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Thaís Muraro

Inner-Cable Mechanisms: A feasibility analysis of cable substitution in planar kinematic chains

Florianópolis
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Supervisor:: Prof. Daniel Martins, Dr. Eng.
Co-supervisor:: Prof. Leonardo Mejia Rincon, Dr. Eng.

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Thaís Muraro

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O presente trabalho em nível de doutorado foi avaliado e aprovado por banca examinadora composta pelos seguintes membros:

Relator: Prof. Rogério Sales Gonçalves, Dr.
Universidade Federal de Uberlândia

Prof. Aníbal Alexandre Campos Bonilla, Dr. Eng.
Universidade do Estado de Santa Catarina - Campus Joinville

Prof. Antonio Carlos Valdiero, Dr. Eng.
Universidade Federal de Santa Catarina

Prof. Henrique Simas, Dr. Eng.
Universidade Federal de Santa Catarina

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Prof. Daniel Martins, Dr. Eng.
Supervisor:

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“Você me pergunta pela minha paixão
Digo que estou encantada como uma nova invenção
Eu vou ficar nesta cidade, não vou voltar pro sertão
Pois vejo vir vindo no vento cheiro de nova estação”
(BELCHIOR, 1976)

RESUMO

A robótica está cada vez mais presente na vida cotidiana e é utilizada para diversos tipos de tarefas. Os robôs aparecem massivamente na indústria, na área médica e hospitalar, e vêm tomando espaço na interação com humanos. A difusão de robôs para as mais variadas atividades poderia ser mais expressiva se o custo, a massa e o tamanho dos dispositivos pudesse ser reduzido sem que se perdesse a funcionalidade e o desempenho. Exatamente de acordo com essas vantagens este trabalho foi desenvolvido. Nesta tese, o problema de minimização de custo, massa e volume é resolvido através do uso conveniente de cabos em sistemas que originalmente possuem todas as suas partes rígidas, sem que haja as perdas de funcionalidade e desempenho antes mencionadas. Nesse sentido, o principal objetivo deste estudo é determinar se existem elos em um dados mecanismo que podem ser substituídos por cabos e quais são eles, a fim de manter o sistema em equilíbrio e permitindo que ele realize determinadas tarefas sem colapsar. Assim, algumas condições são impostas para que a substituição dos elos por cabos de comprimento fixo possa ser bem sucedida. A partir dessas condições os cabos são posicionados e os elos rígidos removidos. Então, a cinemática de posição é realizada a fim de determinar as carcterísticas dos helicoides em função de ângulos e das dimensões conhecidas do mecanismo. Essas posições são utilizadas para escrever as heliforças atuantes no sistema que serão utilizadas no método de Davies para a resolução do modelo estático. Conhecendo as forças envolvidas no sistema, faz-se, então, uma análise vetorial para identificar, dada uma trajetória de um ponto do sistema, o sucesso ou fracasso da substituição. Caso algum cabo não permaneça tensionado durante a trajetória, ele pode ser removido ou a configuração de carregamento pode ser modificada para reverter a situação. A abordagem proposta é mostrada e validada através de estudos de casos nos quais são consideradas diferentes cadeias cinemáticas planares, desde casos mais simples até casos de aplicação real. A substituição proposta gera um tipo de mecanismo (e de sistemas mecânicos em geral) que contém cabos de comprimento fixo não atuados, que não podem ser considerados mecanismos atuados por cabos e tampouco sistemas de tensegridade. Assim, é também proposta uma nomenclatura para esse tipo de mecanismo gerado: os Inner-Cable Mechanisms (ICMs). A fim de incluir os sistemas inner-cable na classificação de sistemas com cabos, uma nova classificação desses sitemas é proposta.

Palavras-chave: Inner-cable Mechanisms. Substituição por cabos. Sistemas de cabos. Sistemas de tensegridade. Método de Davies.

RESUMO EXPANDIDO

Introdução O crescente avanço industrial está sendo causado, principalmente, pela quantidade e diversidade de tarefas que estão sendo realizadas por diferentes robôs industriais. Esses robôs geralmente são grandes e robustos e têm um alto custo de fabricação. Nesse sentido, este trabalho foi motivado pelo interesse em reduzir a massa e, em algumas situações, o tamanho de robôs de grande porte, a fim de reduzir também os custos de fabricação. Para tanto, o método proposto substitui elos rígidos por cabos como uma solução viável para o problema, sendo possível utilizá-lo em diversos sistemas mecânicos, incluindo robôs e mecanismos com cadeias cinemáticas planares. Os estudos acerca da substituição de elos rígidos por cabos começaram com Landsberg e Sheridan, em 1984 (LANDSBERGER, 1984), quando propuseram a substituição dos elos rígidos de uma Plataforma Stewart por cabos acionados por tambores de enrolamento, o que permite variar o comprimento de cada cabo. No entanto, para obter sucesso em tal substituição, foi necessário incluir um elo rígido central conectando a base fixa e a plataforma móvel do sistema, garantindo a tensão nos cabos. A combinação de elos rígidos e cabos também é realizada de forma a gerar uma classe de estruturas de tensegridade. Estruturas de tensegridade são casos muito especiais de treliças (SKELTON et al., 2001). Nelas os cabos e os elos rígidos estão convenientemente posicionados de forma que os elos rígidos não são adjacentes entre si. Ainda, existem os sistemas de tensegridade compostos por estruturas de tensegridade. O tipo mais comum de sistema de tensegridade é o de classe 1, onde não há links adjacentes entre si. O que determina a classe de um sistema desse tipo é a quantidade de elos adjacentes entre si, ou seja, se houver dois elos adjacentes, chama-se Classe 2, com 3 elos adjacentes chama-se classe 3, e assim sucessivamente. Uma característica importante dos sistemas de tensegridade é que as partes rígidas estão sempre sob compressão e os cabos estão sempre sob tensão. Neste trabalho, no entanto, a proposta de substituição de elos rígidos por cabos não contempla a possibilidade de adicionar novos elos ao mecanismo, mantendo as características estruturais e de mobilidade. Além disso, os cabos que substituem os elos rígidos não são acionados por tambores de enrolamento (ou similar) e assim os sistemas gerados não são do tipo atuados por cabos. Nesse sentido, também não podem ser considerados sistemas de tensegridade. Embora o comprimento dos cabos incluídos nos sistemas gerados pelo método proposto também seja fixo como o das estruturas de tensegridade, a mobilidade dos sistemas é diferente de zero e não há condições definidas para os elos, mas sim para os cabos. Portanto, propõe-se uma nomenclatura para esse tipo de mecanismo: *Inner-cable Mechanisms - ICMs*. Uma nova forma de classificação dos sistemas com cabos também é proposta nesta tese, incluindo os ICMs para que eles sejam reconhecidos também como sistemas com cabos. Os ICMs possuem em sua estrutura, um ou mais cabos inextensíveis de comprimento fixo, e esses cabos não são acionados por tambores de enrolamento (ou similar). Neste trabalho, ICMs são gerados por substituição de cabos dentro de cadeias cinemáticas planares, no entanto estende-se a definição de inner-cable também para sitemas mecânicos no geral.

Objetivos

O foco desta tese é a substituição de elos rígidos por cabos em sistemas mecânicos com cadeias cinemáticas planares. O problema desta substituição é manter a funcionalidade do sistema, através do tensionamento dos cabos. Nesse sentido, o objetivo principal é organizar um método de substituição de elos rígidos por cabos, no qual não haja perda de funcionalidade e desempenho do sistema. Para alcançar o objetivo principal, alguns objetivos específicos foram determinados:

- Estabelecer condições necessárias que devem ser impostas ao sistema de forma a viabilizar a substituição de elos rígidos por cabos;
- Validar a substituição para uma determinada tarefa ou trajetória, utilizando o método de Davies e uma análise da tensão em cada cabo;
- Demonstrar a substituição aplicando o método proposto em casos didáticos e também reais;
- Incluir o tipo de mecanismo gerado em uma classificação de sistemas com cabos, propondo, para tanto, uma nomenclatura e uma nova forma de classificação dos sistemas que contém cabos.

Metodologia

A fim de estabelecer um método eficaz para que cabos inextensíveis possam tomar o lugar de elos rígidos em determinados sistemas mecânicos, diversos estudos foram realizados para cumprir os objetivos específicos. O estudo começou com uma revisão aprofundada acerca dos sistemas com cabos, na qual foram identificados dois tipos de sitemas: os sitemas mecânicos atuados por cabos e os sitemas de tensegridades. Durante a pesquisa, percebeu-se que os sitemas gerados através da substituição proposta não estão formalmente definidos na literatura. Por isso, foi proposta uma nomenclatura pra o tipo de sitema e também uma nova forma de classificação dos sistemas com cabos a fim de incluí-los. Então, alguns critérios para realizar a substituição foram definidos, com o objetivo de tornar a proposta mais eficiente. Esses critérios são, de fato, cinco condições necessárias para que a substituição de um ou mais elos rígidos pelo mesmo número de cabos seja factível. Impostas as condições necessárias ao mecanismo, uma análise da tensão no(s) cabo(s) precisou ser executada para validar a substituição que, segundo os estudos, só pode ser realizada se os cabos estiverem sob tensão durante a execução da sua tarefa. Tal análise foi feita utilizando o Método de Davies, ferramenta amplamente difundida e utilizada por diversos pesquisadores, que se utiliza da teoria de helicoides e da teoria de grafos para determinar de uma só vez os esforços de todas as juntas do sistema, no caso estático, incluindo também as forças externas atuantes. Dados os esforços nas juntas, a condição do tensionamento dos cabos foi calculada através do produto interno entre o vetor de direção do cabo e o esforço na junta adjacente a ele para uma dada trajetória ou tarefa executado pelo mecanismo. Finalmente, os gráficos da tensão nos cabos foram apresentados para demonstrar os resultados.

Resultados e Discussão

A vallidação do método proposto se dá através de três sistemas estudados: 1 - um mecanismo de quatro barras; 2 - um mecanismo de Watt, que é avaliado em duas configurações em que cada uma considera um ponto de aplicação de força externa diferente; 3 - um robô paletizador. No primeiro caso, utilizando o método proposto, apenas um elo rígido do mecanismo de quatro barras pôde ser substituido por um cabo. A análise da tensão no cabo foi feita via produto interno entre o vetor de direção do cabo e os esforços na junta adjacente ao cabo, sendo que os esforços nas juntas foram obtidos através do método de Davies, como prevê o método proposto. Mostrou-se um caso (dentre outros que podem existir) em que a substituição foi possível, comprovando através do gráfico da tensão no cabo. O estudo do mecanismo Watt se deu para dois designs diferentes, em que cada uma considera a aplicação da força externa em um elo distinto. Dessa forma, para cada design, as etapas do método foram seguidas e foi possível garantir a substituição de dois elos rígidos em cada caso. O gráfico da tensão nos cabos foi apresentado para quatro configurações de carregamento do primeiro design e 3 configurações de carregamento do segundo design. Assim como no primeiro estudo de caso, foi possível perceber que, para algumas configurações de carregamento, a substituição de elos por cabos não é viável. No entanto, apesar de existirem configurações de carregamento em que a substituição dos dois elos por cabos não é viável, mostrou-se casos em que a substituição de apenas um dos elos poderia ser possível. Por último, o método de substituição de elos rígidos por cabos foi aplicado a um robô paletizador. Nesse caso, impondo as cinco condições necessárias para subsituir elos por cabos, foi possível realizar a substituição de três elos. Então, foi simulada uma situação em que o robô realiza a seguinte tarefa: partindo de um ponto P_1, ele vai até uma carga de 50kg, em P_2, levanta a carga até um ponto P_3, descarrega a carga e retorna para P_1. Para a execução dessa tarafa, o gráfico das tensões nos cabos foi gerado, no qual pode-se ver que tais tensões são sempre positivas, o que valida a substituição proposta.

Considerações Finais

Um novo método para substituição de elos rígidos por cabos em sistemas mecânicos com cadeia cinemática planar foi proposto neste trabalho. Utilizando o método proposto, foram gerados mecanismos que possuem elos rígidos e também cabos inextensíveis; aqui eles são chamados de *Inner-cable Mechanisms* ou *ICMs*. Os ICMs diferem das estruturas de tensegridade por possuírem mobilidade diferente de zero e dos mecanismos acionados por cabos porque seus cabos possuem comprimento fixo e não são acionados por tambores de enrolamento ou outro método similar. Foram determinadas cinco condições necessárias para viabilizar a substituição de elos rígidos por cabos. O recurso teórico utilizado por esta solução é o Método de Davies, baseado na Teoria dos Helicoides e na Teoria dos Grafos. Após determinar do modelo estático pelo método de Davies, uma análise vetorial envolvendo a direção do cabo e o esforço na junta adjacente forneceu informações sobre a tensão nos cabos. Através deste método, é possível identificar se existem elos rígidos que podem ser substituídos por cabos e além disso, quais são eles. Visto que a substituição de elos rígidos por cabos inextensíveis e que não são acionados por tambores de enrolamento, por exemplo, geram um tipo de sistema que não está formalmente descrito na literatura, uma nomenclatura para esse tipo de sistema foi proposta: *Inner-cable system*. Além disso, uma definição para os mecanismos do tipo inner-cable também foi proposta e extendida para os sistemas mecânicos em geral. De fato, apesar de alguns sistemas atuados por cabos e alguns sistemas de tensegridade poderem ser reconhecidos como inner-cable, de acordo com a definição proposta, nem todos os inner-cable poem ser

considerados como um dos dois tipos de sistemas mencionados. Nesse sentido, com vistas a incluir os sistemas inner-cable como sistemas mecânicos com cabos, uma nova classificação foi proposta. A nova classificação refere-se ao comprimento dos cabos do sistema, de forma que foram separados em duas classes: A dos sistemas com cabos de comprimento fixo e a dos sistemas com cabos de comprimento variável. Logo, o método de substituição de elos rígidos por cabos em mecanismos permite a redução de sua massa e consequentemente de seu custo de fabricação. Isso pode tornar a automação de tarefas mais acessível, o que contribui para o avanço tecnológico. A partir dessa ideia, outras situações podem ser estudadas, como a redução do tamanho dos ICMs para uso em tarefas minimalistas, o que os coloca como candidatos a serem utilizados em cirurgia robótica. Além disso, outra substituição pode ser estudada: substituir elos ternários por cabos, utilizando condições como essas, mas outros recursos teóricos. Também, estudos anteriores sobre auto-alinhamento de mecanismos sugerem que substituindo determinados elos por cabos, pode-se gerar um mecanismo auto-alinhado.

Palavras-chave: Inner-cable Mechanisms. Substituição por cabos. Sistemas de cabos. Sistemas de tensegridade. Método de Davies.

ABSTRACT

Robotics is increasingly present in everyday life and is used for various types of tasks. Robots appear massively in industry, in the medical and hospital areas, and have been taking up space in the interaction with humans. The diffusion of robots for the most varied activities could be more expressive if the cost, mass and size of the devices could be reduced without losing functionality and performance. Exactly according to these advantages this work was developed. On this thesis the problem of cost, mass and volume minimization is solved through the convenient use of cables in systems that originally have all their rigid parts, without the aforementioned loss of functionality and performance. In this sense, the main objective of this study is to determine which links can be replaced in order to keeping the system in balance and allowing it to perform certain tasks without collapsing. Thus, some conditions are imposed so that the replacement of links by cables of fixed length can be successful. From these conditions the cables are positioned and the rigid links removed. Then, the position kinematics is performed in order to determine the screw characteristics as a function of known angles and dimensions of the mechanism. These positions are used to write the wrenches acting on the system that will be used in the Davies method to solve the static model. Knowing the forces involved in the system, a vector analysis is then carried out to identify, given a trajectory of a point in the system, the success or failure of the replacement. If any cable does not remain tensioned during the trajectory, it can be removed or some force application criteria can be modified to reverse the situation. The proposed approach is shown and validated through case studies in which different planar kinematic chains are considered, from simple cases to real application cases. The proposed replacement generates a class of mechanisms that contain non-actuated fixed-length cables, which cannot be considered cable-actuated mechanisms or tensegrity systems. Thus, a classification for this type of generated mechanism is also proposed: the Inner-Cable Mechanisms.

Keywords: Inner-cable Mechanisms. Cable replacement. Cable Systems. Tensegrity Systems. Davies Method.

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1R2T	One rotation and two translations
1T	One translation
2R3T	Two rotations and three translations
2T	Two translations
3R3T	Three rotation and three translations
3T	Three translations
CRPM	Completely restrained parallel manipulator
CRPMs	Completely restrained parallel manipulators
CSPM	Cable-suspended parallel mechanism
CSPMs	Cable-suspended parallel mechanisms
DoFs	Degrees of freedom
FCMs	Fully-constrained mechanisms
ICM	Inner-cable mechanism
ICMs	Inner-cable mechanisms
IRPM	Incompletely restrained parallel manipulator
RRPM	Redundantly restrained parallel manipulator
SCPM	Sub-constrained parallel manipulator
SPS	Spherical - Prismatic - Spherical
UPS	Universal - Prismatic - Spherical

LIST OF SYMBOLS

ρ_B	Cable direction vector B
ρ_D	Cable direction vector D
$\$$	Screw
$\mathcal{L}$	First Plücker coordinate
$\mathcal{M}$	Second Plücker coordinate
$\mathcal{N}$	Third Plücker coordinate
$\mathcal{P}^*$	Fourth Plücker coordinate
$\mathcal{Q}^*$	Fifth Plücker coordinate
$\mathcal{R}^*$	Sixth Plücker coordinate
h	Screw pitch
O_{xyz}	Origin of a screw coordinate system
$\vec{S}$	Vector of the screw axis direction
S_0	Point of the screw axis
$\vec{S_0}$	Position vector of the point S_0 in relation to the screw coordinate system
$\$_A$	Action screw
$\hat{\$}$	Unitary screw
ψ	Magnitude of normalized screw
$\vec{u}$	Unitary vector with the direction of the $\vec{S}$
$\hat{\$}_A$	Unitary action screw
l	Number of links
e	Number of direct couplings
G_C	Coupling graph
c	Restrictions of the direct coupling
G_A	Actions graph
ν	Number of strings em G_A
$\hat{A}_N$	Network unitary actions matrix
$[Q_N]_{(k\times C_n)}$	Cuts matrix
$q_N(i,j)$	Elements of the cuts matrix
m	Number of cables
n	Number of degrees of freedom
p	Interconnected compression struts
$\vec{F}$	Force vector present in the adjacent joint to the cable
$\vec{\rho}$	Cable direction vector
Θ	Angle formed by the vectors $\vec{F}$ and $\vec{\rho}$
$\vec{S}_{0_e}$	Position vector of joint e with respect to the origin of the coordinate system
$\hat{\$}_{aF_x}$	Wrench of joint a in x direction
ψ_{aF_x}	Magnitude of the wrench of joint a in x direction

$\hat{\$}_{aF_y}$	Wrench of joint *a* in *y* direction
ψ_{aF_y}	Magnitude of the wrench of joint *a* in *y* direction
$\hat{\$}_{aM_z}$	Wrench of joint *a* in *z* direction
ψ_{aM_z}	Magnitude of the wrench of joint *a* in *z* direction
$\hat{\$}_{bF_x}$	Wrench of joint *b* in x direction
ψ_{bF_x}	Magnitude of the wrench of joint *b* in *x* direction
$\hat{\$}_{bF_y}$	Wrench of joint *b* in *y* direction
ψ_{bF_y}	Magnitude of the wrench of joint *b* in *y* direction
$\hat{\$}_{cF_x}$	Wrench of joint *c* in *x* direction
ψ_{cF_x}	Magnitude of the wrench of joint *c* in *x* direction
$\hat{\$}_{cF_y}$	Wrench of joint *c* in *y* direction
ψ_{cF_y}	Magnitude of the wrench of joint *c* in *y* direction
$\hat{\$}_{dF_x}$	Wrench of joint *d* in *x* direction
ψ_{dF_x}	Magnitude of the wrench of joint *d* in *x* direction
$\hat{\$}_{dF_y}$	Wrench of joint *d* in *y* direction
ψ_{dF_y}	Magnitude of the wrench of joint *d* in *y* direction
$\hat{\$}_{F_{ext}F_y}$	Wrench related to external force
$\psi_{F_{ext}F_y}$	Magnitude of the wrench related to external force
z_a	Third component of the joint position vector *a*
z_b	Third component of the joint position vector *b*
z_c	Third component of the joint position vector *c*
z_d	Third component of the joint position vector *d*
$z_{F\,ext}$	Third component of the external force position vector
λ	Workspace degrees of freedom
Ψ	Vector of the wrenches magnitudes
$\hat{A}_{N_S}$	Matrix that contains only columns referring to secondary variables
Ψ_S	Vector of the secondary variables' magnitudes
$\hat{A}_{N_P}$	Matrix that contains only columns referring to primary variables
Ψ_P	Vector of the primary variables' magnitudes
ρ_E	Cable direction vector *E*
θ	Crank rotation for four-bar and Watt mechanisms
P_1	Starting point for the trajectory of the palletizer robot
P_2	Palletizer robot loading point
P_3	Palletizer robot unloading point
T_{12}	Trajectory between P_1 and P_2
T_{23}	Trajectory between P_2 and P_3
T_{31}	Trajectory between P_3 and P_1

CONTENTS

1 INTRODUCTION

The use of robotic systems is increasingly present in industry and also in everyday life. However, generally, only large industries can benefit from this technological advance because of their purchasing power, since robotic systems, although increasingly common, still have a high manufacturing cost. In addition, because of the robustness of industrial robots, they can still pose risks in their interaction with humans, which corroborates the condition that there is a large physical space for the robot to operate safely.

In this sense, technological advances are not only about the study and innovative projects, but rather the massive dissemination of technology, through its use also by small and medium-sized enterprises. So the more robotic systems are present in the market at affordable prices to consumers, the more robotic systems will be used, further favoring and stimulating research and technological innovations.

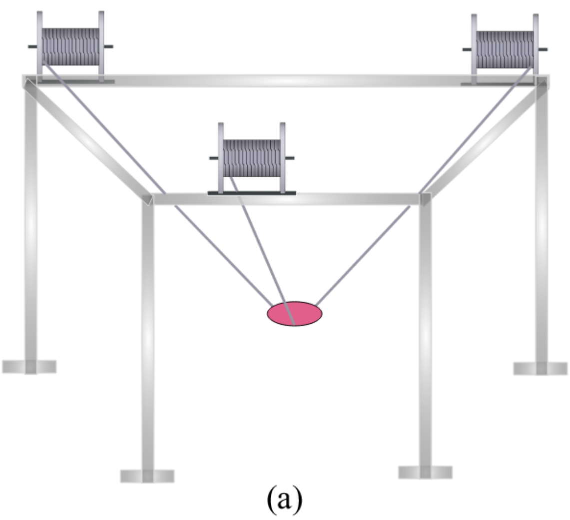
For this reason, reducing the manufacturing costs of robotic systems is one of the motivations of this study. In addition, reducing the mass of the robust parts of the system can also contribute to the reduction of manufacturing costs, as, for example, less powerful engines can be used. Furthermore, considering a robotic system with reduced mass, its interaction with humans may represent lower risk when compared to the same system with higher mass.

Given these considerations, it is understood that one step towards the dissemination of the use of robotic systems is to reduce their manufacturing costs, for example, reducing the mass of certain rigid parts of the system. The proposal for such a reduction, presented in this thesis, is the replacement of rigid parts by cables, maintaining the original functionalities of the system. The replacement of rigid parts by cables can even facilitate the interaction between machine and human, since not all its parts will be rigid.

Systems containing cables have been targets of interest for many decades, mainly because of their advantages. They are generally lighter and with lower production cost compared to systems with all their rigid parts. Besides that, the cable robots can perform high dynamical motion (BRUCKMANN et al., 2008b). They can be designed on huge scales, to act as cranes or, in the case of tensegrity structures, as artistic monuments or even structural parts of civil construction. Still, a great interest in systems with cables, is the fact that it can compare them to the structures of the human body, in which the cables can represent muscles, while rigid parts can represent the bones. There are several studies related to structural issues, workspace, control and their use in the most diverse tasks. However, the main fact regarding any system that contains cables is related to the distribution of cable tensions. In fact, one of the conditions for the effectiveness of a system with cables is that its cables are always tensioned during its operation. The tension value of each cable must be between a minimum value, which ensures that the cable is tensioned and a maximum value, which prevents cable breakage or even engine overload.

The studies on the replacement of rigid links by cables started with Landsberger and Sheridan, in 1984 (LANDSBERGER, 1984), where they proposed the replacement of the rigid links of a Stewart Platform by cables driven by drums, which makes it possible to vary the length of each cable. However, in order to succeed in such a replacement, it was necessary to include a central rigid link connecting the fixed base and the mobile platform of the system, guaranteeing cable tension. So they created the cable driven robots (MERLET, 2013) and since then, many other cable robots were created and are studied to this day. Cable-driven systems can be classified to several ways: according to difference between the number of cables and the degrees of freedom (MING A; HIGUCHI, 1994; VERHOEVEN, 2004), their movements (VERHOEVEN, 2004) or according to type (Fig. 1), with type 1 being those in which the movement of the mobile platform is due to alteration the length of the cables and type 2 those that promote the movement of the platform by changing the position of the cable attachment point (MERLET, 2013). In this work, the proposal to replace rigid links with cables does not include the possibility of adding new links to the mechanism, maintaining structural and mobility characteristics. In addition, the cables that take the place of the rigid links are not driven by winding drums and, thus, the generated systems do not belong to the class of the mechanisms actuated by cables.

Figure 1 – Cable-suspended systems: (a) Type 1; (b) Type 2.



Source: By the author, 2022.

The rigid links and cables combination is also carried out in order to generate a class of structures called tensegrity. Tensegrity structures are very special cases of trusses (SKELTON et al., 2001) and consist of only two different types of straight structural members: rigid parts under compression called struts and cables always under tension (ZHANG, J.; OHSAKI, 2015). The cables and the struts are conveniently positioned so that the struts do not contact each other at their ends and the stability of the system is guaranteed (ZHANG, J.; OHSAKI, 2015). This class of structures also appears in biomechanical models and the artistic world, through sculptures and large monuments (ZHANG, J.; OHSAKI, 2015; SKELTON; DE OLIVEIRA, 2009). In this work, however, although the length of the cables included in the system is also fixed, the mobility of the systems is different from zero and there are no defined conditions for the struts, but for the cables. Besides that, a tensegrity structure has no support, ie, it is free-standing (ZHANG, J.; OHSAKI, 2015) which also makes the generated systems different from the tensegrities as well.

Still, there are the tensegrity systems composed of tensegrity structures. The most common type of a tensegrity system is class 1, where there are no adjacent links to each other. What determines the class of a system of this type is the amount of links adjacent to each other, that is, if there are two adjacent links, it is called Class 2, with 3 adjacent links it is class 3, and so on. An important feature of tensegrity systems is that the rigid parts are always under compression and the cables are always under tension. In this sense, the cables are actuated so that they remain tensioned throughout the movement. Thus, the systems generated through the methodology proposed in this work also differ from the tensegrity systems because they do not have their cables activated.

So, the systems generated in this study by replacing rigid links with cables do not belong to the types or classes mentioned. Thus, we propose in this work a nomenclature for this new type of mechanisms: the *Inner-cable Mechanisms - ICMs*. The Inner-cable mechanisms have, in their structure, one or more inextensible cables with a fixed length, and these cables are not driven by winding drums or similar. In this work, ICMs are generated by cable substitution inside planar kinematic chains.

1.1 CHOSEN WORDS AND TERMS

Some terms used in the writing of this thesis will be presented here. Although these terms are well known and used in engineering, a brief explanation of why these words were chosen is necessary for a good understanding of this research.

- Tension: The word “tension” is being used in colloquial language so that there is consistency with what is studied about cables. Therefore, it should be understood that “tension” refers to the force exerted along the cable. Thus, the unit of tension used here is N.
- Mechanical systems: The term "mechanical systems" will be used to include

mechanisms, manipulators, devices, robots, structures, etc.

- Wire, string, tendon: These words are used in this thesis as synonyms for cable. As much as it is very common to read “cable-driven”, not all authors surveyed use the same word when referring to cable. The intention in using different terms was precisely to maintain coherence between the described system and the author who described it.

1.2 OBJECTIVES AND METHOD

The focus of this thesis is the replacement of rigid links by cables in mechanical systems within planar kinematic chains. The problem with this replacement is to maintain the functionality and original characteristics of the system, through tensioning the cables. In this sense, the main objective is to organize a method of replacing rigid links with cables, in which there is no loss of functionality and performance of the system.

In mechanical systems there are some links that cannot be replaced by cables, as the replacement can cause certain mobility, stability problems, or even the need to include some more part in the system. Therefore, a secondary objective is to establish required conditions for such substitution to be viable.

Subject to the conditions, the substitution can be performed, but even so, it is still necessary to verify if the substitution made is valid for a certain task or position. In this sense, the other objective is to validate the substitution for a given task or trajectory, by using the Davies method and an analysis of the cable tension.

Finally, after validating the replacement of the rigid link by cable, there is a final objective, which is to classify the type of mechanism generated. So, a new class of mechanisms is proposed: the Inner-cable mechanisms.

1.3 CONTRIBUTIONS

This thesis contributes to the replacement of rigid links by cables in mechanisms within planar kinematic chains. Five conditions for correct substitution are presented and discussed and a method, based on screw theory and vector analysis, is presented. This method makes it possible to replace rigid links with cables without changing the mobility of the mechanism and can be applied to mechanical systems in general, as long as they are parallel and have a planar kinematic chain. Furthermore, a new class of mechanisms is proposed: the class of Inner-cable mechanisms, which can be generated through the proposed method.

1.3.1 Contribution to the classification of mechanisms with cables

The following contributions are about the classification of the cable mechanisms:

1. Proposal of a nomenclature for the new type of mechanism: “Inner-cable mechanism” (ICM) and its corresponding definition.
2. Proposal for a new classification of cable systems based on cable length variation: Fixed length cable systems and Variable length cable systems.

1.3.2 Contributions to the replacement of rigid links by cables

The following contributions are about the substitutions of rigid links by cables in mechanisms whit planar kinematic chains.

1. Required conditions for the replacement of rigid links by cables to be feasible.
2. A method that makes it possible to replace binary links by cables in parallel mechanisms with a planar kinematic chains.

1.4 THESIS OUTLINE

Theoretical tools are presented in Chapter 2. It contains the information about the main theoretical tools used to systematize the replacement of rigid links by cables in mechanical systems of rigid structure, which are the screw theory (in 2.1) and the method of Davies (in 2.2), emphasizing the static model (in 2.2.1).

Chapter 3 shows the literature review carried out during the study. Several works were reviewed regarding mechanical systems that contain cables, such as cable-actuated systems (in 3.1.1) and tensegrity systems (in 3.1.2). This chapter also provides a brief review of the cable tension analysis in cable systems (in 3.2).

The Inner-cable mechanisms are presented in Chapter 4. This type of mechanism is defined (in 4.1) and the differences between these and other types of cable systems are presented. Therefore, a new classification of cable mechanisms is proposed: the Inner-cable mechanisms (in 4.2).

Chapter 5 deals specifically with the proposed substitution method. The required conditions for replacing a rigid link by cable (in 5.1) are presented and discussed. The idea used to check the tension in the cables after replacement is also presented and discussed (in 5.2). Finally, the method is proposed and justified (in 5.3).

Case studies are carried out in chapter 6 and their results are shown and discussed. Three different examples are presented, with variations, that explain and exemplify the proposed method in a practical way. The first study is carried out with a four-bar mechanism (in 6.1), the second with the Watt mechanism (in 6.2) and finally, the third with a palletizer robot (in 6.3).

Finally, in chapter 7, the conclusions and complementary discussions about the results obtained are exposed. Besides that, ideas for future work are highlighted.

2 THEORETICAL TOOLS

In this chapter a review of the theoretical-mathematical tools used in the development of this thesis is presented. First, a review is made on the screw theory. Screws are essential for the use of the Davies Method, which is also reviewed in this chapter. Since the development of the thesis uses only the static model of the mechanisms, the review of the Davies method is mainly emphasized in its use for the static model of mechanisms. Finally, a brief review on vector force analysis is presented.

2.1 SCREW THEORY

The screw theory, formulated by Mozzi in 1763 and systematized by Ball (BALL, 1876), is an important tool used to represent the instantaneous state of motion, in the kinematic model, and the instantaneous state of actions, in the static model, of a rigid body in the space (H., 2006). It only came to be considered a geometric entity from the mid-19th century onwards, when Julius Plücker proposed the following homogeneous coordinates for a line (axis):

$$\$ = \left\{ \begin{array}{c} \mathcal{L} \\ \mathcal{M} \\ \mathcal{N} \\ --- \\ \mathcal{P}^* \\ \mathcal{Q}^* \\ \mathcal{R}^* \end{array} \right\} \tag{1}$$

where \$ denotes the screw, $\mathcal{L}$, $\mathcal{M}$, $\mathcal{N}$, $\mathcal{P}^*$, $\mathcal{Q}^*$ and $\mathcal{R}^*$ are the homogeneous Plücker coordinates.

A pitch h and a screw axis (directed line) completely determine a screw. In fact, let O_{xyz} the origin of a coordinate system, $\vec{S}$ a vector of the screw axis direction and S_0 a point of the screw axis. $\vec{S_0}$ is defined as the position vector of that point relative to the origin of the coordinate system. Then the screw is written as

$$\$ = \left\{ \begin{array}{c} \mathcal{L} \\ \mathcal{M} \\ \mathcal{N} \\ --- \\ \mathcal{P}^* \\ \mathcal{Q}^* \\ \mathcal{R}^* \end{array} \right\} = \left\{ \begin{array}{c} \vec{S} \\ ----- \\ \vec{S}_0 \times \vec{S} + h\vec{S} \end{array} \right\}. \tag{2}$$

In this formation, the screw is said to be in axial form. In axial form, if used to represent the state of velocities it is called a *twist*. A twist represents the state of motion of a rigid body and is used in the kinematic analysis of mechanisms. In this case, the first three coordinates represent the angular velocity of the body and the other three coordinates represent its linear velocity. On the other hand, if it is written as

$$\$ = \left\{ \begin{array}{c} \mathcal{L} \\ \mathcal{M} \\ \mathcal{N} \\ --- \\ \mathcal{P}^* \\ \mathcal{Q}^* \\ \mathcal{R}^* \end{array} \right\} = \left\{ \begin{array}{c} \vec{S}_0 \times \vec{S} + h\vec{S} \\ ----- \\ \vec{S} \end{array} \right\}, \tag{3}$$

so it is in radial form. In a radial form, if used to represent the state of forces, it is called *wrench*. A wrench represents the state of actions of a rigid body, being used in the static analysis of mechanisms. In this form the first three components indicate the moments and the last three, the resultant force. In statics, (application of screws in this thesis) a screw is usually denoted by $\$_A$, as it is an action screw, that is, it represents the relative actions between bodies. In both cases, if the screw axis is represented by a unit vector, the screw is said to be normalized and is then denoted by $\hat{\$}$ and has a magnitude ψ associated:

$$\$ = \psi\hat{\$}. \tag{4}$$

In the case of the wrench, the magnitude has units of force. In fact, normalizing $\vec{S}$, we have

$$\vec{S} = ||\vec{S}||\vec{u}, \tag{5}$$

where $\vec{u}$ is a unit vector with the direction of force and $||\vec{S}||$ is the magnitude of the force. Then the unit wrench can be written as

$$\hat{\$}_A = ||\vec{S}|| \left\{ \begin{array}{c} \vec{S}_0 \times \vec{u} + h\vec{u} \\ ----- \\ \vec{u} \end{array} \right\}, \tag{6}$$

where the magnitude of the wrench is

$$\psi = ||\vec{S}||. \tag{7}$$

Still, there are special cases of wrenches that need to be evaluated. Such cases occur when the screw pitch is zero or when the pitch is assumed to be infinite. In the first case, when $h = 0$, the Eq. (6) is

$$\hat{\$}_A = \psi \left\{ \begin{array}{c} \vec{S}_0 \times \vec{u} \\ ----- \\ \vec{u} \end{array} \right\}, \tag{8}$$

which means that there are only forces acting on the system.

In the second case, when h is assumed to be infinite, it means that the resultant force is zero, that is, the wrench represents pure moment. So the wrench is written as

$$\hat{\$}_A = \psi \left\{ \begin{array}{c} \vec{u} \\ — \\ \vec{0} \end{array} \right\}. \tag{9}$$

For kinematics, these analyzes are similar, using the axial shape of the screw, Eq. (2), and the magnitude with the corresponding unit. However, only the static model will be performed in this thesis using the Davies method, therefore, only wrenches will be used. For this reason twists are not the focus of this section.

2.2 DAVIES' METHOD

The Davies Method was based on the equations of Davies, (1981) (DAVIES, 1981, 1983a, 1983b; H., 1983; DAVIES, 1995, 2000; H., 2006). It is a powerful way to solve mechanical systems statically and kinematically. It is supported by important theories such as Screw Theory, Graph Theory, and the Kirchhoff Cut-set Laws. This approach allows all efforts on all joints of the mechanism and also on the end-effector to be determined at once, including the efforts caused by the action of external forces, which is one of the advantages of the method. The kinematic and static models are obtained through the solution of the linear system and the main difference between the resolutions of the kinematic model and the static model is that in the first one uses the freedom in the mechanism joints and the Kirchhoff circulation laws, while in the second one uses constraints and nodal laws (DAVIES, 2000).

This approach is widely used by researchers to solve the kinematic and static models of mechanisms with rigid links (CAZANGI et al., 2008; FRANTZ et al., 2017), but it has also been validated for the resolution of systems actuated by cables (MURARO et al., 2015; MURARO, T.; MARTINS; SACHT, 2017), soft-robots (FRANTZ et al., 2020) and with gears (SOUZA et al., 2019). It has also been used to solve problems involving force capability (MEJIA RINCON et al., 2016; FRANTZ et al., 2020; HERRERA PINEDA et al., 2017), cooperative robots (FRANTZ et al., 2018; HERRERA PINEDA et al., 2017), vehicular dynamics (MORENO CONTRERAS; VIEIRA; MARTINS, Daniel, 2018; GUILLERMO; FLÓREZ; RAMÓN, 2022), among others. In this thesis, only the static approach solution will be used, but the approach to the kinematic analysis can be researched in several works, such as (DAVIES, 1981; H., 1983, 2006; CAZANGI et al., 2008) and other references.

2.2.1 The Davies method for statics analysis

The Davies method for statics consists of nine detailed steps (CAZANGI et al., 2008; MURARO et al., 2015), but it will be presented here in a summarized form, considering four macro steps:

1. Graphic representation;
2. Determination of the wrenches;
3. Determination of system matrices;
4. Resolution of the linear system.

In the first stage, the schematic representation of the mechanism is made, conveniently positioning the reference system. Thus, it is possible to graphically represent the kinematic chain determining the number of links l and direct couplings e. Then, is defined the coupling graph G_C represents it, given the following correspondence: each of the l links is represented by a vertex and each of the e direct couplings is represented by a directed edge. External forces and moments are also represented by directed edges. As the graph vertices are numbered according to the links in the kinematic chain, it is common to direct the edges in the ascending order of vertices. Then, each edge of G_C is replaced by c edges in parallel, which represent the c restrictions of the direct coupling that it represents, thus generating the actions graph G_A. Still in this step, you must conveniently choose a spanning tree for the actions graph, and then the ($\nu = l - 1$) strings and the ($k = e - l + 1$) cuts referring to this choice will also be determined.

Then, in the second macro step, all movement restrictions imposed by the system are specified, as well as those that come from external actions, that is, the wrench corresponding to each movement restriction is calculated according to the screw theory (BALL, 1876). According to the screw theory a wrench must be written in radial form and it depends on the following vector parameters: initial position S_0 and direction S and the scalar parameter: the step h. In a wrench, the first three components represent moments and the last three components represent forces in each direction.

The third macro step is the determination of the network unitary actions matrix $\hat{A}_N$, the main matrix of the method. For $\hat{A}_N$ to be generated, it is also necessary to determine the cuts matrix $[Q_N]_{(k \times C_n)}$, where C_n is the total number of system constraints, which is given as follows: $q_N(i,j)$=0 if edge j does not belong to cut i, $q_N(i,j)$=1 if edge j is in the same direction as cut i and $q_N(i,j)$=-1 if the edge j is in the opposite direction of the cut i. Thus it is possible to generate the network unitary actions matrix $\hat{A}_N$ replacing each element $q_N(i,j)$ with a column vector given by multiplying the element $q_N(i,j)$ by the corresponding constraint j.

Finally, the last macro step is the resolution of the homogeneous linear system whose coefficient matrix is $\hat{A}_N$. If the matrix $\hat{A}_N$ is square, the system is completely solved. But if this does not happen, it is necessary to separate the variables into primary and secondary ones, in which the primary variables are those imposed by the system, that is, with known values and the secondary variables are those to be determined. In this case, the system to be solved is non-homogeneous and the matrix of coefficients has only columns referring to secondary variables, while the matrix of independent terms is the sum of the opposite of the columns referring to primary variables. Thus, the solution will be given in matrix form, through a column vector, in which each row corresponds to one of the secondary variables determined.

3 LITERATURE REVIEW

In this chapter the types of systems with cables found in the literature are presented and some relevant papers are reviewed. The mechanical systems covered in this chapter are those actuated by cables and the tensegrity systems. The classification criteria for cable systems are also presented in this chapter.

3.1 MECHANICAL CABLE SYSTEMS

The use of robotic systems in several commercial and industrial sectors promotes the growing technological advance in which the world is inserted. However, most of these systems are still quite robust and have high cost, mass and dimension. Thus, enabling the use of these systems in differentiated and economically disadvantaged sectors is an important point to spread the technology and transform the world scenario even more. According to the literature, the systems that are able to combine the advantages of cost, mass and volume, in addition to having greater speed in the execution of tasks, are robotic systems actuated by cables (VERHOEVEN, 2004; BRUCKMANN et al., 2008a). But these systems also have disadvantages, especially in relation to control, which must be performed in real time, always maintaining the tension of the cables during a trajectory. There are also other disadvantages related to the tensioning of cables that involve their structural characteristics.

Tensegrity systems, in turn, also appear as a lighter, more flexible and adaptable option, however their robustness depends on the geometry of the system, which is usually complex and has many elements (ZHANG, J.; OHSAKI, 2015). In addition, tensegrity systems have elastic properties that make them return to their original shape after deforming due to the application of some force (SKELTON; DE OLIVEIRA, 2009). Considering, then, the disadvantages of cable-actuated systems, and tensegrity systems, it is understood that using cable instead of rigid links, without needing to drive them or add elements, is a possible solution to reduce cost, mass and size of robotic systems.

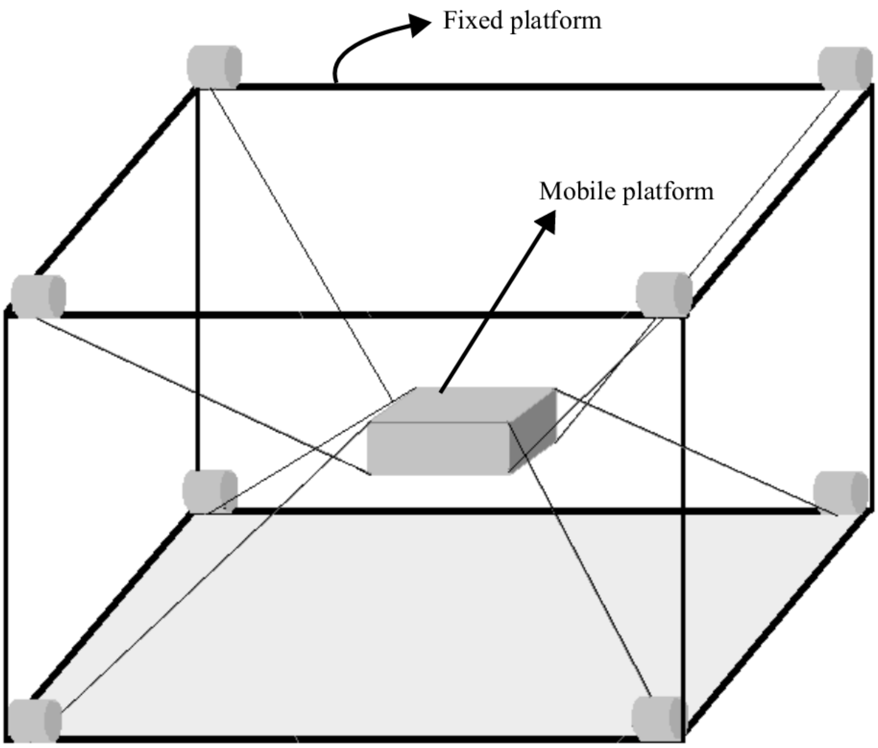
In this sense, this section presents a review about the systems with cables found in literature. Examples, advantages, problems and relevant works about systems actuated by cables and tensegrity systems are shown.

3.1.1 Mechanical Systems Actuated by Cables

As well as the parallel mechanisms of rigid structure, the ones actuated by cables have also been the subject of much interest and study in the last decades. Cable-actuated robots have a parallel structure and their constructive characteristics are similar to those of a Stewart platform (1965), however, flexible cables take the place of rigid links (BRUCKMANN et al., 2008a). They present some advantages in relation to the classic parallel robots (BRUCKMANN et al., 2008a), since they are lighter, that is, easy to move, and they can have a large number of cables, which increases the load supported and often the safety of the cargo handled. Besides that, the cable robots can perform high dynamical motion (BRUCKMANN et al., 2008b).

A robot actuated by cables (Fig. 2) consists of a mobile platform, where the end effector is positioned, a base or fixed platform, whose purpose is to sustain the load moved and give the necessary rigidity to the robot, the cables, which allow the movement of the mobile platform, and the motors or actuators that drive the movement of cables (MURARO et al., 2015). This type of robot has been studied more in the last decades, mainly because of its large workspace compared to classic parallel robots (ZHANG, Z. et al., 2022) and its structural simplicity (BOSSCHER et al., 2007), allied to the reduced production cost (BARRETTE; GOSSELIN, C. m. M., 2005; MERLET, 2004). Its application is more geared towards situations where rigid and heavy manipulators are not the best choice or for tasks where accuracy is not that important (VERHOEVEN; HILLER; TADOKORO, 1998).

Figure 2 – Basic structure of a cable driven robot.



Source: By the author, 2022.

Although this type of robot has similar characteristics to classic parallel robots, there are certain important differences (BRUCKMANN et al., 2008a). Some of them can be characterized as advantages, such as the fact that the cables can be wound up very quickly, while the moving mass of the robot is very small. This allows the robot to have very high acceleration and speeds at the end-effector (BRUCKMANN et al., 2008a). Also because the mass of the moving parts - the cables - is smaller, these robots become more energy efficient and thus suitable for moving heavier loads, even acting as cranes (POTT et al., 2013). In addition, by increasing the number of cables, you can modify the workspace, increase the load capacity or even improve the safety of what is transported. Thus, the use of a greater number of cables than the number of degrees of freedom of the end-effector is allowed. If the position of these cables is favorable, the end-effector of the mechanism can still overcome some obstacles. Also as advantages, we highlight the high capacity of handled weight, the ease of transport and the cost of construction (BARRETTE; GOSSELIN, C. m. M., 2005), and also the possibility of repositioning the connection points of the cables with the fixed base, changing its configuration (MERLET, 2004). These systems can be applied in several areas, such as civil construction (BOSSCHER et al., 2007), radio telescope (QIAN et al., 2020) (Fig. 3), camera positioning systems (SPIDERCAM, 2022; CONE et al., 1985) (Fig 4) or solar panels, structures for muscle rehabilitation (NUNES et al., 2012), on gigantic or microscale, depending on the type of task to be performed.

Figure 3 – Radio telescope FAST.

Source: Available on Carlos Serrano (2022).

The motion control of this type of robot is also not trivial, since there may be traction redundancy, that is, there may be more cables than controllable degrees of

Figure 4 – Spidercam.

Source: Available on Spidercam (2022).

freedom, and thus the distribution of traction in the cables must be evaluated (TRAVI, 2009) and the way its elasticity influences movement (BRUCKMANN et al., 2008a). Furthermore, in cable-suspended mechanisms there are problems regarding the trajectory of the mobile platform, which cannot be completely controlled (GOSSELIN, C., 2014), since they are generally under-actuated. The kinematics of these manipulators are also complex and to control them it is very important to find a strategy where real-time computation is efficient (TRAVI, 2009).

The construction of robots actuated by cables, even for commercial purposes as is the case of Spidercam (SPIDERCAM, 2022; CONE et al., 1985), and the theoretical studies that involve them have also developed a lot. The basic theory has been widely studied in this century mainly by Verhoeven, Merlet, Pott, Bruckmann and Gosselin (VERHOEVEN; HILLER; TADOKORO, 1998; VERHOEVEN, 2004; MERLET, 2004, 2013; POTT et al., 2013; POTT; BRUCKMANN, 2018; BRUCKMANN et al., 2008a, 2008b; GOSSELIN, C., 2014). In the last decade, complete reviews of the state of the art and studies on cable actuated systems have been published: in 2013 by Merlet (MERLET, 2013), 2014 by Gosselin (GOSSELIN, C., 2014), 2018 by Pott (POTT; BRUCKMANN, 2018), 2022 by Zhang (ZHANG, Z. et al., 2022) and others.

3.1.1.1 Classification of Cable Actuated Systems

Cable actuated systems have already been classified according to different factors and by different authors. Ming and Higuchi separated them in 1994 (MING A; HIGUCHI, 1994) into Completely Restrained Parallel Manipulator (CRPM) and Incompletely Restrained Parallel Manipulator (IRPM). Ten years later, Verhoeven (VERHOEVEN, 2004) and Riechel (RIECHEL, A. et al., 2004) separated the two classes, adding the Redundantly Restrained Parallel Manipulator (RRPM) and Sub-constrained Parallel Manipulator (SCPM) categories. The classification was made according to the relationship between the number of cables m and the number of degrees of freedom n of the manipulator:

- *Completely Restrained Parallel Manipulator - CRPM*:

$$m = n + 1. \tag{10}$$

  The position of the mobile platform is determined by the kinematic constraint defined by the traction of the cables. See Fig. 5(b).

- *Sub-constrained Parallel Manipulator - SCPM*:

$$m < n. \tag{11}$$

  It is rarely used because it cannot achieve a stable tensegrity structure and the end effector has uncontrollable DoFs (ZHANG, Z. et al., 2022).

- *Incompletely Restrained Parallel Manipulator - IRPM*:

$$m = n. \tag{12}$$

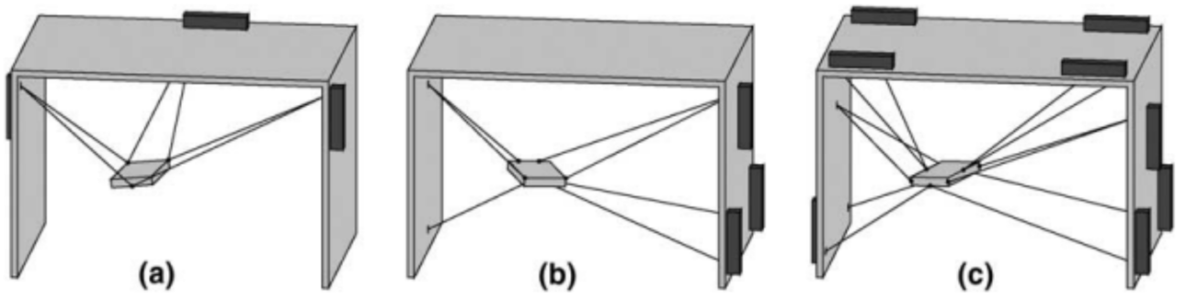
  In addition to the kinematic constraint defined by the cable, a dynamic equation is needed to position the platform. However, if gravity is considered a virtual cable, it can be considered a fully constrained cable driven parallel with limited acceleration (Fig. 5(a)).

- *Redundantly Restrained Parallel Manipulator - RRPM*:

$$m > n + 1. \tag{13}$$

  The position of the mobile platform in this type of manipulator is completely determined by the number of cables, therefore having more than one redundant cable. See Fig. 5(c).

Figure 5 – Classification by number of cables: (a) IRPM; (b) CRPM and (c) RRPM.



Source: Available on Hiller et al. (2005).

Verhoeven, in 1998 (VERHOEVEN; HILLER; TADOKORO, 1998), classified the CRPMs according to the number of controllable degrees of freedom and the type of end-effector movement, as shown in the Table 1.

Table 1 – The classes of CRPMs.

DoFs	*m*	*n*	**Type of motion**
1T	1	2	linear motion of a body
2T	2	3	planar motion of a point
1R2T	3	4	planar motion of a body
3T	3	4	spatial motion of a point
2R3T	5	6	spatial motion of a line
3R3T	6	7	spatial motion of a body

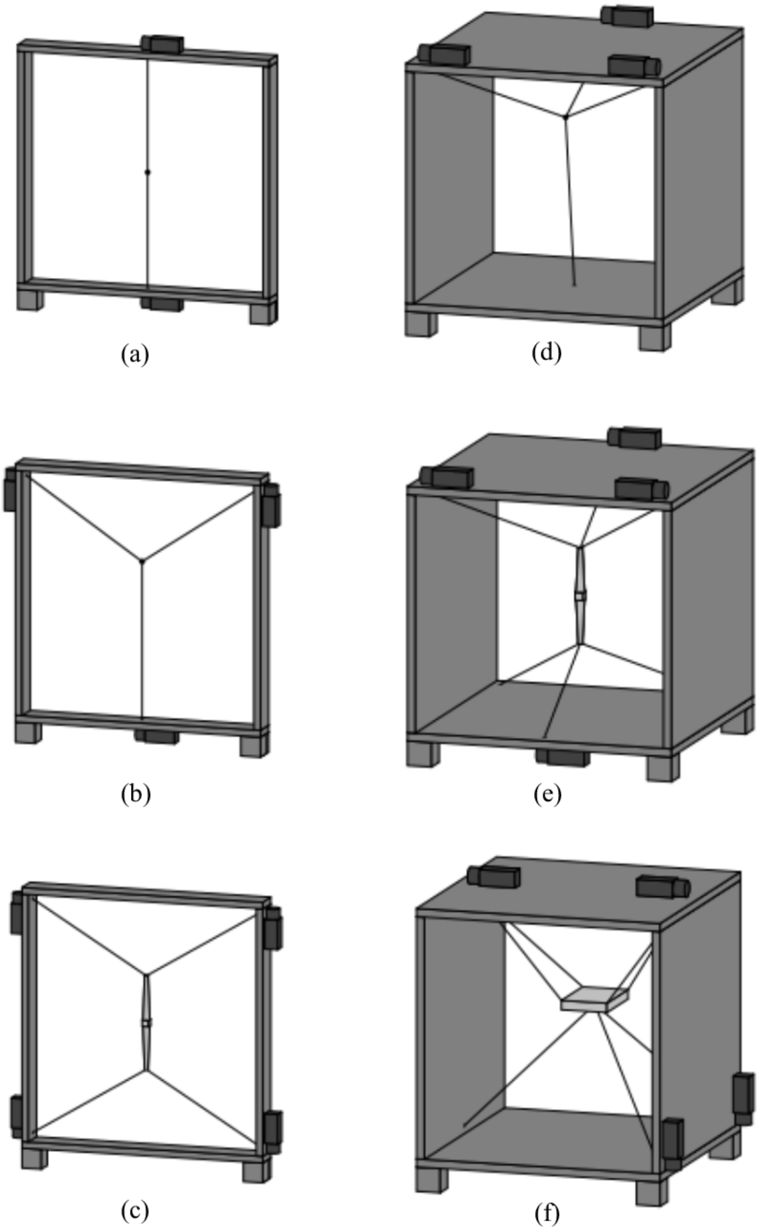
Source: (VERHOEVEN; HILLER; TADOKORO, 1998).

In 2004, Verhoeven (VERHOEVEN, 2004) also classified the cable-driven Stewart platforms according to the number of controllable degrees of freedom and the type of end-effector movement, as shown in the Table 2 and in the Fig. 6, which presents cable configurations that provide the described movements. In fact, this classification is the same as the one made in 1998. However, here it does not consider the number of cables, which makes it a general classification, considering only the degrees of freedom of the end-effector.

Merlet, in 2013 (MERLET, 2013), classified cable actuated systems into two types: Type 1 and Type 2, as shown Fig. 1. What differentiates the two types is the way in which the movement of the mobile platform takes place. In type 1, the movement of the mobile platform is generated by modifying the length of the cables which is caused by the winding drums. In type 2, the movement of the mobile platform occurs by changing the point of attachment of the cables to the fixed platform.

Motivated by the performance of these devices, which requires cables to be always tensioned, Gosselin (GOSSELIN, C., 2014) deals only with two classes of cable-

Figure 6 – Combinations of end-effector DoFs which a tendon-based Stewart platform: (a) Linear motion of a body; (b) Planar motion of a point; (c) Planar motion of a body; (d) Spatial motion of a point; (e) Spatial motion of a bean; (f) Spatial motion of a body.



Source: Available on Verhoeven (2004).

actuated mechanisms: fully-constrained mechanisms - FCMs and cable-suspended parallel mechanisms - CSPMs. What exactly differentiates the two classes is the tension in the cables, which in the first is generated by the device itself – by the other cables – and in the second by the action of gravity, as the cable system of Fig. 5a). The difference is visually perceptible through the way the cables are positioned, both in relation to the

Table 2 – The combinations of end-effector DoFs which a tendon-based Stewart platform.

DoFs	End-effector DoFs	Type of motion
1T	1	linear motion of a body
2T	2	planar motion of a point
1R2T	3	planar motion of a body
3T	3	spatial motion of a point
2R3T	5	spatial motion of a bean
3R3T	6	spatial motion of a body

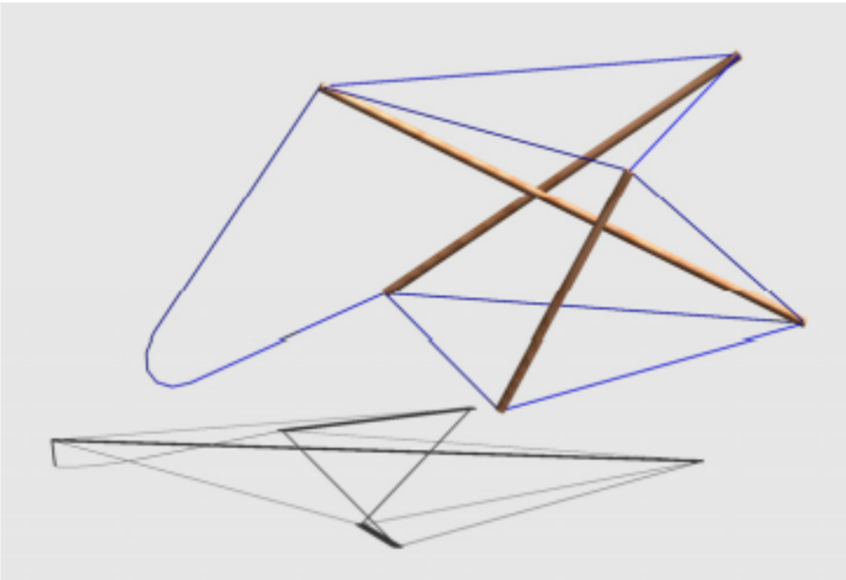
Source: (VERHOEVEN, 2004).

attachment points to the mobile platform and to the fixed platform, as can be seen in the Fig. 5, in which Fig. 5(b) and Fig. 5(c) represent FCMs, while the Fig. 5(a) represents a CSPM.

3.1.2 Tensegrity Systems

Although there are indications about the creation of tensegrity structures in the 1920s (Fig. 7), the first patents were registered, according to Gómez-Jáuregui (GÓMEZ-JÁUREGUI, 2009), in the 60s by Richard Buckminster Fuller (FULLER, 1962), David Georges Emmerich (EMMERICH, 1963) and Kenneth D. Snelson (SNELSON, 1965), in that chronological order.

Figure 7 – "Gleichgewichtkonstruktion" or “Structure-Sculpture” by Karl Ioganson, first proto-tensegrity system, 1920.

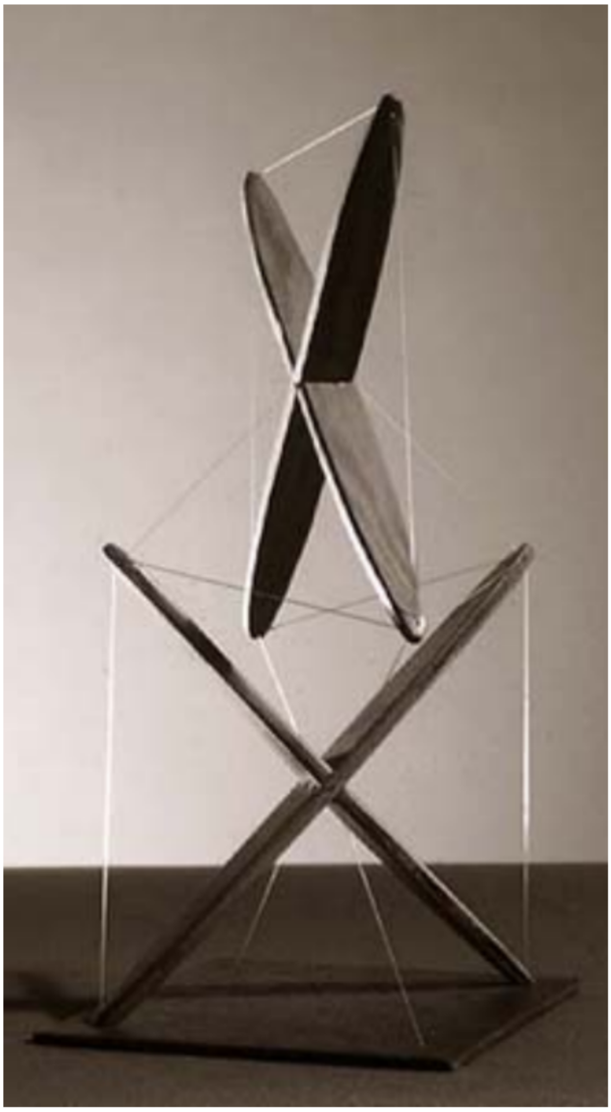


Source: Available on Gómez-Jáuregui (2009).

On the other hand, according to Bansod (BANSOD; NANDANWAR; BURŠA,

2014) and Zang (ZHANG, J.; OHSAKI, 2015) the module-X (see Fig. 8) built by Snelson gave rise to the tensegrity principle. This simple structure consists of two X-shaped wooden struts suspended in the air by stretched nylon cables.

Figure 8 – "X-column" by Snelson, his first tensegrity art piece.



Source: Available on Gómez-Jáuregui (2009).

Even with many studies, works and decorative commercial products (see Fig. 9) involving tensegrities, there is no strict definition of tensegrity structures that is accepted by all people (ZHANG, J.; OHSAKI, 2015). According to Bansod (BANSOD; NANDANWAR; BURŠA, 2014), probably the most complete definition of tensegrity structures is presented by Gómez-Jáuregui (GÓMEZ-JÁUREGUI, 2004) as “Tensegrity is a structural principle based on the use of isolated components in compression inside a net of continuous tension, in a way that the compressed members (usually known as bars/struts) do not touch each other and the prestressed tensioned members (usually known as cables/tendons) delineate the system spatially". However, according to Zhang, (ZHANG, J.; OHSAKI, 2015) it is generally accepted that a tensegrity structure should have the following characteristics:

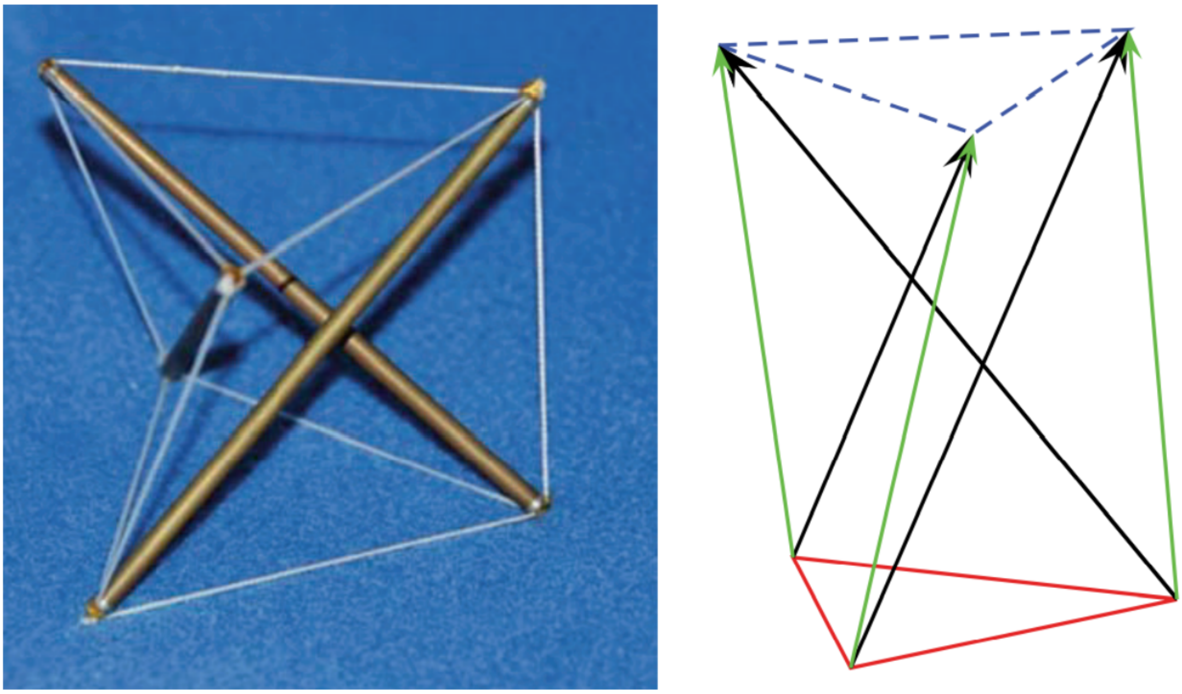
Figure 9 – Decorative tensegrity structure.

Source: Available on vegdecor (2022).

- The structure is free-standing, without any support.
- The structural members are straight.
- There are only two different types of structural members: struts carrying compression and cables carrying tension.
- The struts do not contact with each other at their ends.

Tensegrity structures (Fig. 10) are similar in appearance to conventional truss structures, however, their members carry forces (prestresses) even when no load is applied. This requires the limbs to be balanced in order to maintain their balance. Furthermore, most tensegrity structures are naturally unstable and it is precisely the introduction of prestresses that makes them stable (ZHANG, J.; OHSAKI, 2015). Tensegrities are usually modeled with frictionless joints, disregarding the self-weight of cables and struts (BANSOD; NANDANWAR; BURŠA, 2014).

Figure 10 – Perspective view and diagram of a regular minimal tensegrity prism.



Source: Available on Skelton and De Oliveira (2009).

Inspired by the natural world, tensegrity structures combine characteristics of rigid structures and soft structures making robots more versatile, realistic, and compatible for human interaction. However, the combination of soft materials and rigid materials, despite being a solution to improve the performance of robots, presents considerable challenges for the design, construction and control of robots based on tensegrity structures (LIU et al., 2022). Furthermore, when elementary tensegrity modules are conveniently joined they can compose even more complex tensegrity structures. As the structure is large, it will not support loads greater than the critical ones and the bars may start to collide or touch each other (BANSOD; NANDANWAR; BURŠA, 2014).

Bansod, in 2014 summarized the characteristics of tensegrity structures, also highlighting some advantages and disadvantages. Most of the highlighted advantages are related to its spatial arrangement, which prevents torsion and buckling and provides load distribution throughout the structure, causing there to be no critical points of weakness and also absorbing seismic shocks and vibrations, which makes them applicable as sensors or actuators. In addition, according to him, tensegrity structures are resilient (depending on the assembly and material used), lighter compared to others with the same resilience and very economical. However, there are also disadvantages related to its spatial arrangement due to the manufacturing complexity. Furthermore, they have relatively high deflections and low material efficiency compared to conventional continuous structures. And when it comes to tensegrity robots, it can be said that their design space is still not well understood, and precise control is very difficult due to the complicated interactions between components (LIU et al., 2022), as their behavior cannot be predicted. Considering the behavior of any of its components separately (BANSOD; NANDANWAR; BURŠA, 2014).

Tensegrities can be classified according to the number of bars adjacent to each other, denoting Class 1, Class 2, Class 3 and so on (SKELTON; DE OLIVEIRA, 2009; LIU et al., 2022). They can also be divided into three main categories, according to geometric characteristics: Tensegrity Prism (T-prism), Diamond Tensegrity (T-diamond) and Zig-zag Tensegrity (Z-type) (BANSOD; NANDANWAR; BURŠA, 2014). Tensegrity robots, mostly with class 1 structures, are classified according to the reference objects (SKELTON; DE OLIVEIRA, 2009; LIU et al., 2022). The main classes are prismatic tensegrity robots, spherical tensegrity robots, musculoskeletal humanoid tensegrity robots, and bio-inspired tensegrity robots.

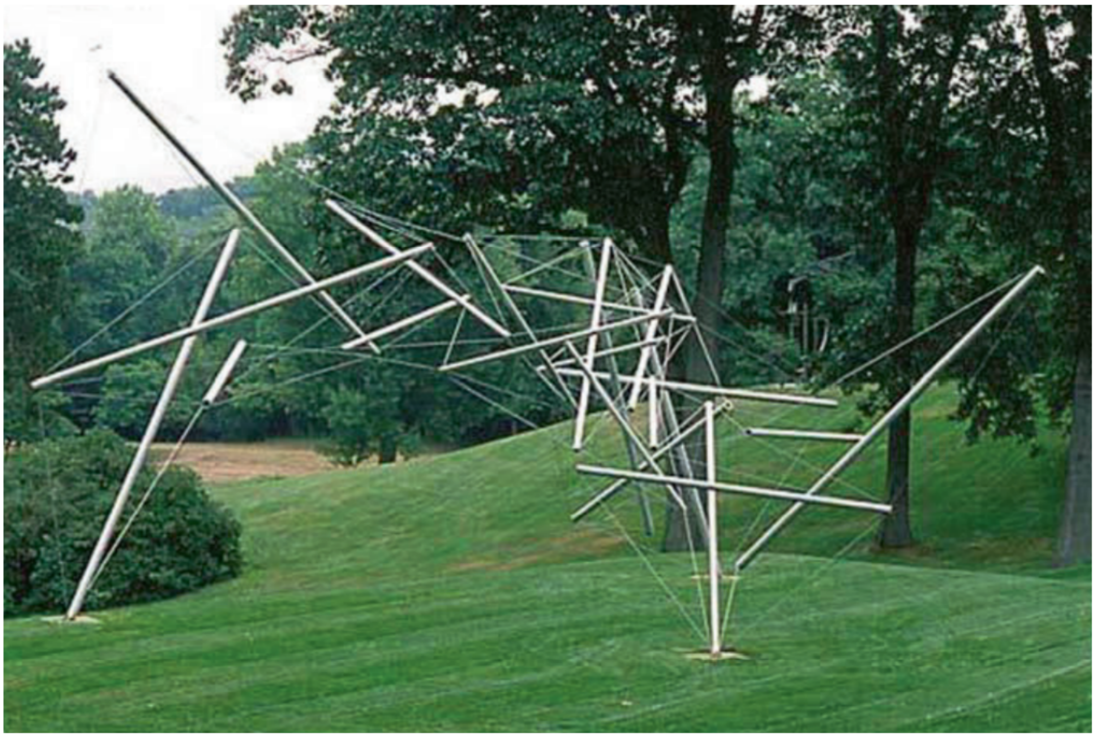
Although tensegrity structures have been around for (almost) a century, their application in robotics is still relatively recent (about a decade). Research is mainly focused on basic theories and there are few tensegrity robots to date (mostly class 1), as most of what has been developed has not been applied in practice in engineering fields (LIU et al., 2022). Most applications are representations of nature, including on the nanoscale (SKELTON et al., 2001; BANSOD; NANDANWAR; BURŠA, 2014; LIU et al., 2022), or are in art (Fig. 11) and architecture (Fig. 12) (SKELTON et al., 2001; BANSOD; NANDANWAR; BURŠA, 2014; LIU et al., 2022; SNELSON, 2022). Thus, the development of tensegrity robots is still an open field. However, it is believed that tensegrity robots may have potential for applications in several areas such as space exploration, inspection and rescue, rehabilitation, assistive technology, bionic and deployable robots (LIU et al., 2022).

3.1.2.1 Classification of Tensegrity Systems

Tensegrity systems are generally classified according to two distinct criteria: according to the number of connecting bars (SKELTON; DE OLIVEIRA, 2009; LIU et al., 2022) or according to their general shapes or reference objects (SKELTON; DE OLIVEIRA, 2009; BANSOD; NANDANWAR; BURŠA, 2014; ZHANG, J.; OHSAKI, 2015; LIU et al., 2022).

A tensegrity structure as described in 3.1.2 has some fundamental characteristics and one of them is that “The struts do not contact with each other at their ends”. However, any tensegrity with this characteristic is called class 1 tensegrity, precisely

Figure 11 – Class 1 tensegrity structure in art.



Source: Available on Snelson (2022).

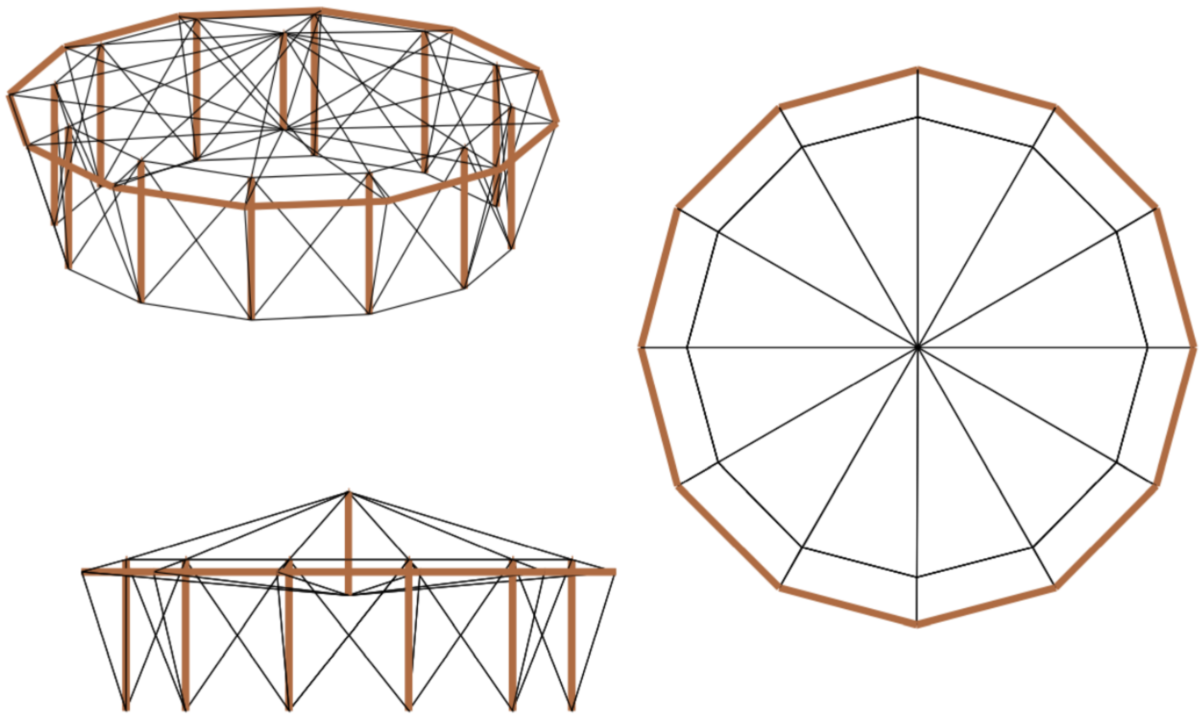
because there is no direct connection between the struts (Fig. 10). Thus, a tensegrity is said to be of class *p* if it has a maximum of *p* interconnected compression struts (SKELTON; DE OLIVEIRA, 2009; LIU et al., 2022). In this sense, in the class 2 tensegrity structure the connections between struts must be two to two. If the tensegrity is class 3 then there is at least one set of three struts that connect to each other and so on. See a example for Class 2 in Fig. 12.

According to its appearance and geometric construction, a tensegrity system (BANSOD; NANDANWAR; BURŠA, 2014) can also be classified into three classes: T-prism, T-diamond and Z-type.

A Tensegrity prism can be thought of as a twisted prism consisting of two faces twisted relative to each other (BANSOD; NANDANWAR; BURŠA, 2014). They are composed of p struts and usually at least $3m$ strings that form m-sided polygons at the top and bottom, such as triangular tensegrity prism with 3 struts and 9 strings, and quadrilateral tensegrity prism with 4 struts and 12 strings (LIU et al., 2022). The first T-prism was invented by Karl Ioganson in Moscow in 1921 (Fig: 10).

A diamond tensegrity is so classified as each of its stems is surrounded by a diamond shape of four tendons that are supported by two adjacent struts (BANSOD; NANDANWAR; BURŠA, 2014). This is a class 2 tensegrity structure, but there is a string-to-string connection (joint) where the vertical and horizontal strings intersect

Figure 12 – A class 2 tensegrity shelter.



Source: Available on Skelton and De Oliveira (2009).

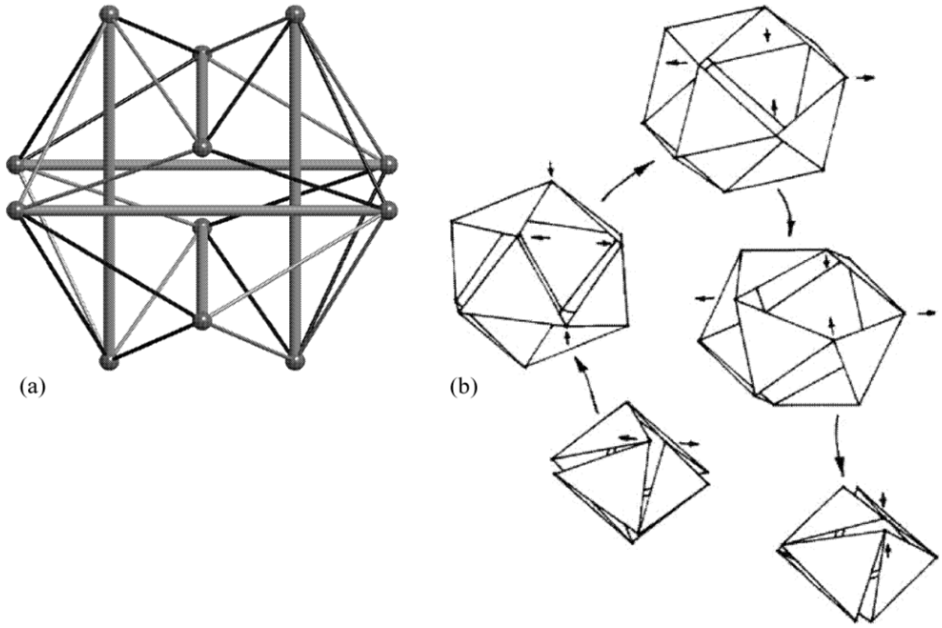
(SKELTON; DE OLIVEIRA, 2009). The tensegrity icosahedron also known as the T-icosahedron depicted in Fig. 13 is a classic example of diamond tensegrity. These tensegrities are characterized by the fact that each triangle of tendons is connected to the adjacent one by means of a strut and two interconnected tendons. It was first displayed by Buckminster Fuller in 1949 and is one of the few tensegrities that exhibit mirror symmetry (BANSOD; NANDANWAR; BURŠA, 2014).

A Zig-zag tensegrity (Z-type) or tensegrity tetrahedron (see Fig. 14) also has 6 supports like the T-diamond, but the main difference between them is that the T-tetrahedron has four tendinous triangles, while the T-icosahedron (see Fig: 13) has eight of them. In general, Z-configuration zig-zag structures are simpler and less rigid due to fewer tendons than their rhombic-configuration diamond counterparts.

Tensegrity robots are classified according to their reference objects (LIU et al., 2022). They can be divided into prismatic tensegrity robots, spherical tensegrity robots, musculoskeletal humanoid tensegrity robots, and bio-inspired tensegrity robots.

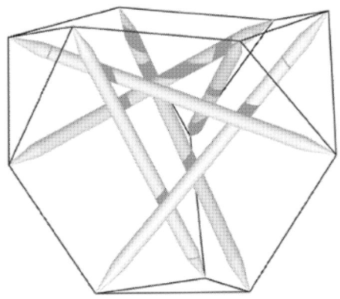
As the name implies, prismatic tensegrity robots are designed based on T-prism structures. As triangular and quadrilateral prisms have the simplest tensegrity structures, the prismatic tensegrity robots introduced in the literature start with this type of structure (LIU et al., 2022). Several prismatic tensegrity robots can be seen in Fig. 15.

Figure 13 – Diamond tensegrity: (a) T-icosahedron and (b) its corresponding transformation from and back to doubled-up octahedron.



Source: Available on Bansod, Nandanwar, and Burša (2014).

Figure 14 – Z-type configuration: T-Tetrahedron.

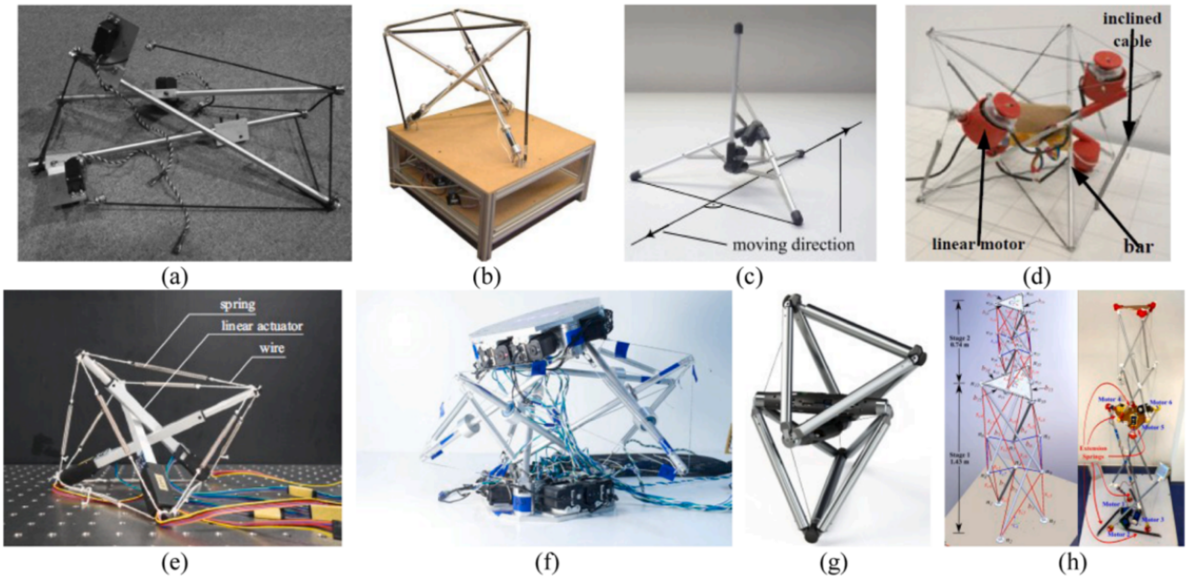


Source: Available on Bansod, Nandanwar, and Burša (2014).

Spherical tensegrity robots are shaped like a sphere. By changing the position of the center of mass, spherical tensegrity robots are able to roll under the actions of gravitational energy and tensile potential, making them suitable as mobile robotic platforms (LIU et al., 2022). A spherical tensegrity robot is usually cable driven (LIU et al., 2022). Figure 16 shows some examples of spherical tensegrity robots.

Humanoid musculoskeletal tensegrity robots (see Fig. 17) are representations of human body parts that are made up of bones, muscles, tendons, organs, and fascia, a type of soft tissue made up of connective tissue that accounts for about 20% of

Figure 15 – Prismatic tensegrity robots: (a) Triangular tensegrity prism; (b) 3-strut prismatic tensegrity robot; (c) Uniaxial tensegrity robot(d) quadrilateral tensegrity prism-based robot; (e) Triangular prism tensegrity robot; (f) 6-strut tensegrity robot TenseBot as flight simulator; (g) Duct Climbing tetrahedral Tensegrity robot; (h) Two-stage stacked tensegrity manipulator.



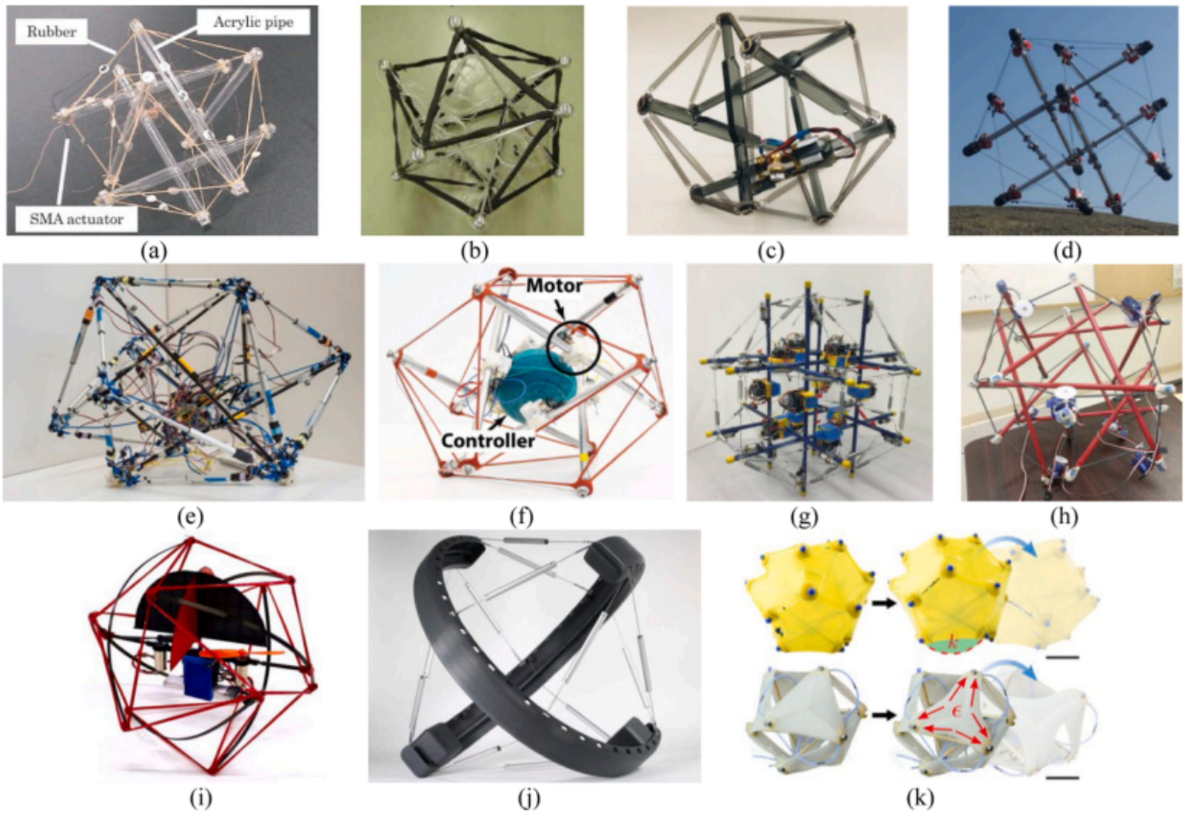
Source: Available on Liu et al. (2022).

the body's weight. Fascia surrounds the muscles, tendons and organs, and connects them with the bones, forming a network of continuous tension throughout the body with the bones suspended in it, which is exactly the characteristic of a tensegrity (LIU et al., 2022) structure. Therefore, humanoid musculoskeletal tensegrity robots provide a better framework to explain why all biological structures are structurally stable and have flexible adaptability to avoid injury (LIU et al., 2022).

Figure 18 shows some biologically inspired tensegrity robots. They describe musculoskeletal structures of animals but can be used for different purposes. For example, some fish are capable of efficient, highly maneuverable and silent swimming movements, making them ideal candidates as bio-inspired autonomous underwater vehicles. Vertebrates, including amphibians, reptiles, birds, and mammals, have the ability to change the stiffness of the spine to increase flexibility or load-bearing capacity, which inspired the development of artificial columns of variable stiffness (LIU et al., 2022).

Tensegrity robots, like tensegrity structures in general, can be classified according to more than one criterion. So, for example, the tensegrity system shown in Fig. 18a) is a class 3 Bio-inspired tensegrity robot and the tensegrity system in Fig. 15a) is a class 1 prismatic tensegrity robot. Similarly, the tensegrity structure shown in Fig. 13 is a class 1 T-diamon.

Figure 16 – Spherical tensegrity robots: (a) Cable-actuated by shape memory; (b)cable-actuated by pneumatic actuators; (c) cable-actuated by motors; (d) SUPER-ball v2 robot; (e) TT-2 robot; (f) TT-4mini robot; (g) T12-R robot; (h) 12-strut spherical tensegrity robot; (i) Soft tensegrity; (j) Compact shape morphing tensegrity robot; (k) Membrane-driven spherical tensegrity robot.



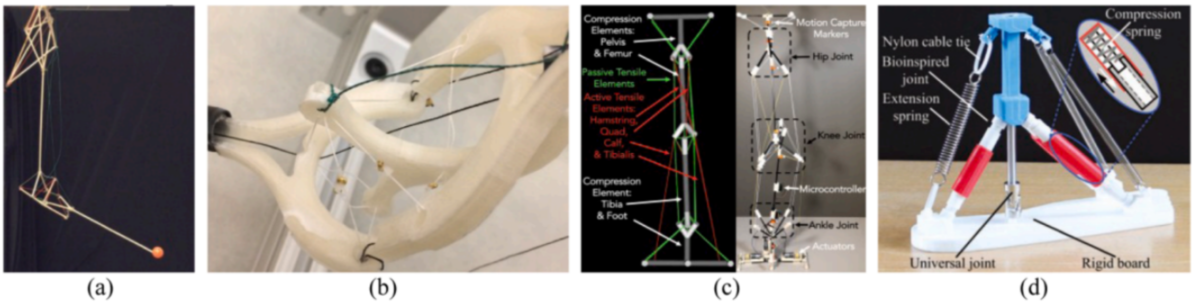
Source: Available on Liu et al. (2022).

3.2 CABLE TENSION IN MECHANICAL SYSTEMS

Cables in robotic systems require permanent tension. A sagged cable is unable to do any task nor to assist in the movement, resistance, or even balance of the system. When one of the cables is not pulled, there may be malfunctions, positioning errors, excessive wear on some joints, or even accidents, depending on the type of system. Thus, a crucial problem in cable systems is their workspace, which is necessary to keep cables tensioned during a given trajectory (MERLET, 2013; CUI et al., 2019). Another issue of tension distribution in cable systems is about their maximum and minimum values in which if the tension value is low, the cable is saggy and unbalances the system (light magenta regions). On the other hand, if the tension value is too high, the cable may break (orange regions). Figure 19 describes these tensions.

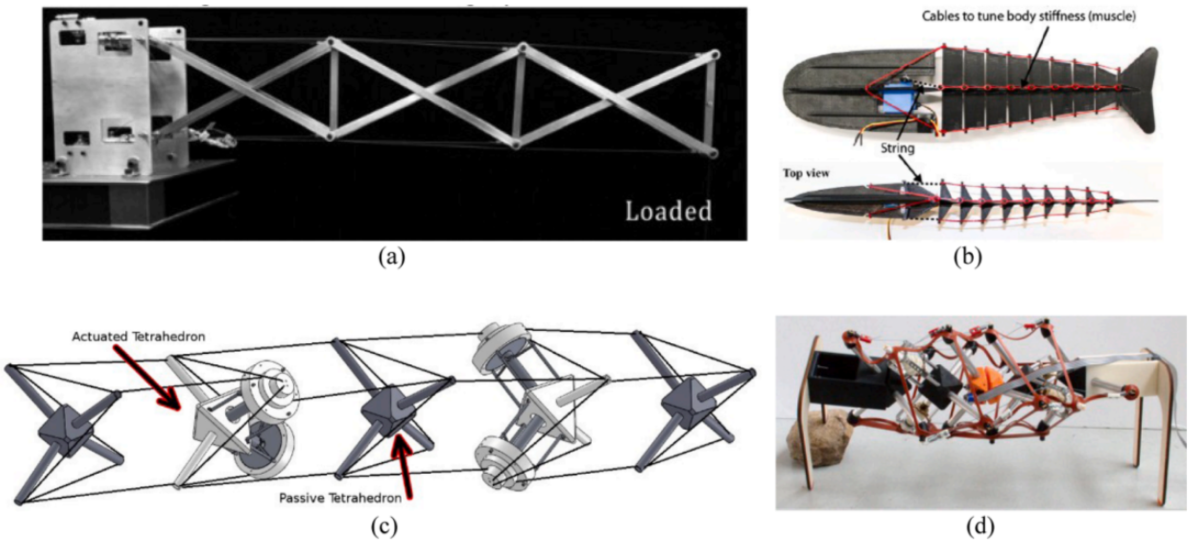
Several works found in the literature discuss cable tension (BOSSCHER; RIECHEL,

Figure 17 – Humanoid musculoskeletal tensegrity robots: (a) Modular humanoid tensegrity robotic arm; (b) Wrist mechanism by stacking up two class 1 tensegrity structures sequentially; (c) Tensegrity structure-based lower extremity prototype; (d) Foot mechanism.



Source: Available on Liu et al. (2022).

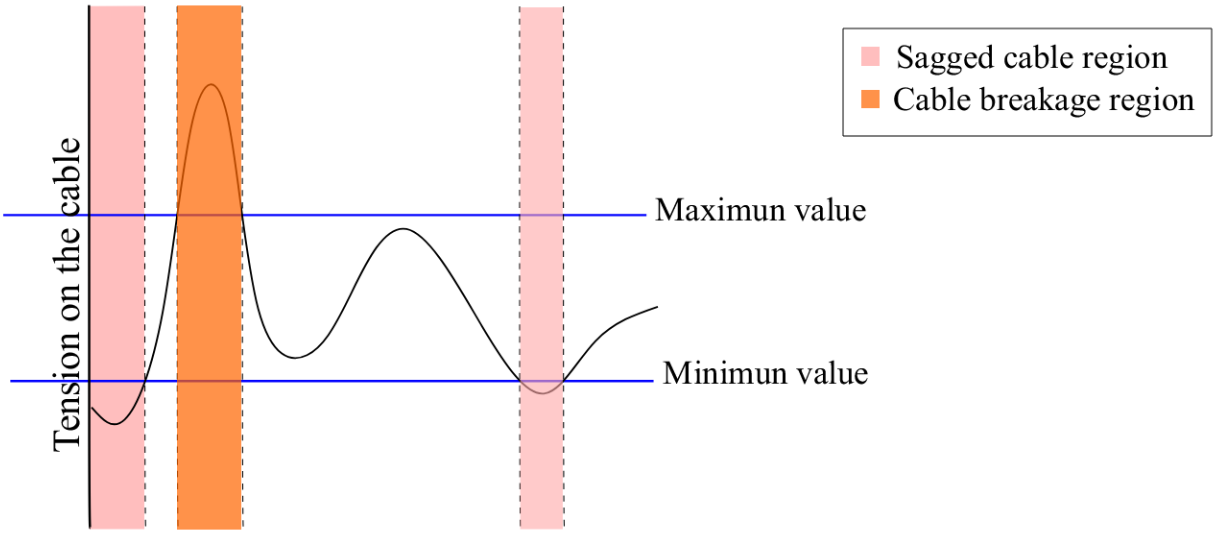
Figure 18 – Bio-inspired tensegrity robots: (a) High performance artificial pectoral fin; (b) Tensegrity robotic fishes; (c) Assistive Spine (ULTRA Spine), for quadruped robots; (d) Quadruped robot Laika.



Source: Available on Liu et al. (2022).

A. T.; EBERT-UPHOFF, 2006; GOUTTEFARDE; MERLET; DANEY, 2007; PHAM et al., 2009; MERLET, 2013; MURARO et al., 2015; MURARO, T.; MARTINS; SACHT, 2017; CUI et al., 2019; SKELTON et al., 2001; WILLIAMSON; SKELTON; HAN, 2003; ZHANG, J.; OHSAKI, 2015), but few works deal with the study of cable tension using a network search approach (MURARO et al., 2015; MURARO, T.; MARTINS; SACHT, 2017). In this thesis, the tensions of the cables will be treated in a vectorial way, evaluating the

Figure 19 – Regions of the maximum and minimum tensions.



Source: By the author, 2022.

efforts in the joints of the mechanisms in relation to the external forces applied.

4 MECHANISM PROPOSAL: INNER-CABLE MECHANISM

As presented in Section 3.1 of this work, the literature review was carried out on mechanical systems with cables for the studies of this thesis. From this review, it was possible to analyze two types of systems: the cable-driven systems and the tensegrity systems. According to what was presented in the 3.1.1 and 3.1.2 subsections, it can be seen that most tensegrity robots are also actuated by cables, except for those whose mobility is zero, which in this case do not have any type of acting. Furthermore, in the recently published review by Liu (LIU et al., 2022), 28 tensegrity robots were analyzed. Of these, only four are not actuated by cables: the first has linear actuators and elastic cables (MIRATS-TUR; CAMPS, 2011) and is shown in the Fig. 20; the second can be seen in Fig. 15(e), it has struts and cables with variable length (YAGI et al., 2019); and the last two are of the spherical type, that is, they roll without changing the general external shape, one (Fig. 16(i)) relies on movement through the change of the center of mass (MINTCHEV et al., 2018) and the other (Fig. 16(k)) with pneumatic membrane actuators (BAINES; BOOTH; KRAMER-BOTTIGLIO, 2020).

Figure 20 – Two snapshots from the vertical trajectory of the structure: (a) initial configuration (b) final configuration.

(a)

(b)

Source: Avaliable in Mirats-Tur and Camps (2011).

Therefore, it was found that mechanical systems with non-zero mobility and that combine fixed length non-actuated cables and rigid links are not formally defined in the literature. Thus, in the next subsections, a nomenclature for this type of mechanism will be proposed, as well as a classification of the cable mechanisms in which it is included.

4.1 INNER-CABLE MECHANISM (ICM)

In this section, we propose the following definition for the Inner-cable mechanism:

Any mechanism composed of rigid links, actuated joints and inextensible cables not driven by pulleys or winding drums (or similar) can be considered an Inner-cable mechanism (ICM).

In this case, given any mechanism with rigid links, the replacement of at least one of its links by an inextensible cable makes it an Inner-cable mechanism. Thus, in addition to the mechanisms generated through the method of replacing rigid links with cables proposed in this work, the mechanisms actuated by cables Type 2, 1(b), according to Merlet's classification (MERLET, 2013) can also be considered Inner-cable mechanisms, because the movement of the mobile platform occurs by changing the point of attachment of the cables to the fixed platform, without changing the cable length.

It is important to note that although we consider here the mechanisms actuated by type 2 cables as inner-cable mechanisms, the opposite is not true, since the inner-cables mechanisms described here cannot be considered mechanisms actuated by cables, since they are structurally different, that is, they do not necessarily have a mobile platform or end-effector whose movement is exclusively due to the movement of the cables.

Likewise, it is possible to refer to inner-cable robots, inner-cable manipulators or, simply, inner-cable systems. In this sense, the concept of inner-cable is understood as shown in Fig. 21.

Note that in the Venn diagram shown in Fig 21 the intersection between cable-driven, tensegrity and inner-cable is empty. In fact, for there to be at least one element at this intersection, suppose there is a tensegrity structure that satisfies the following conditions:

a) the movement of its mobile platform or end-effector takes place through the activation of the cables (to belong to the cable-driven set);

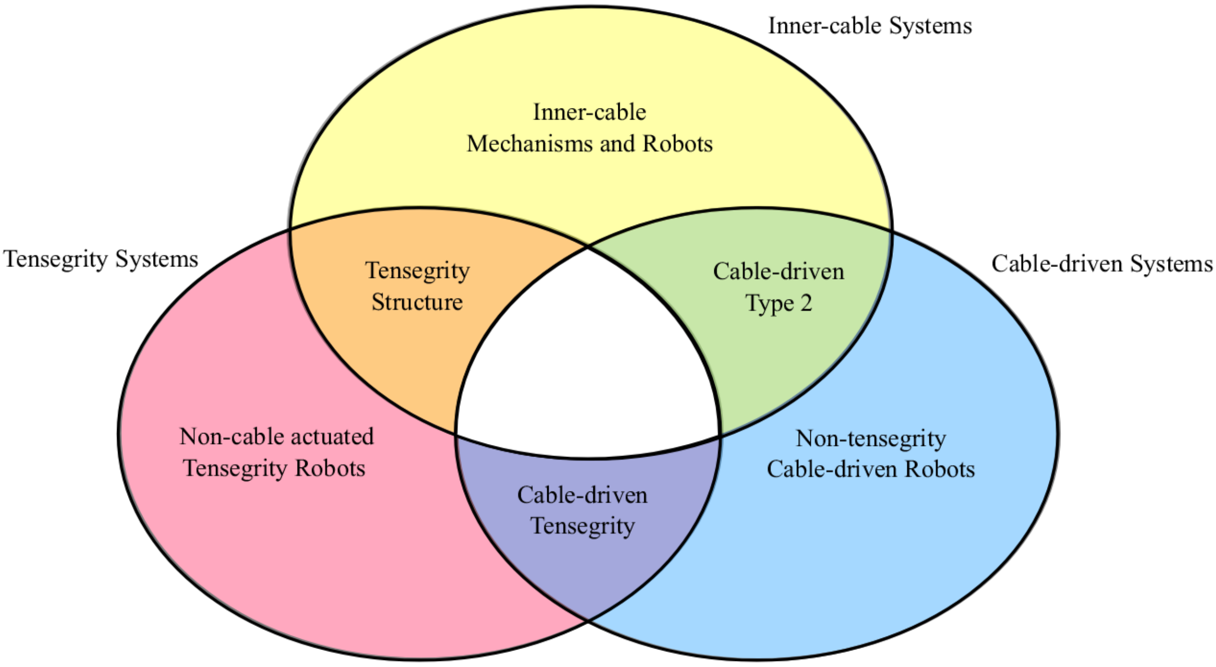
b) their cables do not change the length (to belong to the inner-cable set).

Under these conditions, the tensegrity structure starts to have some type of activation, thus ceasing to be a structure, which is a contradiction. It is verified, therefore, that the intersection between tensegrity, cable-driven and inner-cables is empty.

4.2 PROPOSAL FOR CLASSIFICATION OF CABLE SYSTEMS

As presented in the mechanics with cables section, cable systems have already been described and classified by several researchers. However, the classification of sys-

Figure 21 – Insertion and interposition of the inner-cable mechanism in the set of cable systems.



Source: By the author, 2022.

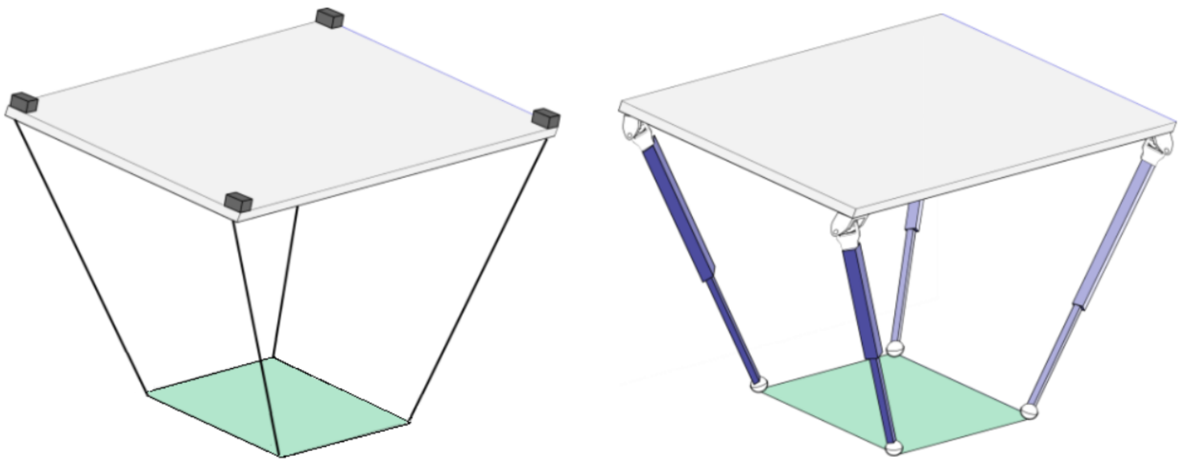
tems always separates them into cable-driven or tensegrity. In this section, a new form of classification is proposed, which includes all systems that contain cables, including those that are not cable-actuated but that are also not considered tensegrity.

The classification proposed here is according to the cable length variation. In fact, as presented in Chapter 3, a cable can have a fixed or variable length, and the variation of its length can occur through pulleys or winding drums, or it can also occur due to the elasticity of the cable.

This classification can also be given through the representation of the cables for modeling purposes, because, when there is a variation in the cable length, it is common to represent it as a set of spherical-prismatic-spherical (*SPS*) joints, or replacing one or both spherical joints by universal joints in order to ignore the rotation around the cable itself (MURARO et al., 2015; MURARO, T.; MARTINS; SACHT, 2017). This fact is described in Fig. 22. In the case of tensegrity structures, for example, in which the cable does not change in length, they are commonly modeled with frictionless joints, and the self-weight of the cables and struts is neglected (BANSOD; NANDANWAR; BURŠA, 2014).

In this sense, the proposed classification considers all systems that contain cables in their construction, from structures to mobile devices. In each cable system, the cables included in the mechanisms that compose them are considered. In this way, it is

Figure 22 – Cable-driven mechanism: (a) Cable suspended mechanism, (b) Cable representation through the UPS joint set.



Source: Adapted from Muraro et al. (2015).

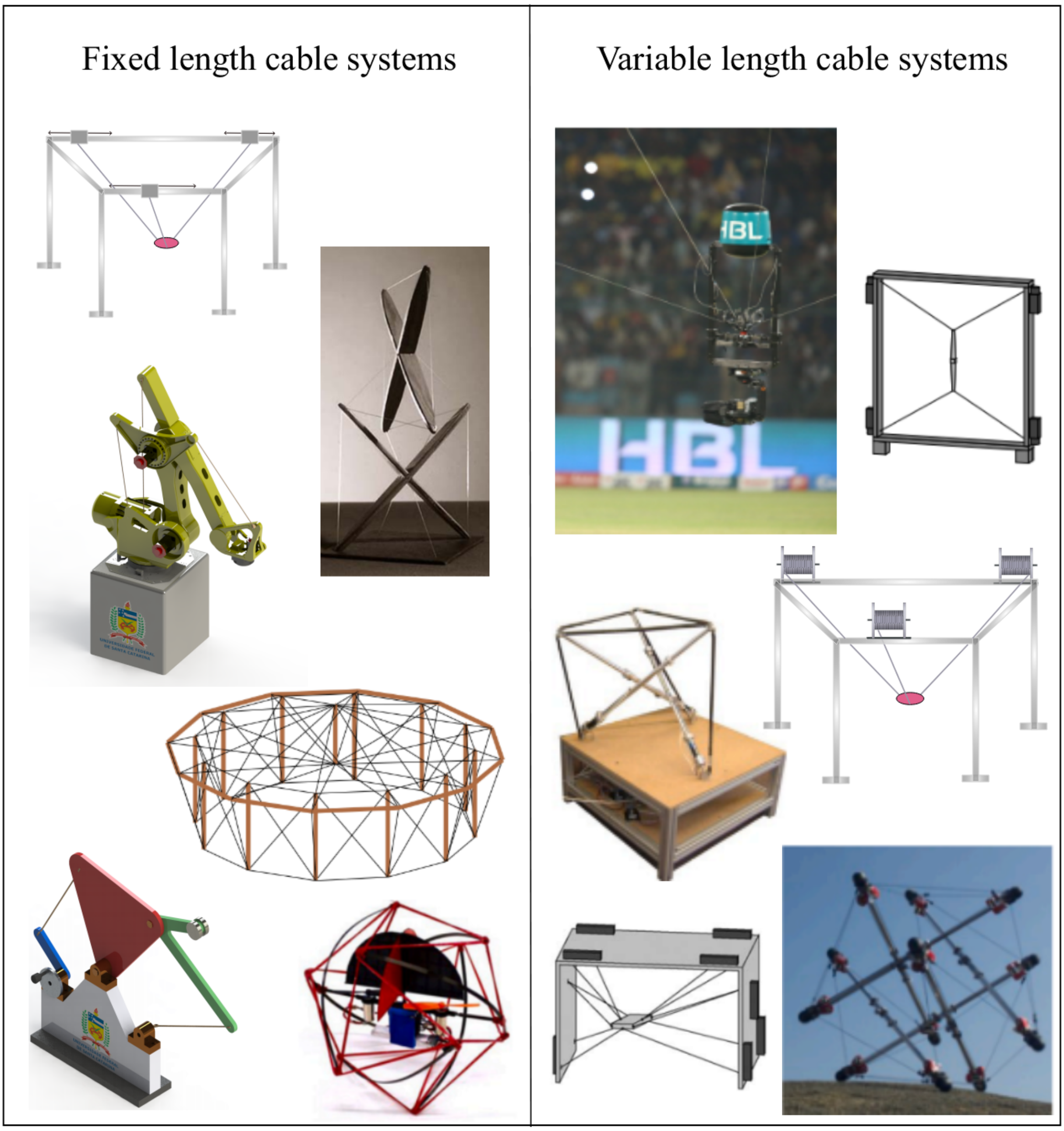
possible to include the Inner-cable mechanisms. The proposed classification considers only two classes: the Fixed length cable systems class and the Variable length cable systems class, as described below.

Classification of cable systems according to cable length variation:

- ***Fixed length cable systems***: this class includes systems that contain cables whose cables do not change their length. In this sense, such cables cannot be wound by winding pulleys or drums, nor can they be elastic. This class includes inner-cable mechanisms, tensegrity structures and type 2 cable-actuated systems. Here, some types of tensegrity robots can also be inserted, which are based on tensegrity structures, but their action takes place, for example, by changing the position of the center of mass and not changing its configuration or cable and struts lengths, as is the case with the robot shown in Fig. 16(i).
- ***Variable length cable systems***: this class includes systems whose cables are driven or elastic, allowing for variation in their length. This class includes type 2 cable-actuated systems and tensegrity systems and robots whose cables need to change their length in order to perform the desired movement.

Some systems presented in the course of this thesis are shown in Fig. 23 exemplifying the proposed classification.

Figure 23 – Examples for classification according cable length variation.



Source: By the author, 2022.

5 PROPOSED METHOD

This chapter presents and discusses the proposed method for replacing binary links in mechanisms with planar kinematic chains, which culminates in the generation of Inner-cable mechanisms. Before replacing the links with cables, it is necessary to analyze the mechanism and disregard, using the five proposed conditions, the links that cannot be replaced due to the constructive characteristics of the mechanism.. Likewise, after the replacement is completed, it is necessary to assess the tension of the cables that have been added and make the necessary adjustments to keep their tension within an acceptable range, when possible. When a sagged cable is found, it is possible to choose not to replace this link or change the direction and/or position of the force applied in order to pull the cable.

5.1 DISCUSSION ABOUT THE REQUIRED CONDITIONS FOR REPLACING RIGID LINKS WITH CABLES

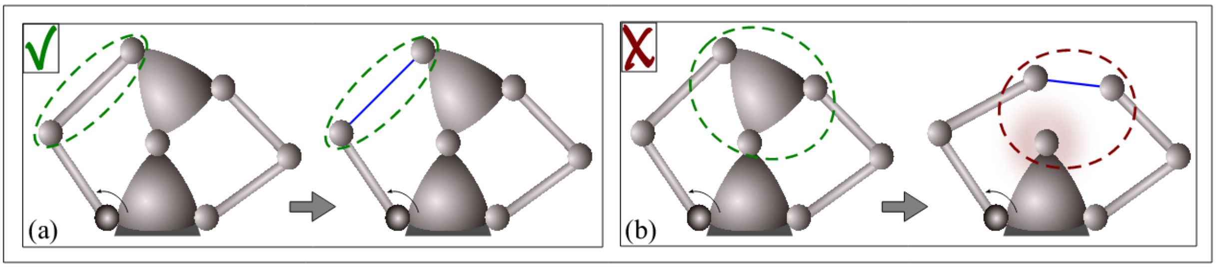
To replace rigid links with cables in any mechanical system, it is first necessary to evaluate the conditions to which the cables will be submitted, as they must remain in tension in any task or movement performed by the system. Furthermore, imposing certain conditions reduces the number of cases to be studied, which makes the method more efficient. Thus, conditions were determined in order to exclude possibilities that would certainly present tensioning problems. Such conditions are numbered as shown below:

1. A cable can only replace a binary link;
2. A cable cannot replace the base;
3. A cable cannot replace a link where a force is applied to the environment;
4. A cable cannot replace a link adjacent to an actuated joint;
5. Two cables cannot have a direct coupling.

The first condition, shown in Fig. 24, imposes that the replacement must be done only in binary links (Fig. 24(a)). The replacement of ternary and quaternary links is apparently more complex and needs further studies, as shown in Fig. 24(b).

Conditions 2, and 3 prevent all or most of the system load from being supported by a cable (Fig. 25(a) and Fig. 25(c) respectively) and also prevent the required task from being performed by a cable (Fig. 25(b)). In fact, according to condition 2, the base must remain a rigid link in order to fully support the total load of the system. Otherwise, as Fig. 25(a) suggests, the system may collapse. Figure 25(b) illustrates the situation in which the end-effector (in a manipulator) is exactly on the link that was replaced by a cable, which should not happen, according to condition 3, so that it does not change the system tension conditions. That is, applying an external force in a direction

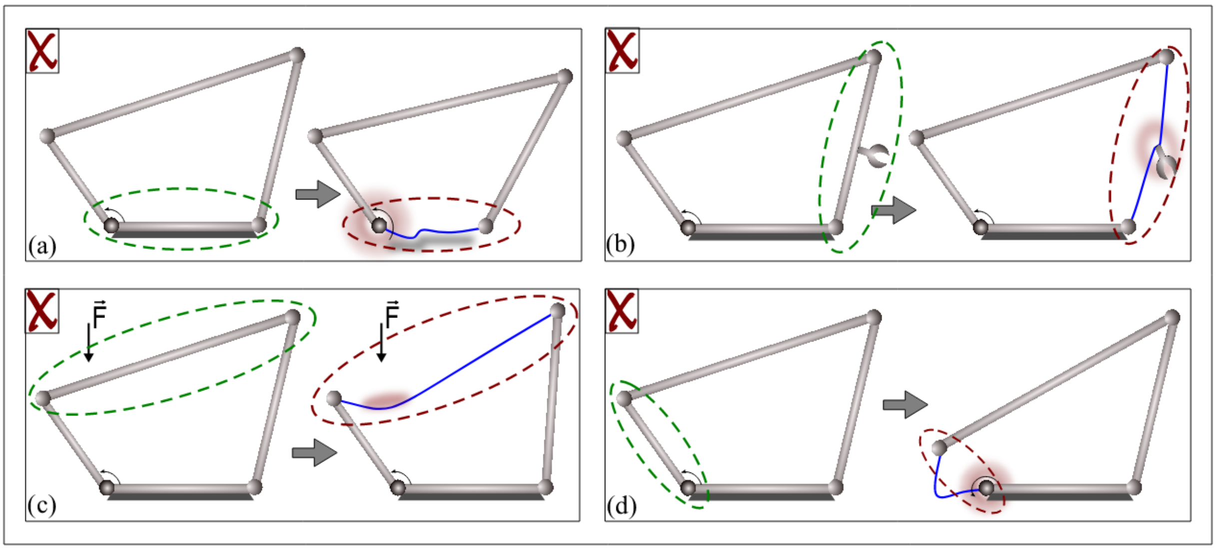
Figure 24 – First required condition: (a) Feasible substitution; (b) Unfeasible substitution.



Source: By the author, 2022.

that is not the same as the cable has a high chance of system collapse, as it cannot be guaranteed that the tensioning conditions remain unchanged, as is assumed in Figs. 25(b) and 25(C). Also, as the method was proposed for systems with rotating joints, actuating a cable in this way would likely compromise the movement caused by the motor drive, as the cable would not respond in the desired way (Fig. 25(d)) and, therefore, condition 4 was imposed.

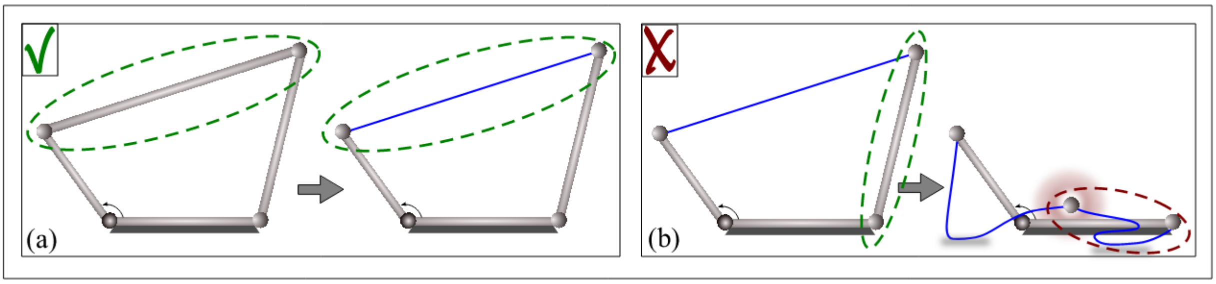
Figure 25 – Cases prevented by conditions 2, 3 and 4: (a) Condition 2; (b) and (C) Condition 3; (d) Condition 4.



Source: By the author, 2022.

Finally, condition 5 prevents two cables from being connected only by a passive joint. Reject condition 6 means that there are more possibilities for replacements that, at the end of the study, need to be discarded, or because two cables are not able to pull each other, maintaining an angle required by the system between them (Fig. 26(b)). In other words, the predictability is that the two adjacent cables line up, so that the passive joint between them loses its function, which in some cases alters the mobility of the system, making it impossible to perform the required task.

Figure 26 – Last required condition: (a) Feasible substitution - One cable; (b) Unfeasible substitution - Two cables with a direct coupling.



Source: By the author, 2022.

It is important to explain that the five conditions presented are only necessary conditions, not sufficient, for the replacement of a given link by a cable to be viable. Given a rigid frame mechanism, the five conditions help to exclude links that constructively should not be replaced by cables. Thus, even considering these conditions, after carrying out the desired replacement, it is still necessary to check the tension of the cables of the final system.

5.2 DISCUSSION ABOUT CABLE TENSION USING VECTORS

After using the conditions defined above to exclude links, one or more links (among those that remain) are selected, as appropriate, to be replaced by cables, still observing the fifth condition. Then, the statics of the system is solved using the Davies Method, in order to determine the efforts in the joints. Finally, the tension of the cables that will replace the selected rigid links is evaluated, making a vector analysis based on the projection of the force vector present in the adjacent joint, $\vec{F}$, into the possible cable direction vector, $\vec{\rho}$. Thus, if the projection vector forms an angle between 90° and 270° with the vector $\vec{\rho}$, it means that the cable is in tension, otherwise, the cable is not in tension. This question can be evaluated through the sign of the internal product between $\vec{F}$ and $\vec{\rho}$, as shown mathematically, as follows:

Let $\vec{F}$ be a force acting on the system by tensioning a cable in the direction of $\vec{\rho}$. So the projection of $\vec{F}$ on $\vec{\rho}$ is opposite to that of $\vec{\rho}$.

In fact, given the vector $\vec{\rho}$, the orthogonal projection of a vector $\vec{F}$ onto $\vec{\rho}$ is given by:

$$Proj_{\vec{\rho}}\vec{F} = \frac{\vec{F} \cdot \vec{\rho}}{||\vec{\rho}||^2}\vec{\rho}. \tag{14}$$

Therefore, the direction of vector $Proj_{\vec{\rho}}\vec{F}$ in relation to vector $\vec{\rho}$ is given by the sign of the scalar $\frac{\vec{F}\cdot\vec{\rho}}{||\vec{\rho}||^2}$. As

$$||\vec{\rho}||^2 > 0, \tag{15}$$

then just the sign of the internal product $\vec{F} \cdot \vec{\rho}$ is sufficient to determine the direction of the $Proj_{\vec{\rho}}\vec{F}$. Writing the internal product in terms of cosine, we have

$$\vec{F} \cdot \vec{\rho} = ||\vec{F}||||\vec{\rho}|| \cos\Theta, \tag{16}$$

where θ is the angle formed by the vectors $\vec{F}$ and $\vec{\rho}$. Likewise, as

$$||\vec{F}||||\vec{\rho}|| > 0, \tag{17}$$

then

$$sign(\vec{F} \cdot \vec{\rho}) = sign(\cos\Theta), \tag{18}$$

where $sign(y)$ considers only the sign of the scalar y, where $sign(y) = 1$ if $y > 0$, $sign(y) = -1$ if $y < 0$ and $sign(y) = 0$ if $y = 0$. It is known that if

$$90^\circ < \Theta < 270^\circ, \tag{19}$$

then

$$\cos\Theta < 0. \tag{20}$$

That is,

$$\vec{F} \cdot \vec{\rho} < 0, \tag{21}$$

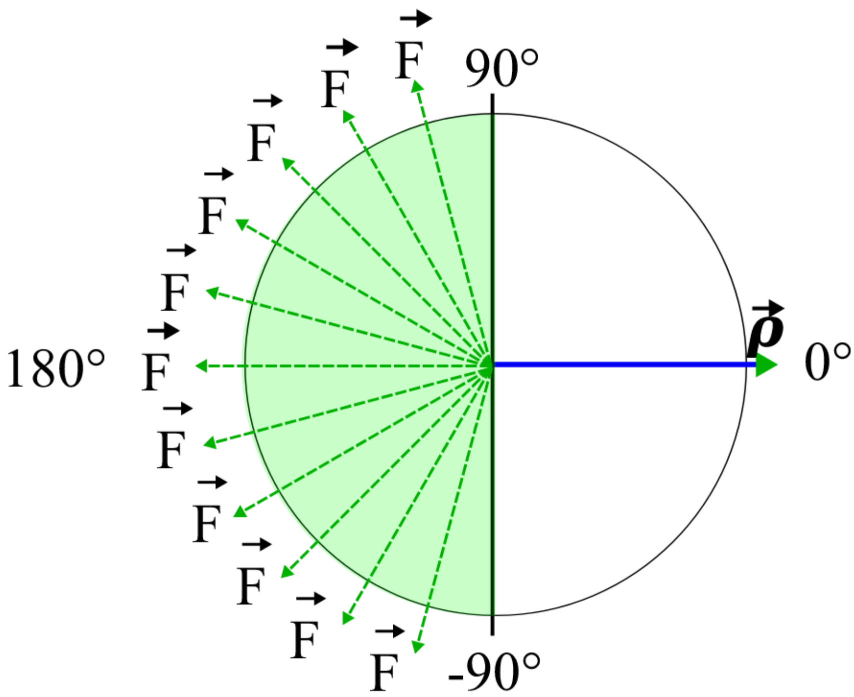
and therefore the cable direction vector and the force at the joint adjacent to the cable are opposites, which suggests that the cable is under tension, as illustrate in Fig. 27.

However, for the convenience and elegance of the method, the cable will be considered tensioned if

$$-\vec{F} \cdot \vec{\rho} > 0, \tag{22}$$

where "·" is the internal product.

Figure 27 – Cable tension condition.



Source: By the author, 2022.

5.3 METHOD FOR REPLACING RIGID LINKS WITH CABLES

The method proposed in this thesis for replacing rigid links with cables in mechanisms with a planar kinematic chain includes not only the systematization of the actions that lead to the final objective, but also proposes five conditions that must necessarily be observed for the method to become efficient. Considering the conditions proposed in 5.1, the method can be used to replace only one link of a given mechanism or multiple links. However, the fifth condition, which refers to two cables, should only be evaluated when replacing two or more links of a given mechanism, or when the mechanism already has a cable. In fact, note that it does not make sense when there is only one cable in the mechanism. Thus, the steps that define the proposed method are:

1. Check if any of the links already represents a cable;
2. In positive case, identify the binary links that satisfies the five required conditions; if not, identify the binary links that satisfies the first four required conditions;
3. Select, among the links identified in step 2, the links that will be replaced by cables;
4. Apply the Davies Method to statics;

5. Calculate $-\vec{F} \cdot \vec{\rho} > 0$ for the cables included in the system, during a given trajectory or task;
6. For multiple cables, intersect the tension intervals on each cable to obtain the feasible interval;
7. Adjust, if necessary, the load configuration according to the expected result and return for the fifth step.

To replace another link with a cable, the method can be used again and so on depending on the number of links to be replaced.

It is important to note that for the situation where two or more links are replaced by cables, there may be intervals in which only one of the cables is tensioned and thus this interval cannot be considered feasible. In cases where this happens, it is necessary to vary the application point of the forces and/or the magnitude of the forces (vary the load configuration) so that all cables remain in tension during the required task. If even varying the magnitude or the force application point, the cable does not remain in tension, it means that the link cannot be replaced by a cable. If you still want to add one more cable to the system, go back to the second step to determine another link that can be replaced and repeat the other steps. That is, the method facilitates a correct replacement of rigid links by cables, however it does not guarantee that a given rigid link can be replaced.

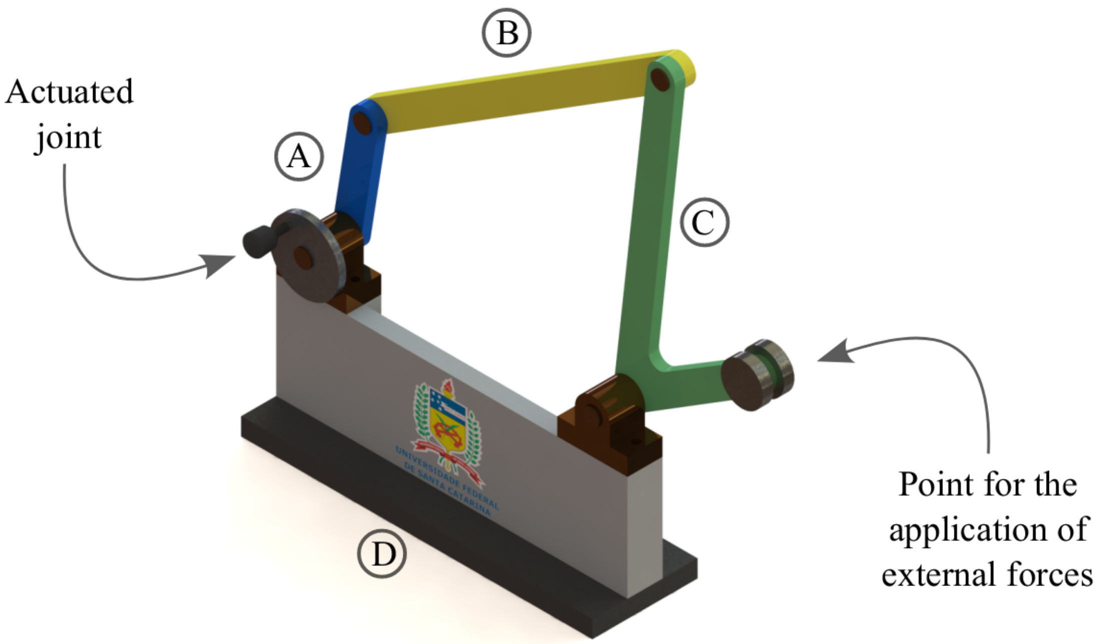
6 APPLICATIONS AND RESULTS

In order to validate the proposed method, study cases of three mechanisms are presented in this Chapter: 1 - a four-bar mechanism; 2 - a Watt mechanism; 3 - the mechanism of a palletizer robot. In each study case, at least one rigid body was replaced by a cable by the proposed method and the tension conditions are evaluated in order to determine if the substitution is feasible. Each study case is individually described below including some posture variations.

6.1 FOUR BAR MECHANISM

In this first study case, a four bar mechanism as shown in Fig. 28 was studied. A variable point for the application of external forces was considered on the output link. The joint between the fixed link and the crank is considered a reactive joint that guarantees the static equilibrium of the system.

Figure 28 – First study case: Four bar mechanism.



Source: By the author, 2022.

Applying the proposed method, we have the following steps:

1 - Check if any of the links already represents a cable.

In the first step of the method, it is verified that there are no links representing cables.

2 - In positive case, identify the binary links that satisfies the five required conditions; If not, identify the binary links that satisfies the first four required conditions.

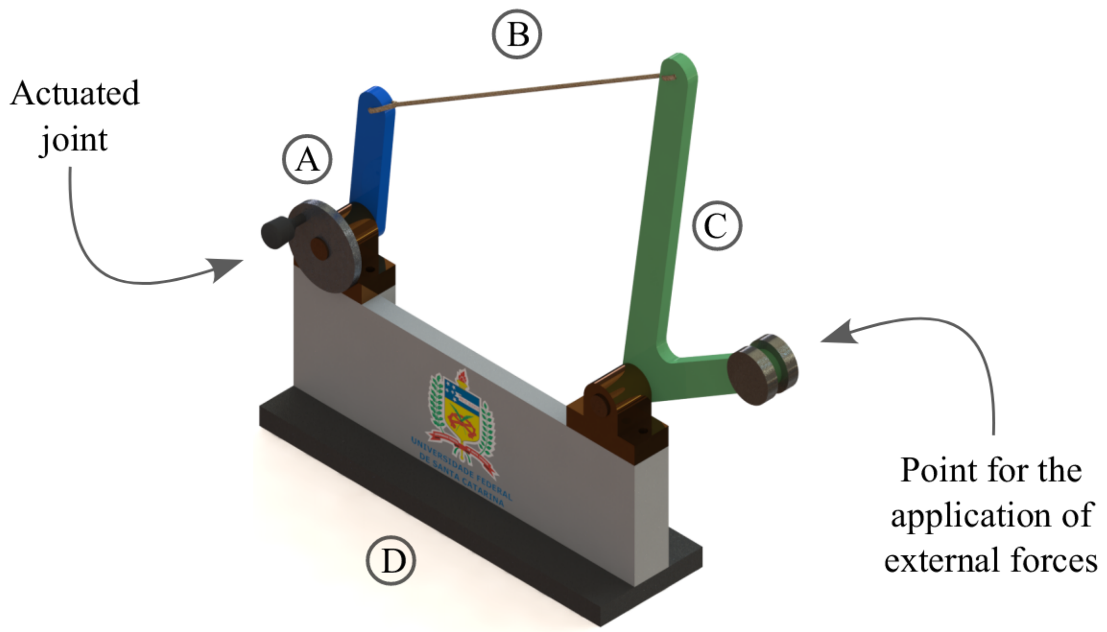
As there is no link representing a cable, it is necessary to use the first four conditions to make the replacement of a link by cable feasible. Observing these conditions, note that:

- Condition 1 is naturally satisfied, as all links in the four-bar mechanism are binary;
- Condition 2 excludes link D;
- Condition 3 excludes the link C, where the force is being applied;
- Condition 4 excludes link A.

3 - Select, among the links identified in step 2, the links that will be replaced by cables.

Thus, the only link that can be replaced by a cable is link B, as shown in Fig. 29.

Figure 29 – First study case: Four bar Inner-cable mechanism.

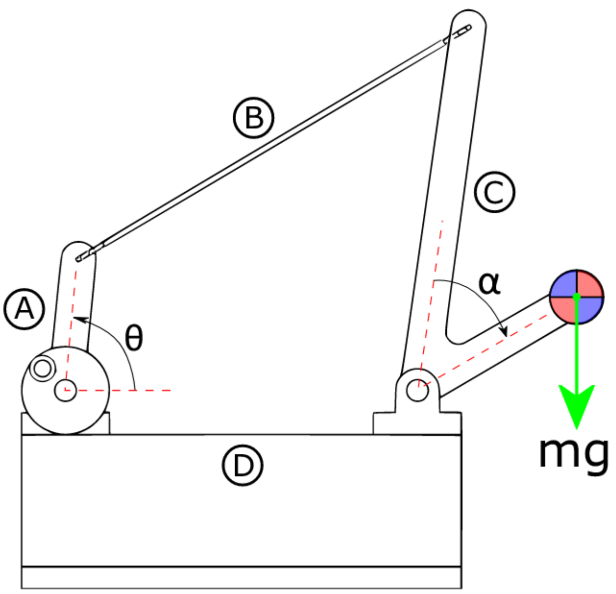


Source: By the author, 2022.

Figure 30 shows the functional representation of the studied four-bar mechanism in which it is possible to observe in a better way the existence of a point for the application of external forces.

4 - Apply the Davies Method to statics.

Figure 30 – Four bar mechanism - Functional representation.



Source: By the author, 2022.

Davies' method will be applied here to solve the static model of the four bar mechanism in order to determine the stresses on the joints. The 4 macro steps explained in the section 2.2.1 will be followed.

a) Graphic representation.

The schematic representation of the mechanism is made, conveniently positioning the reference system (Fig. 31(a)). In the schematic representation also is possible to determine the number of links *n* and direct couplings *e*. Figure 31(b) shows the actions graph. In it, each vertex represents a link and each set of *c* directed edges between two links represents the direct coupling between these links, where *c* is the number of constraints of each direct coupling. As the graph vertices are numbered according to the links, the direct the edges are in the ascending order of vertices. Also, the cuts (K_1, K_2 and K_3) and the strings, which are the dotted edges, were represented in the action graph.

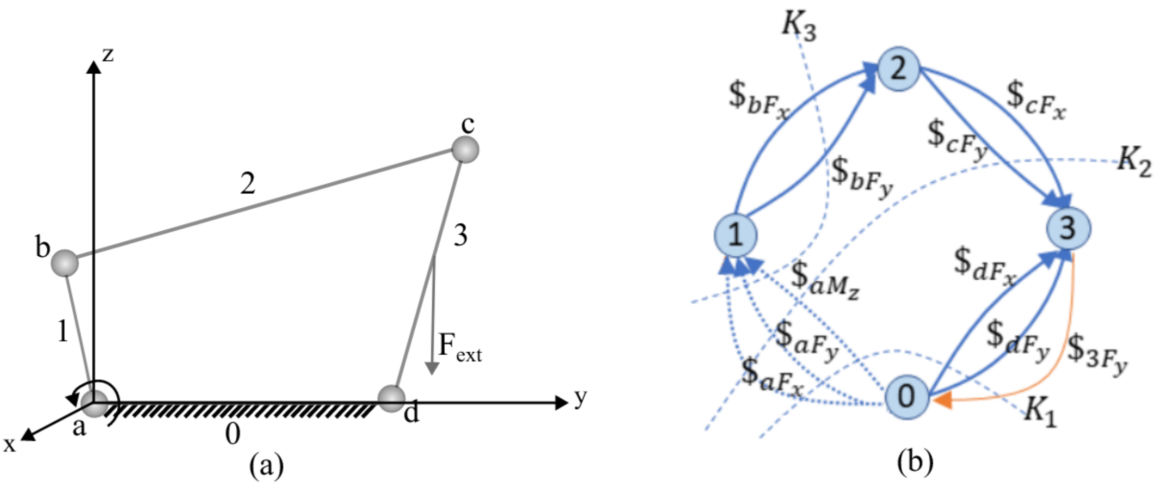
b) Determination of the wrenches.

Here, the characteristics of the action screws are determined as follow:

$$\vec{S}_{0_e} = (x_e, y_e, z_e), \tag{23}$$

for each direct coupling e, $\vec{S}$ is $\hat{i}$, $\hat{j}$ or $\hat{k}$, and the pitch $h \to \infty$ only for $\hat{\$}_{aM_z}$, where there is pure moment. For the other wrenches, $h = 0$. Thus each unitary wrench can be written:

Figure 31 – Four bar mechanism: (a) Schematic representation; (b) Actions graph with cuts.



Source: By the author, 2022.

$$\hat{\$}_{aF_x} = \psi_{aF_x} \begin{Bmatrix} 0 \\ z_a \\ -y_a \\ --- \\ 1 \\ 0 \\ 0 \end{Bmatrix}, \hat{\$}_{aF_y} = \psi_{aF_y} \begin{Bmatrix} -z_a \\ 0 \\ x_a \\ --- \\ 0 \\ 1 \\ 0 \end{Bmatrix}, \hat{\$}_{aM_z} = \psi_{aM_z} \begin{Bmatrix} 0 \\ 0 \\ 1 \\ --- \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{24}$$

$$\hat{\$}_{bF_x} = \psi_{bF_x} \begin{Bmatrix} 0 \\ z_b \\ -y_b \\ --- \\ 1 \\ 0 \\ 0 \end{Bmatrix}, \hat{\$}_{bF_y} = \psi_{bF_y} \begin{Bmatrix} -z_b \\ 0 \\ x_b \\ --- \\ 0 \\ 1 \\ 0 \end{Bmatrix}, \tag{25}$$

$$\hat{\$}_{cF_x} = \psi_{cF_x} \begin{Bmatrix} 0 \\ z_c \\ -y_c \\ --- \\ 1 \\ 0 \\ 0 \end{Bmatrix}, \hat{\$}_{cF_y} = \psi_{cF_y} \begin{Bmatrix} -z_c \\ 0 \\ x_c \\ --- \\ 0 \\ 1 \\ 0 \end{Bmatrix}, \tag{26}$$

$$\hat{\$}_{dF_x} = \Psi_{dF_x} \begin{Bmatrix} 0 \\ z_d \\ -y_d \\ --- \\ 1 \\ 0 \\ 0 \end{Bmatrix}, \hat{\$}_{dF_y} = \Psi_{dF_y} \begin{Bmatrix} -z_d \\ 0 \\ x_d \\ --- \\ 0 \\ 1 \\ 0 \end{Bmatrix}, \hat{\$}_{F_{ext}F_y} = \Psi_{F_{ext}F_y} \begin{Bmatrix} -z_{F\,ext} \\ 0 \\ -x_{F\,ext} \\ --- \\ 0 \\ -1 \\ 0 \end{Bmatrix}. \quad (27)$$

We must remember that z_a, z_b z_c, z_d and $z_{F\,ext}$ are equal to zero, since λ=3 is being considered. Thus, each wrench has only three components, excluding the first two and the last component of each wrench.

c) Determination of system matrices.

In this step, the matrices of the system are determined. The cuts matrix is as follows:

$$Q_N = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ 1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (28)$$

where each column refers to one of the previously written wrenches. Thus, the network unitary actions matrix is:

$$\hat{A}_N = \begin{pmatrix} -y_a & x_a & 1 & 0 & 0 & 0 & 0 & -y_d & x_d & x_{p_3} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ -y_a & x_a & 1 & 0 & 0 & y_c & -x_c & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ y_a & -x_a & -1 & -y_b & x_b & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (29)$$

d) Resolution of the linear system.

The homogeneous linear system obtained is:

$$\hat{A}_N \Psi = 0, \quad (30)$$

where Ψ is the vector of the wrenches magnitudes, that is:

$$\Psi = \begin{pmatrix} \Psi_{aF_x} \\ \Psi_{aF_y} \\ \Psi_{aM_z} \\ \Psi_{bF_x} \\ \Psi_{bF_y} \\ \Psi_{cF_x} \\ \Psi_{cF_y} \\ \Psi_{dF_x} \\ \Psi_{dF_y} \\ \Psi_{F_{ext}F_y} \end{pmatrix}. \tag{31}$$

Note that the coefficient matrix is not square. Therefore, to completely determine the system, the strategy is to separate the variables (magnitudes) into primary and secondary ones, with the secondary ones being those to be determined, in this case, the efforts on the joints. Making the separation, the system is:

$$\hat{A}_{N_S}\Psi_S = -\hat{A}_{N_P}\Psi_P. \tag{32}$$

were $\hat{A}_{N_S}$ is the matrix that contains only columns referring to secondary variables, Ψ_S is the vector of the secondary variables' magnitudes, $\hat{A}_{N_P}$ is the matrix that contains only columns referring to primary variables and Ψ_P is the vector of the primary variables' magnitudes.

Therefore, the linear system to be solved is non-homogeneous and its solution provides the efforts in all joints of the mechanism.

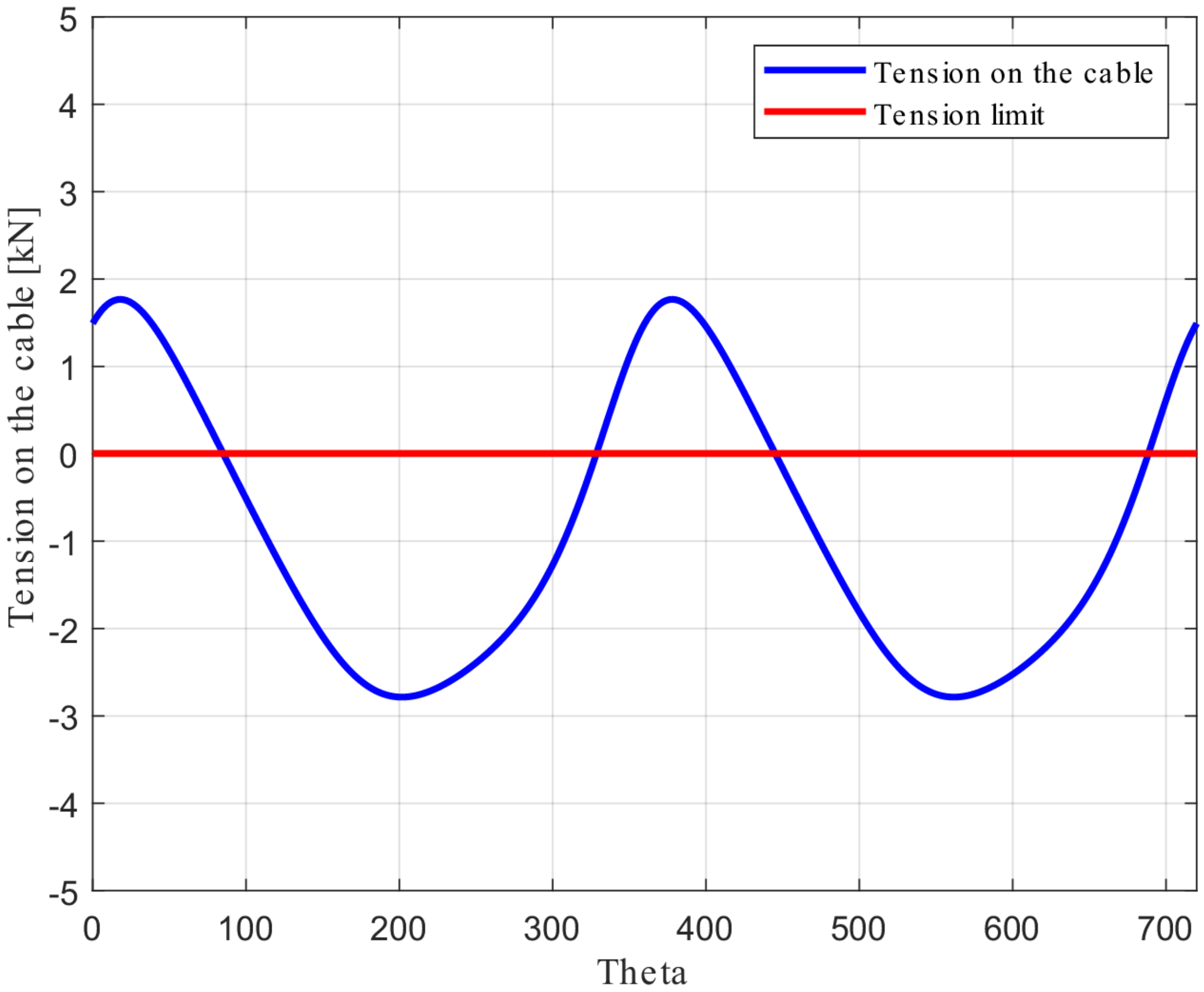
5 - Calculate $-\vec{F} \cdot \vec{\rho} > 0$ for the cable included in the system, during a given trajectory or task.

Mechanisms with mobility 1 have an advantage in relation to this step of the method: the trajectory defined to perform the calculation of the tension in the cable added to the system, can be exactly the complete workspace of a manipulator generated from the four-bar mechanism. As the four-bar mechanism has mobility 1, the evaluated trajectory is the workspace itself.

From the joint efforts calculated in the previous step, the internal product $-\vec{F} \cdot \vec{\rho}$ was calculated for two complete turns of the crank. The following information indicated in Fig. 30 was considered: A and L measuring 1m, B and D measuring 4m and C measuring 2m. The base of the mechanism was considered to be parallel to the surface and the angle α taken equal to zero. For this situation, the graph of tensions in the cable shown in Fig. 32 was obtained during the rotations.

For this posture it is possible to check, graphically, that there are intervals of θ that do not satisfy $-\vec{F}\cdot\vec{\rho} > 0$. Then, it is necessary to move to the next step of the method.

Figure 32 – Four bar mechanism: Result of $-\vec{F} \cdot \vec{\rho}$ for initial posture.



Source: By the author, 2022.

6 - For multiple cables, intersect the tension intervals on each cable to obtain the feasible interval.

This step does not apply in this case because only one link was chosen to be replaced.

7 - Adjust, if necessary, the load configuration according to the expected result and return for the fifth step.

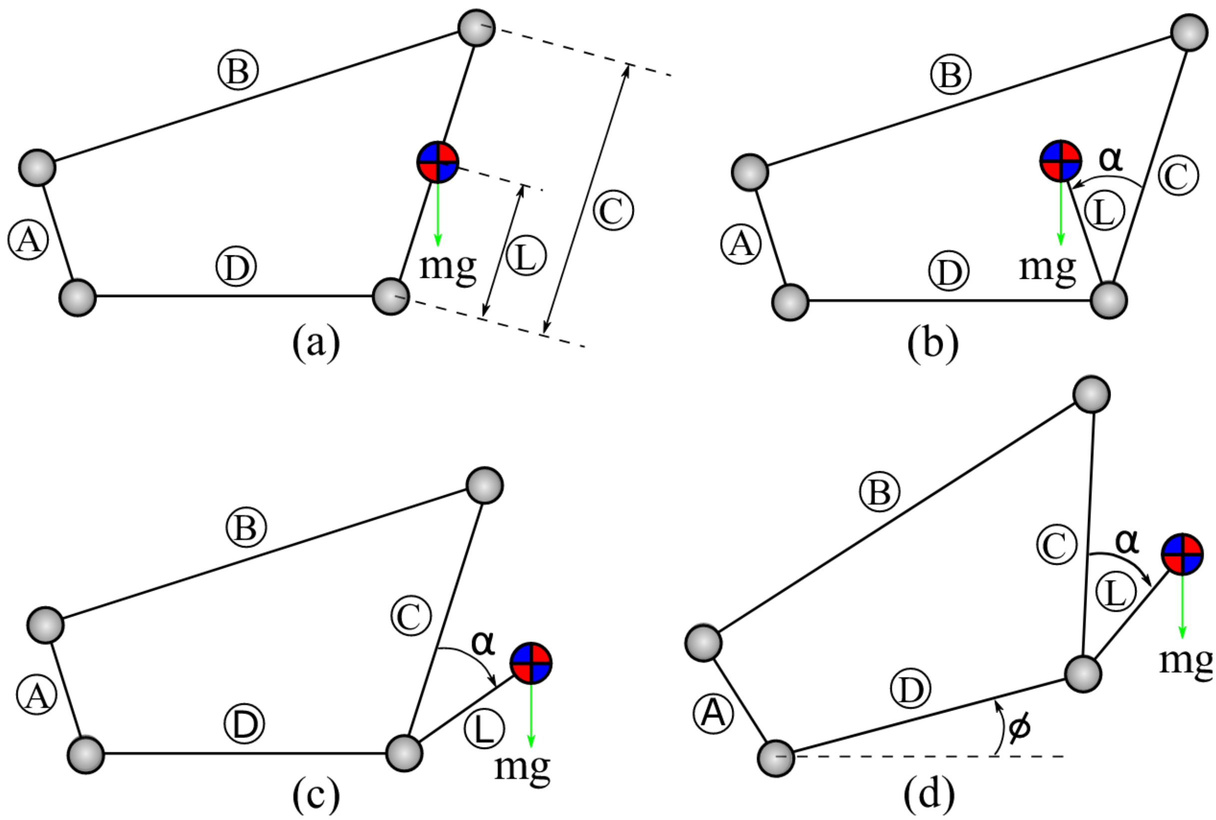
Thus, another four load configurations were applied in order to validate the influence of this variable on the tension of the cable. In each load configuration the position of the point for the application of the external forces is changed and the orientation for the local reference system is rotated by an angle (φ) for postures 4 and 5. Table 3 presents the structural information used in this study for all simulation purposes (including the initial posture: posture 1). Columns A, B, C, D and L are the dimensions, in meters. All the simulations were done considering two complete rotations of the mechanism's crank ($[0^\circ, 720^\circ]$). Figure 33 shows the five postural variations studied.

Table 3 – Structural information about four bar mechanism.

load configuration	A	B	C	D	L	α	φ	θ
1	1	4	2	4	1	0°	0°	$[0^\circ, 720^\circ]$
2	1	4	2	4	1	20.5°	0°	$[0^\circ, 720^\circ]$
3	1	4	2	4	1	-48°	0°	$[0^\circ, 720^\circ]$
4	1	4	2	4	1	-37.2°	23.3°	$[0^\circ, 720^\circ]$
5	1	4	2	4	1	-69.5°	23.3°	$[0^\circ, 720^\circ]$

Source: By the author, 2022.

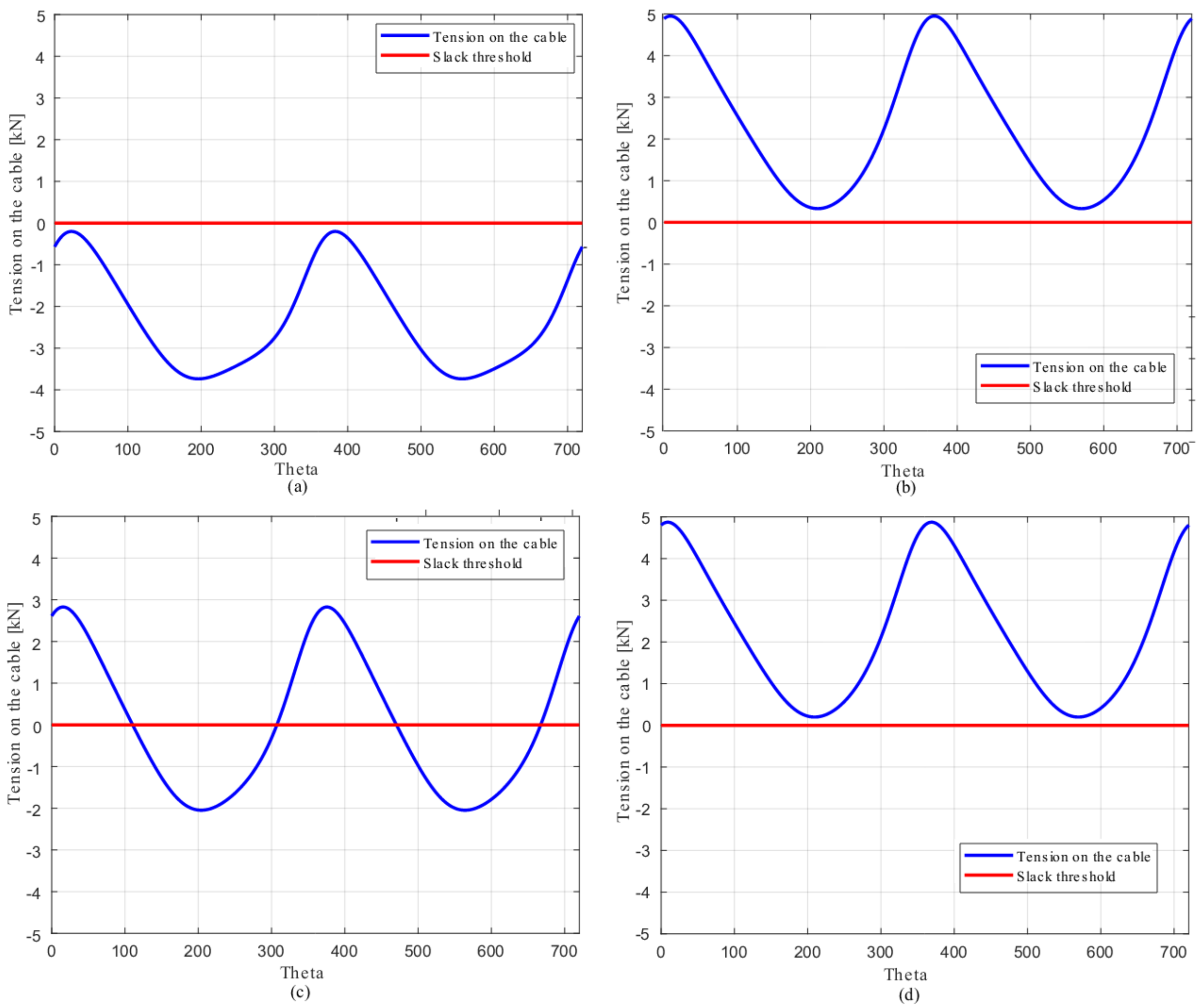
Figure 33 – Four bar mechanisms load configurations studied: (a) load configuration 1; (b) load configuration 2; (c) load configuration 3; (d) load configurations 4 and 5.



Source: By the author, 2022.

Figure 34 shows the cable tension for each one (of the four) load configurations studied in this step. In the Fig. 34(a) it can be seen that load configuration 2 does not satisfy, for any interval of θ, the condition $-\vec{F} \cdot \vec{\rho} > 0$. In Fig. 34(c), there are intervals below the tension limit, which means that in the load configuration 4 it is not possible to keep the cable tensioned during the entire rotation. However, the load configurations 3 and 5, whose graphics are shown in Fig. 34(b) and 34(d) are feasible, since the cable remains tensioned throughout the movement.

Figure 34 – Four bar mechanisms' feasible regions diagram, considering maximum and minimum cable tension: (a) load configuration 2; (b) load configuration 3; (c) load configuration 4; (d) load configuration 5.

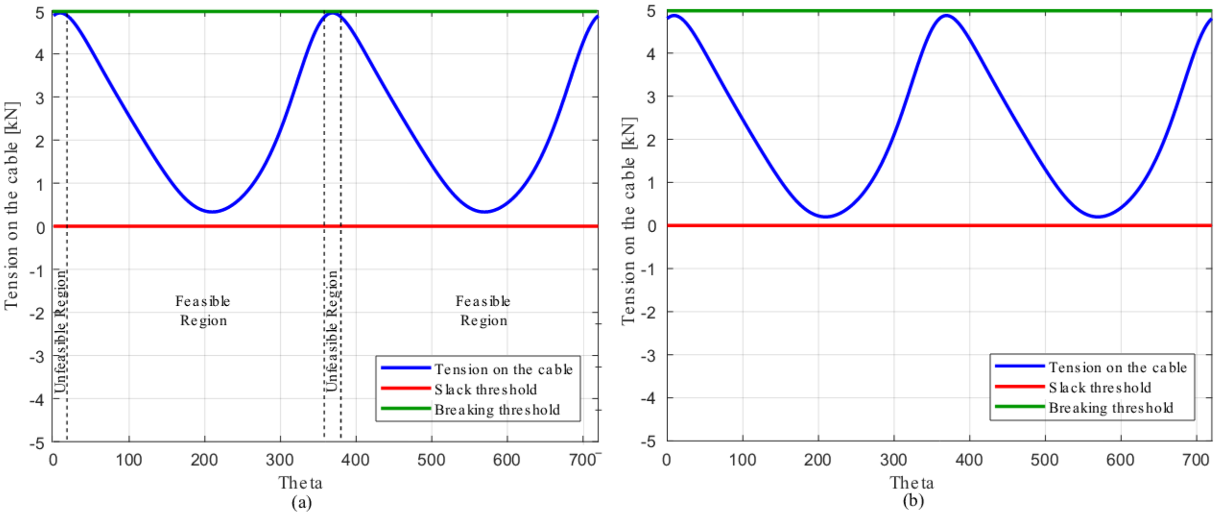


Source: By the author, 2022.

Therefore, only using the four bar mechanism in load configurations 3 and 5 can the replacement of the rigid link by the cable be considered. However, if we still consider an upper limit for the tension in the cable, that is, a value of tension for which the cable breaks, we can have the case shown in Fig. 35.

Therefore, load configuration 5 allows the replacement of link B by a cable, considering the tension limits in the cable as in Fig. 35. However, several other load configurations can be evaluated, according to the need.

Figure 35 – Four bar mechanisms' feasible regions diagram: (a) load configuration 3; (b) load configuration 5.



Source: By the author, 2022.

6.2 WATT MECHANISM

For the second mechanism studied, the Watt mechanism, two different designs were considered. The two designs differ by the place of application of the external force. In both cases, two rigid links were replaced by cables.

This is also a case of a mechanism with mobility 1. Therefore, the trajectory considered to evaluate the tension in the cables, from step 5 of the method, is also the complete workspace of a manipulator generated from the Watt mechanism.

6.2.1 Watt mechanism - First design

In this second case, a Watt mechanism as shown in Fig. 36 was studied. For this mechanism, a mobile point for the application of external forces was considered on the output link. The input joint is considered a reactive joint keeping the static equilibrium of the system.

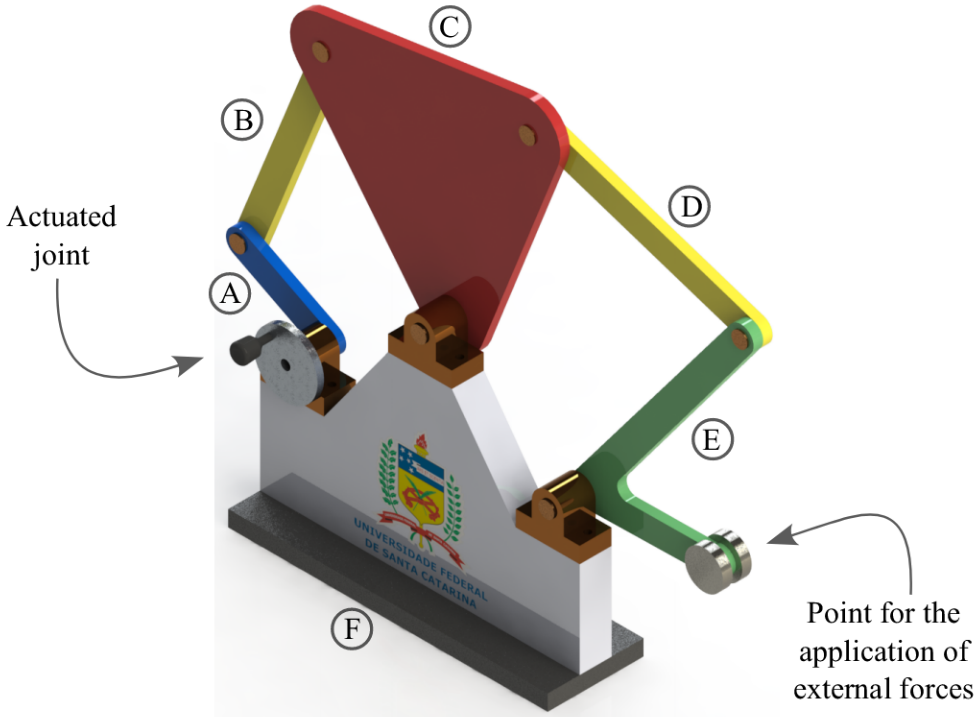
Applying the proposed method, we have the following steps:

1. **Check if any of the links already represents a cable**;

   In the first step of the method, it is verified that there are no links representing cables.

2. **In positive case, identify the binary links that satisfies the five required conditions; If not, identify the binary links that satisfies the first four required conditions.**

Figure 36 – Watt mechanism - First design.



Source: By the author, 2022.

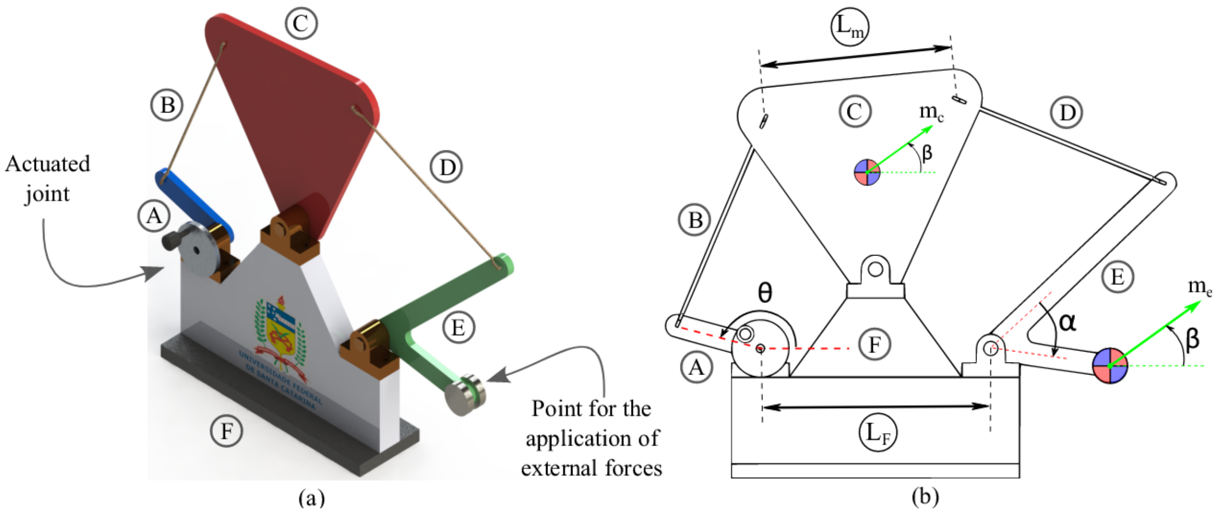
As there is no link representing a cable, it is necessary to use the five necessary conditions to make the replacement of a hard link by cable feasible. Observing these conditions, note that:

- Condition 1 excludes ternary links C and F;
- Condition 2 also excludes link F;
- Condition 3 excludes the link E, where the force is being applied;
- Condition 4 excludes link A.

3. **Select, among the links identified in step 2, the links that will be replaced by cables, *observing condition 5*;**

   There are only two possible links to be replaced: B and D. Condition 5 is satisfied by links B and D, as they do not form a direct coupling. In this sense, both links can be replaced by cables, according Fig. 37(a). Figure. 37(b) shows the functional representation of the studied mechanism in which it is possible to observe in a better way the existence of a point for the application of external forces.

Figure 37 – Watt mechanism - First design: (a) Watt inner-cable mechanism; (b) Functional representation.



Source: By the author, 2022.

4. **Apply the Davies Method to statics.**

   The schematic representation of the mechanism and the actions graph with cuts can be seen in Figs. 38(a) and 38(b). It is possible to verify in the action graph that for this case the static model needs 16 wrenches. Thus, as the Davies method is applied in order to obtain the forces present in the joints adjacent to the cables, and the focus of this thesis is the application of the method, for convenience, the equation will not be described here. The results were obtained by implementing the Davies method in the Matlab software.

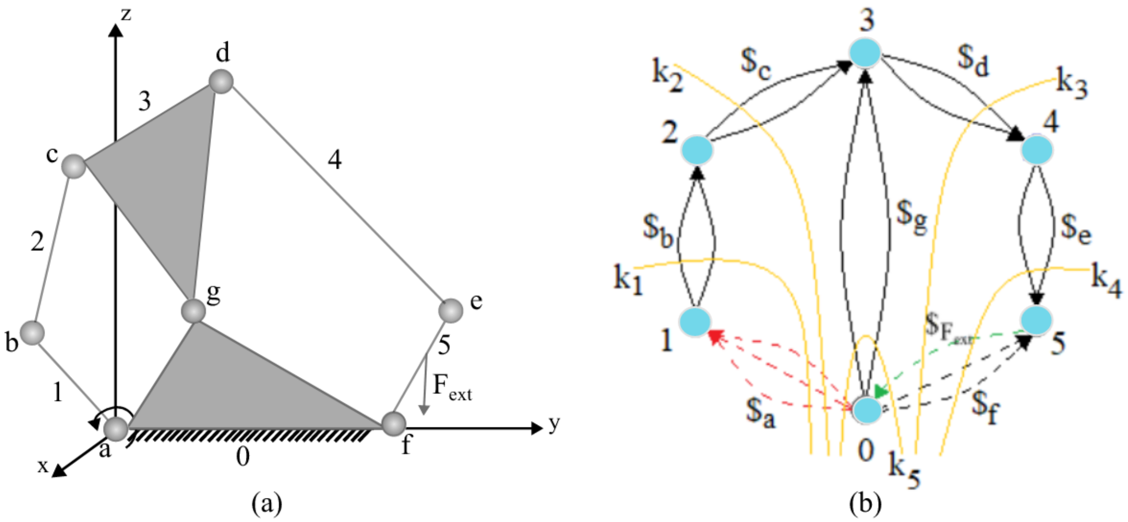
5. **Calculate $-\vec{F} \cdot \vec{\rho} > 0$ for all cables included in the system, during a given trajectory or task.**

   From the joint efforts calculated in the previous step, the internal products $-\vec{F} \cdot \vec{\rho}_B$ and $-\vec{F} \cdot \vec{\rho}_D$ were done considering two complete rotations of the mechanism's crank ($[0^\circ, 720^\circ]$) and the direction of the external force is $\beta = 270^\circ$. The graph of the tensions in the cables as a function of θ obtained is shown in Fig. 39.

6. **For multiple cables, intersect the tension intervals on each cable to obtain the feasible interval**;

   Although cable D is in good tensioning condition, it is possible to notice that cable C has tension failures during some parts of the rotation. The feasible

Figure 38 – Watt mechanism - First design: (a) Schematic representation; (b) Actions graph.



Source: By the author, 2022.

regions are shown in Fig. 40. In this case, one possibility is to replace only the D link with cable. However, the proposal for this case is to replace the two possible links by cable. In this sense, we proceed to the next step of the method.

7. **Adjust, if necessary, the load configuration according to the expected result and return for the fifth step.**

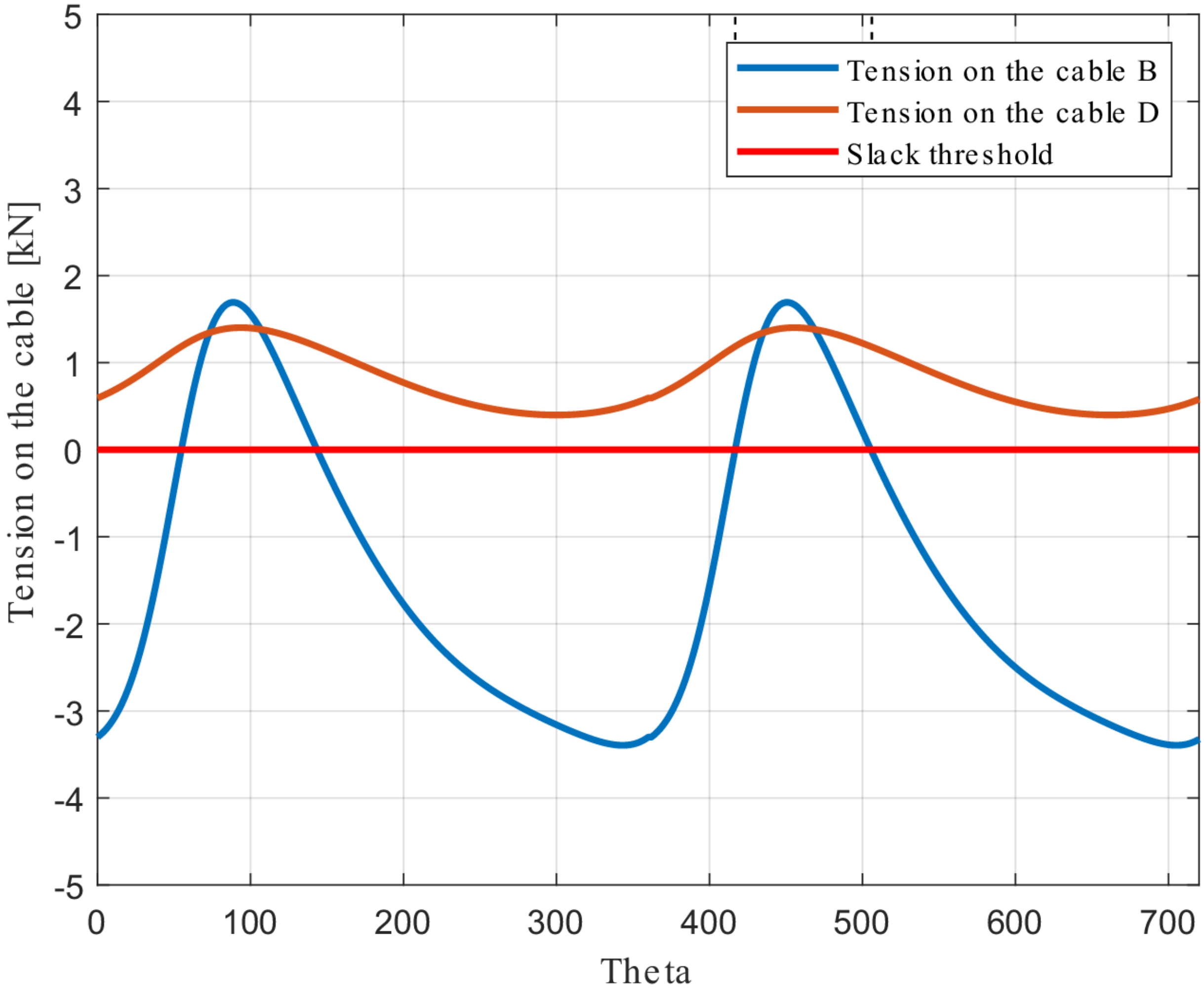
   In the same way, the internal products $-\vec{F} \cdot \vec{\rho}_B$ and $-\vec{F} \cdot \vec{\rho}_D$ were done considering two complete rotations of the mechanisms' crank ($[0°, 720°]$) and the direction of the external force (β) is decreasing to 90º in each posture from initial posture. Table 4 presents the structural information used in this study for all simulation purposes (including initial posture: posture 1). The simulations for the four studied postures are shown in Fig. 41.

Table 4 – Structural information about Watt mechanism - First design.

Load configuration	A	B	D	E	L_f	L_m	α	β	θ
1	1	4	4	4	3	3	$0°$	$270°$	$[0°, 270°]$
2	1	4	4	4	3	3	$0°$	$180°$	$[0°, 270°]$
3	1	4	4	4	3	3	$0°$	$90°$	$[0°, 270°]$
4	1	4	4	4	3	3	$0°$	$0°$	$[0°, 270°]$

Source: By the author, 2022.

Figure 39 – Watt mechanism - First design: Internal products $-\vec{F}\cdot\vec{\rho_B}$ and $-\vec{F}\cdot\vec{\rho_D}$ - Initial load configuration.



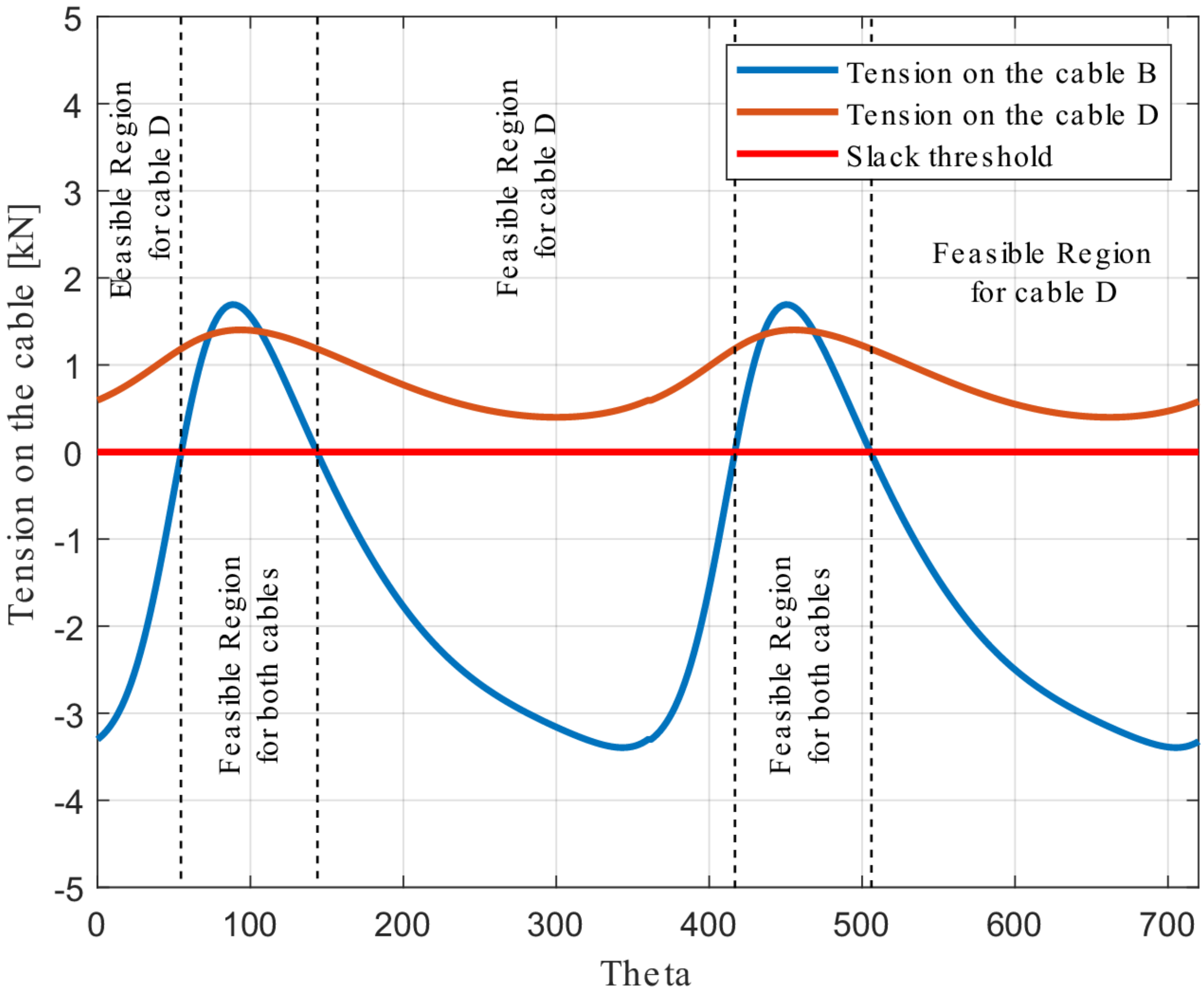
Source: By the author, 2022.

The Fig. 41 shows only one load configuration (Fig. 41(d)) in which both cables are tensioned and another (Fig. 41(b)) in which both cables are not tensioned, ie, a completely feasible and a completely unfeasible load configurations. On the other hand, Fig. 41(a) and Fig. 41(c) present load configurations with feasible and infeasible regions for the intersection between tensions in each cable. These regions are indicated in the graphs.

6.2.2 Watt mechanism - Second design

In the second design of the Watt mechanism, the intention was to change the point of application of the external force, which is now being applied to link D, as shown in Fig. 42, and also to verify the possibility of replacing two rigid links by cables. In this sense, observing the new design and the five conditions required for replacing rigid

Figure 40 – Watt mechanisms' feasible regions diagram - First design: Initial load configuration.



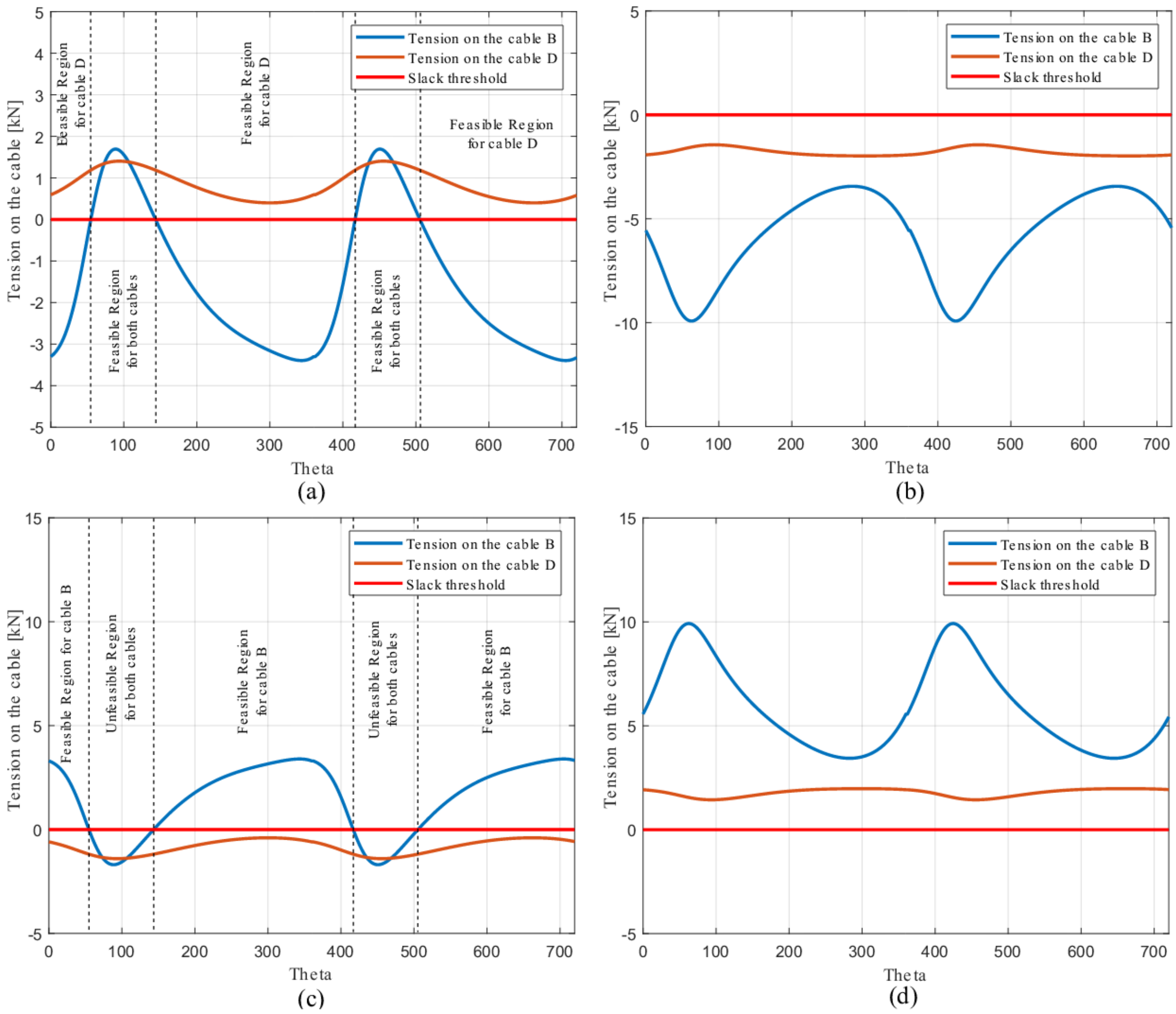
Source: By the author, 2022.

links by cables, only links B and E are now candidates to be replaced by cables. The functional representation and the generated Watt inner-cable mechanism are shown in Fig. 43.

From the joint efforts calculated using Davies method, the internal products $-\vec{F} \cdot \vec{\rho}_B$ and $-\vec{F} \cdot \vec{\rho_E}$ were done considering two complete rotations of the mechanism's crank ($[0^\circ, 720^\circ]$) and the direction of the external force (β) is considering 210°. Using the second configuration of the Watt mechanism in the initial posture, the two cables are loose during the entire rotation of θ. Under these conditions, the replacement of links B and E cannot be performed. In this way, a new posture was evaluated. The new stance considers $\beta = 120^\circ$. The graphics of cable tensions as a function of θ for postures 1 and 2 are shown in Fig. 44.

In load configuration 2, even cable E being in good tensioning conditions, it is

Figure 41 – Watt inner-cable mechanisms' feasible regions diagram - First design: (a) Load configuration 1, (b) Load configuration 2, (c) Load configuration 3, (d) Load configuration 4.

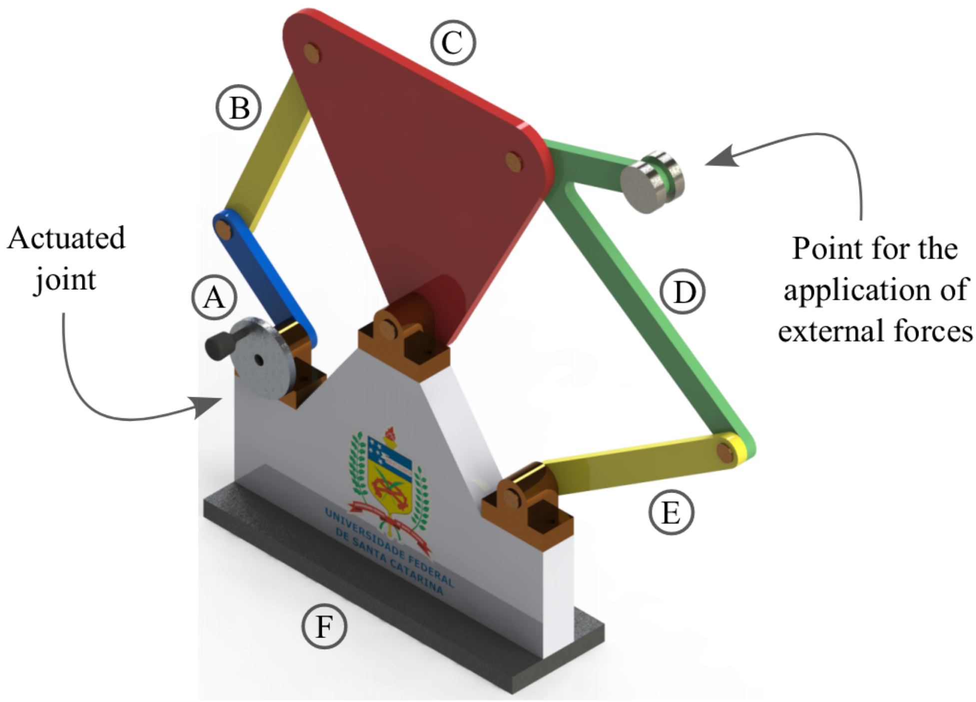


Source: By the author, 2022.

possible to see that cable B is loose during the entire rotation (Fig. 44). In this case, as there are no feasible regions for cable B, the intersection between the feasible regions is empty. As in the initial load configuration of the first design of the Watt mechanism, it would be feasible to replace only the link whose cable is capable of maintaining tension during the entire rotation of θ. However, here we also want to evaluate the possibility of replacing the two links with cables. In this sense, it is necessary to evaluate a new load configuration.

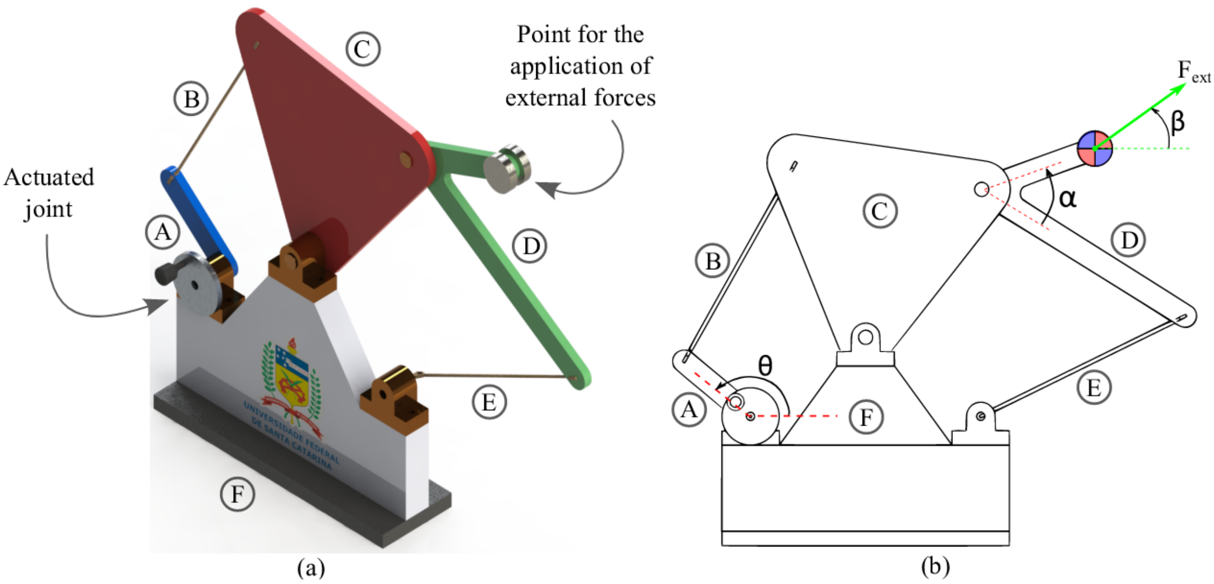
Considering, then, $\beta = 30^{\circ}$, the graph of tensions in cables B and E for the rotation from $[0^{\circ} \leq \theta \leq 720^{\circ}]$ is shown in Fig. 45. For this load configuration, the two cables remain tensioned during the entire rotation of θ, so this is a possible load configuration for replacing the two links with cables.

Figure 42 – Watt mechanism - Second design.



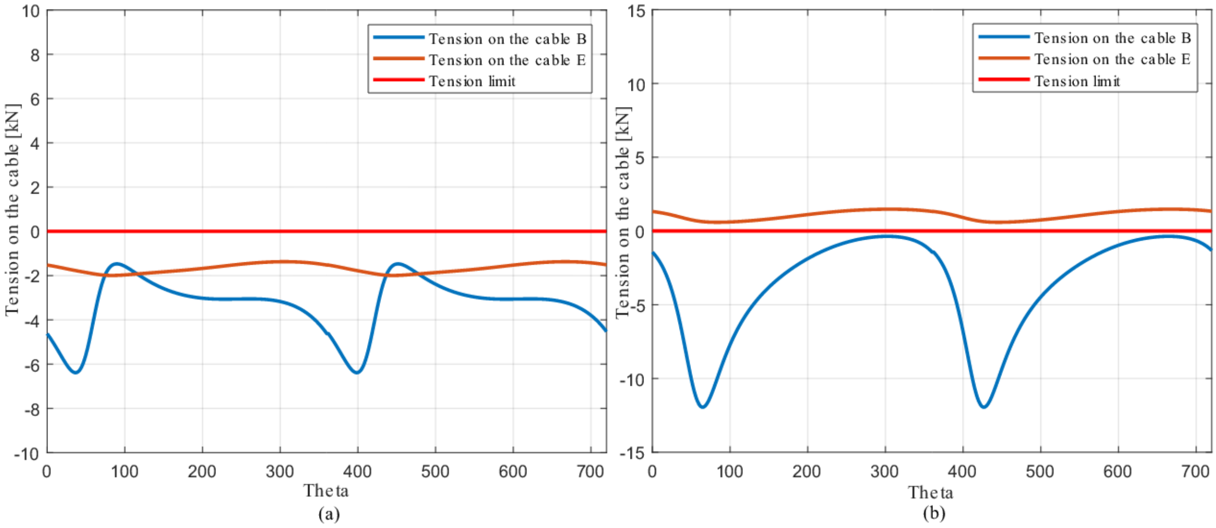
Source: By the author, 2022.

Figure 43 – Watt mechanism - Second design: (a) Watt inner-cable mechanism; (b) Functional representation.



Source: By the author, 2022.

Figure 44 – Watt mechanism - Second design: Internal products $-\vec{F} \cdot \vec{\rho}_B$ and $-\vec{F} \cdot \vec{\rho}_E$. (a) Load configuration 1; (b) Load configuration 2.



Source: By the author, 2022.

The information about second design of Watt mechanism are shown in Tab. 5.

Table 5 – Structural information about Watt mechanism - Second design.

Load conofiguration	A	B	D	E	L_f	L_m	α	β	θ
1	1.5	4	4	4	3	3	0°	210°	[0°, 270°]
2	1.5	4	4	4	3	3	0°	120°	[0°, 270°]
3	1.5	4	4	4	3	3	0°	30°	[0°, 270°]

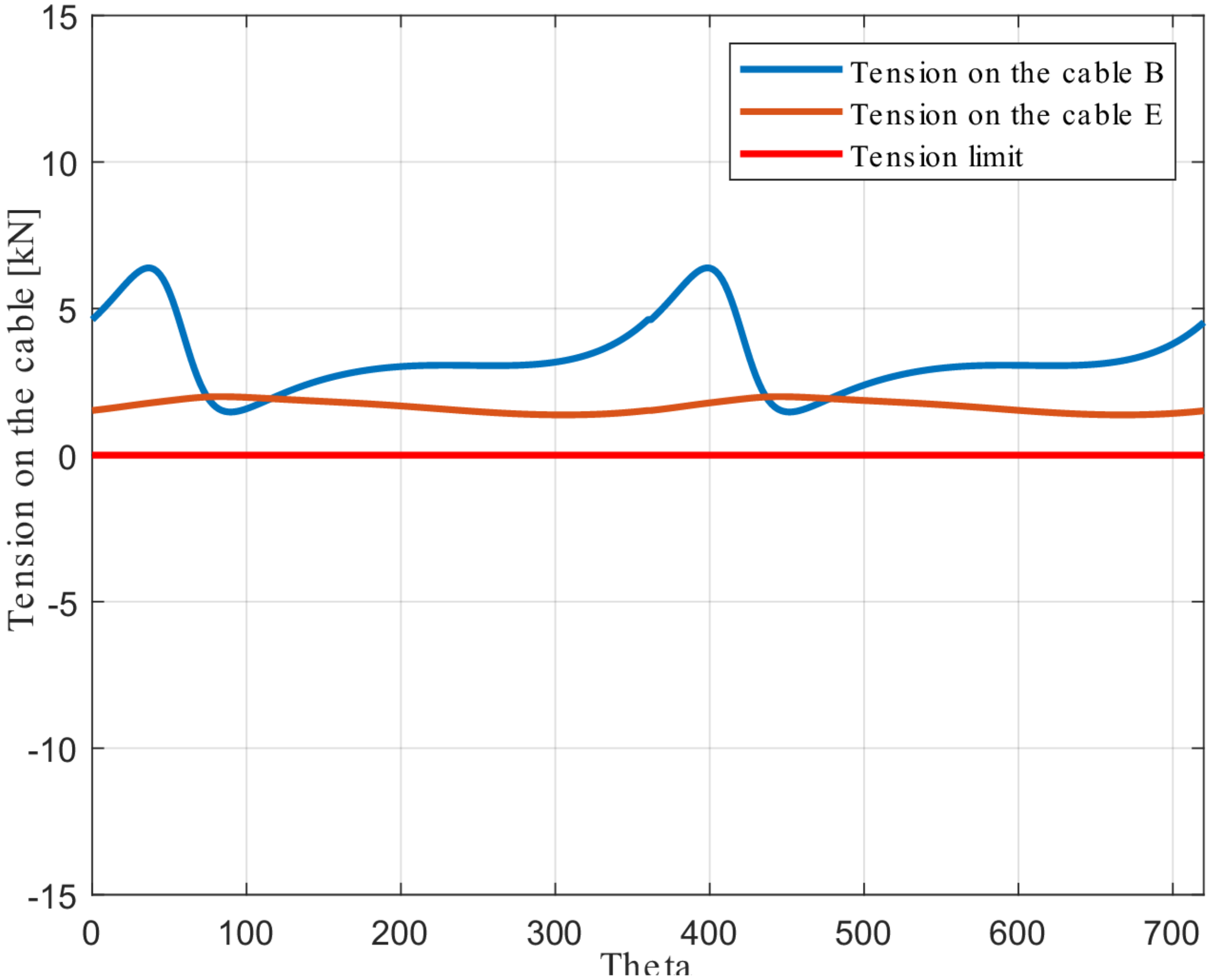
Source: By the author, 2022.

6.3 PALLETIZER ROBOT

The last case presented in this thesis is about a palletizer robot.This case is interesting to demonstrate the application of the method in real cases, that is, in mechanisms that compose industrial robots, for example. Figure. 46 shows the original robot and also the robot with three cables instead of three links. Thus, we can call the robot in Fig. 46(b) Inner-cable robot, as proposed in chapter 5 of this thesis.

For this study a trajectory ($P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow P_1$) was simulated by palletizer robot, repeating the task for carrying a load. From P_2 to P_3 was considered a load of 50kg. Figure 47 graphically shows the task performed by the robot: starting at P_1, it moves to P_2, where it takes a load of 50kg and carries it to P_3. It unloads to P_3 and returns unloaded to P_1. The trajectory between the points P_1, P_2 and P_3 are straight lines that are indicated in the figure by T_{12}, T_{23} and T_{31}.

Figure 45 – Watt mechanism - Second design: Internal products $-\vec{F} \cdot \vec{\rho}_B$ and $-\vec{F} \cdot \vec{\rho}_E$ - Feasible load configuration.

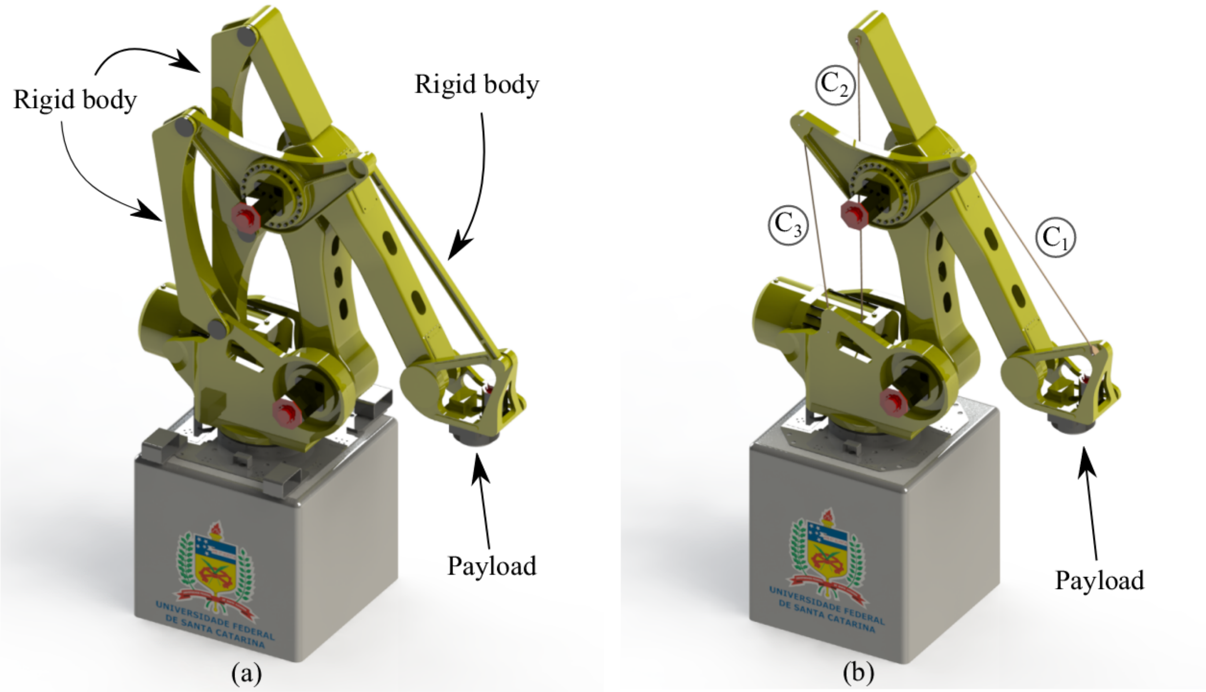


Source: By the author, 2022.

In the simulation, the three cables C_1, C_2 and C_3 (Fig. 47) remained with positive tension during the complete trajectory. In order to show the feasible solution, the tension values in function of a point on the trajectory for the three cables are shown in Fig. 48.

This case study was inspired by the ABB Palletizer Robot IRB 760 (ROBOTICS, 2023). According to the manufacturer's specifications, the robot has a total mass of 2300kg. The mass of the three links that were hypothetically replaced by cables in this case study is 14.326kg, according to information taken from CAD models available on the manufacturer's website (ROBOTICS, 2023), which represents approximately 0.62% of the robot's total mass. Thus, if each link is replaced by a 5mm galvanized steel cable, whose minimum effective breaking load is 2450kgf, according to catalog specifications, available on the manufacturer's website (SIVA, 2023), the total mass of the cables would be approximately 0.69kg. In this sense, the reduction in the total mass

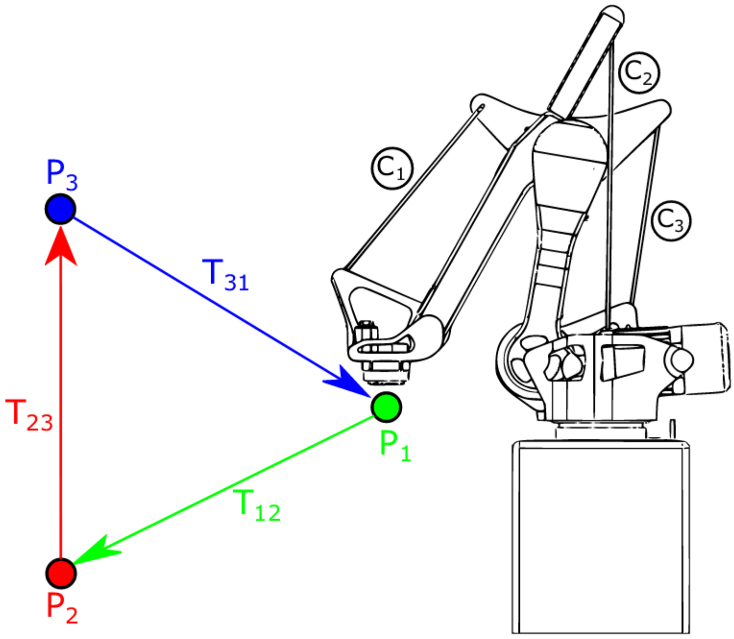
Figure 46 – Third study case: (a) Rigid palletizer manipulator; (b) Inner-cable palletizer manipulator.



Source: By the author, 2022.

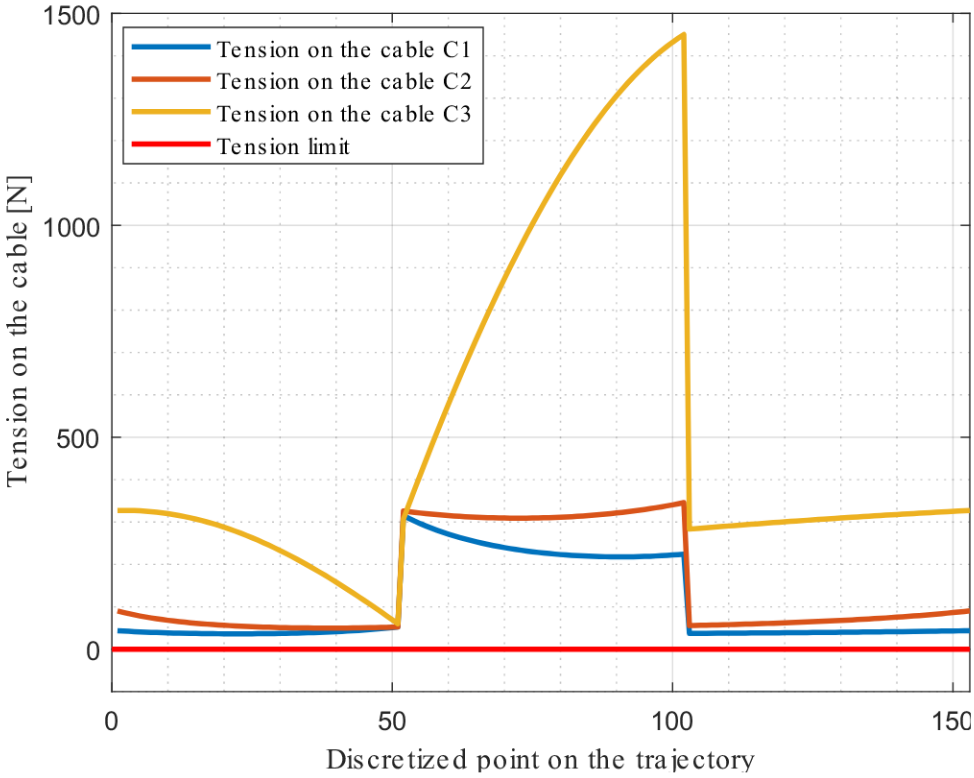
of the robot would be approximately 0.6%. On the other hand, the mass reduction of replaced links is around 95%, which is a very expressive value.

Figure 47 – Third study case: Palletizer manipulator - Functional representation.



Source: By the author, 2022.

Figure 48 – Palletizer Robot's feasible regions diagram.



Source: By the author, 2022.

7 CONCLUSION

A new method for replacing rigid links by cables in mechanical systems with a planar kinematic chain is proposed in this thesis. Five required conditions were determined and discussed in order to make the cable substitution feasible. The theoretical resource used by this solution is the Davies Method, based on Screw Theory and Graph Theory and a vectorial analysis.

The new method of replacing rigid links with cables was proposed in order to facilitate the feasible replacement of rigid links with cables, and can be used cyclically in cases where two or more links are to be replaced. It is worth noting that the required conditions determined to make the replacement of rigid links by cables were designed for the general case, that is, so that more than one link can be replaced. In the case where only one link is to be replaced, only the first four conditions need to be considered (provided that all other links are rigid).

In the proposed method, after evaluating the static model by the Davies method, a vector analysis involving the direction of the cable and the effort in the adjacent joint provides information about the tension on the links or cables according to Eq. (22). Through this method it is possible to identify if there are rigid links that can be replaced by cables and, moreover, where they are. It is also important to point out that the method or conditions presented in this thesis do not guarantee that a given rigid link can, in fact, be replaced by a cable. What the method provides is the ease of identifying whether or not a rigid link can be replaced by a cable in a given mechanism loading configuration.

Using the proposed method, mechanisms that have rigid links and cables were generated; here they are called Internal Cable Mechanisms (ICMs). In this work, any mechanism composed of rigid links, actuated joints and inextensible cables not driven by pulleys or winding drums (or similar) can be considered an ICM. However, this definition can be extended to mechanical systems in general, as used in section 6.3, where an inner-cable palletizer robot was generated using the proposed method.

Furthermore, a new classification of mechanical cable systems has been proposed based on the length of the cables in the system, referring to them as fixed or variable length cables. This classification was proposed in order to include the inner-cable systems in some classification, since until now they have not been cited and classified rigorously. Thus, two classes were proposed that cover all types of cable systems: Variable length cable systems and Fixed length cable systems, in which inner-cable systems are inserted.

Finally, the method of replacing rigid links by cables in mechanisms with a planar kinematic chains allows, as a great advantage, self-alignment of mechanisms. In addition, it allows the reduction of mass and consequently of the manufacturing cost of the systems in which replacement is possible. This can make task automation more accessible, which contributes to technological advancement.

7.1 SUGGESTION FOR FUTURE WORK

Since the method is proposed in order to facilitate the feasible replacement of rigid binary links by cables, the studies can be extended by optimizing the loading configurations. Thus, by replacing a rigid link with a cable, it would be possible to predict the optimal loading configuration, if any, for a given replacement. Likewise, an analysis can be made for the entire workspace and not just for a given trajectory, in the case of mechanisms that have mobility greater than 1.

Besides that, other situations can be studied, such as reducing the size of the generated mechanisms for use in minimalist tasks. So, the size reduction would place ICMs as candidates to be used in robotic surgery.

Also, another substitution can be studied: change ternary links by cables, using conditions like these but other theoretical resources.

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