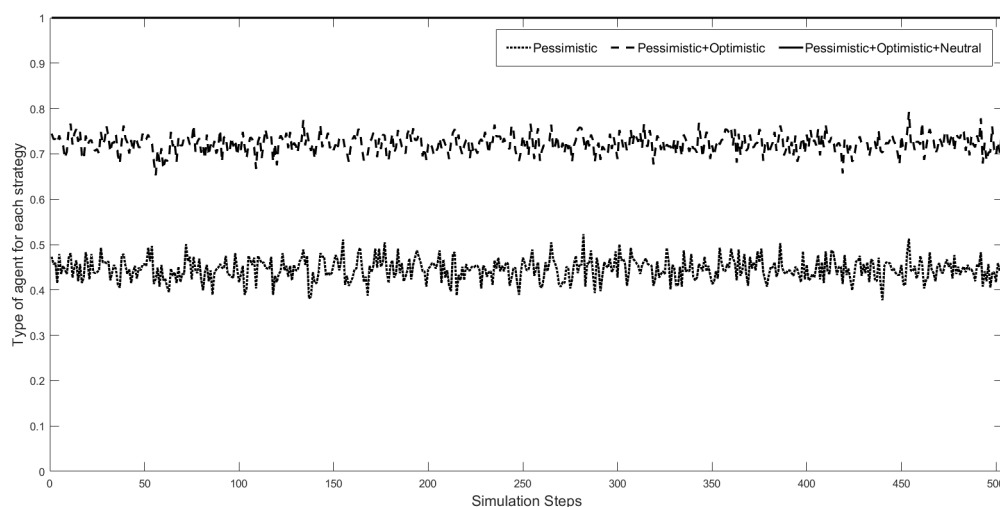
distribution of the three types of unemployment expectations across workers in the initial period and using the set of parameter values found in the calibration of the model and shown in Table 3.2. We ran each simulation with 566 steps (periods) and the time interval composed of the first 62 periods was disregarded as the transient interval, so only the dynamics in the last 504 periods are shown in the following figures and are used in the causality tests. All the results shown were generated with the same random number seed.

Figure 3.5 shows the distribution of perceptions throughout the simulations. We can see the persistence of heterogeneity in the choice of perceptions. Furthermore, no perception proved to be dominant over the others throughout the simulations. Compared to the distribution of expectations generated in the first essay (see Figure 1.3), there is no significant difference in the distribution of perceptions when considering the effect of networks on the formation of perceptions.

Figures 3.6 and 3.7 show the dynamics of the balance of perceptions and the simulated unemployment rate. In the Figures reported, we normalize the BS by 200 to have both the BS and the unemployment rate varying in the same range. Remember that the increase in the BS occurs when the proportion of pessimistic agents increases in relation to the proportion of optimistic agents. Similar to the model reported in the first essay, the formation of interactive expectations also generates moments of optimism (pessimism) accompanied by low (high) unemployment.

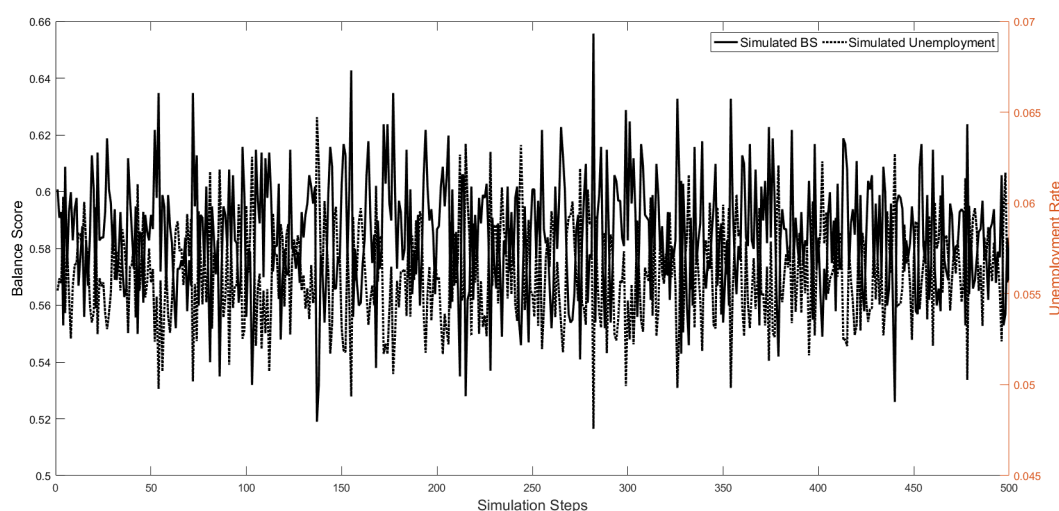
Although visually there seems to be a correlation between the unemployment rate and the BS, the temporal causality test did not point in this direction, as can be seen in Table 3.3. The p -value of the Granger causality test was greater than 0.1 both

Figure 3.5 – Simulated proportion of neutral, optimistic and pessimistic workers.



Source: Own elaboration.

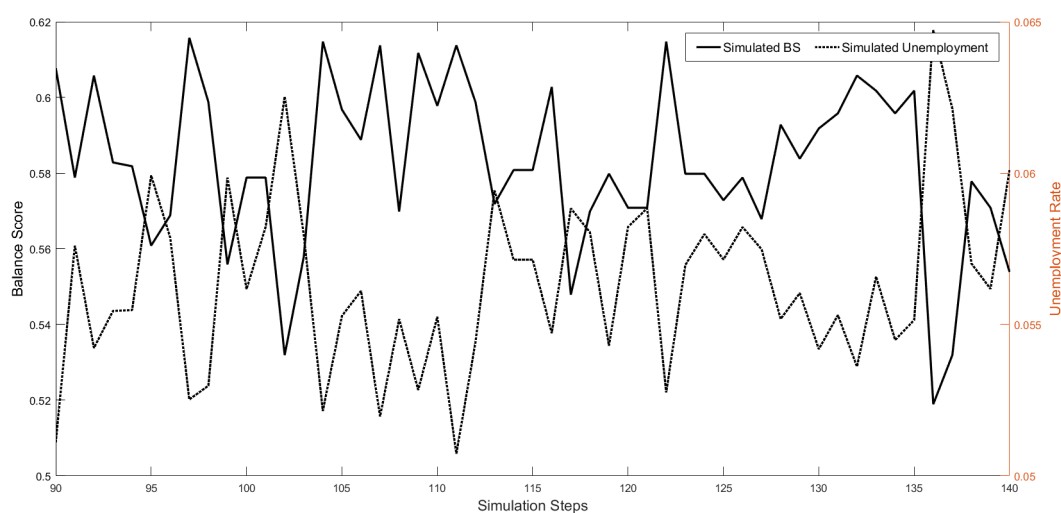
Figure 3.6 – Simulated Normalized BS and simulated unemployment rate over the last 504 simulation steps.



Source: Own elaboration.

in the sense that the unemployment rate causes the BS and in the sense that the BS causes the unemployment rate. Therefore we do not have enough statistical evidence to defend this hypothesis of temporal causality when considering interactive expectations in this model. The Granger causality test was carried out using the number of lags provided by the Akaike information criterion (AIC), i.e. 12 lags. In addition, the Vector Auto Regression (VAR) model used for the test was estimated with the variables in level since the Augmented Dickey-Fuller (ADF) unit root test indicated that both series are

Figure 3.7 – Simulated Normalized BS and simulated unemployment rate between simulation steps 90 and 140.



Source: Own elaboration.

stationary at the 5% significance level.

Table 3.3 – Granger causality test for simulated unemployment and BS.

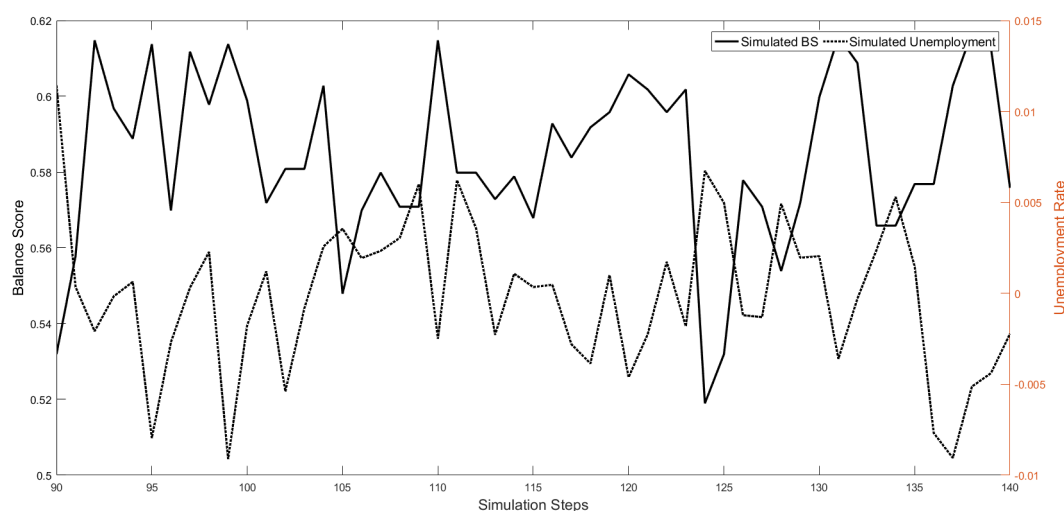
Null Hypothesis	p-value
Unemployment rate does not Granger cause BS	0.5
BS does not Granger cause unemployment rate	0.2

Source: Own elaboration.

Another analysis made in the first essay that we also replicate here is the dynamics of the BS with the yearly change in the unemployment rate. Figure 3.8 shows this dynamic for steps 90 to 140. Again, we normalize the BS by 200 to have both BS and the unemployment rate varying in the same interval. It can be seen that a similar dynamic to that found between the BS and the unemployment rate is present in the visual analysis of the yearly change in the unemployment rate and the BS. In other words, moments of high (low) unemployment are accompanied by moments of high (low) BS.

To further investigate this dynamic, we perform the Granger causality test. The Information criteria AIC e FPE reported 24 lags, while HQ and SC reported 12 lags. when estimating the model with 12 lags, the p-value of the Granger causality test for the yearly change in unemployment Granger causes BS is equal to 0.19, so it is not significant. However, we found significant Granger causality from the BS to the unemployment rate, which can also be found in the empirical data. Table 3.4 shows the result for both the simulated and the empirical data.

Figure 3.8 – Simulated Normalized BS and simulated yearly change in the unemployment rate between simulation steps 90 and 140.



Source: Own elaboration.

Table 3.4 – Granger causality test for BS and yearly change in unemployment.

Null Hypothesis	p-value
BS does not Granger causes a yearly change in unemployment rate simulated data	0.00
BS does not Granger causes a yearly change in unemployment rate observed data	0.00

Source: Own elaboration.

3.3 INTERACTIVE EXPECTATION IN GNR MODEL AUGMENTED BY EXPECTATIONS

In this section, we present an extension of the GNR model to incorporate expectations formed through social interaction. The general structure of the ABM proposed by Guerini, Napoletano, and Roventini (2018), which we also use in this section as the baseline model, was presented in section 2.2. To avoid repetition, we will skip this presentation and go straight to the structure of expectations formation proposed to analyze the dynamics of interactive expectations and macroeconomic variables generated by the model.

In this section, we continue to use the European survey as a motivation, but we have changed the perspective to the formation of expectations about the unemployment rate. This was done because the European empirical research that analyzes expectations as an important driver of economic activity uses expectations about unemployment and the unemployment rate actually observed as a proxy for the level of economic activity. This is due to the fact that the unemployment rate series is subject to only minor revisions (see Girardi (2014, p. 316), Leduc and Sill (2013, p.1353) and

Lehmann and Weyh (2016)).²

To collect unemployment expectations from consumers in the European Union (EU), the EU Programme of Business and Consumer Surveys (BCS) asks monthly: “How do you expect the number of people unemployed in this country to change over the next 12 months?”. Consumers have the following possible answers: “increase sharply”, “increase slightly”, “remain the same”, “fall slightly”, “fall sharply”, “don’t know”.

Inspired by the BCS Survey, we assume that households in our ABM can be very pessimistic, slightly pessimistic, neutral, slightly optimistic, or very optimistic depending on their belief about whether the unemployment rate will increase sharply, increase slightly, remain the same, fall slightly, or fall sharply, respectively, over the next 12 months.

Considering these five types of perceptions, we assume the same structure of expectation formation used in the second essay (see section 2.3). Following what was proposed in that essay, we assume that agents can choose between the five aforementioned perceptions based on their own experiences in the labor market and the perceptions of other households. Formally, we consider that in each period t , a household (worker) $h \in \{1, 2, \dots, H\} \subset \mathbb{R}$ can assume a type $\tau_h \in \mathcal{T} = \{vo, so, n, sp, vp\}$, with the subscripts standing for very optimist, slightly optimist, neutral, slightly pessimist and very pessimist, respectively, about the unemployment rate over the next 12 periods.

In section 2.3, we presented the utility or payoff function of the household h . This function was shown to be additively decomposed into two components. The first component is deterministic and denoted by $U^d(\tau_{h,t})$, which is associated with the observed motivations of the household. The second component is random and denoted by $\varsigma(\tau_{h,t})$, which refers to the unobservable motivations of the household. Formally:

$$U(\tau_{h,t}) = U^d(\tau_{h,t}) + \varsigma(\tau_{h,t}). \quad (3.13)$$

Moreover, we consider that each household’s deterministic utility comprises a private and social component. Considering those components, the deterministic utility can be written as follows:

$$U^d(\tau_{h,t}) = V(\tau_{h,t}) + \psi S(\tau_{h,t}), \quad (3.14)$$

where $V(\tau_{h,t})$ is the private utility, $S(\tau_{h,t})$ is the social utility and $\psi \in \mathbb{R}_{++}$ stands for the weight of social utility over deterministic utility. We call this parameter the *social influence weight*.

Also remembering from the second essay, we assume that the private utility is associated with the employment history of the h -th household over the past 12 periods, referred to as the employment indicator, and defined as follows:

$$\mathcal{E}_{h,t-1} = \iota_{t-1} \xi_{h,t-1} + \iota_{t-2} \xi_{h,t-2} + \dots + \iota_{t-12} \xi_{h,t-12}, \quad (3.15)$$

² These articles were reviewed in subsection 1.1.2.

where $\iota_{t-\ell} \in \mathbb{R}_+$ is the weight of past employment situation, with $\sum_{\ell=1}^{12} \iota_{t-\ell} = 1$ and $\xi_{h,t-\ell} \in [0, 1] \subset \mathbb{R}$ is the number of hours worked by household h at $t-\ell$. In the study conducted by Guerini, Napoletano, and Roventini (2018), it was assumed that each worker offers inelastically one hour of work. This implies that for every household h , the employment indicator $\mathcal{E}_{h,t-1} \in (0, 1) \subset \mathbb{R}$ is at its highest when the household has its labor supply employed for all $\ell = 1, 2, \dots, 12$. Conversely, the employment indicator for households is at its lowest when the household is unemployed for all $\ell = 1, 2, \dots, 12$.

Furthermore, as in the first and second essay of this dissertation, it is assumed here that the weight of past employment, $\iota_{t-\ell}$, on the macroeconomic indicator decays geometrically with time lag ℓ by a factor $q \in (0, 1) \subset \mathbb{R}$.

We also assume that an agent's private utility for a particular choice or perception depends on whether their employment indicator has improved, worsened or remained unchanged. If an agent faces a worsening in their employment indicator, their private utility of pessimism is higher than that of neutrality, to a great extent, of optimism. Conversely, if they see an improvement in their employment indicator, their private utility of optimism is higher than that of neutrality and, to a great extent, of pessimism. If there is no change in the indicator, the private utility of neutrality is the highest. Additionally, we assume that if an agent's employment indicator worsens, the private utility of a very pessimistic perception must be greater than that of a slightly pessimistic perception. Similarly, if an agent's employment indicator improves, the private utility of a very optimistic perception should be greater than that of a slightly optimistic perception.

Considering those assumptions about agents' preferences raised above, we can define the private utility as follows:

$$V(\tau_{h,t}) = \begin{cases} \mathcal{E}_{h,t-1} - \mathcal{E}_{h,t-2}, & \text{if } \tau_{h,t} = vO, \\ (\mathcal{E}_{h,t-1} - \mathcal{E}_{h,t-2})^2, & \text{if } \tau_{h,t} = sO, \\ -(\mathcal{E}_{h,t-1} - \mathcal{E}_{h,t-2})^2, & \text{if } \tau_{h,t} = n, \\ -(\mathcal{E}_{h,t-1} - \mathcal{E}_{h,t-2})^2, & \text{if } \tau_{h,t} = sP, \\ -(\mathcal{E}_{h,t-1} - \mathcal{E}_{h,t-2}), & \text{if } \tau_{h,t} = vP. \end{cases} \quad (3.16)$$

Furthermore, we assume that the social component of the deterministic utility depends on the perception of the agent's neighborhood. For this purpose, We consider that the H households are in a small-world network G represented as a graph (\mathcal{H}, g) that consists of a set of finite nodes $\mathcal{H} = \{1, \dots, H\} \subset \mathbb{N}$ and a real-valued $H \times H$ matrix g , where $g_{i,j}$ represents the relation between nodes $i \in \mathcal{H}$ and $j \in \mathcal{H}$.

If household i and j are connected, it means that the choice of perception of household i is socially influenced by the perception of household j , just as the choice of perception of household j is also socially influenced by the choice of perception of household i . In this case, $g_{i,j} = 1$. If household i and j are not connected, $g_{i,j} = 0$. In

this essay, we consider that each household $h \in \mathcal{H}$ is connected to its four closest neighbors. We define the impact of the perception of neighbor j on the social utility of each perception τ_h as follows:

$$s_j(\tau_{h,t} = vo) = \begin{cases} 1, & \text{if } \tau_{j,t} = vo, \\ -1/4, & \text{if } \tau_{j,t} = so, \\ -1/2, & \text{if } \tau_{j,t} = n, \\ -3/4, & \text{if } \tau_{j,t} = sp, \\ -1, & \text{if } \tau_{j,t} = vp. \end{cases} \quad (3.17)$$

$$s_j(\tau_{h,t} = so) = \begin{cases} -1/4, & \text{if } \tau_{j,t} = vo, \\ 1, & \text{if } \tau_{j,t} = so, \\ -1/4, & \text{if } \tau_{j,t} = n, \\ -1/2, & \text{if } \tau_{j,t} = sp, \\ -3/4, & \text{if } \tau_{j,t} = vp. \end{cases} \quad (3.18)$$

$$s_j(\tau_{h,t} = n) = \begin{cases} -1/2, & \text{if } \tau_{j,t} = vo, \\ -1/4, & \text{if } \tau_{j,t} = so, \\ 1, & \text{if } \tau_{j,t} = n, \\ -1/4, & \text{if } \tau_{j,t} = sp, \\ -1/2, & \text{if } \tau_{j,t} = vp. \end{cases} \quad (3.19)$$

$$s_j(\tau_{h,t} = sp) = \begin{cases} -3/4, & \text{if } \tau_{j,t} = vo, \\ -1/2, & \text{if } \tau_{j,t} = so, \\ -1/4, & \text{if } \tau_{j,t} = n, \\ 1, & \text{if } \tau_{j,t} = sp, \\ -1/4, & \text{if } \tau_{j,t} = vp. \end{cases} \quad (3.20)$$

$$s_j(\tau_{h,t} = vp) = \begin{cases} -1, & \text{if } \tau_{j,t} = vo, \\ -3/4, & \text{if } \tau_{j,t} = so, \\ -1/2, & \text{if } \tau_{j,t} = n, \\ -1/4, & \text{if } \tau_{j,t} = sp, \\ 1, & \text{if } \tau_{j,t} = vp. \end{cases} \quad (3.21)$$

Considering n_h as the set of neighbors of households h , the value of the social utility of a given perception of that household in period t is:

$$S(\tau_{h,t}) = \frac{1}{4} \sum_{j \in n_h} s_j(\tau_{h,t}) \quad (3.22)$$

As it has already been explained in subsection 1.2.2.1, due to the random element in the utility function, it is only possible to determine the likelihood of each agent's choice, which is consistent with the discrete choice literature. To calculate the probability density function of the vector of random variables $\vec{\zeta}_h$, which comprises random variables $\zeta(\tau_h)$, we use the logit specification. Using the utility function in equation (3.13), we establish the probability that a household h in period t perceives the future unemployment rate as $\tau_{h,t} \in \mathcal{T}$ by applying the following logistic cumulative distribution function:

$$Prob(\tau_{h,t}) = \frac{1}{1 + \sum_{\tau'_{h,t-1} \in \mathcal{T}, \tau'_{h,t-1} \neq \tau_{h,t-1}} e^{-v\{[V(\tau'_{h,t-1}) + \psi(S(\tau'_{h,t-1}))] - [V(\tau_{h,t-1}) + \psi(S(\tau_{h,t-1}))]\}}}, \quad (3.23)$$

where $v \in \mathbb{R}_+$ is the *intensity of choice*. The value of v determines the relative weight of the deterministic component over the random component of the utility function to determine the propensity of a household h to hold the perception τ_h about the future unemployment rate. The higher the value of v , the higher will be the weight of the deterministic component.

To define the perception of a household h in any period $t \geq 2$, a random number $r_{h,t} \in [0, 1] \subset \mathbb{R}$ is generated from a uniform distribution. Since $\sum_{\tau \in \mathcal{T}} Prob_{\tau,t} = 1$ for all period $t \in \mathbb{N}$, we follow the rules as specified in Table 3.5.

Table 3.5 – Algorithm to choose a perception about the future economic activity in every period $t \geq 2$

Possible cases	Perception of household h
$r \leq Prob(\tau_{h,t} = vp)$	Very pessimist
$Prob(\tau_{h,t} = vp) \leq r \leq Prob(\tau_{h,t} = vp) + Prob(\tau_{h,t} = sp)$	Slightly pessimist
$Prob(\tau_{h,t} = vp) + Prob(\tau_{h,t} = sp) \leq r \leq Prob(\tau_{h,t} = vp) + Prob(\tau_{h,t} = sp) + Prob(\tau_{h,t} = n)$	Neutral
$Prob(\tau_{h,t} = vp) + Prob(\tau_{h,t} = sp) + Prob(\tau_{h,t} = n) \leq r \leq Prob(\tau_{h,t} = vp) + Prob(\tau_{h,t} = sp) + Prob(\tau_{h,t} = n) + Prob(\tau_{h,t} = so)$	Slightly optimist
$Prob(\tau_{h,t} = vp) + Prob(\tau_{h,t} = sp) + Prob(\tau_{h,t} = n) + Prob(\tau_{h,t} = so) \leq r$	Very optimist

Source: Own elaboration.

In this extension of the GNR model, we consider how households perceive future unemployment rates, which could affect their current consumption preferences. Our hypothesis is that optimistic households tend to have a positive bias towards current consumption, while pessimistic households have a negative bias. To account for this effect, we include the influence of household type/perception in period t on its desired consumption, as defined in equation (2.5). This means that the desired consumption now takes into account the household's perception of the future unemployment rate, as follows:

$$\hat{c}_{h,t} = \tilde{c}_h + \alpha \frac{\Delta A_{h,t}}{P_{t-1}} + \beta(\bar{c}_{h,t-1} - c_{h,t-1}) + \zeta E(\tau_{h,t}), \quad (3.24)$$

where the variable $E_{h,t} \in (-0.2, 0.2) \subset \mathbb{R}$, which here on we will call *consumption perception bias*, is a bias towards more or less consumption of the household h whose perception (type) is $\tau_{h,t} \in \mathcal{T}$ in period t .

All other variables affecting consumption being equal, household consumption should be lower for pessimistic households than for neutral and, to a greater extent, optimistic households. For this purpose, for each household h , we choose the consumption perception bias as a random scalar drawn from the uniform distribution in the interval $(0, 0.1) \subset \mathbb{R}$, $(0.1, 0.2) \subset \mathbb{R}$, $(-0.1, 0) \subset \mathbb{R}$ and $(-0.2, -0.1) \subset \mathbb{R}$ if $\tau_{h,t} = so$, $\tau_{h,t} = vo$, $\tau_{h,t} = sp$ and $\tau_{h,t} = vp$, respectively. For those households that are neutral, the consumption perception bias is taken as null. Table 3.6 summarizes the possible values for the consumption perception bias.

Table 3.6 – Range in which the consumption perception bias will be contained for possible perceptions of the future economic situation

Ranges for the consumption perception bias
$E(\tau_{h,t} = vp) \in (-0.2, -0.1) \subset \mathbb{R}$
$E(\tau_{h,t} = sp) \in (-0.1, 0) \subset \mathbb{R}$
$E(\tau_{h,t} = n) = 0$
$E(\tau_{h,t} = so) \in (0, 0.1) \subset \mathbb{R}$
$E(\tau_{h,t} = vo) \in (0.1, 0.2) \subset \mathbb{R}$

In the next section, we show the calibration strategy used to find the values of the parameters that bring the simulated series closer to the empirical data and the emergent properties of this model using that set of parameter values.

3.3.1 Computational implementation and calibration strategy

To implement the proposed model computationally, we decided to increase the number of agents proposed by the benchmark model, which is 200 households, since the influence of networks can be better modeled when a larger network is considered. We decided to implement the model with $H=500$ agents (households). In the initial period of the simulation, we consider that each type of expectation (very pessimistic, slightly pessimistic, neutral, slightly optimistic, and very optimistic) is adopted by 1/5 of the agents, i.e., each strategy is adopted by 100 agents. The initial condition for the other variables in the model is their respective full employment equilibrium levels. In addition, we calibrated only the model in the decentralized search and matching scenario since this, as expected, showed a better fit to the empirical data compared to the centralized scenario.

Our calibration strategy consists of finding the parameters related to the formation of perceptions, that is, the intensity of choice ν , the social influence weight ψ , the

probability of rewriting p and the decay rate of the employment indicator q that minimize the distance between the observed and the simulated time series. More precisely, we look for the combination of values that provides the best fit of the model to the empirical data by finding the combination of parameter values that minimizes the sum of squares of the deviations of the simulated data from the respective observed data. For the parameters of the model, but those related to the expectation formation process, we use those proposed by Guerini, Napoletano, and Roventini (2018), which we report in Table 3.7.

Table 3.7 – Parameter Values.

Parameter	Value	Meaning
F	20	Number of firms
α	0.4	Sensitivity to economic effects
β	0.4	Sensitivity to social effects
γ	0.4	Sensitivity to price/wage indexation
ϑ	0.5	Percentage of retained profit
φ	5	Sensitivity of labor demand to real wage
ρ^{LM}	2	Matching efficiency in the labor market
ρ^{GM}	2	Matching efficiency in the goods market

Source: Own elaboration with information provided by Guerini, Napoletano, and Roventini (2018).

For the calibration, we use the following three empirical data from Italy: (i) balance of perceptions about the future unemployment rate from BCS, which is available on a monthly basis;³ (ii) the monthly rate of change of the Harmonized Index of Consumer Prices (HICP); and (iii) the seasonally adjusted monthly unemployment rate.⁴ For all those three empirical time series, we use data from February 1996 to January 2020, so we have 288 observations in total.⁵

The balance of the unemployment expectations is the aggregation of the perceptions about the future unemployment rate. It is calculated as the percentage of respondents who believe the unemployment rate will increase minus the percentage who believe the unemployment rate will decrease, with different weights depending on

³ Available on https://economy-finance.ec.europa.eu/economic-forecast-and-surveys/business-and-consumer-surveys/download-business-and-consumer-survey-data/time-series_en

⁴ HICP and unemployment rate data are available on https://ec.europa.eu/eurostat/databrowser/explore/all/all_themes?lang=en&display=list&sort=category. Balance data is available on https://economy-finance.ec.europa.eu/economic-forecast-and-surveys/business-and-consumer-surveys/download-business-and-consumer-survey-data/time-series_en

⁵ We did not use more recent data, as there are missing observations on the balance of perceptions in 2020

the level of increase/decrease believed, that is:

$$B_t = 100[(\rho_t^- + 0.5\rho_t) - (\theta_t^+ + 0.5\theta_t)], \quad (3.25)$$

where $B_t \in [-100, 100] \in \mathbb{R}$ is the balance in period t , ρ_t^- is the proportion of respondents who believe the unemployment rate will sharply increase (very pessimistic), ρ_t is the proportion of respondents who believe the unemployment rate will slightly increase (slightly pessimistic), θ_t is the proportion of respondents who believe the unemployment rate will slightly decrease (slightly optimistic) and θ_t^+ is the proportion of respondents who believe that unemployment rate will sharply decrease (very optimistic).

We proceed with the calibration by selecting the combination of parameter values that minimizes the following objective function:

$$\sum_{t=1}^T [(B_t^{observed} - B_t^{simulated})^2 + (u_t^{observed} - u_t^{simulated})^2 + (\pi_t^{observed} - \pi_t^{simulated})^2], \quad (3.26)$$

where $T = 288$ is the total number of periods, $B^{observed}$ is the observed balance, $B^{simulated}$ is the simulated balance, $u^{observed}$ is the observed unemployment rate, $u^{simulated}$ is the simulated unemployment rate, $\pi_t^{observed}$ is the observed inflation and $\pi_t^{simulated}$ is the simulated inflation. Except for the q parameter, which was also calibrated in this test, the other parameters used in the simulations are described in Table 2.4. Besides, the model was run for 500 periods, and the first 212 simulations were discarded, so we had a total of 288 simulated observations of inflation, unemployment and balance.

We use the function *fminsearchbnb* from MATLAB to solve that minimization problem. The procedure used in this function was explained in the first essay of this dissertation. We defined the following plausible ranges within which the *fminsearchbnd* algorithm could search for the parameter values: $0 \leq v \leq 10$, $0 \leq \psi \leq 2$, $0 \leq p \leq 1$ and $0 \leq q \leq 1$.

In the *fminsearchbnb* function, you need to provide an “initial guess” so that the algorithm starts looking for the combination of parameters that minimizes the objective function. To avoid selecting a local minimum, we tested 30 initial parameter combinations, and the initial combination that generated the lowest value for the function after minimization was $\psi = 1.04$, $\beta = 6.85$, $p = 0.25$, $q = 0.11$. When we calibrated this combination of initial parameter values, the combination of parameters ultimately selected by the *fminsearchbnd* function is reported in Table 3.8.

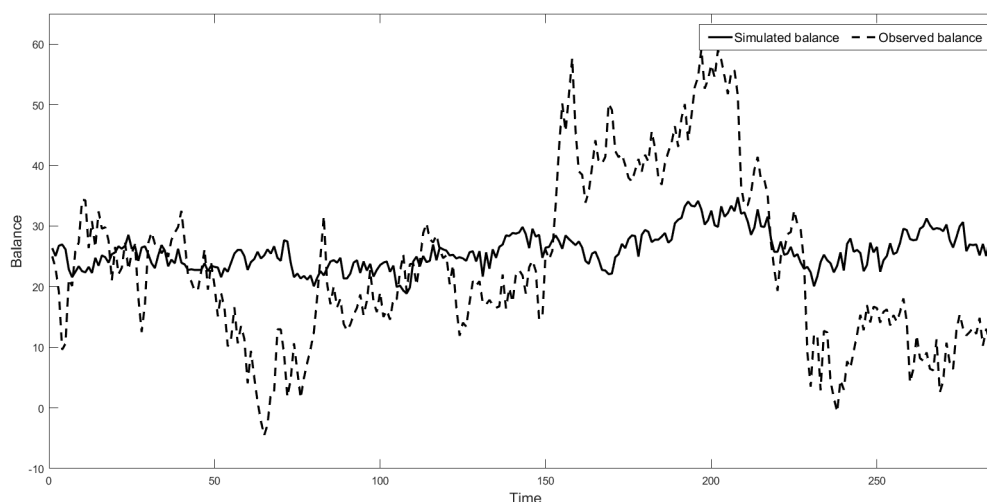
Figures 3.9, 3.10, and 3.11 show the observed and simulated balance of unemployment expectations, unemployment rate, and inflation rate, respectively. Although the model manages to follow the balance of expectations and the inflation rate on average, the unemployment rate seems somewhat distant from the observed one.

Table 3.8 – Calibrated parameter Values.

Parameters	Calibrated values
Intensity of choice (ν)	8.04
Social influence weight (ψ)	0.85
Probability of rewriting (ρ)	0.28
The decay rate of the employment indicator (q)	0.06

Source: Own elaboration.

Figure 3.9 – Observed and simulated balance in the GNR model augmented by interactive expectations.



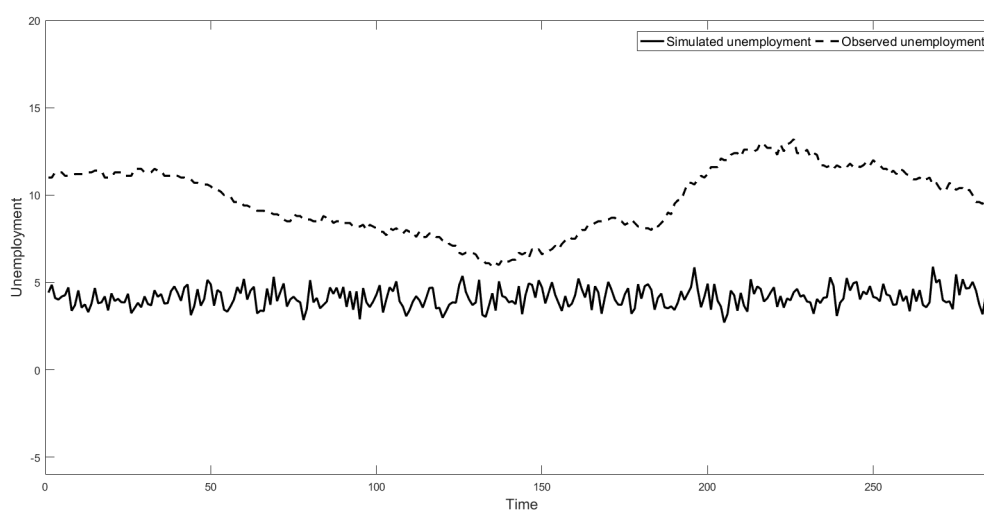
Source: Own elaboration.

3.3.2 Emergent properties

In this subsection, we provide some emergent properties that we could extract from the simulations. All the properties found below were generated using the parameters reported in Tables 3.7 and 3.8 and with the same random number seed. In addition, we ran the simulation with 500 periods and the time interval composed of the first 212 steps was disregarded. Figure 3.12 shows the co-evolution of the simulated unemployment rate and the simulated unemployment expectations for all the simulation steps and Figure 3.13 shows the same property for the simulations steps 100 to 150.

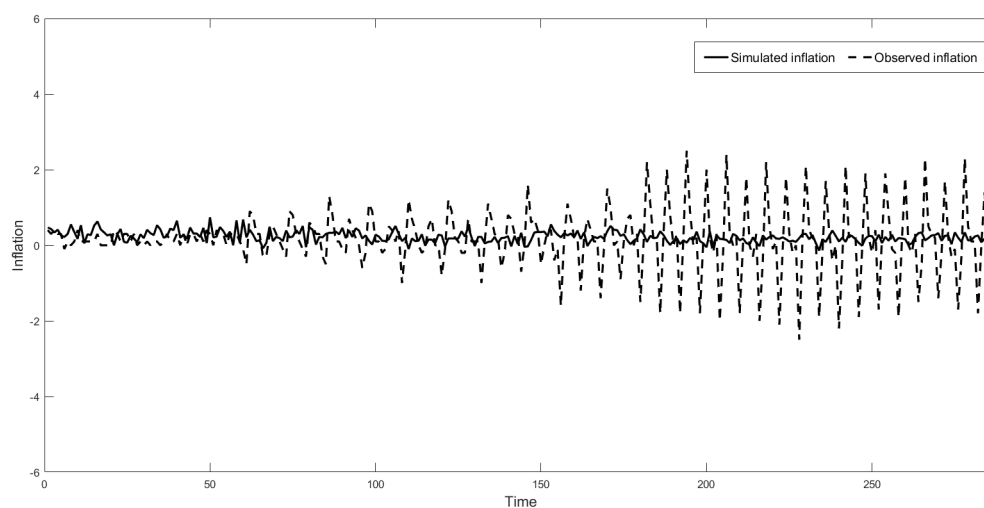
On visual inspection, we can realize that periods of pessimism, reflected by a higher balance, are generally followed by periods of rising unemployment. Note that this is in line with the Keynesian analysis of self-fulfilling movements in expectations. It follows from equation (3.24) that the perception of good times ahead for the labor market increases households' desired consumption, which tends to increase firms'

Figure 3.10 – Observed and simulated unemployment by the GNR model augmented by expectations.



Source: Own elaboration.

Figure 3.11 – Observed and simulated inflation in the GNR model augmented by interactive expectations.



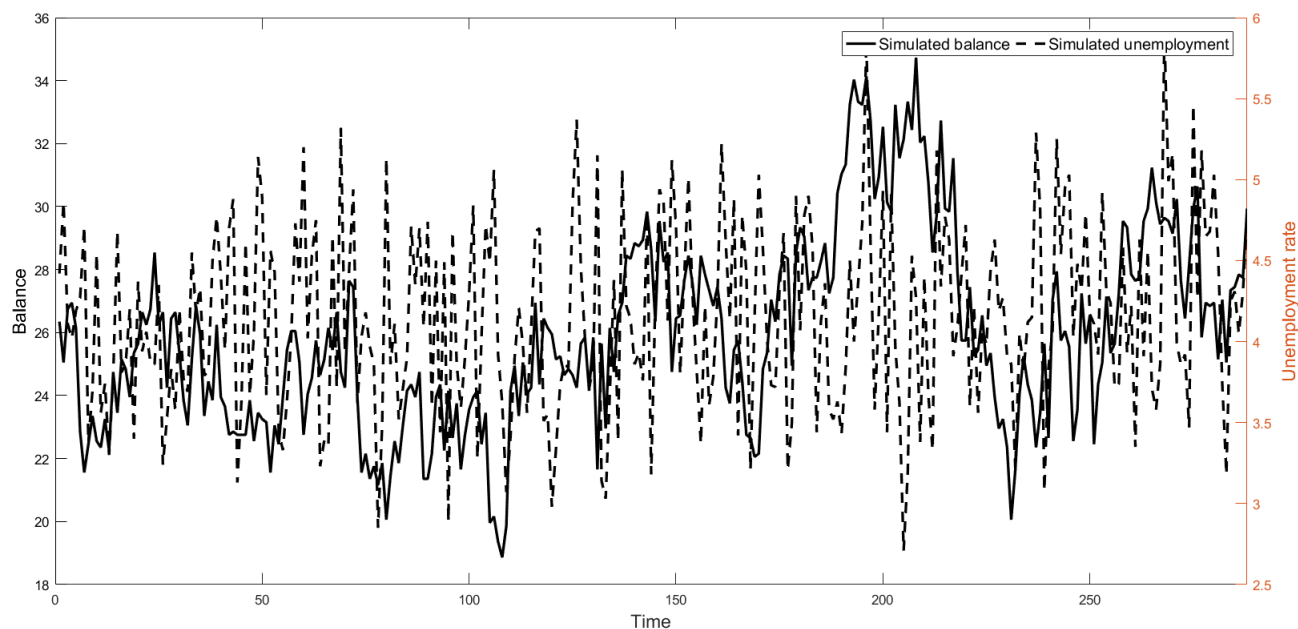
Source: Own elaboration.

desired output in the event of excess demand (see equation 2.3).

Figure 3.14 shows the observed time series of balance and unemployment rate. Due to the long cycles of rise and decline in the observed unemployment rate, the moments of high (low) balance values followed by moments of high (low) unemployment rate values are not as clear as in the simulated case.

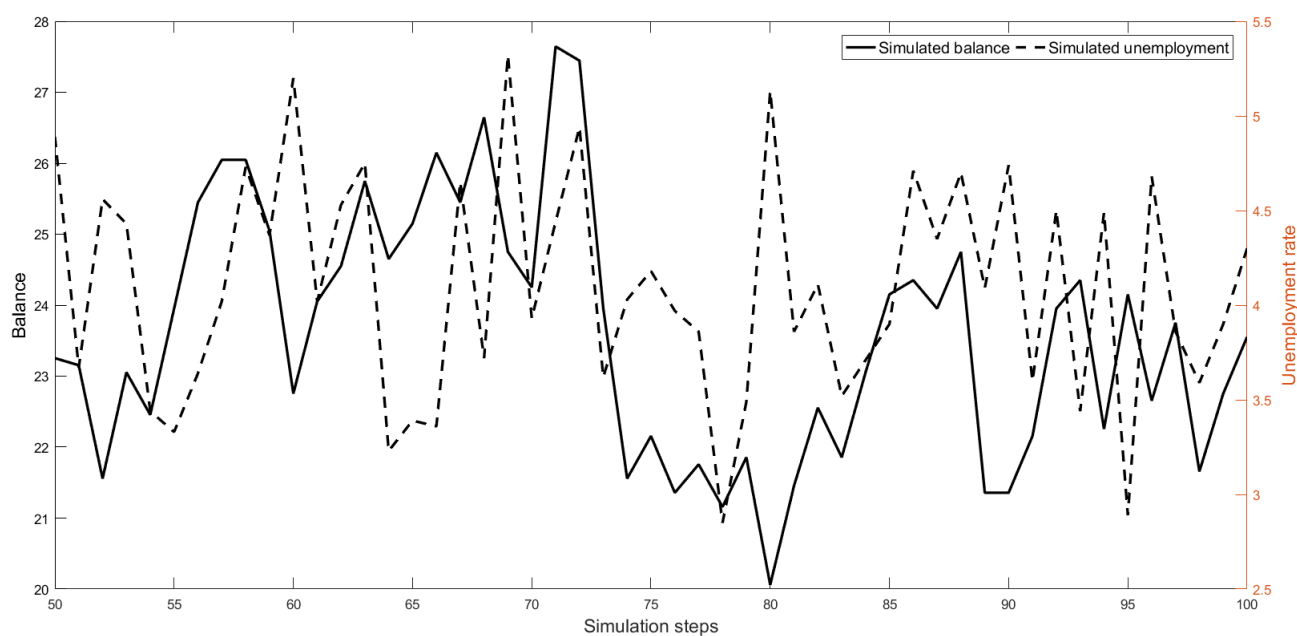
On the other hand, this pattern can be much clearer if we look at the time series

Figure 3.12 – Simulated co-evolution of the balance and the unemployment rate in the GNR model augmented by interactive expectations.



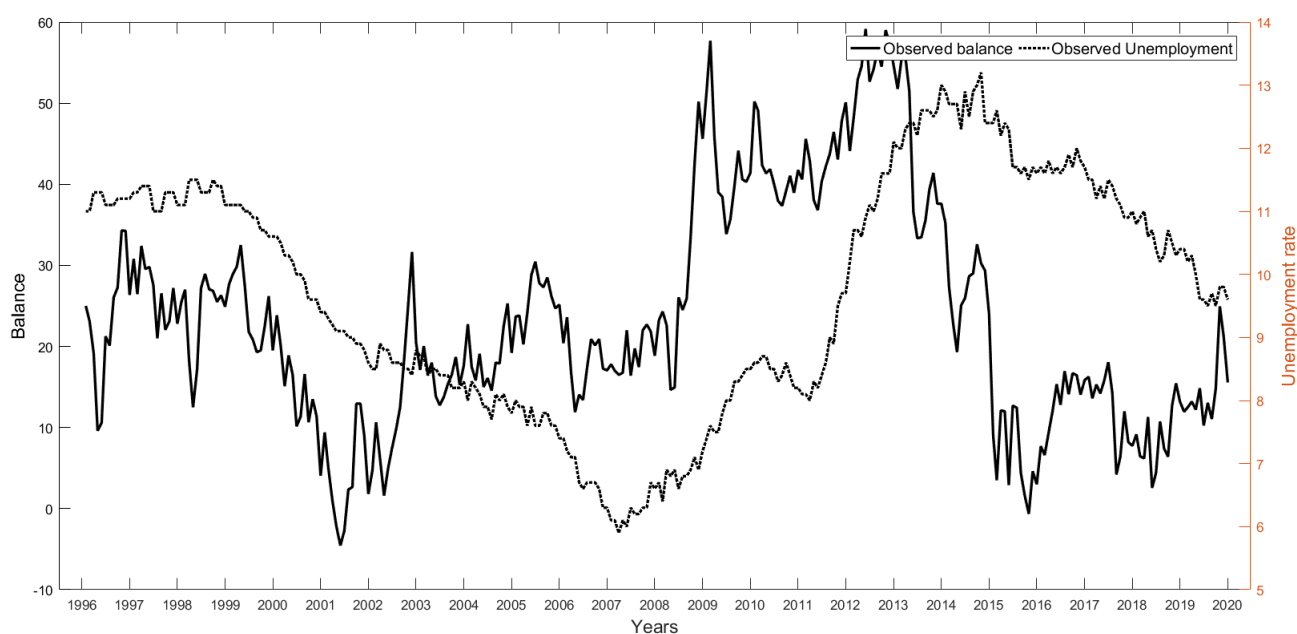
Source: Own elaboration.

Figure 3.13 – Simulated balance and unemployment rate in the GNR model augmented by interactive expectations in simulation steps 50 and 100.



Source: Own elaboration.

Figure 3.14 – Observed balance and unemployment rate.



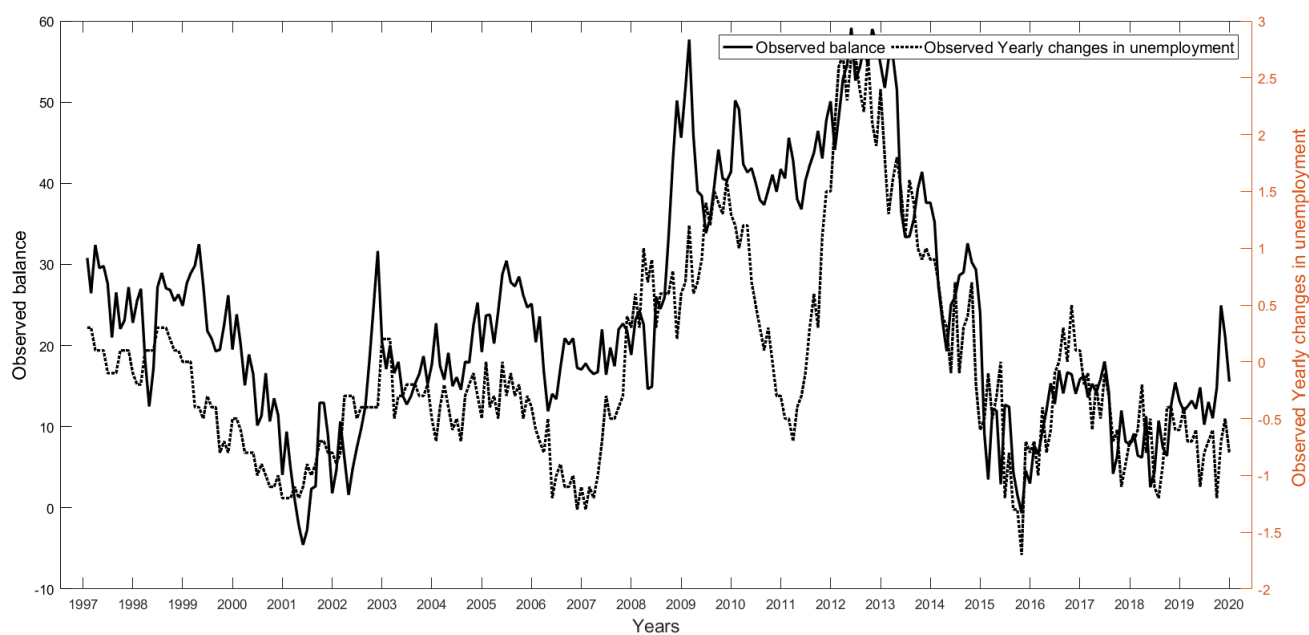
Source: Own elaboration.

of yearly changes in the unemployment rate. A yearly change in the unemployment rate is defined as the difference between the unemployment rate for a given month and the value of the unemployment rate for the same month of the previous year. In Figure 3.15, we can see the observed yearly changes in the unemployment rate together with the observed balance. It is now clear that there are moments of optimism (pessimism) together with lower (higher) unemployment rates. As the annual change in the unemployment rate provides information on the rate 12 months ago, there is also a backward-looking component in this formation. Therefore, this is an indication that there is also a backward-looking component in the formation of expectations.

In addition, Figures 3.16 and 3.17 show the same time series in steps 100 to 150 of the simulation and for all the periods of the simulation, respectively. We can see that a similar pattern to the empirical data occurs with the simulated data. That is, we observe moments of pessimism (optimism) accompanied by an increase (reduction) in unemployment.

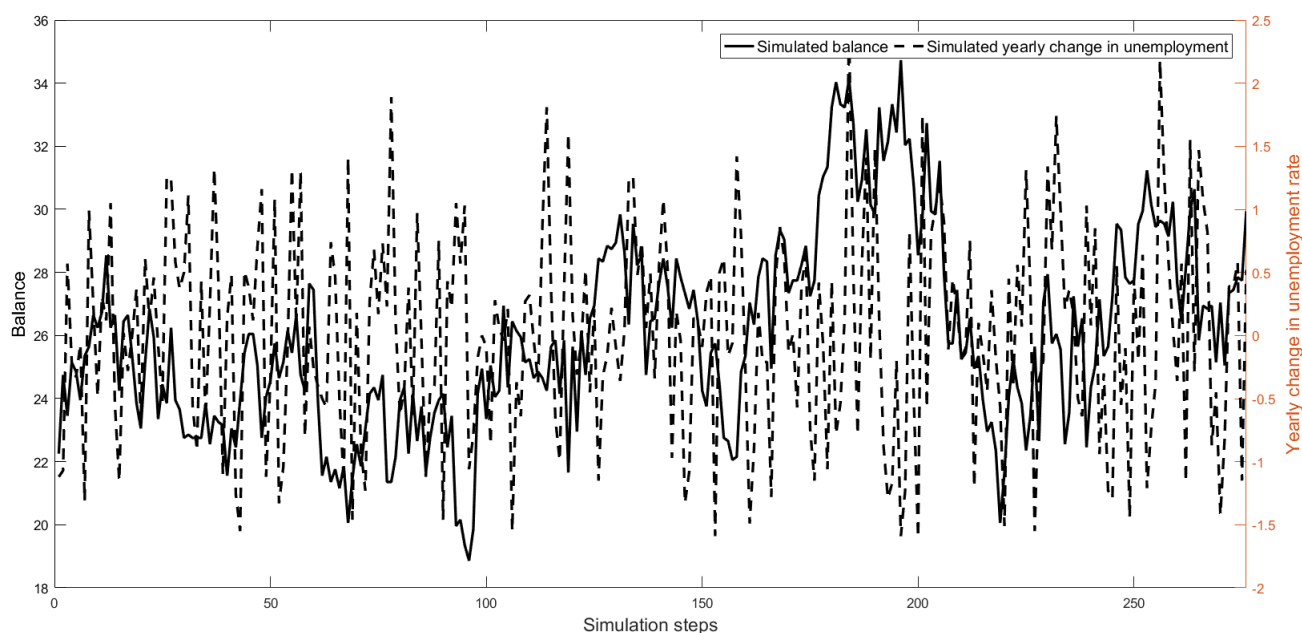
To further investigate the feedback between expectations and the observed unemployment rate, we make an analysis of temporal causality, by means of the Granger causality test. We perform the test on both empirical and simulated data. A similar causality analysis for the empirical data was carried out by Girardi (2014). The author only reports the causality between the yearly changes in the unemployment rate and the balance. Here, we replicate the causality test for the yearly changes in the unem-

Figure 3.15 – Observed balance and yearly change in unemployment rate.



Source: Own elaboration.

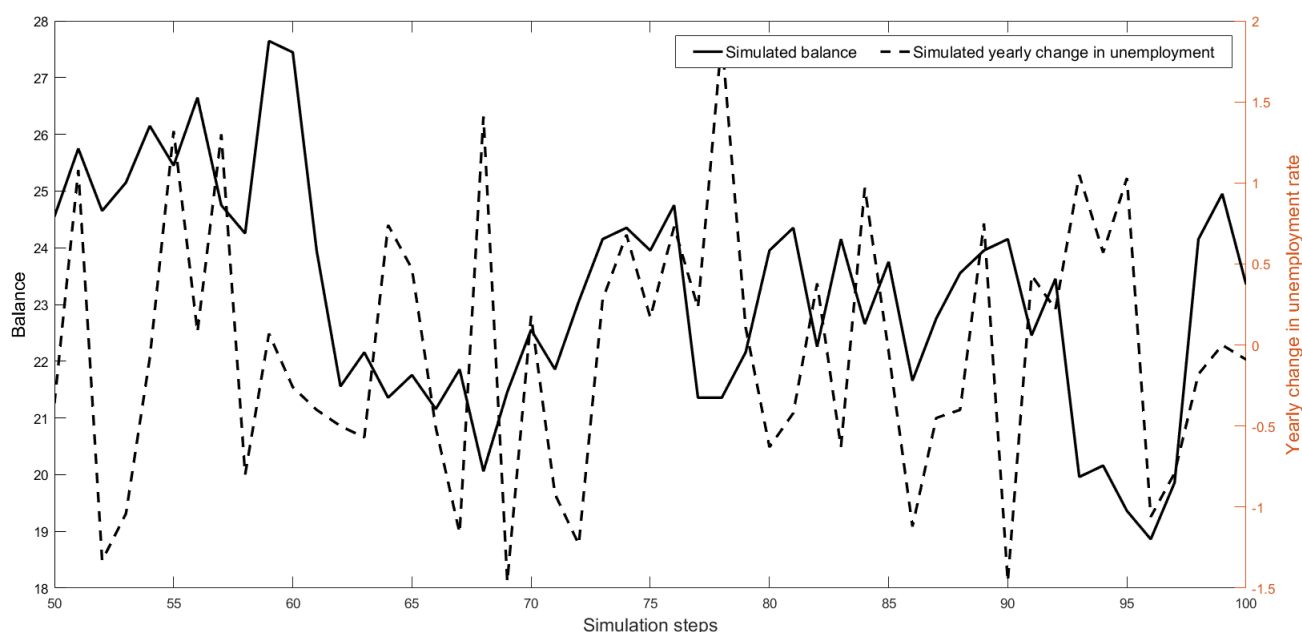
Figure 3.16 – Simulated balance and yearly change in the unemployment rate.



Source: Own elaboration.

ployment rate and the balance, using more recent data, and we also test for the Granger causality between the unemployment rate (without taking yearly change) and the bal-

Figure 3.17 – Simulated balance and yearly change in the unemployment rate between steps 50 to 100.



Source: Own elaboration.

ance of perceptions. The statistical tests of the empirical time series were conducted using the same frequency and period as those used to calibrate the ABM.

Starting by presenting the results for the empirical data of unemployment rate and balance, the ADF test indicated that both the time series are non-stationary, as the p-value of the test was 0.77 and 0.65, respectively. For this reason, we conducted the causality test using both series in the first difference. The AIC and FPE information criteria indicated a VAR model with 7 lags, while the HQ and SC criteria indicated a VAR model with 2 lags. Our findings indicate that both the balance and the unemployment rate Granger caused each other when using a VAR model with 7 lags. Similarly, when using a VAR model with 2 lags, we found statistical evidence that the balance Granger causes the unemployment rate at a significance level of 1%, while the unemployment rate Granger causes the balance only at a 10% significance level. All those results are presented in Appendix B.1.

To analyze whether this statistical evidence of temporal causality also occurs in the simulated data, we carried out the same test for the simulated data. The ADF test for the balance showed that the null hypothesis of non-stationarity can only be rejected at a significance level of 10%. The null hypothesis of non-stationarity for the unemployment rate was rejected at a significance level of 1%. As 10% is a reasonable significance level, we decided to analyze both variables in level. When estimating the model with 3 lags, as proposed by AIC criteria, the Granger causality test indicated

evidence of causality at 10% significance for both the unemployment rate Granger causing the balance and the balance Granger causing the unemployment rate. The test results are shown in Table 3.9.

Table 3.9 – Granger causality test for simulated unemployment and BS using 3 lags in the VAR model.

Null Hypothesis	p-value
Unemployment rate does not Granger-cause BS	0.00
BS does not Granger-cause unemployment rate	0.08

Source: Own elaboration.

Interestingly, the VAR parameter estimated in lag 3 indicates a positive impact of the BS on the unemployment rate, significant at the 10% significance level. This result is an indication that self-fulfilling movements in expectations are an emergent property of the model. The results of the VAR model are presented in Table 3.10.

Table 3.10 – VAR estimation of the unemployment equation.

	Estimated parameter	Standard error	t value	P value
Constant	0.03	0.00	6.65	0.00
B_{t-1}	0.02	0.02	0.7	0.48
u_{t-1}	0.13	0.06	2.2	0.03
B_{t-2}	-0.03	0.03	-1.05	0.28
u_{t-2}	-0.03	0.06	-0.46	0.64
B_{t-3}	0.03	0.02	1.65	0.09
u_{t-3}	-0.02	0.06	-0.42	0.64

Source: Own elaboration.

We can now turn to analyze the Granger causality between yearly changes in the unemployment rate and the balance of perceptions. The time series of the yearly changes in the unemployment rate and the time series of the balance of expectations were both non-stationary, so we used their first differences. Based on the information criteria, we selected a VAR model with 12 lags and 24 lags. The causality test showed that both the balance of perceptions Granger causes the yearly change in the unemployment rate and the yearly change in the unemployment rate Granger causes the balance of perception, with statistical evidence at a 5% significance level. All the results are available in the appendix B.2.

When performing this same statistical analysis for the simulated data in level, the information criteria indicated VAR models with 14 and 3 lags. As these models are quite different in terms of lags, we applied the causality test to both estimated models. For the model estimated with 3 lags, statistical evidence of Granger causality was found in both directions of causality, while for the model estimated with 14 lags, Granger causality

Table 3.11 – Granger causality test for simulated yearly change in unemployment rate and BS using 3 lags in the VAR model.

Null Hypothesis	p-value
Yearly change in the unemployment rate does not Granger-cause BS	0.00
BS does not Granger-cause unemployment rate	0.03

Source: Own elaboration.

was found only in the direction of the yearly change in the unemployment rate causing the balance of perceptions. Those results can be seen in Tables 3.11 and 3.12.

Table 3.12 – Granger causality test for simulated yearly change in unemployment rate and BS using 14 lags in the VAR model.

Null Hypothesis	p-value
Yearly change in the unemployment rate does not Granger-cause BS	0.15
BS does not Granger-cause unemployment rate	0.03

Source: Own elaboration.

3.4 CONCLUDING REMARKS

In this essay, we analyze the formation of interactive expectations in macroeconomic ABMs with two closures. In the dynamic version of the efficiency wage model augmented by expectations, proposed in section 3.2, we analyze the co-evolution of interactive expectations and the unemployment rate actually observed by means of an expectations-enhanced efficiency wage model. It is interesting to note that the parameter related to the intensity of choice has decreased considerably in relation to the model with global interaction, presented in the first essay, which indicates that the intensity of choice decreases when agents interact locally when forming their perceptions in the proposed model. The value found in the calibration for the rewriting probability indicates that the interaction network for the proposed model is close to a small-world network. Furthermore, in the analysis of the model's dynamics, it was seen that moments of pessimism about the future unemployment rate are followed by an increase in the unemployment rate. On the other hand, no statistical indication of temporal causality was found for these variables, but statistical evidence of temporal causality was found between yearly changes in the unemployment rate and the balance score (BS) of perceptions.

In the GNR model augmented by expectations proposed in section 3.3, our hypothesis is that households' perceptions about the future unemployment rate influence the macroeconomy through the consumption channel. We find that this model could generate endogenous waves of optimism and pessimism, represented by the variation in the balance of perceptions. The Granger causality test indicated that the temporal

causality between the balance of perceptions and the unemployment rate, reported in the empirical literature, can be generated as an emergent property of the model. Furthermore, through a VAR analysis, a significant positive impact of the balance of perceptions on the unemployment rate was found, which indicates that self-fulfilling movements in expectations are being reflected in this model.

FINAL REMARKS

In this dissertation, we presented the formation of expectations with the hypothesis of bounded rationality interacting agents in macroeconomic models with different closures.

In the first essay, we develop a dynamic version of an efficiency wage model augmented by expectations. The main result is that the temporal causality between pessimistic expectations and the level of unemployment, observed in the empirical literature, emerges in an efficiency-wage model with the assumption that agents form their expectations based on their own experiences in the labor market and that they have limited rationality in their decision-making process. We also found that the persistence of heterogeneity of expectations is an emergent property of the computational model developed. In addition, we show statistical evidence that the temporal causality relationship between the unemployment rate observed and that expected by agents only emerges in a situation where the intensity of choice is strictly greater than zero and the weight of social utility is at most two times greater than the weight of private utility.

In the second essay, we expand on a macroeconomic ABM that considers agents' decision rules based on heuristics. In this model, we hypothesized that household expectations impact macroeconomic activity through the consumption channel. The main findings indicate that in an environment considering the existence of a central planner in both the labor and goods markets, the economy only returns to full employment equilibrium levels for high levels of rationality and social influence on perception choice. Furthermore, we observe that in an environment without the presence of a central planner, the economy may persistently deviate from the full-employment steady-state level to low values of the intensive of choice parameter and social impact on agent decision-making.

Finally, in the last essay, we consider a network structure in shaping households' perceptions in the efficiency-wage ABM, presented in the first essay, and in the ABM with a Keynesian closure, presented in the second essay. For the efficiency-wage ABM, we found that the parameter related to the intensity of choice is lower in the version of the ABM efficiency wage model augmented by interactive expectations than in the ABM efficiency wage model augmented by expectations proposed in the first essay. In other words, by adding networks to the agents' choice process, the choice process becomes less responsive to the observed incentives. In addition, we found that the persistence of heterogeneity of expectations and the temporal causality between the balance score and the yearly change in the unemployment rate were emergent properties of the model. For the model with a Keynesian closure, the main conclusion was that an expectation-formation environment with bounded rationality and social interaction could reflect self-fulfilling movements in the level of economic activity.

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APPENDIX A – ESSAY 1

A.1 STATISTICS FOR THE CAUSALITY TEST BETWEEN THE UNEMPLOYMENT RATE AND BS USING DATA FOR THE US.

Table A.1 – ADF test empirical data.

Variables	P-value ADF test
BS	0.01
Unemployment rate	0.07

Table A.2 – Information criterion empirical data.

Information criterion	AIC	HQ	SC
Number of lags	6	2	2

Table A.3 – Granger causality test with 2 lags for the empirical data.

Null hypothesis	p-value of the test
Unemployment rate does not Granger-cause BS	0.00
BS does not Granger-cause unemployment rate	0.00

Table A.4 – Granger causality test with 6 lags for the empirical data.

Null hypothesis	p-value of the test
Unemployment rate does not Granger-cause BS	0.00
BS does not Granger-cause unemployment rate	0.00

A.2 STATISTICS FOR THE CAUSALITY TEST BETWEEN BS AND YEARLY CHANGE IN THE UNEMPLOYMENT RATE EMPIRICAL DATA.

Table A.5 – ADF test yearly.

Variables	P-value ADF test
BS	0.01
Unemployment rate	0.07

Table A.6 – Information criterion empirical data.

Information criterion	AIC	HQ	SC
Number of lags	6	2	2

Table A.7 – Granger causality test with two lags for the empirical data.

Null hypothesis	p-value of the test
Unemployment rate does not Granger-cause BS	0.00
BS does not Granger-cause unemployment rate	0.00

Table A.8 – Granger causality test with 6 lags for the empirical data.

Null hypothesis	p-value of the test
Unemployment rate does not Granger-cause BS	0.00
BS does not Granger-cause unemployment rate	0.00

APPENDIX B – ESSAY 3

B.1 STATISTICS FOR THE CAUSALITY TEST BETWEEN THE UNEMPLOYMENT RATE AND THE BALANCE OF EXPECTATIONS USING DATA FROM ITALY.

Table B.1 – ADF test empirical data in level.

Variables	P-value ADF test
Balance	0.65
Unemployment rate	0.77

Table B.2 – Information criterion empirical data.

Information criterion	AIC	HQ	SC	FPE
Number of lags	7	2	2	7

Table B.3 – Granger causality test with 2 lags for the empirical data.

Null hypothesis	p-value of the test
Unemployment rate does not Granger-cause balance	0.07
Balance does not Granger-cause unemployment rate	0.00

Table B.4 – Granger causality test with 7 lags for the empirical data.

Null hypothesis	p-value of the test
Unemployment rate does not Granger-cause balance	0.00
Balance does not Granger-cause unemployment rate	0.00

B.2 STATISTICS FOR THE CAUSALITY TEST BETWEEN THE BALANCE OF EXPECTATIONS AND YEARLY CHANGE IN THE UNEMPLOYMENT RATE EMPIRICAL DATA.

Table B.5 – ADF test empirical data in level.

Variables	P-value ADF test
Balance	0.67
Yearly change in unemployment rate	0.4

Table B.6 – Information criterion empirical data.

Information criterion	AIC	HQ	SC	FPE
Number of lags	24	12	1	13

Table B.7 – Granger causality test with 12 lags for the empirical data.

Null hypothesis	p-value of the test
Yearly change in the unemployment rate does not Granger cause balance	0.05
Balance does not Granger cause a yearly change in unemployment rate	0.00

Table B.8 – Granger causality test with 24 lags for the empirical data.

Null hypothesis	p-value of the test
Yearly change in the unemployment rate does not Granger cause balance	0.49
Balance does not Granger cause a yearly change in unemployment rate	0.05