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Three essays on monetary and fiscal policy interactions

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Coorientador: Guilherme Valle Moura, Dr.

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THREE ESSAYS ON MONETARY AND FISCAL POLICY INTERACTIONS

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"Blessed are the young, for they shall inherit the national debt."
Herbert Hoover

Resumo

Esta tese consiste em três ensaios acerca das interações entre políticas fiscal e monetária.
No primeiro ensaio, um modelo Novo Keynesiano padrão é extendido para incorporar a restrição orçamentária do governo quando gastos públicos são financiados por impostos distorcivos e/ou emissão de dívidas de longo prazo, e a existência de participação limitada no mercado de ativos. Dada a inabilidade de comprometimento a um plano ótimo, a discrição na condução das políticas na presença da dívida do governo faz com que o problema de viés inflacionário seja dependente de estado e, também, cria um viés de estabilização da dívida. Além disso, a presença de participação limitada no mercado de ativos agrava as distorções na economia. Como resultado disto, a fração de agentes com restrição de liquidez impacta os valores de equilíbrio de longo prazo das variáveis macroeconômicas. Adicionalmente, a resposta ótima frente a choques pode ser radicalmente diferente para valores distintos do nível do estoque da dívida pública e da fração de consumidores rule-of-thumb. Por fim, níveis mais altos de dívida causam um efeito redistributivo levando a aumentos nas desigualdades entre agentes no estado estacionário.
O segundo ensaio analisa as dependências de estado nas interações estratégicas entre um banco central independente e conservador com relação à inflação e uma autoridade fiscal benevolente. Um modelo Novo Neynesiano padrão é extendido para incluir política fiscal e dívida pública nominal e os efeitos de formuladores de política independentes, discricionários e não-cooperativos são analisados. A principal contribuição deste trabalho é resolver o equilíbrio discricionário dos jogos de política usando métodos globais e não-lineares de solução. Os resultados obtidos sugerem que sob um jogo de liderança fiscal, a delegação de política monetária para uma autoridade conservadora pode funcionar como um dispositivo de disciplina fiscal, reduzindo tanto o viés inflacionário quanto o viés de estabilização. As consequências em termos de bem estar são praticamente inócuas neste caso. No entanto, um jogo simultâneo de política além de aumentar o hiato entre inflação observada e a meta é também associado a maiores perdas de bem estar. A perda em termos de bem estar é uma função crescente do grau de conservadorismo monetário.
O ensaio final identifica a estrutura de liderança no jogo entre as autoridades fiscal e monetária na economia brasileira do pós-metas de inflação. Um modelo Novo Keynesiano de pequena escala é aumentado para incluir política fiscal e estimado usando métodos Bayesianos. Assume-se que as autoridades fiscal e monetária podem agir de maneira estratégica sob discrição em um ambiente não-cooperativo e compara-se três formas de jogos de política: (i) jogo simultâneo; (ii) liderança fiscal; e (iii) liderança monetária. Os resultados obtidos evidenciam um forte suporte empírico em favor da hipótese de que a autoridade fiscal brasileira age como um líder de Stackelberg e podem contribuir para melhorias no design de política na economia brasileira.

Palavras-chaves: Política fiscal; Política monetária; Consistência temporal; Dívida pública.

Resumo Expandido

Introdução

A crise financeira global de 2008 e seus desdobramentos reacenderam o debate acerca das interações entre políticas fiscal e monetária. Muitos países responderam ao recente ambiente de incertezas com ações conjuntas de política econômica que desafiam a prescrição de política macroeconômica convencional que isola políticas fiscal e monetária. Para estimular um ambiente macroeconômico débil, pacotes fiscais ambiciosos foram lançados, causando um aumento consistente nas razões dívida-PIB.

Os efeitos da crise ressonaram não apenas nas finanças públicas, mas também na disponibilidade de crédito para as famílias, sugerindo um aumento na fração da população sem acesso ao mercado de ativos. Este cenário de aumento na razão dívida-PIB e na restrição no acesso ao mercado de ativos motiva o primeiro ensaio desta tese, dado que a presença de consumidores com restrição de liquidez na economia pode ter implicações profundas para a prescrição de políticas macroeconômicas e que o nível da dívida pública, por sua vez, pode causar efeitos de redistribuição entre os agentes que possuem acesso ao mercado de ativos e aqueles com restrição de liquidez.

A prescrição de política macroeconômica convencional prediz que as distorções causadas pela falta de comprometimento a uma regra de política ótima pode ao menos mitigada delegando-se a condução da política monetária para um banco central independente e conservador com relação à inflação. A maior parte da literatura nesta área, entretanto, abstrai do comportamento fiscal. No entanto, as interações entre autoridades fiscal e monetária independentes podem implicar em resultados macroeconômicos muito distintos daqueles emergentes nas análises de políticas isoladas. Estudos recentes que examinam as interações estratégicas entre autoridades independentes mostram que a proposta de delegação da política monetária, quando a autoridade fiscal é benevolente mas age de forma estratégica, normalmente está associada a aumentos no viés de estabilização, levando a perda de bem estar. Apesar de robustos à especificação do modelo e à escolha de instrumentos fiscais, os resultados obtidos são dependentes do nível de dívida pública em torno do qual a economia é linearizada. Estas não-linearidades na forma com que políticas fiscal e monetária interagem na presença da dívida do governo como variável de estado são o objeto de estudo do segundo ensaio desta tese.

Por fim, a economia brasileira do pós-crise apresenta um caso atrativo para o estudo das interações entre políticas fiscal e monetária. No curto período de 2014 a 2016, o país apresentou uma rápida deterioração fiscal, revertendo metade da queda na dívida pública obtida previamente, posicionando-se entre as economias mais endividadas do mundo. Esta falta de disciplina fiscal combinada com um sistema "míope" de metas de inflação

pode ter criado pressões inflacionárias no final de 2015. No terceiro ensaio, portanto, os jogos de política na economia brasileira do pós-metas de inflação de 1999 são analisados. A identificação da estrutura de jogo prevalente entre autoridades independentes que agem de maneira estratégica é fundamental para melhorias no design de políticas econômicas.

Objetivos

O primeiro ensaio da presente tese explora as interações entre políticas fiscal e monetária quando uma fração da população de agentes não tem acesso ao mercado de ativos e o formulador de política econômica age de maneira discricionária. As relações entre participação limitada no mercado de ativos e o nível de dívida nominal e seus potenciais efeitos redistributivos e sobre o bem estar das famílias sob uma política consistente temporalmente são analisados.

O principal objetivo do segundo ensaio é analisar as dependências de estado nas interações estratégicas entre um banco central independente e conservador com relação à inflação e uma autoridade fiscal benevolente em um modelo novo Keynesiano padrão extendido para incluir política fiscal e dívida pública nominal quando o formulador de política econômica segue uma política discricionária.

O terceiro ensaio desta tese apresenta uma análise empírica das interações estratégicas entre as políticas fiscal e monetária e tem como principal objetivo identificar o regime de liderança prevalente que caracteriza o jogo entre as duas autoridades na economia brasileira após a implementação do regime de metas de inflação em 1999.

Metodologia

Os modelos desenvolvidos no presente trabalho pertencem à classe de modelos dinâmicos e estocásticos de equilíbrio geral (DSGE). As interações entre políticas fiscal e monetária são estudadas em modelos novo Keynesianos convencionais extendidos para incluir política fiscal e a dívida nominal do governo. Assume-se que tanto política monetária quanto fiscal são conduzidas de maneira ótima e consistente temporalmente.

De forma mais específica, no primeiro ensaio um modelo novo Keynesiano de dois agentes (TANK[1]) é desenvolvido. Este modelo é extendido para incorporar a restrição orçamentária do governo quando os gastos públicos são financiados por impostos distorcivos e/ou emissão de dívida de longo prazo, e a existência de participação limitada no mercado de ativos. A hipótese de discrição na condução das políticas ótimas combinada com a presença da dívida pública como variável de estado faz com que os problemas de vieses inflacionário e de estabilização da dívida sejam dependentes de estado o que, por sua vez, implica em endogeneidade do estado estacionário. A existência de um estado estacionário endógeno faz com que os métodos tradicionais de linearização convencionalmente utilizados para solução de modelos DSGE seja inapropriado. Portanto, o equilíbrio discricionário do modelo é calculado utilizando-se métodos globais e não-lineares de solução.

No segundo ensaio desta tese um modelo novo Keynesiano convencional é extendido para incluir política fiscal e dívida pública nominal. Assume-se que a política macroeconômica é conduzida de maneira ótima sob discrição por duas autoridades independentes: um ban-

[1] Two-agent New Keynesian model.

co central conservador com relação à inflação e uma autoridade fiscal benevolente. Os dois formuladores de política econômica podem agir de maneira estratégica e potencialmente não-cooperativa e dois tipos de jogos são considerados: (i) um jogo simultâneo (jogo de Nash); e (ii) um jogo de liderança fiscal (autoridade fiscal age como um líder de Stackelberg). O equilíbrio discricionário dos jogos de política é obtido utilizando-se métodos globais e não-lineares de solução.

No ensaio final do presente trabalho um modelo novo Keynesiano de pequena escala é aumentado para incluir política fiscal e estoque nominal da dívida pública. Assume-se que as autoridades fiscal e monetária podem agir de maneira estratégica sob discrição em um ambiente não-cooperativo e três formas distintas de jogos de política são comparados: (i) jogo simultâneo; (ii) liderança fiscal; e (iii) liderança monetária. A solução do modelo é obtida utilizando-se métodos lineares quadráticos de expectativas racionais e, por fim, os modelos são estimados via métodos Bayesianos de estimação de modelos DSGE.

Resultados e discussão

Os principais resultados do primeiro ensaio podem ser assim sumarizados:

- Aumentos na fração de consumidores com restrição de liquidez estão associados a reduções consideráveis tanto na taxa de inflação de estado estacionário quanto na razão dívida-PIB.
- A presença de participação limitada no mercado de ativos enfraquece a tendência crescente da razão dívida-PIB de estado estacionário causada pela emissão de dívida pública de maturidades mais longas.
- As respostas ótimas dos instrumentos de política frente a choques adversos no markup das firmas é fortemente dependente tanto da fração de agentes com restrição de liquidez quanto do nível de dívida pública.
- Aumentos na restrição de participação no mercado de ativos representam ganhos de bem estar tanto para agentes otimizadores quanto para agentes com restrição de liquidez. No entanto, os ganhos de bem estar para agentes Ricardianos são superiores aos ganhos observados para agentes rule-of-thumb e, portanto, as desigualdades no estado estacionário também são maiores.
- Mudanças para um novo cenário com níveis mais elevados de endividamento levam a efeitos redistributivos, causando aumentos nas desigualdades de estado estacionário.

Os principais resultados do segundo ensaio podem ser assim descritos:

- O grau de conservadorismo monetário tem efeitos monotônicos sobre os valores de estado estacionário das variáveis macroeconômicos, independente da estrutura do jogo de política.
- As dinâmicas ótimas em resposta a choques no markup das firmas são sensíveis à estrutura do jogo considerado. Sob um regime de liderança fiscal, um banco central conservador

impõe um forte viés de estabilização da dívida sobre a autoridade fiscal e também reduz o problema de viés inflacionário, reduzindo a volatilidade das variáveis macroeconômicas relevantes. Um banco central conservador e independente sob um jogo simultâneo, por sua vez, leva a um aumento do viés de estabilização.

- Análises de bem estar mostram que, apesar de o conservadorismo monetário ser relativamente inócuo quando a autoridade fiscal age como um líder de Stackelberg, as perdas em termos de bem estar num jogo simultâneo são uma função crescente do grau de conservadorismo monetário.

Por fim, os resultados das estimativas empíricas no terceiro ensaio sugerem que há uma forte evidência em favor do regime de liderança fiscal na economia brasileira após a implementação do regime de metas de inflação em 1999. Este resultado é compatível com nossa crença *a priori* dado que decisões de política fiscal são feitas em uma frequência muito menor do que decisões de condução de política monetária. Portanto, a autoridade monetária brasileira ao agir como um seguidor no jogo de política cumpre um papel de disciplinador da política fiscal. Além disso, as estimativas dos objetivos de política mostram que não há evidências de conservadorismo nem com relação à inflação nem com relação ao produto por parte tanto da autoridade fiscal quanto monetária. O principal objetivo da política monetária, como esperado, é a estabilização da inflação, enquanto o da autoridade fiscal é a suavização dos seus instrumentos de política.

Considerações finais

As questões abordadas no presente trabalho são particularmente relevantes no cenário macroeconômico atual de elevado endividamento do setor público das principais economias mundiais. As interações entre políticas fiscal e monetária tornaram-se cada vez mais relevantes a partir da crise financeira global de 2008 e seus desdobramentos. As relações entre consolidação fiscal, independência e problemas de coordenação na condução de políticas fiscal e monetária discricionárias, nível de endividamento e participação limitada no mercado de ativos são alguns dos problemas analisados nesta tese. Os resultados obtidos sugerem que a prescrição de política macroeconômica convencional nem sempre é a resposta ótima frente a choques adversos, evindenciando a potencial fragilidade no design de políticas que desconsidere o papel da política fiscal.

O presente trabalho pode ser extendido para explorar inúmeras outras questões relevantes, por exemplo: considerações acerca de economias abertas e dívida soberana; questões no design de política econômica em uma união monetária com diversas autoridades fiscais; análises de políticas de redistribuição de renda; entre outras.

Palavras-chaves: Política fiscal; Política monetária; Consistência temporal; Dívida pública.

Abstract

This thesis consists of three essays on the interactions of fiscal and monetary policies.
In the first essay, we build on a standard New Keynesian model to incorporate the government's budget constraint, where public expenditures are financed by distortionary taxation and/or issuing of long-term debt, and the existence of limited asset markets participation. Without the ability to commit to an optimal plan, discretionary policies in the presence of government debt yield the inflationary bias problem state-dependent and also creates a debt stabilization bias. Moreover, the presence of limited asset markets participation deepens the distortions in the economy. As a result of that, the size of the share of liquidity constrained agents impacts the long-run equilibrium values of relevant macroeconomic variables. Furthermore, the optimal response to shocks can be radically different for different values of government debt levels and fraction of rule-of-thumb consumers. Finally, higher levels of public debt causes a redistribution effect leading to rises in steady state inequalities amongst agents.
The second essay addresses the state dependencies in the strategic interactions between an inflation conservative central bank and a benevolent fiscal authority. Building on a standard New Keynesian model extended to include fiscal policy and nominal government debt, we consider the effects of independent, discretionary and possibly non-cooperative policymakers, rather than joint optimal policies. The main contribution of this work is to solve for the discretionary equilibrium of the policy games using nonlinear global solution techniques. We found that under a fiscal leadership policy game, delegating monetary policy to a conservative authority can function as a device for fiscal discipline and reduces both the stabilization and level inflationary biases. The consequences in terms of welfare are mostly harmless in this case. Nonetheless, a simultaneous move policy game not only increases the gap between actual inflation and the target rate, in comparison with the cooperative setup, but is also associated with higher welfare losses. The loss in terms of welfare is an increasing function of the degree of monetary conservatism.
The third essay identifies the leadership structure of the game played by monetary and fiscal authorities in the Brazilian economy after the inflation targeting regime in 1999. A stylized small-scale New Keynesian model augmented with fiscal policy is estimated using Bayesian methods. We assume that monetary and fiscal authorities can act strategically under discretion in a non-cooperative setup and compare three different forms of games: (i) simultaneous move; (ii) fiscal leadership; and (iii) monetary leadership. We find strong empirical support for the hypothesis that the Brazilian fiscal authority acts as a Stackelberg leader. The results obtained can shed some light to the improvement of policy design in the Brazilian economy.

Keywords: Fiscal policy; Monetary policy; Time-consistency; Government debt.

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Chapter 1

Optimal monetary and fiscal policy with limited asset markets participation and government debt

Abstract

Building on a standard New Keynesian model, the model economy is augmented to incorporate the government's budget constraint, where public expenditures are financed by distortionary taxation and/or issuing of long-term debt, and the existence of limited asset markets participation. Without the ability to commit to an optimal plan, discretionary policies in the presence of government debt yield the inflationary bias problem state-dependent and also creates a debt stabilization bias. Moreover, the presence of limited asset markets participation deepens the distortions in the economy. As a result of that, the size of the share of liquidity constrained agents impacts the long-run equilibrium values of relevant macroeconomic variables. Furthermore, the optimal response to shocks can be radically different for different values of government debt levels and fraction of rule-of-thumb consumers. Finally, higher levels of public debt causes a redistribution effect leading to rises in steady state inequalities amongst agents.

Keywords: Fiscal policy; Monetary policy; Limited asset markets participation; Time-consistency; Government debt.

1.1 INTRODUCTION

The onset of the global financial crisis in 2008 and its developments have reignited the debate concerning monetary and fiscal policy interactions that dates back, at least, to Sargent and Wallace (1981). Many countries reacted to the recent turmoil with joint policy actions that challenge the conventional macroeconomic policy prescription that insulates monetary and fiscal policies. To stimulate a weak macroeconomic environment, ambitious fiscal packages were launched causing a steadily increase in public debt-to-output ratios in advanced countries, as shown in Figure 1.A.1.

The effects of the crisis resonated not only on public finances but also on the availability of credit to households. The empirical evidence on credit standards reported by Albonico and Rossi (2017) shows a sharp reduction of availability of credit to households in the economies of the Euro area and an even stronger decline for the US, see Figure 1.A.2. This suggests that the fraction of the population who have no access to asset markets has increased during the period of the crisis. Nevertheless, limited asset markets participation seems to be an important feature of most economies even during normal times. A vast empirical literature has provided some evidence that the share of liquidity constrained households lies roughly between 30% and 50% of the agents population[1].

This scenario of increasing government debt-to-GDP ratios and limited asset markets participation mainly motivates the current work, given that the presence of liquidity constrained consumers on the economy can have profound implications for macroeconomic policy prescriptions and that the level of debt, by its turn, may cause redistribution effects among those agents who have access to asset markets and those who are liquidity constrained.

Considering the aforementioned background, this paper studies jointly optimal and time-consistent monetary and fiscal policy in the presence of limited asset markets participation. In order to do so, we follow Leeper, Leith, and Liu (2019) in constructing a standard New Keynesian model extended to include optimally chosen distortionary taxation and government spending, when government debt can be issued with a more realistic maturity structure, but furthers their analysis by allowing that a fraction of households cannot smooth consumption intertemporally and, thus, consume all of their current disposable labor income at each period, in the tradition of Galí, López-Salido, and Vallés (2004).

Our main results can be summarized as follows:

1. Increases in the share of liquidity constrained consumers are associated with substantial decreases in both the steady state rate of inflation and public debt-to-output ratio. Limited asset markets participation constitutes another source of distortion in the model economy. When the fraction of agents with no access to asset markets is higher, it raises per capita profits earned by optimizing households. This strengthens the state-dependent inflationary bias problem and the debt stabilization bias. Therefore, the discretionary policymaker has a stronger incentive to influence the endogenous inflationary bias and mitigate the costs of distortionary taxation through reductions in the level of government debt.

2. The presence of limited asset markets participation weakens the upward trend in steady state debt-to-GDP ratio caused by the issuing of longer-term government debt. Although a longer maturity structure of the debt pushes the public debt-to-output ratio up through reductions in the debt stabilization bias, when the fraction of liquidity constrained rises this effect is lower as this pushes the bias in the opposite direction. Therefore, the net effect is reduced and the upward trend in the steady state debt-to-output as maturity increases is flatter.

[1] See, e.g., Campbell and Mankiw (1989), Muscatelli, Tirelli, and Trecroci (2004), Di Bartolomeo, Rossi, and Tancioni (2011) and Albonico, Paccagnini, and Tirelli (2014, 2016).

3. The optimal responses of policy instruments to a shock to the firms markup is largely dependent on both the share of liquidity constrained agents in the model economy and the size of government debt. Our numerical results show that for empirically plausible values of the fraction of rule-of-thumb consumers, when the public debt is low enough, the conventional policy prescription is reversed. In this case, the degree of activism of fiscal policy is enhanced and monetary policy accommodates the shock by ensuring that the real return on debt falls, yielding an inverse aggregate demand logic, as in Bilbiie (2008). However, when public debt breaches a given threshold, the conventional assignment of policies is restored.

4. Increases in the restriction on asset markets participation, when distortionary labor taxes are the main tool for fiscal stabilization, represent welfare gains for both optimizing and liquidity constrained agents. This is due to the fact that a higher share of liquidity constrained agents leads to a lower level of government debt in the long run equilibrium, which allows a smaller income tax rate. Rule-of-thumb agents, thus, experience a boost in consumption. For optimizing agents, additional to this effect, their financial wealth also rises. Therefore, although the welfare for both types of agents increases, steady state inequalities are also higher.

5. Finally, when there are shifts to a macroeconomic scenario of high level of government debt, for a given share of limited participation, there is a redistribution effect causing inequalities in the steady state to rise. The reason is the same presented in Mankiw (2000): higher levels of debt are associated with raises in labor income taxes. In our model economy, taxes fall equally among agents reducing their consumption. Nevertheless, optimizing agents experience a rise in their financial wealth that more than offsets the welfare losses caused by higher taxes. Thus, while liquidity constrained consumers suffer a welfare lost, there are gains for Ricardian agents, raising the long run equilibrium inequalities.

The paper is structured in the following way. Section 1.2 briefly reviews the related literature. The model economy is described on Section 1.3 and the optimal time-consistent policy problem in Section 1.4. The solution method and the model calibration are discussed in Section 1.5 and the numerical results are presented in 1.6. Section 1.7 concludes.

1.2 RELATED LITERATURE

The current work is related to several branches of optimal monetary and fiscal policy and limited asset markets participation (LAMP henceforth) literatures. This section brings a brief overview of those works that are most closely related in terms of topics and methods.

There is a vast literature exploring the implications on monetary policy that the presence of limited asset markets participation, in the tradition of Campbell and Mankiw (1989), may cause. This strand of research tends to focus on the determinacy properties and policy design in standard New Keynesian models augmented by non-Ricardian agents. Extending a New

Keynesian model with sticky prices to include rule-of-thumb consumers, Galí, López-Salido, and Vallés (2004) found that the Taylor rule principle may not be a good policy guideline when some consumers are not able to smooth consumption intertemporally. If the central bank follows a Taylor rule responding to current inflation, the Taylor principle is strengthened when the share of rule-of-thumb consumers is high enough. Nevertheless, if the rule is set to respond to expected future inflation, rather than current, the policy rule may need to violate the Taylor principle, turning passive, in order to ensure a determinate equilibrium. Bilbiie (2008) by introducing limited asset markets participation to a standard dynamic general equilibrium model shows that this 'inverted Taylor principle' holds in general, regardless of whether the rule is specified in terms of current or expected inflation. In his model, when the share of liquidity constrained agents is high enough and/or the elasticity of labor supply is low, prevails an equilibrium where aggregate demand is positively related to real interest rates, i.e., the IS curve is upward sloping. Under this inverted aggregate demand logic, as he labeled, an optimal welfare-maximizing discretionary monetary policy has to follow a passive rule (lower real interest rate in response to a higher inflation) to guarantee uniqueness. In a similar vein, Di Bartolomeo and Rossi (2007) study the effectiveness of monetary policy in the presence of LAMP. Their main result is that, although an increase in the share of rule-of-thumb consumers reduces monetary policy effectiveness through consumption intertemporal allocation, the behavior of those same agents of reacting to changes in current disposable income in a Keynesian way supports a more effective policy in a way that more than offsets the former negative effect. Ascari, Colciago, and Rossi (2017) show that the inverted aggregate demand logic of Bilbiie (2008) heavily relies on the assumption of nominal wage flexibility. Introducing wage stickiness to the model can restore the conventional policy prescription.

The intertwine between limited asset markets participation and fiscal policy mainly focus on attempts to overturn the inability of standard representative agents, based in the permanent income hypothesis, to replicate the empirical evidence that government expenditure shocks have positive effects on private consumption. Examples of that are, *inter alios*, Galí, López-Salido, and Vallés (2007) and Furlanetto (2011).

Most closely related to this work, there is also a literature on monetary and fiscal policy interaction allowing for the presence of non-Ricardian consumers. Kirsanova et al. (2007) study the importance of fiscal policy stabilization in a monetary union using an open-economy model with simple fiscal policy rules, rather than completely optimal, and non-Ricardian consumers based on the overlapping-generations framework of Blanchard-Yaari (YAARI, 1965; BLANCHARD, 1985). Also considering the perpetual youth structure of Blanchard-Yaari in a New Keynesian model, Chadha and Nolan (2007) explore joint fiscal and monetary rules used in stabilization policy. Their main finding is that conducting a stabilization policy requires not only a monetary policy that adopts the Taylor principle, but also a fiscal policy that accounts for automatic stabilizers. Still building on the perpetual youth model of Blanchard-Yaari, Leith and Thadden (2008) study determinacy properties of simple rules-based fiscal and monetary policy in the presence of non-Ricardian agents, which breaks the Ricardian equivalence. Analysis of local steady-state dynamics shows that stabilisation policies are dependent on the level of government debt. Unlike the present work, all studies mentioned use optimal exogenous policy rules. Albonico and Rossi (2017) investigate the effects that the presence of limited asset markets participation have on optimal monetary and fiscal policies when those are conducted by independent authorities who can act strategically. They assume that the government runs a balanced budget by levying on appropriate lump-sum taxes and that government debt plays no role. Rigon and Zanetti (2018) extend this framework to consider a microfounded welfare function to study optimal discretionary monetary policy and its interaction with fiscal policy in a New Keynesian model with non-Ricardian agents and government debt. In their linear-quadratic model they find that the welfare relevance of debt stabilization is proportional to the debt-to-GDP ratio, whose steady state value is exogenously given. When government debt is introduced to the model, it becomes a relevant state variable which needs to be accounted for, as Leith and Thadden (2008) pointed out. Linearization around a steady state becomes problematic as now the steady state becomes endogenous and, thus, global techniques as applied in this work are more well suited.

Aside from the literature exploring the implications of limited asset markets participation, this work also connects to a more general literature on optimal fiscal and monetary policy. Under the scope of the latter, there is a vast number of works in the tradition of Barro (1979) and Lucas and Stokey (1983) which tend to focus on real or flexible price economies where the policymaker operates under full-commitment (or Ramsey policies). Barro (1979) shows that, when the policymaker is not able to issue state-contingent debt or to use inflation surprises to hedge debt against shocks, debt and distortionary taxes are smoothed over time following a random walk process. In Lucas and Stokey (1983), by its turn, government can issue real state-contingent debt which can isolate government's finances from the effects of shocks and, thus, taxes are flat and inherit the properties of disturbances, unlike the tax-smoothing result of Barro (1979). In both the aforementioned works, the long-run level of debt depends on its initial conditions and, therefore, fail to explain why the public debt is a sizable share of output. Aiyagari et al. (2002) show that a departure from the assumption of complete markets in a model otherwise identical to Lucas and Stokey (1983) makes this issue even worse, as it now government accumulate assets rather than liabilities.

The inability of those models to allow for debt accumulation can be overturned through departures from the assumption of benevolent planners under full-commitment. The political economy literature shows that when there is a political disagreement, which is a limitation to the ability to commit, among different policymakers, this can lead to an inefficiently high level of government debt (PERSSON; SVENSSON, 1989; ALESINA; TABELLINI, 1990). This gave rise to a literature focusing on optimal time-consistent fiscal policy in real models. The assumption of lack of commitment by policymakers yields their optimal behavior characterized by generalized Euler equations that involves the derivatives of some equilibrium decision rules and, thus, linearization around a deterministic steady state becomes a complicate matter as now this steady state is endogenous and a priori unknown. This fact claims for the application of different numerical techniques in order to solve the model, see Klein and Ríos-Rull (2003), Krusell, Martin, and Rios-Rull (2006), Ortigueira (2006) and Klein, Krusell, and Ríos-Rull

(2008). Debortoli and Nunes (2013), using a numerical algorithm that is similar to the one used in the present work, find that the lack of commitment is enough to ensure that the government debt converges to a specific and determinate level of steady state (a striking different result from the full commitment case). Nevertheless, the economy often converges to a steady state characterized by no debt accumulation at all and, therefore, lack of commitment *per se* cannot help to explain why the level of debt is so high in several developed economies.

Finally, the work that is most closely related to ours, both in topics and numerical methods, was developed by Leeper, Leith, and Liu (2016, 2019). In their paper they study jointly optimal and time-consistent monetary and fiscal policy in a standard New Keynesian model with optimally chosen distortionary taxation and government spending, augmented to include government debt with a more realistic maturity structure. Their model is solved using non-linear projection methods, more specifically, using Chebyshev polynomials. Their main result is that the standard inflationary bias problem, under discretion, becomes state-dependent and it is exacerbated when the level of public debt is high and/or debt is short-term. But this also creates a debt stabilization bias as now the policymaker incentive to return government debt to its steady state value is higher. Those considerations can change radically the optimal response to shocks in New Keynesian models. In this work, we build on the models of Leeper, Leith, and Liu (2016, 2019) to incorporate another source of departure from Ricardian equivalence, alongside distortionary taxation. That is, we extended their work to allow for limited asset markets participation.

1.3 THE MODEL

Building on a standard New Keynesian model, our model economy is augmented to incorporate the government's budget constraint, where government spending is financed by distortionary taxation and/or issuing of long-term debt, and the existence of limited asset market participation.

1.3.1 Households

The model economy is populated by a continuum of infinitely-lived households of unit mass. A fraction $1 - \lambda$ is represented by optimizing households who are forward looking, can smooth consumption and have access to asset markets: we refer to this subset of consumers as 'optimizing' or 'Ricardian' agents[2] (GALÍ; LÓPEZ-SALIDO; VALLÉS, 2004, 2007). The remaining λ share of households is composed of agents who do not own any assets, cannot smooth consumption and, therefore, display a "hand-to-mouth" behavior of consuming all their current disposable labor income at each period: 'liquidity constrained' (BILBIIE, 2008) or 'rule-of-thumb' consumers[3] (GALÍ; LÓPEZ-SALIDO; VALLÉS, 2004, 2007). As it is typically assumed in the LAMP literature, the fraction λ is an exogenously given constant, meaning that

[2] Alternative nomenclatures found in the literature are: 'asset holders' in Bilbiie (2008), 'savers' in Mankiw (2000), 'market participants' or 'active' in Nisticò (2016).

[3] Or 'non-traders' as in Alvarez, Lucas, and Weber (2001), 'spenders' in Mankiw (2000) and 'financially inactive' in Nisticò (2016).

agents cannot change their type over time[4].

All households derive utility from private consumption, C_t^j, the provision of public goods, G_t, and disutility from labor supply, N_t^j, where $j \in \{o, r\}$[5]. Both optimizing and liquidity constrained agents share the same preference structure defined by the discount factor $\beta \in (0, 1)$ and period utilities that take the following separable form:

$$u(C_t^j, G_t, N_t^j) = \frac{(C_t^j)^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^j)^{1+\varphi}}{1+\varphi}, \tag{1.1}$$

where $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution between private consumption, $\sigma_g > 0$ is the inverse of the intertemporal elasticity of substitution between public consumption, $\chi > 0$ is a scaling parameter, and $\varphi > 0$ is the inverse of the Frisch labor supply elasticity.

Ricardian households

Ricardian agents seek to maximize the expected utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^o)^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^o)^{1+\varphi}}{1+\varphi} \right), \tag{1.2}$$

subject to the sequence of budget constraints:

$$P_t C_t^o + P_t^M \frac{B_t^M}{1-\lambda} \leq \frac{\Xi_t}{1-\lambda} + (1 + \rho P_t^M) \frac{B_{t-1}^M}{1-\lambda} + (1-\tau_t) W_t N_t^o, \tag{1.3}$$

where P_t is the nominal price index, Ξ_t is the Ricardian's share of profits in the monopolistically competitive firms, W_t is the nominal wage set in a competitive labor market, and τ_t is a wage income tax rate. Following Woodford (2001), optimizing households buy government bonds B_t^M in period t at price P_t^M that are actually a portfolio of many bonds paying an exponential decaying coupon of ρ^j monetary units $j+1$ periods after they were issued[6], where $0 \leq \rho < \beta^{-1}$. If prices are stable, a measure of the duration of such bond is given by $(1-\beta\rho)^{-1}$. The budget constraint (1.3) states that the total financial wealth of Ricardian agents in period t plus its consumption spending cannot exceed the sum of financial wealth brought into the period and the after-tax nonfinancial income.

It is important to emphasize that policymakers cannot levy on any kind of lump-sum tax-financed subsidy that would enable him to offset distortions associated with monopolistic competition, a typical, but unrealistic, assumption in New Keynesian models (LEEPER; LEITH; LIU, 2016, 2019).

Necessary and sufficient conditions for household optimization require the household's

4 A possible way to generalize this framework is to allow for stochastic transition between the two types of agents, see Nisticò (2016). We leave the introduction of this Markov switching process to future research.

5 Henceforth, to keep the notation of Galí, López-Salido, and Vallés (2004, 2007), superscripts "o" and "r" denote, respectively, variables of optimizing and rule-of-thumb households.

6 Note that this simple structure nests the cases of consols ($\rho = 1$) and the standard single period bonds ($\rho = 0$).

budget constraints to bind with equality. Defining Ricardian consumers' wealth as $D_t \equiv (1+\rho P_t^M)\frac{B_{t-1}^M}{1-\lambda}$, the no-Ponzi game constraint can be written as:

$$\lim_{T\to\infty} \mathbb{E}_t \left[R_{t,T}^M \frac{D_T}{P_T} \right] \geq 0, \tag{1.4}$$

where $R_{t,T}^M \equiv \prod_{s=t}^{T-1} \left(\frac{1+\rho P_{s+1}^M}{P_s^M} \frac{P_s}{P_{s+1}} \right)$ for $T \geq 1$ and $R_{t,t}^M = 1$, see Preston and Eusepi (2011). This solvency constraint means that Ricardian agents do not overaccumulate debt, and must hold with equality in equilibrium.

First-order conditions (FOCs) associated with Ricardian agents' optimization problem are given by:

$$P_t^M = \beta \mathbb{E}_t \left\{ \left(\frac{C_t^o}{C_{t+1}^o} \right)^\sigma \frac{1}{\Pi_{t+1}} (1+\rho P_{t+1}^M) \right\}, \tag{1.5}$$

$$(1-\tau_t)\left(\frac{W_t}{P_t}\right) = (N_t^o)^\varphi (C_t^o)^\sigma, \tag{1.6}$$

where $\Pi_{t+1} \equiv P_t/P_{t+1}$ is the gross inflation rate.

For later use, the stochastic discount factor is defined as:

$$Q_{t,t+1} \equiv \beta \left(\frac{C_t^o}{C_{t+1}^o} \right)^\sigma \left(\frac{P_t}{P_{t+1}} \right),$$

where $\mathbb{E}_t [Q_{t,t+1}] = R_t^{-1}$ is the inverse of the short-term nominal interest rate, R_t, which is the monetary policy's instrument.

Equation (1.5) gives the optimal allocation across time, and prices the declining payoff consols. If $\rho = 0$, the model reduces to the single period bonds case, and the price of these bonds will be given by $P_t^M = R_t^{-1}$. Otherwise, the presence of long term bonds introduces the term structure of interest rates to the model.

The latter optimality condition (1.6) states that the marginal rate of substitution between consumption and leisure equals the after-tax real wage rate.

Liquidity constrained households

Rule-of-thumb consumers are unable to smooth their consumption path over time. At each period they solve a static problem, i.e., they maximize their period utility:

$$\frac{(C_t^r)^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^r)^{1+\varphi}}{1+\varphi}, \tag{1.7}$$

subject to the constraint that all of their labor income net of taxes is consumed:

$$P_t C_t^r = (1-\tau_t) W_t N_t^r. \tag{1.8}$$

The associated first-order condition yields:

$$(1-\tau_t)\frac{W_t}{P_t} = (N_t^r)^\varphi (C_t^r)^\sigma. \tag{1.9}$$

Note from the liquidity constrained consumers' problem that they do not intertemporally substitute consumption in response to changes in interest rates.

As in Albonico and Rossi (2017), firms are indifferent with respect to the type of agent they employ. Therefore, the equilibrium on the competitive labor market requires:

$$(N_t^o)^\varphi (C_t^o)^\sigma = (N_t^r)^\varphi (C_t^r)^\sigma, \tag{1.10}$$

that is, the marginal rates of substitution between consumption and leisure of Ricardian and liquidity constrained consumers are equalized.

Aggregation

Aggregate private consumption and labor supply are defined as follows:

$$C_t \equiv \lambda C_t^r + (1-\lambda) C_t^o, \tag{1.11}$$

$$N_t \equiv \lambda N_t^r + (1-\lambda) N_t^o. \tag{1.12}$$

1.3.2 Firms and price-setting

There is a continuum of monopolistically competitive firms, indexed by $j \in [0,1]$, producing differentiated intermediate goods. A typical firm in the intermediate good sector produces a differentiated consumption good subject to a linear production function:

$$Y_t(j) = N_t(j). \tag{1.13}$$

Each firm faces a demand curve for their product that is given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} Y_t, \tag{1.14}$$

where $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$ and $P_t = \left[\int_0^1 P_t(j)^{1-\epsilon_t} dj\right]^{\frac{1}{1-\epsilon_t}}$. The elasticity of substitution between varieties, ϵ_t, is assumed to be a time-varying exogenous AR(1) process given by:

$$\ln(\epsilon_t) = (1-\rho_\epsilon)\ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon_{t-1}) + \sigma_\epsilon \varepsilon_t, \qquad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,1), \tag{1.15}$$

where $0 \leq \rho_\epsilon < 1$. The assumption of a stochastic elasticity of substitution allows for variations in the desired price markups and, hence, it is a device to introduce markup shocks to the model, as in Beetsma and Jensen (2004).

Following Rotemberg (1982), sluggish price adjustment is introduced by assuming that firms face a resource cost for adjusting their nominal prices that is quadratic in price changes and proportional to the nominal level of activity. This quadratic cost is defined for a monopolistic firm j as[7]:

$$\eta_t(j) \equiv \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi^* P_{t-1}(j)} - 1\right)^2 Y_t, \tag{1.16}$$

where $\phi \geq 0$ measures the degree of nominal price stickiness and Π^* is the chosen inflation target.

The problem facing firm j is to maximize the discounted value of nominal profits:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{z=0}^{\infty} Q_{t,t+z}\Xi_{t+z}(j),$$

where nominal profits are defined as,

$$\Xi_t(j) \equiv P_t(j)Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi^* P_{t-1}(j)} - 1\right)^2 P_t Y_t,$$

subject to the linear production function (1.13), the demand schedule for their product (1.14), and the quadratic adjustment costs in changing prices (1.16).

The first-order condition for a symmetric equilibrium implies the following Rotemberg version of the nonlinear New Keynesian Phillips curve (NKPC):

$$\frac{\Pi_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*} - 1\right) = \beta \mathbb{E}_t\left[\left(\frac{C_t^o}{C_{t+1}^o}\right)^{\sigma} \frac{Y_{t+1}}{Y_t}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right)\right] + \phi^{-1}((1-\epsilon_t) + \epsilon_t mc_t), \tag{1.17}$$

relating current inflation to future expected inflation and to the level of activity.

Defining the real marginal cost of production as $mc_t \equiv W_t/P_t$ and combining it with equations (1.6) and (1.9), this expression can be rewritten as $mc_t = (1-\tau_t)^{-1}[\lambda(N_t^r)^{\varphi}(C_t^r)^{\sigma} + (1-\lambda)(N_t^o)^{\varphi}(C_t^o)^{\sigma}]$.

1.3.3 Government

The government comprises a single policymaker who coordinates monetary and fiscal policies aiming to maximize welfare of both Ricardian and liquidity constrained consumers. Monetary policy uses nominal interest rate on short-term nominally riskless discount bonds, R_t, as its instrument. Fiscal policy's control variables are the level of government consumption, G_t, and distortionary labor income taxes, τ_t.

The level of aggregate government expenditures on the provision of public goods takes the same form as private consumption:

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right)^{\frac{\epsilon_t}{\epsilon_t - 1}},$$

[7] We consider the Rotemberg (1982) pricing approach, rather than Calvo (1983) pricing, because this reduces the number of endogenous state variables. In the latter, price dispersion becomes an additional endogenous state variable, which complicates matters when solving the model through nonlinear methods. However, as Leith and Liu (2016) and Sims and Wolff (2017) conclude, the form of nominal inertia adopted is not innocuous for higher order of approximations.

such that government's demand for individual goods is given by:

$$G_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} G_t.$$

Government expenditures are financed by levying distortionary taxation on labor income at the rate τ_t, and by issuing long-term bonds, B_t^M. The government solvency constraint can then be written, in real terms, as:

$$P_t^M b_t = (1 + \rho P_t^M)\frac{b_{t-1}}{\Pi_t} - \tau_t w_t N_t + G_t, \tag{1.18}$$

where $b_t \equiv B_t^M / P_t$ denotes real debt and $w_t \equiv W_t/P_t$ are real wages.

The nominal nature of debt widens the interaction between monetary and fiscal policies. Monetary nominal interest rate policy decisions can affect the government budget in the following ways: (i) influencing directly the nominal return on government's instruments; (ii) an indirect effect on the real market value of outstanding government debt, via changes in the price level; and (iii) on a sticky-price environment, the real effects of monetary policy can change the size of the tax base (LEEPER; LEITH; LIU, 2016).

The maturity of the government debt also plays an important role. If all government debt is comprised of one-period debt, that is $\rho = 0$, by equation (1.18) adjustments on the ex-post real return on bonds are only possible through changes in current period inflation Π_t, which can be costly in a sticky-price economy. A longer maturity of government debt, $0 < \rho < 1$, introduces another source of adjustments in the ex-post real return. Changes in the bond price, P_t^M, which depends on future inflation, can help to achieve the necessary adjustments on real debt return at a smaller cost.

1.3.4 Market Clearing

Goods market clearing requires, for each good j,

$$Y_t(j) = C_t(j) + G_t(j) + \eta_t(j),$$

such that, in a symmetrical equilibrium,

$$Y_t \left[1 - \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*} - 1\right)^2\right] = C_t + G_t. \tag{1.19}$$

Alongside, market clearing condition in the bonds market requires that the portfolio of long-term bonds held by households evolves according to the government's budget constraint.

Before analyzing the optimal time-consistent policy problem, the competitive rational expectations equilibrium is defined as follows:

Definition 1.1 (Competitive equilibrium). *A competitive rational expectations equilibrium consists of a plan $\{C_t^o, C_t^r, N_t^o, N_t^r, \Pi_t\}_{t=0}^{\infty}$ satisfying: (i) households' budget constraints (1.3) and (1.8); (ii) optimality conditions (1.5), (1.6), (1.9) and (1.17); (iii) the production function $Y_t = N_t$, and aggregations (1.11) and (1.12); (iv) equilibrium in the competitive labor market (1.10); (v) the government's budget constraint (1.18); (vi) the market clearing condition (1.19); and (vii) the no-Ponzi-game condition (1.4), given the government policies $\{R_t, G_t, \tau_t, b_t\}_{t=0}^{\infty}$, prices $\{w_t, P_t^M\}_{t=0}^{\infty}$, the exogenous process $\{\epsilon_t\}_{t=0}^{\infty}$ and an initial level of government debt b_{t-1}.*

1.4 JOINT OPTIMAL TIME-CONSISTENT POLICY

Fiscal and monetary policies are assumed to be conducted by a single policymaker constrained to act in a time-consistent manner. Acting under discretion, the policymaker is unable to commit to any particular future plan and, instead, reoptimizes his responses at each period. However, the presence of an endogenous state variable in the form of government debt yields the optimal discretionary policy history dependent and, hence, policy actions made today can have effects on future expectations through the debt stock that the policy bequeaths to the future.

Following Albonico and Rossi (2017), we assume that the welfare criterion adopted by the policymaker is to maximize a weighted average of Ricardian and liquidity constrained consumers' utilities. That is, the policy under discretion is described by a set of decision rules $\{C_t^o, C_t^r, N_t^o, N_t^r, Y_t, \Pi_t, b_t, \tau_t, G_t\}$ which maximize the value function:

$$V(b_{t-1}, \epsilon_t) = \max \left\{\lambda u(C_t^r, G_t, N_t^r) + (1-\lambda)u(C_t^o, G_t, N_t^o) + (\delta\beta)\mathbb{E}_t[V(b_t, \epsilon_{t+1})]\right\}, \tag{1.20}$$

subject to the budget constraint of rule-of-thumb consumers (1.8), the equalization condition of marginal utilities between Ricardian and liquidity constrained consumers (1.10), aggregate conditions (1.11)-(1.12), the production function (1.13), the New Keynesian Phillips curve (1.17), the resource constraint (1.19), and the government's budget constraint (1.18). The parameter $0 \leq \delta \leq 1$ captures the possibility that the policymaker may suffer from a degree of myopia, that is, it can discount the future more heavily than society does[8].

Defining the following state-dependent auxiliary functions to capture future expectations:

$$\begin{aligned}
M(b_t, \epsilon_{t+1}) &\equiv (C_{t+1}^o)^{-\sigma} Y_{t+1} \frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right), \\
L(b_t, \epsilon_{t+1}) &\equiv (C_{t+1}^o)^{-\sigma}(\Pi_{t+1})^{-1}(1 + \rho P_{t+1}^M),
\end{aligned}$$

the Lagrangian for the policy problem can, after some algebraic manipulations, be written as:

$$\begin{aligned}
\mathcal{L} \;=\; & \left\{\lambda\left[\frac{(C_t^r)^{1-\sigma}}{1-\sigma} + \chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^r)^{1+\varphi}}{1+\varphi}\right] + (1-\lambda)\left[\frac{(C_t^o)^{1-\sigma}}{1-\sigma} + \chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^o)^{1+\varphi}}{1+\varphi}\right] + (\delta\beta)\mathbb{E}_t[V(b_t, \epsilon_{t+1})]\right\} \\
+ \; & \mu_{1t}\left[Y_t\left(1 - \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*} - 1\right)^2\right) - \lambda C_t^r - (1-\lambda)C_t^o - G_t\right] \\
+ \; & \mu_{2t}\left[(1-\epsilon_t) + \epsilon_t(1-\tau_t)^{-1}[(N_t^o)^{\varphi}(C_t^o)^{\sigma}] - \phi\frac{\Pi_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*} - 1\right) + \beta\phi(C_t^o)^{\sigma}Y_t^{-1}\mathbb{E}_t[M(b_t, \epsilon_{t+1})]\right] \\
+ \; & \mu_{3t}\left[\beta(C_t^o)^{\sigma} b_t \mathbb{E}_t[L(b_t, \epsilon_{t+1})] - \frac{b_{t-1}}{\Pi_t}(1 + \rho\beta(C_t^o)^{\sigma}\mathbb{E}_t[L(b_t, \epsilon_{t+1})]) + \left(\frac{\tau_t}{1-\tau_t}\right)[(N_t^o)^{\varphi}(C_t^o)^{\sigma}]Y_t - G_t\right] \\
+ \; & \mu_{4t}[Y_t - \lambda N_t^r - (1-\lambda)N_t^o] \\
+ \; & \mu_{5t}[(N_t^r)^{\varphi}(C_t^r)^{\sigma} - (N_t^o)^{\varphi}(C_t^o)^{\sigma}] \\
+ \; & \mu_{6t}[C_t^r - (N_t^r)^{1+\varphi}(C_t^r)^{\sigma}]
\end{aligned} \tag{1.21}$$

where the bond pricing equation (1.5) was used to eliminate the current value of the bond in the government's budget constraint. As Leeper, Leith, and Liu (2019) emphasize, although the

[8] The parameter δ can be interpreted as an exogenous probability of the policymaker being voted out of office

policymaker optimizes with respect to all endogenous variables, it is not acting as a social planner. Instead, it is influencing the decentralized equilibrium by choosing its policy instruments, aiming to maximize the objective function subject to the time-consistency constraint.

The implied set of first-order conditions is given by:

$$\begin{aligned}
C_t^o: \quad & (1-\lambda)(C_t^o)^{-\sigma} - \mu_{1t}(1-\lambda) + \sigma\mu_{2t}\left[\epsilon_t(1-\tau_t)^{-1}(N_t^o)^{\varphi}(C_t^o)^{\sigma-1} + \beta\phi(C_t^o)^{\sigma-1}Y_t^{-1}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]\right] \\
& +\sigma\mu_{3t}\left[\beta(C_t^o)^{\sigma-1}b_t\mathbb{E}_t[L(b_t,\epsilon_{t+1})] - \rho\beta\frac{b_{t-1}}{\Pi_t}(C_t^o)^{\sigma-1}\mathbb{E}_t[L(b_t,\epsilon_{t+1})] + \left(\frac{\tau_t}{1-\tau_t}\right)(N_t^o)^{\varphi}(C_t^o)^{\sigma-1}Y_t\right] \\
& -\sigma\mu_{5t}(N_t^o)^{\varphi}(C_t^o)^{\sigma-1} = 0, \\
C_t^r: \quad & \lambda(C_t^r)^{-\sigma} - \lambda\mu_{1t} + \sigma\mu_{5t}(N_t^r)^{\varphi}(C_t^r)^{\sigma-1} + \mu_{6t}[1-\sigma(N_t^r)^{1+\varphi}(C_t^r)^{\sigma-1}] = 0, \\
N_t^o: \quad & (\lambda-1)(N_t^o)^{\varphi} + \varphi\mu_{2t}\epsilon_t(1-\tau_t)^{-1}(N_t^o)^{\varphi-1}(C_t^o)^{\sigma} + \varphi\mu_{3t}\left(\frac{\tau_t}{1-\tau_t}\right)(N_t^o)^{\varphi-1}(C_t^o)^{\sigma}Y_t \\
& -(1-\lambda)\mu_{4t} - \varphi\mu_{5t}(N_t^o)^{\varphi-1}(C_t^o)^{\sigma} = 0, \\
N_t^r: \quad & \lambda(N_t^r)^{\varphi} + \lambda\mu_{4t} - \varphi\mu_{5t}(N_t^r)^{\varphi-1}(C_t^r)^{\sigma} + (1+\varphi)\mu_{6t}(N_t^r)^{\varphi}(C_t^r)^{\sigma} = 0, \\
G_t: \quad & \chi G_t^{-\sigma_g} - \mu_{1t} - \mu_{3t} = 0, \\
Y_t: \quad & \mu_{1t}\left(1-\frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*}-1\right)^2\right) - \beta\phi\mu_{2t}(C_t^o)^{\sigma}Y_t^{-2}\mathbb{E}_t[M(b_t,\epsilon_{t+1})] + \mu_{3t}\left(\frac{\tau_t}{1-\tau_t}\right)(N_t^o)^{\varphi}(C_t^o)^{\sigma} + \mu_{4t} = 0, \\
\Pi_t: \quad & -\phi\mu_{1t}\frac{Y_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*}-1\right) - \mu_{2t}\frac{\phi}{\Pi^*}\left(2\frac{\Pi_t}{\Pi^*}-1\right) + \mu_{3t}\frac{b_{t-1}}{\Pi_t^2}(1+\rho\beta(C_t^o)^{\sigma}\mathbb{E}_t[L(b_t,\epsilon_{t+1})]) = 0, \\
\tau_t: \quad & \mu_{2t}\epsilon_t + \mu_{3t}Y_t = 0, \\
b_t: \quad & -(\delta\beta)\mathbb{E}_t\left[\mu_{3t+1}\frac{1}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right] + \beta\phi\mu_{2t}(C_t^o)^{\sigma}Y_t^{-1}\mathbb{E}_t[M_b(b_t,\epsilon_{t+1})] \\
& +\mu_{3t}\left[\beta(C_t^o)^{\sigma}\mathbb{E}_t[L(b_t,\epsilon_{t+1})] + \beta(C_t^o)^{\sigma}b_t\mathbb{E}_t[L_b(b_t,\epsilon_{t+1})] - \rho\beta\frac{b_{t-1}}{\Pi_t}(C_t^o)^{\sigma}\mathbb{E}_t[L_b(b_t,\epsilon_{t+1})]\right] = 0,
\end{aligned}$$

where $X_b(b_t,\epsilon_{t+1}) \equiv \partial X(b_t,\epsilon_{t+1})/\partial b_t$ for functions $X \in \{L, M\}$, and we have used the envelope theorem on the first-order condition for government debt to obtain:

$$\frac{\partial V(b_{t-1},\epsilon_t)}{\partial b_{t-1}} = -\mu_{3t}\frac{1}{\Pi_t}(1+\rho P_t^M).$$

Optimality condition for inflation reveals another possible source of inflationary bias, besides the standard bias associated with discretionary policies. The first term captures the costs, in terms of the resource constraint, of raising inflation, while the second highlights the positive effects on the output-inflation trade-off, given expectations, in the New Keynesian Phillips curve when the economy is at a suboptimal position. The last term in the first-order condition captures another benefit of raising inflation, the negative effect it has on the real market value of government debt. Ricardian agents will perceive that a higher debt is an incentive for government to introduce inflation surprises in an attempt to reduce the debt burden, in doing so they are expected to raise their inflationary expectations, $\mathbb{E}_t[M_b(b_t,\epsilon_{t+1})] > 0$, until that incentive is offset. Leith and Wren-Lewis (2013) labeled this the "debt stabilization bias".

The first-order condition for debt can be rewritten as:

$$P_t^M\mu_{3t} - (\delta\beta)\mathbb{E}_t\left[\tfrac{\mu_{3t+1}}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right] - \mu_{3t}\left\{\phi\beta\epsilon^{-1}\mathbb{E}_t[M_b(b_t,\epsilon_{t+1})] - \left[\left(b_t-\rho\tfrac{b_{t-1}}{\Pi_t}\right)\mathbb{E}_t[L_b(b_t,\epsilon_{t+1})]\right]\right\} = 0, \quad (1.22)$$

in the following period (STEHN; VINES, 2008).

the first two terms of the equation is a version of the standard tax-smoothing argument of Barro (1979), requiring that marginal costs of taxation are smoothed over time. Under commitment, only these two terms would appear, implying that the steady state of debt will follow a random walk (LEEPER; LEITH; LIU, 2016).

Given that the policymaker is constrained to act in a time-consistent manner, its decisions can affect future variables through the level of debt it bequeaths. This is captured by the partial derivatives of debt in the last term of equation (1.22)[9]. As in Leeper, Leith, and Liu (2019), the numerical results obtained in this work yield: (i) $M_b(b_t, \epsilon_{t+1}) > 0$, which means that, as previously discussed, inflation expectations rise when debt levels are higher. Hence, with nominal rigidity, the policymaker has an incentive to reduce debt and inflation, deviating from tax smoothing; and (ii) $L_b(b_t, \epsilon_{t+1}) < 0$, implying a negative relationship between debt level and bond prices. A higher level of debt leads to lower bond prices, given that it raises inflation. With lower prices on government bonds, the policymaker needs to issue more bonds to finance its debt, but at the same time it also means that it needs to pay less to buy back the existing debt stock. As the maturity of the debt increases, the latter effect rises relative to the former, and may even be reversed, reducing the incentive to lower debt levels that the policymaker is facing.

The nonlinear system of first-order conditions previously described seems to be largely unaffected by the fraction of liquidity constrained consumers in the economy. However, this apparent invariance is deceptive. Increases in the share of rule-of-thumb consumers, λ, are, ceteris paribus, associated with higher per capita profits in the sector of monopolistically competitive firms, increasing the monopolistic distortion (ALBONICO; ROSSI, 2017). Therefore, the presence of liquidity constrained agents constitutes an additional distortion in the economy. These distortions, alongside the degree of myopia of the policymaker and the maturity of debt it issues, as we shall see in the numerical results, are crucial in determining the long run equilibrium rate of both inflation and debt-to-GDP ratio.

1.5 NUMERICAL METHODS AND CALIBRATION

This section outlines the numerical method used to solve for the discretionary equilibrium and the calibration of parameters.

1.5.1 Solution method

For the model described in sections 1.3 and 1.4, the equilibrium policy functions cannot be computed analytically and, thus, numerical methods are necessary. Linearization around the steady state, a common approach in economics, is not feasible since the presence of a Generalized Euler Equation yields that steady state endogenous and, therefore, *a priori* unknown. Following Leeper, Leith, and Liu (2016, 2019), we resort on a global approximation method to solve

[9] The presence of partial derivatives of policy functions makes this relation a generalized Euler equation - as labeled by Krusell, Kuruscu, and Smith (2002) - which needs to be solved numerically, given that, in general, the form of these functions are unknown.

for the time consistent equilibrium of the model[10]. More specifically, the policy functions are approximated by Chebyshev polynomials and the nonlinear system of equations is iterated until a set of time-invariant equilibrium of policy rules mapping the vector of state variables to the optimal decisions is reached[11]. The numerical algorithm is detailed in Appendix 1.B.

1.5.2 Calibration

The model is calibrated to a quarterly frequency. The baseline parameterization is summarized on Table 1.1, which is in line with Leeper, Leith, and Liu (2016) and Albonico and Rossi (2017). The discount factor for Ricardian households is set to 0.995, implying an annual real interest rate of 2%. The intertemporal elasticity of substitution between private consumption, σ, and public consumption, σ_g, are equal to one half ($\sigma = \sigma_g = 2$). The baseline value of φ is set to be consistent with a Frisch elasticity of labor supply of one-third (i.e., $\varphi = 3$). The elasticity of substitution between varieties is assumed to be $\bar{\epsilon} = 21$, to be consistent with a markup of 5% as in Siu (2004). The scaling parameter $\chi = 0.055$ is calibrated to ensure that, in the steady state, the government spending-to-GDP ratio, G/Y, is approximately 19%. The annual inflation target, Π^*, is assumed to be 2%, a value which is in line with the adopted by most inflation targeting economies. The coupon decay parameter ρ is calibrated to 0.9598 to ensure that, given the discount factor of Ricardian households β and the inflation target Π^*, the average term to maturity of debt corresponds to 5 years, a duration compatible with data from most OECD countries (see Eusepi and Preston (2013)). The Rotemberg price adjustment cost parameter, $\phi = 32.5$, implies that on average firms re-optimize prices approximately every six months[12] - an empirically plausible value. The degree of myopia of the policymaker is set to $\delta = 0.9899$, which implies a time horizon of approximately 25 years. The parameters characterizing the cost-push exogenous process are given by $\rho_\epsilon = 0.95$ and $\sigma_\epsilon = 0.01$.

Following Albonico and Rossi (2017), the fraction of liquidity constrained consumers can assume three alternative values: $\lambda \in \{0; 0.3; 0.5\}$. When $\lambda = 0$, all consumers of the model are optimizing agents and, hence, the model is the standard representative agent New Keynesian model augmented to include the government's budget constraint. The empirical literature on limited asset market participation reports that the fraction of rule-of-thumb consumers lies between 30% and 50% of the agents population. Campbell and Mankiw (1989) and Muscatelli, Tirelli, and Trecroci (2004) estimated values for λ in the neighborhood of 50%, the empirical microeconomic evidence surveyed in Mankiw (2000) is in line with those findings. Di Bartolomeo, Rossi, and Tancioni (2011) found that, on average, the fraction of non-Ricardian agents for

[10] Time consistency problems, in general, can be treated as dynamic games. In problems of this kind, multiplicity of equilibria often arises. By using polynomial approximations, we are focusing only on continuous equilibria. See Judd (2004) for a discussion on the existence, uniqueness and alternative computational approaches to deal with problems of this kind.

[11] For textbook treatments on the numerical techniques involved see Judd (1998), Miranda and Fackler (2004) and Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016).

[12] Given the equivalence between Calvo and Rotemberg pricing for linearized models, $\phi = \frac{(\epsilon-1)\theta}{(1-\theta)(1-\beta\theta)}$, where θ is the fraction of firms that keep their prices unchanged on Calvo model, see Leith and Liu (2016) and Sims and Wolff (2017).

Parameter	Value	Description
β	0.995	Quarterly discount factor (household)
δ	0.9899	Degree of myopia (policymaker)
σ	2	Relative risk aversion coefficient
σ_g	2	Relative risk aversion coefficient (government spending)
φ	3	Inverse Frisch elasticity of labor supply
χ	0.055	Scaling parameter (government spending)
λ	varying	Fraction of liquidity constrained consumers
ρ	0.9598	Debt maturity structure (5 years)
Π^*	2%	Annual inflation target
ϕ	32.5	Rotemberg adjustment cost coefficient
$\bar{\epsilon}$	21	Elasticity of substitution between varieties
ρ_ϵ	0.95	AR-coefficient of cost-push shock
σ_ϵ	0.01	Standard deviation of cost-push shock

Table 1.1 – Calibration

the G7 countries is 26%. More recently, Albonico, Paccagnini, and Tirelli (2014, 2016) report values of the share of liquidity constrained consumers in between 25% and 53% for the Euro area, and 47% for the US. Those findings motivate our choice for the values of λ.

1.6 NUMERICAL RESULTS

This section reports the numerical results obtained in the solution of the optimal discretionary policy formerly described. Subsection 1.6.1 explores the long-run equilibrium properties under different parameterizations. Optimal dynamics in response to a markup shock through impulse response functions are considered in Subsection 1.6.2. The role of the maturity structure of the government debt in the presence of limited asset markets participation is discussed in Subsection 1.6.3. Lastly, in Subsection 1.6.4 introduces the welfare effects of LAMP and higher levels of public debt.

1.6.1 Steady State

Table 1.2 summarizes the steady state values under different parameterizations of the model. Under the benchmark calibration, when all households on the economy are Ricardian agents, the combination of a myopic policymaker alongside a government debt with a maturity structure of five years[13] implies a positive debt-to-GDP ratio, $\frac{bP^M}{4Y}$, of 55.6% and an annualized rate of inflation in the steady state of 5.76%, which outweighs the established target of 2%. The magnitude of the inflationary bias, that causes the overshooting of inflation with respect to the target, is determined by an inefficiently low equilibrium level of output that gives the discretionary policymaker an incentive to generate surprise inflation to push output closer to

[13] When the discount factor of both policymaker and households are equal and/or debt is short-term, numerical results show that the steady state debt-to-GDP ratio is negative, which means that the government accumulates assets rather than issues liabilities. This is in line with much of the literature, see the discussion on Section 1.2 and Debortoli and Nunes (2013) and Leeper, Leith, and Liu (2016, 2019).

the efficient steady state level. In standard analyses of the inflationary bias problem, the solely source of inefficiency is the degree of monopolistic competition that distorts the steady state. However, in the presence of government debt and distortionary taxation, this inflationary bias problem becomes state-dependent, since at higher levels of debt or taxes, the inefficiency is accentuated and, therefore, increasing the desire to induce surprise inflation.

Variable	Benchmark calibration			$\phi = 50$			$\frac{\bar{\epsilon}-1}{\bar{\epsilon}} = 6\%$		
	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$
$\frac{bP^M}{4Y}$	55.6%	12.6%	-9.0%	64.9%	7.8%	-22.6%	40.4%	-3.6%	-26.4%
$(\Pi^4 - 1)$	5.76%	4.88%	4.45%	5.29%	4.44%	3.97%	5.52%	4.59%	3.88%
$(R^4 - 1)$	7.90%	7.01%	6.57%	7.42%	6.55%	6.08%	7.65%	6.70%	5.98%
Y	1.029	1.031	1.033	1.028	1.032	1.034	1.027	1.030	1.031
G/Y	19.3%	19.4%	19.4%	19.3%	19.4%	19.5%	19.3%	19.4%	19.5%
τ	21.4%	20.6%	20.2%	21.6%	20.5%	20.0%	21.4%	20.5%	20.1%
C^o	0.829	0.844	0.860	0.829	0.844	0.859	0.827	0.844	0.862
C^r	-	0.800	0.803	-	0.800	0.805	-	0.794	0.798
C	0.829	0.831	0.832	0.829	0.831	0.832	0.827	0.829	0.830
N^o	1.029	1.020	1.009	1.028	1.021	1.011	1.027	1.017	1.005
N^r	-	1.058	1.056	-	1.057	1.056	-	1.059	1.058
$\frac{\Xi}{1-\lambda}$	0.048	0.069	0.097	0.047	0.069	0.097	0.057	0.082	0.116
V^o	-352.7	-346.2	-339.2	-352.8	-346.3	-339.9	-352.8	-345.5	-337.6
V^r	-	-367.7	-366.2	-	-367.4	-365.3	-	-369.7	-368.1
V	-352.7	-352.6	-352.7	-352.8	-352.6	-352.6	-352.8	-352.7	-352.9

Table 1.2 – Steady state: Benchmark calibration, price flexibility and monopolistic competition.

The presence of liquidity constrained consumers constitutes another source of distortion to the model economy. As Albonico and Rossi (2017) pointed out and our numerical results show, when the fraction of rule-of-thumb agents, λ, increases, per capita profits earned by optimizing agents, $\frac{\Xi}{1-\lambda}$, also rises[14], pushing the monopolistic distortion even further. A more pronounced monopolistic distortion strengthens the state-dependent inflationary bias, for a given level of debt. Optimizing agents, aware of this higher inflationary bias, raise their inflationary expectations accordingly. As a result, the debt stabilization bias also increases - the discretionary policymaker has now a stronger incentive to influence the endogenous inflationary bias and to mitigate the costs of distortionary taxation through reductions in the level of debt. Therefore, increases in the share of rule-of-thumb consumers on the economy are associated with substantial decreases in both the steady state rate of inflation and debt-to-GDP ratio - the latter even turns negative when $\lambda = 0.5$. Since government spending-to-GDP ratio, G/Y, is mainly unresponsive to variations in λ, lower levels of debt can be financed by lower tax rates. Other model variables are changed only marginally.

As a departure from the benchmark calibration, we consider firstly an increase in the rigidity of prices, ϕ, meaning that inflation is now more costly and that the efficacy of monetary

[14] This increase in the financial wealth of optimizing agents ends up boosting Ricardian consumption in the steady state and, thus, has distributional consequences, see Subsection 1.6.4.

policy in affecting the real side of the economy is enhanced. Consequently, the policymaker has now a stronger incentive to reduce the state-dependent inflationary bias problem. As a result, in general, the equilibrium steady state rate of inflation and debt-to-GDP ratio are lower when prices are less flexible. The Ricardian agents economy, $\lambda = 0$, highlights the fact that equilibrium values are highly dependent on both the magnitude of the government debt stock and its maturity. Under the benchmark calibration, the policymaker faces a high level of debt stock. Once the rigidity of prices rises, the incentive to boost activity through inflation lessens (the inflation bias problem is reduced). Moreover, now it is more costly for the policymaker to use current inflation as a device to reduce the debt burden (steady state equilibrium inflation rate reduces to 5.29%). Therefore, due to the mix of a long-term maturity alongside a high level of debt stock, the steady state debt-to-GDP ratio rises instead - to 64.9% - as this lowers bond prices and, thus, makes it more cheaper for the government to buy back the existing debt stock[15].

Lastly, a more pronounced monopolistic distortion is considered by increasing the markup by one percentage point. Under a greater monopolistic distortion, the inefficiencies of the economy are higher and, hence, the inflationary bias problem is also higher for a given level of debt. As a consequence of that, the policymaker faces a greater incentive to influence the endogenous inflationary bias by reducing the level of debt. By raising the debt stabilization bias, the government ensures that the debt-to-GDP ratio and the inflation rate of equilibrium are lower.

To summarize, the effects that the existence of limited asset market participation have on the jointly optimal monetary and fiscal policy are robust to different parameterization scenarios. Increases in the share of rule-of-thumb consumers, by raising the per capita financial wealth of optimizing agents, deepens the degree of monopolistic distortions in the economy. In doing so, strengthens both the state-dependent inflationary and debt stabilization biases leading to a reduction in the steady state values of debt-to-GDP ratio and inflation rate. Therefore, the fraction of liquidity consumers in the economy is a key driver of the equilibrium rate of inflation and debt stock, alongside the debt maturity and the degrees of myopia of the policymaker[16] and monopolistic competition (LEEPER; LEITH; LIU, 2019). It is important to note that, while the overall value function, V, is largely unaffected by the share of rule-of-thumb agents, the same is not true for their individual counterparts - V^o and V^r - suggesting that the degree of access to asset markets can have distributional consequences (see Subsection 1.6.4).

[15] To test the robustness of these results, we considered the following alternative parameterizations of the model: (i) single-period debt ($\rho = 0$); and (ii) a low degree of myopia ($\delta = 0.995$) - which implies a lower level of debt in the steady state. An increase in the sluggishness of prices causes steady debt-to-GDP ratio falls from 10.8% to 9% in the single-period scenario, and from 4.7% to 2.5% when the policymaker is more patient. Inflation rate also falls: from 3.8% to 3.6% when $\rho = 0$, and from 3.7% to 3.5% when $\delta = 0.995$. This corroborates the results obtained.

[16] The roles of debt maturity and degree of myopia are discussed, respectively, in Subsections 1.6.3 and 1.6.4.

1.6.2 Optimal dynamics in response to a markup shock

Figure 1.1 displays the optimal dynamics of the key macroeconomic variables in response to a positive markup shock through impulse response functions (IRFs henceforth). All variables are measured as percentage deviation from the steady state. Under the benchmark calibration, we consider the cases in which all agents are Ricardians ($\lambda = 0$) and when the fraction of liquidity constrained consumers is $\lambda = 0.3$ and $\lambda = 0.5$[17].

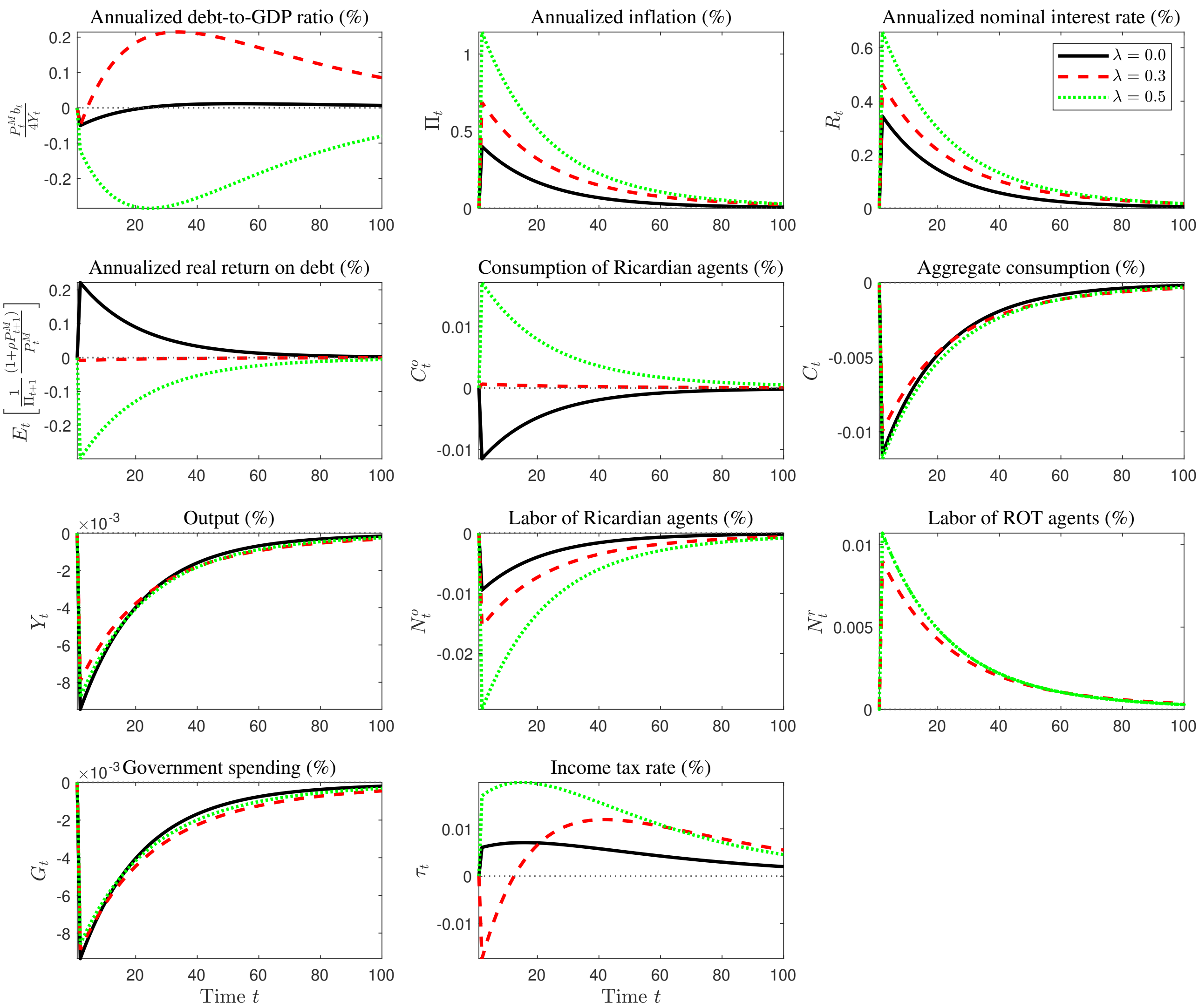


Figure 1.1 – IRFs to a positive markup shock under the benchmark calibration with $\lambda = 0$ (black solid line), $\lambda = 0.3$ (red dashed lines) and $\lambda = 0.5$ (green dotted lines). All variables measured as percentage deviation from steady state.

Before turning to the analysis of IRFs for each scenario, it is convenient to make some general observations. The existence of real imperfections in the form a cost-push shock in our model economy breaks the 'divine coincidence' and, thus, the optimizing discretionary policymaker faces a policy tradeoff between stabilization of inflation and output. In all the cases

[17] It is important to note that all cases considered are associated with a different level of government debt in

considered, following a decrease of one standard deviation, σ_ϵ, in the elasticity of substitution between intermediate goods, which implies an increase in firms markup, the optimal response made by the policymaker acting time-consistently is to accommodate only partially the inflationary consequences of the shock and, in doing so, allowing a fall in output.

Ricardian agents economy. When all households can smooth consumption intertemporally, $\lambda = 0$, the optimal dynamics in response to the markup shock are mainly driven by a desire to reduce debt levels through higher distortionary wage income tax rates. Although, in theory, an income tax cut, in form of a wage subsidy, could offset the effects of the shock, this would deteriorate fiscal finances leading to an increase in the government debt. Since in the Ricardian agents economy steady state level is already high, the policymaker's optimal response is to raise taxes as a more effective way to mitigate the inflationary consequences of the shock. Given that the fiscal side is mainly concerned with debt stabilization, by running fiscal surpluses, monetary policy is tightened to help ensure that inflation rate returns to its steady state.

Limited asset markets participation ($\lambda = 0.3$). The presence of limited asset markets participation amplifies the inflationary consequences of a markup shock. As can be seen in the IRFs, when the fraction of liquidity constrained agents in the economy is set to 30%, the initial surge in inflation is higher than it was on the Ricardian agents case, reflecting the fact that when distortions are deepened, the discretionary policymaker is more willing to accept a higher rate of inflation to temper the fall in output. The most striking result is that monetary policy's optimal response to the shock is to set the nominal interest rate in a manner which leads to an almost pegged real rate of return. This, in turn, leaves the burden of stabilization of the economy against the consequences of the disturbance almost entirely to fiscal policy. The reason for this is that while variations in the real rate of return affect solely the decisions of Ricardian households, fiscal instruments have a direct impact on both agents in the model economy and, thus, constitute a more effective way in mitigating the inflationary consequences caused by the rise in markup. Moreover, numerical results show that government expenditures are hardly used as an instrument of either macroeconomic or fiscal stabilization[18]. This is due to the facts that: (i) government consumption enters directly in the utility functions of both agents; (ii) under the benchmark calibration, the elasticity of substitution for government consumption is low; and (iii) public expenditures crowds out private consumption (GALÍ; LÓPEZ-SALIDO; VALLÉS, 2007). Therefore, the key instrument used to stabilize both inflation and government debt is the distortionary tax rate.

The immediate response of the optimal policy mix following the shock is to lower distortionary taxes. By cutting taxes, the policymaker offsets the inflationary consequences of the shock, since this lowers real marginal costs of production. As a direct consequence of tax reductions, government debt levels initially rise. Nevertheless, time consistency constraint requires that debt has to return to its steady state level and, thus, after some time, there is an over-

the steady state and that those different conditions can have different implications to the optimal dynamics.

[18] Government spending movements are largely in line with variations in output, yielding a stable ratio of government consumption-to-GDP.

shooting in labor taxes to ensure that the upward trend in public debt is reversed and starts to converge to its long run equilibrium value.

To summarize, the coexistence of limited asset markets participation and a low steady state level of debt-to-GDP, in the case of $\lambda = 0.3$, gives fiscal policy a much more active role in macroeconomic stabilization. Under an almost fixed real rate of return, the burden of both offsetting the inflationary consequences of a markup shock and debt stabilization rests entirely on distortionary taxation.

Limited asset markets participation ($\lambda = 0.5$). When the fraction of liquidity constrained consumers is increased even further to half of the model economy's population, the optimal responses of the policy mix to a markup shock are changed. In this particular case, the steady state level of public debt is negative (as shown in Subsection 1.6.1). With the government accumulating a stock of assets, rather than liabilities, the policymaker now faces a tradeoff: while inflation surprises boosts the output moving it closer to the efficient level, it also deteriorates the real value of those assets. The discretionary policymaker, then, has to balance those opposing forces when deciding how to move its instruments in response to the shock. The optimal IRFs to a markup shock, thus, imply decreases on the rate of real return and the net stock of nominal assets held by the government and a surge in distortionary tax rates. Fiscal instruments are optimally chosen to reduce the stock of assets in order to compensate the losses on their real value impulged by the hike in the inflation rate. Note that although this could be obtained by tax cuts, as in the previous case, now half of the consumers population spends all of their disposable income and, therefore, tax cuts would exacerbate the inflationary pressures, demanding an even higher deaccumulation of assets. Monetary policy accommodates this more active role of fiscal policy by lowering the rate of real return, displaying the inverse aggregate demand logic of Bilbiie (2008). Nevertheless, as we shall see, this passive-like behavior of monetary policy is dependent on the steady state level of government debt.

Remark 1.1. Numerical results show that the optimal time-consistent policy mix's choice of instruments on macroeconomic and fiscal stabilization to the shock is largely dependent on the share of rule-of-thumb consumers in the model economy. For values of λ greater than 0.3, the conventional macroeconomic prescription is reversed and monetary policy's behavior resembles a passive policy while the activism of fiscal policy is enhanced. Nonetheless, those results are also dependent on the level of government debt. Figure 1.A.3 shows the non-linearities implied by the policy decisions as functions of lagged debt. The optimal decision rules suggest that for higher levels of public debt the conventional policy assignments are restored. In other words, once debt breaches some threshold, monetary policy responds to the shock by assuring that the real rate of return on debt rises and, thus, the conventional slope of aggregate demand is obtained.

Remark 1.2. We conjecture that those results rely on the assumption of a joint discretionary policy. In this case, a single policymaker has full control of all policy instruments in the economy. However, nowadays most economies delegate monetary policy to an instrument independent central bank and fiscal policy to the finance ministry or the legislature. It would be interesting to expand the present analysis to the case where there are interactions between independent authorities. We leave that for future research.

1.6.3 The role of maturity structure of debt

The effects of the maturity structure of debt on the optimal policy mix under the presence of limited asset market participation are considered in Table 1.3. Setting all the parameters at their values on the baseline parameterization, we fix the fraction of liquidity constrained consumers at $\lambda = 0.3$ and allow the coupon decay parameter to assume values in the following set $\rho \in \{0; 0.7588; 0.9598; 0.9849\}$ corresponding, respectively, to 1 quarter, 1 year, 5 years and 10 years debt maturity.

Variable	1 qtr maturity $\rho = 0$	1 yr maturity $\rho = 0.7588$	5 yr maturity $\rho = 0.9598$	10 yr maturity $\rho = 0.9849$
$\frac{bP^M}{4Y}$	0.49%	6.44%	12.6%	19.8%
$(\Pi^4 - 1)$	4.14%	4.47%	4.88%	5.35%
$(R^4 - 1)$	6.25%	6.58%	7.01%	7.48%
Y	1.032	1.031	1.031	1.031
G/Y	19.4%	19.3%	19.4%	19.4%
τ	20.4%	20.4%	20.6%	20.8%
C^o	0.844	0.845	0.844	0.844
C^r	0.802	0.801	0.800	0.798
C	0.831	0.832	0.831	0.830
N^o	1.021	1.020	1.020	1.020
N^r	1.057	1.057	1.058	1.058
$\frac{\Xi}{1-\lambda}$	0.070	0.069	0.069	0.069
V^o	-346.3	-346.2	-346.2	-346.1
V^r	-366.9	-367.3	-367.7	-368.2
V	-352.5	-352.5	-352.6	-352.7

Table 1.3 – Steady state: the role of debt maturity structure ($\lambda = 0.3$).

The first column of the table considers the conventional assumption found in the literature that debt's duration is only single period. Given the model parameterization, when $\rho = 0$ debt is of quarterly maturity. Under this assumption, the steady state debt-to-GDP ratio is almost zero and the annualized inflation rate reaches 4.14%.

As previously discussed in Section 1.4, a longer maturity of debt reduces the debt stabilization bias and in doing so allows the policymaker to sustain a higher debt-to-GDP ratio in the steady state. This can be seen at Table 1.3 as we gradually rise debt maturity to one, five and ten years. The results show that the steady state debt-to-GDP ratio is an increasing function of the debt maturity structure, reflecting the inverse relationship between longer-term debt and the debt stabilization bias. As the level of debt drives the endogenous inflationary bias problem, the steady state rate of inflation follows the upward trend, going rising from 4.14% (in annual terms) under quarterly maturity to 5.35% when debt maturity is 10 years.

In a similar model, Leeper, Leith, and Liu (2019) show that when debt maturity rises from one quarter to ten years, debt-to-GDP ratio exhibits an accentuated increase from -11.1% to 53.6%. The presence of limited asset market participation in the economy makes the steady state debt stock less sensitive to variations in maturity. Our numerical results show that $\frac{bP^M}{4Y}$

rises from 0.49% to 19.8%, over the same range. The reason for this flatter upward trend when the economy deviates from the Ricardian agents scenario is the following: notwithstanding the fact that an increase in the maturity of debt reduces the debt stabilization bias, when the share of liquidity constrained consumers in the economy rises, this leads to a more pronounced monopolistic distortion, pushing the bias in the opposite direction. Therefore, in a LAMP model the reduction in the debt stabilization bias is lower in magnitude, yielding a flatter upward trend in the steady state debt-to-GDP ratio as the maturity increases[19].

1.6.4 Welfare analysis

In this subsection we consider the welfare effects that variations on the fraction of liquidity constrained consumers and on the degree of myopia of the policymaker impose on the model economy. A closer inspection of Table 1.2 shows that different values of the share of limited asset market participation, λ, have distinct impacts on the value functions of optimizing, V^o, and rule-of-thumb agents, V^r, suggesting distributional consequences. Furthermore, as Mankiw (2000) pointed out, a higher level of debt increases steady state inequality between agents. We address this issue by varying the degree of myopic behavior of the policymaker, δ, given that this affects the steady state level of government debt.

We follow Adam and Billi (2008) and Albonico and Rossi (2017) in adopting a measure for the utility losses associated to different parameterization scenarios. The percentage loss in terms of consumption of alternative scenarios is calculated with respect to the deterministic steady state of some given benchmark parameterization. Let V^B denotes the utility for the benchmark's deterministic steady state, given by:

$$V^B = \frac{1}{(1-\beta)}[\lambda u(C^r, G, N^r) + (1-\lambda)u(C^o, G, N^o)], \tag{1.23}$$

where $u(C^j, G, N^j)$ for $j \in \{o, r\}$ is the period utility defined in equation (1.1). If V^A is the value function of an alternative scenario evaluated at the deterministic steady state, $u(C^j_A, G_A, N^j_A)$, the permanent reduction in private consumption, μ^A, that would imply the benchmark deterministic steady state to be welfare equivalent to the alternative scenario is implicitly defined by the following expression:

$$V^A = \frac{1}{1-\beta}[\lambda u(C^r(1+\mu^A), G, N^r) + (1-\lambda)u(C^o(1+\mu^A), G, N^o)]. \tag{1.24}$$

The same formulae are used to evaluate the value functions of each type of consumer, i.e., $V^B_j = u(C^j, G, N^j)/(1-\beta)$, and:

$$V^A_j = \frac{1}{1-\beta}[u(C^j(1+\mu^A), G, N^j)].$$

Firstly we look at the welfare effects that variations in the fraction of liquidity constrained consumers have under the baseline calibration detailed in Table 1.1, assuming $\lambda = 0.3$

[19] When we set λ to zero and, thus, consider the case in which all households are Ricardians, debt-to-GDP ratio rises from 4.71% when debt maturity is of single period to 73.7% under 10 years debt maturity. If half

as a benchmark scenario. The welfare losses (or gains) for each type of consumer relative to their benchmark values, in terms of consumption equivalents, under different specifications for λ are reported in Table 1.4.

	RAE ($\lambda = 0$)	$\lambda = 0.5$
Ricardians, V^o	-2.69	3.04
Liquidity Constrained, V^r	-	0.59
Total, V	-0.03	-0.03

Table 1.4 – Welfare losses in consumption equivalents (percentages).

Numerical results suggest that while total welfare is only marginally affected by variations in λ, reflecting the fact that the policymaker aims to maintain the overall welfare of the economy, increases in the share of rule-of-thumb agents represent welfare gains for both types of consumers. Optimizing agents experience a welfare a welfare gain when λ rises due to the fact that their financial wealth is enhanced allowing a boost in consumption. In comparison with the benchmark scenario, the Ricardian welfare gain is of 3.04% when λ rises to half of the population, and there is a welfare loss of 2.69% when $\lambda = 0$ and the model collapses to the standard Ricardian agents economy (RAE). Welfare of rule-of-thumb agents also raises when λ increases. The reason for that is, since a higher fraction of liquidity constrained consumers reduces the steady state level of debt, this allows a smaller income tax rate compared to the benchmark scenario and, thus, consumption of rule-of-thumb agents rise. Note, however, that this is the only source of gains for liquidity constrained consumers, while Ricardian agents can support an even higher level of consumption since their financial wealth also increases. Therefore, a higher degree of limited asset market participation brings welfare gains for both Ricardian and rule-of-thumb agents but, at the same time, increases the steady state inequality.

Turning now to the distributional effects that the level of government debt has on the economy, Table 1.5 shows the welfare losses in percentage terms of consumption equivalents of a high government debt level scenario in relation to the baseline calibration of Section 1.5. Reported results distinguishes between total, Ricardian and liquidity constrained welfare and between different values of λ.

	RAE ($\lambda = 0$)	$\lambda = 0.3$	$\lambda = 0.5$
Total	-0.44	-0.33	-0.27
Ricardians	-0.44	0.04	0.29
Liquidity Constrained	-	-1.14	-0.78

Table 1.5 – Level of debt and welfare losses in consumption equivalents (percentages)

A more myopic behavior of the policymaker - lowering the value of δ - serves to render the equilibrium steady state of debt-to-GDP ratio higher. Under the alternative scenario considered in this exercise, δ is set to 0.9808 which implies a time horizon for the policymaker of approximately 13 years. This raises the steady state value of debt to 113% in the Ricardian agents economy, 55.9% when $\lambda = 0.3$ and 20.7% in the case of $\lambda = 0.5$. The logic runs as follows: reductions in the policy maker's time horizon yield the debt stabilization bias smaller once he is less inclined to incur the costs of debt reduction in order to achieve long-term benefits (LEEPER; LEITH; LIU, 2019).

The numerical results reported in Table 1.5 show that when there is a switch from the benchmark case to the high government debt level scenario, total welfare shrinks for all values of λ. In the Ricardian agents economy, $\lambda = 0$, this reduction is caused by the fall in consumption of optimizing agents, as a higher level of debt means a higher level of taxation to help the financing of this debt. Nevertheless, when the share of rule-of-thumb consumers in the economy rises, there is an associated increase in the financial wealth of Ricardian agents as now per capita profits are higher. The rise in the financial wealth more than offsets the losses caused by higher tax rates and, thus, the welfare of Ricardian consumers rises in both cases when $\lambda = 0.3$ and $\lambda = 0.5$. Since liquidity constrained households do not have access to asset markets, they experience a pronounced fall in consumption induced by the higher taxation prevailing under higher levels of debt. Therefore, welfare of rule-of-thumb agents decreases as the level of debt rises.

To summarize, the level of government debt influences the distribution of income and consumption in our model economy in the presence of limited asset market participation. Higher levels of debt mean a higher level of taxation. The distortionary labor income tax rate falls both on optimizing and liquidity constrained consumers, but only Ricardian agents receive interest payments on government bonds. Therefore, when the level of debt increases there is a redistribution effect raising the steady state inequalities in the model economy. This corroborates the results found by Mankiw (2000) in his savers-spenders model.

1.7 CONCLUSION

The recent global financial crisis and its developments were associated with a steadily increase in the public debt-to-GDP ratios and sharp reductions on the availability of credit to households in advanced economies. To cope with this scenario, the present work studied jointly optimal and time-consistent monetary and fiscal policies in the presence of limited asset markets participation. A standard New Keynesian model is augmented to incorporate the government's budget constraint, where government spending is financed by distortionary taxation and/or issuing of long term debt, and the existence of a share of consumers who do not smooth consumption intertemporally. A single policymaker has full control of all policy instruments and is constrained to act in a time-consistent manner. The inability to commit yields the inflationary bias problem state-dependent and also creates an incentive to bring back government debt to its steady state, following a shock. In order to deal with the state-dependencies caused by discretionary policy, the model is solved using global solution and

of the consumers' population is liquidity constrained, $\lambda = 0.5$, numerical results show an increase in the net stock of nominal assets (rather than liabilities) from 1.09% to 9.2%.

nonlinear techniques.

We found that the presence of limited asset markets participation, by increasing the distortions in the model economy, impacts the long-run equilibrium values of relevant macroeconomic variables. More specifically, increases in the share of liquidity constrained agents are associated with lower steady state rate of inflation and public debt-to-GDP ratio. Moreover, when a fraction of consumers cannot smooth consumption intertemporally, the upward trend in the steady state debt-to-output ratio caused by the issuing of longer-term government debt is weakened. The optimal responses of stabilization policy in face of a shock to the firms' markup are largely dependent on both the size of government debt and the fraction of rule-of-thumb consumers. Finally, the share of liquidity constrained agents and the size of debt both cause redistribution effects amongst agents and, therefore, influence steady state inequalities.

The current work can be extended in several ways, which include:

1. To take capital accumulation explicitly into consideration. As Galí, López-Salido, and Vallés (2004) point out, in disregarding capital accumulation, the only difference in behavior across Ricardian and liquidity constrained agents is the fact that optimizing households earn dividend incomes from the ownership of firms. In this case, ultimately, all shares of profits have to be held by Ricardian consumers in equal proportions.

2. To consider redistributive fiscal policies. In the current work we disregard the possibility for fiscal authority to rely on different fiscal transfers across households. Albonico and Rossi (2015) show that fully redistributive fiscal policy eliminates the extra inflationary bias caused by the presence of limited asset markets participation, but at the cost of reducing Ricardian agents' welfare. Redistributive policies can also be useful in tempering the rise of steady state inequalities caused by higher levels of government debt.

3. To consider stabilization issues when macroeconomic policy is delegated to independent and potentially strategic fiscal and monetary authorities.

Appendices

1.A FIGURES

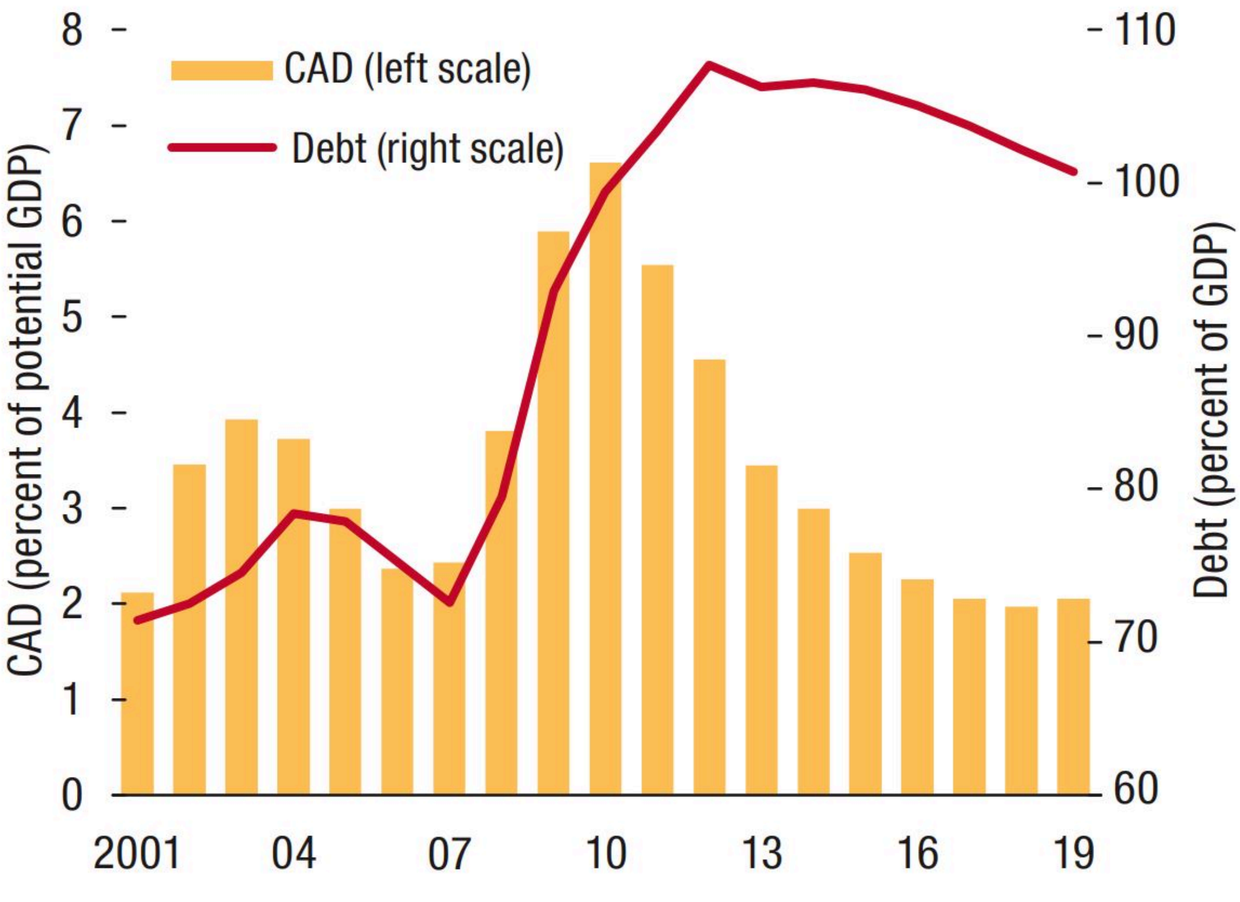
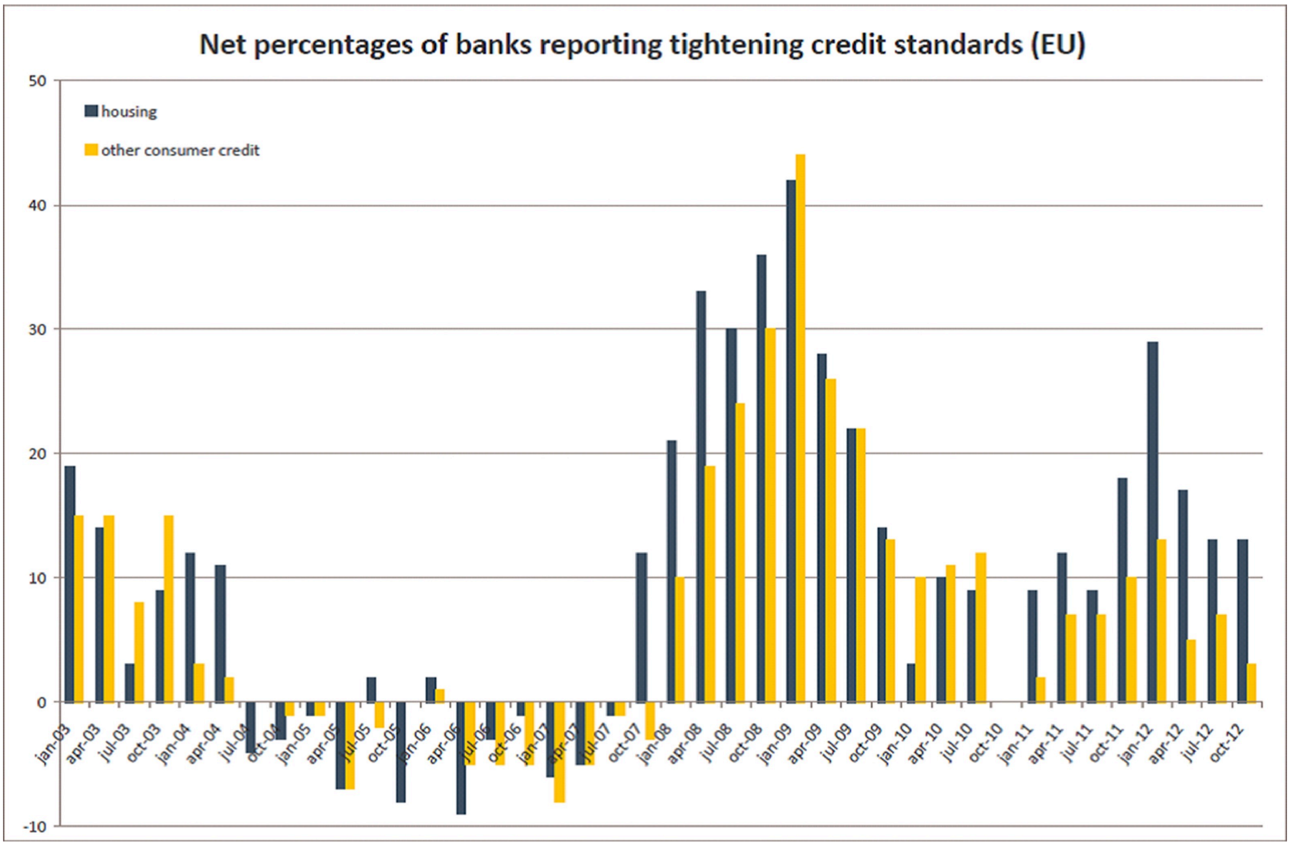
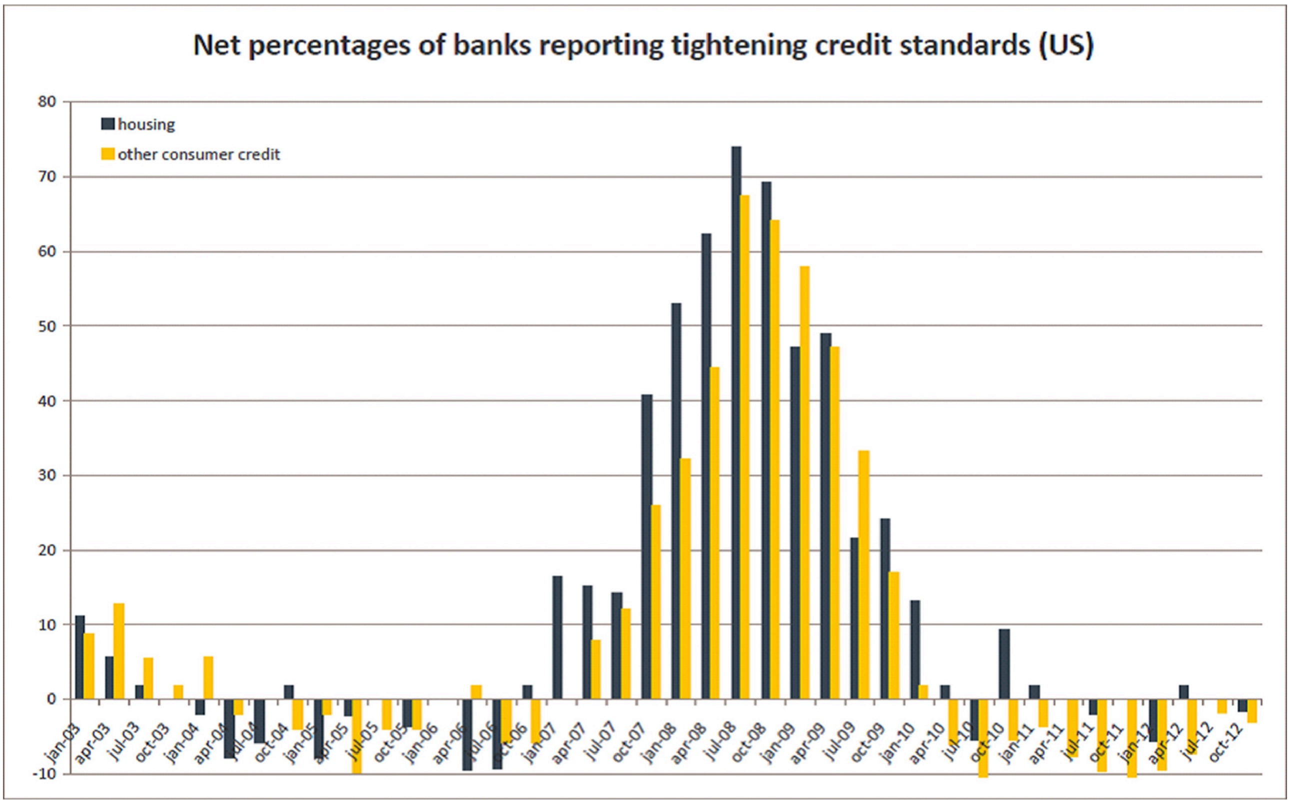


Figure 1.A.1 – Debt-to-GDP ratio and cyclically adjusted deficit (CAD) as percentage of potential GDP in advanced economies. Source: International Monetary Fund (2014).



(a) Credit standards in the EU economy.



(b) Credit standards in the US economy

Figure 1.A.2 – Net percentages of banks reporting tightening credit standards in the EU and US economies. Source: Albonico and Rossi (2017).

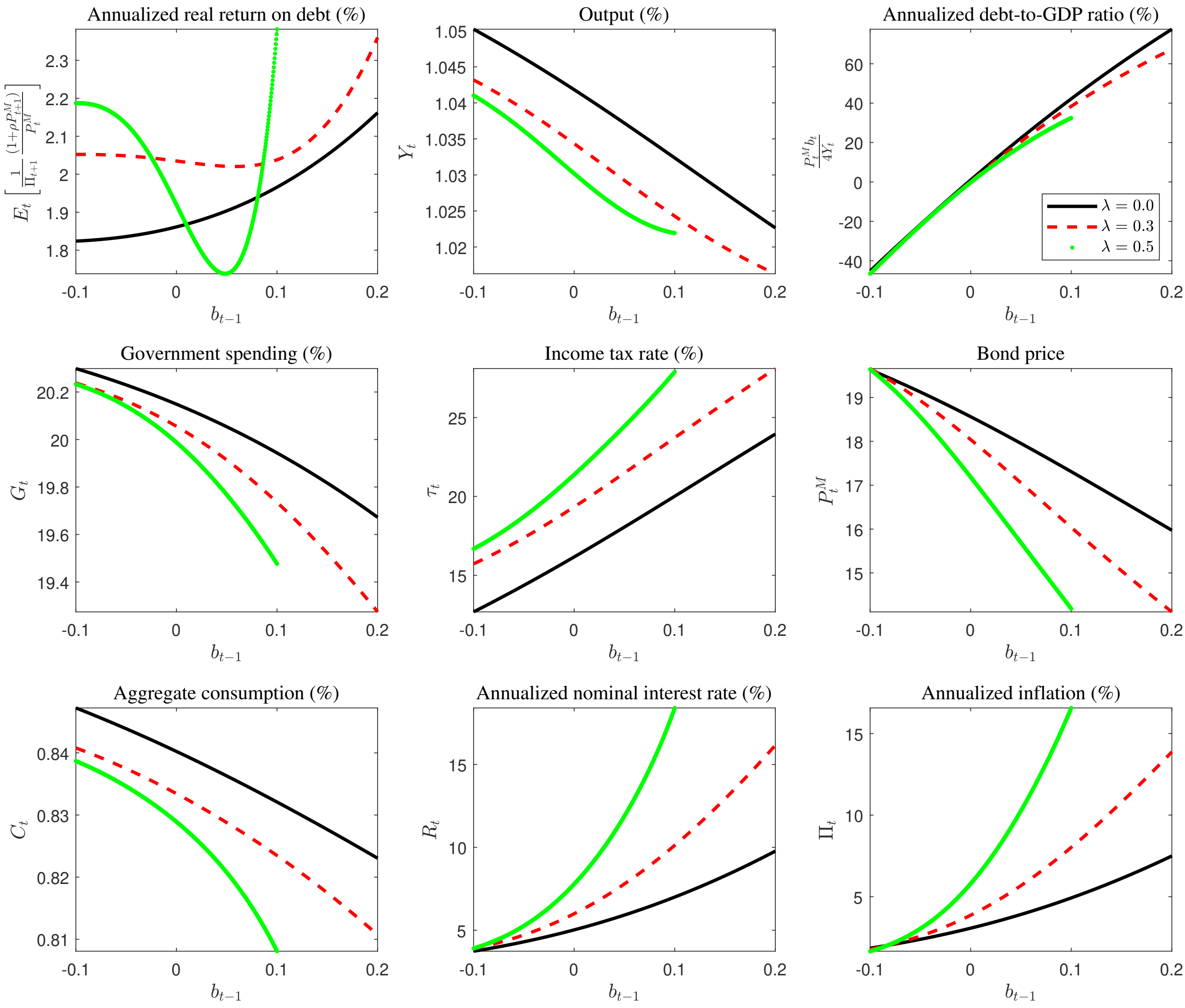


Figure 1.A.3 – Policy rules as functions of lagged debt under the benchmark calibration. The grid for the elasticity of substitution between varieties is fixed at $\bar{\epsilon}$.

1.B NUMERICAL ALGORITHM

In this appendix we outline the Chebyshev collocation method with time iteration employed to solve the model described in the paper. This method can be framed under a more general approach, namely, projection methods. Projection methods handle DSGE models by building a basis function, indexed by fixed coefficients, that approximates a policy function in order find the vector of coefficients which minimizes some given residual function. For textbook treatments of the numerical methods, see Miranda and Fackler (2004), Judd (1998) and Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016). Our exposition heavily relies on the description of the techniques found in Leeper, Leith, and Liu (2019).

Define the vector of states $s_t = (b_{t-1}, \epsilon_t)$, where b_{t-1} is the real stock of debt, which is endogenous, and ϵ_t is the elasticity of substitution between varieties, assumed to be an exogenously given process. The law of motion for each state variable is given by:

$$
\begin{aligned}
P_t^M b_t &= (1+\rho P_t^M)\frac{b_{t-1}}{\Pi_t} - \tau_t w_t N_t + G_t, \\
\ln(\epsilon_t) &= (1-\rho_\epsilon)\ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon_{t-1}) + \sigma_\epsilon \varepsilon_t, \quad \varepsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1),
\end{aligned}
$$

where $0 \le \rho_\epsilon < 1$.

We approximate the following 16 functional equations associated with 10 endogenous variables and 6 Lagrange multipliers, $\{C_t^o(s_t), C_t^r(s_t), N_t^o(s_t), N_t^r(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), P_t^M(s_t), G_t(s_t), \mu_{1t}(s_t), \mu_{2t}(s_t), \mu_{3t}(s_t), \mu_{4t}(s_t), \mu_{5t}(s_t), \mu_{6t}(s_t)\}$. In order to collect the policy functions of endogenous variables, we define a function $X : \mathbb{R}^2 \to \mathbb{R}^{16}$, where:

$$\begin{aligned} X(s_t) &= \big(C_t^o(s_t), C_t^r(s_t), N_t^o(s_t), N_t^r(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), P_t^M(s_t), G_t(s_t), \\ &\quad \mu_{1t}(s_t), \mu_{2t}(s_t), \mu_{3t}(s_t), \mu_{4t}(s_t), \mu_{5t}(s_t), \mu_{6t}(s_t)\big). \end{aligned}$$

In doing so, the equilibrium conditions of the model can be rewritten compactly as:

$$\Gamma\left(s_t, X(s_t), \mathbb{E}_t[Z(X(s_{t+1}))], \mathbb{E}_t[Z_b(X(s_{t+1}))]\right) = 0,$$

where $\Gamma : \mathbb{R}^{2+16+3+3} \to \mathbb{R}^{16}$, and:

$$Z(X(s_{t+1})) = \begin{bmatrix} Z_1(X(s_{t+1})) \\ Z_2(X(s_{t+1})) \\ Z_3(X(s_{t+1})) \end{bmatrix} \equiv \begin{bmatrix} M(b_t, \epsilon_{t+1}) \\ L(b_t, \epsilon_{t+1}) \\ (\Pi_{t+1})^{-1}(1+\rho P_{t+1}^M)\mu_{3t+1} \end{bmatrix},$$

with:

$$\begin{aligned} M(b_t, \epsilon_{t+1}) &= (C_{t+1}^o)^{-\sigma} Y_{t+1} \frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right), \\ L(b_t, \epsilon_{t+1}) &= (C_{t+1}^o)^{-\sigma}(\Pi_{t+1})^{-1}(1+\rho P_{t+1}^M), \end{aligned}$$

and:

$$Z_b(X(s_{t+1})) = \begin{bmatrix} \frac{\partial Z_1(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_2(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_3(X(s_{t+1}))}{\partial b_t} \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial M(b_t, \epsilon_{t+1})}{\partial b_t} \\ \frac{\partial L(b_t, \epsilon_{t+1})}{\partial b_t} \\ \frac{\partial[(\Pi_{t+1})^{-1}(1+\rho P_{t+1}^M)\mu_{3t+1}]}{\partial b_t} \end{bmatrix}.$$

Taking the derivatives yields:

$$\begin{aligned} M_b(b_t, \epsilon_{t+1}) &\equiv \frac{\partial M(b_t, \epsilon_{t+1})}{\partial b_t} = -\sigma (C_{t+1}^o)^{-\sigma-1} Y_{t+1} \frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right)\frac{\partial C_{t+1}^o}{\partial b_t} \\ &+ (C_{t+1}^o)^{-\sigma}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right)\frac{\partial Y_{t+1}}{\partial b_t} + (C_{t+1}^o)^{-\sigma}\frac{Y_{t+1}}{\Pi^*}\left(\frac{2\Pi_{t+1}}{\Pi^*} - 1\right)\frac{\partial \Pi_{t+1}}{\partial b_t}, \end{aligned}$$

and

$$\begin{aligned} L_b(b_t, \epsilon_{t+1}) &\equiv \frac{\partial L(b_t, \epsilon_{t+1})}{\partial b_t} = -\sigma (C_{t+1}^o)^{-\sigma-1}(\Pi_{t+1})^{-1}(1+\rho P_{t+1}^M)\frac{\partial C_{t+1}^o}{\partial b_t} \\ &- (C_{t+1}^o)^{-\sigma}(\Pi_{t+1})^{-2}(1+\rho P_{t+1}^M)\frac{\partial \Pi_{t+1}}{\partial b_t} + \rho (C_{t+1}^o)^{-\sigma}(\Pi_{t+1})^{-1}\frac{\partial P_{t+1}^M}{\partial b_t}. \end{aligned}$$

As in Leeper, Leith, and Liu (2019), we are relying on the Interchange of Integration and Differentiation Theorem and assuming that $\mathbb{E}_t[Z_b(X(s_{t+1}))] = \partial \mathbb{E}_t[Z(X(s_{t+1}))]/\partial b_t$. To solve the model, we then use projection methods to find a vector-valued function X that Γ maps to some "approximately" zero function.

In order to easy notation, we follow the convention in the literature by using $s(b, \epsilon)$ to denote the current state of the economy, and s' to represent the next period state.

The Chebyshev collocation algorithm to solve the nonlinear system describing the model economy can be described as follows:

1. Define the collocation nodes and the space of linearly independent basis functions to approximate the policy functions:

   - Choose an order of approximation[20] (i.e., the polynomial degrees) n_b and n_ϵ for each dimension of the state space $s = (b, \epsilon)$, then there are $N_s = (n_b + 1) \times (n_\epsilon + 1)$ nodes. Let $S = (S_1, S_2, \ldots, S_{N_s})$ denote the set of collocation nodes.
   - Compute the $n_b + 1$ and $n_\epsilon + 1$ zeros of the Chebyshev polynomials of order $n_b + 1$ and $n_\epsilon + 1$ as:

$$\begin{aligned} z_b^i &= \cos\left(\frac{(2i-1)\pi}{2(n_b+1)}\right), \quad i = 1, 2, \ldots, n_b + 1. \\ z_\epsilon^i &= \cos\left(\frac{(2i-1)\pi}{2(n_\epsilon+1)}\right), \quad i = 1, 2, \ldots, n_\epsilon + 1. \end{aligned}$$

   Chebyshev polynomials have the convenient property that they are smooth and bounded between $[-1, 1]$. Besides, their roots are quadratically clustered toward ± 1.

   - Given that the domain of Chebyshev polynomials is $[-1, 1]$ and that the state variables of our DSGE model are different, we should use some form of linear translation. In order to do that, we compute the collocation points ϵ_i as:

$$\epsilon_i = \frac{\epsilon_{\max} + \epsilon_{\min}}{2} + \frac{\epsilon_{\max} - \epsilon_{\min}}{2} z_\epsilon^i = \frac{\epsilon_{\max} - \epsilon_{\min}}{2}(z_\epsilon^i + 1) + \epsilon_{\min}, \quad i = 1, 2, \ldots, n_\epsilon + 1,$$

   which map $[-1, 1]$ onto $[\epsilon_{\min}, \epsilon_{\max}]$. Similarly, the collocation points b_t are:

$$b_i = \frac{b_{\max} + b_{\min}}{2} + \frac{b_{\max} - b_{\min}}{2} z_b^i = \frac{b_{\max} - b_{\min}}{2}(z_b^i + 1) + b_{\min}, \quad i = 1, 2, \ldots, n_b + 1,$$

   mapping $[-1, 1]$ onto $[b_{\min}, b_{\max}]$. Note that:

$$S = \{(b_i, \epsilon_j) | i = 1, 2, \ldots, n_b + 1, j = 1, 2, \ldots, n_\epsilon + 1\},$$

   are the tensor grids, with $S_1 = (b_1, \epsilon_1), S_2 = (b_1, \epsilon_2), \ldots, S_{N_s} = (b_{n_b+1}, \epsilon_{n_\epsilon+1})$.

   - The space of the approximating functions, denoted as Ω, is a matrix of two-dimensional Chebyshev polynomials. More specifically,

$$\Omega(S) = \begin{bmatrix} \Omega(S_1) \\ \Omega(S_2) \\ \vdots \\ \Omega(S_{n_\epsilon+1}) \\ \vdots \\ \Omega(S_{N_s}) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_1))) & T_0(\xi(b_1)T_2(\xi(\epsilon_1))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_1))) \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_2))) & T_0(\xi(b_1)T_2(\xi(\epsilon_2))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_2))) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_{n_\epsilon+1}))) & T_0(\xi(b_1)T_2(\xi(\epsilon_{n_\epsilon+1}))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1}))) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & T_0(\xi(b_{n_b+1})T_1(\xi(\epsilon_{n_\epsilon+1}))) & T_0(\xi(b_{n_b+1})T_2(\xi(\epsilon_{n_\epsilon+1}))) & \cdots & T_{n_b}(\xi(b_{n_b+1})T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1}))) \end{bmatrix}_{N_S \times N_S}$$

[20] In this work we have used polynomials of 11^{th} degree for each state variable.

where $\xi(x) = 2\frac{x-x_{\min}}{x_{\max}-x_{\min}} - 1$ is a linear translation mapping the domain of $x \in [x_{\min}, x_{\max}]$ onto $[-1, 1]$, and T defines the Chebyshev polyomials recursively, with $T_0(x) = 1, T_1(x) = x$, and the general $n+1$-th order polynomial is given by:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x).$$

- Then, at each node $s \in S$, the policy functions $X(s)$ are approximated by $X(s) = \Omega(s)\Theta_X$, where

$$\Theta_X = [\theta^{c^o}, \theta^{c^r}, \theta^{n^o}, \theta^{n^r}, \theta^{y}, \theta^{\Pi}, \theta^{b}, \theta^{\tau}, \theta^{\tilde{p}}, \theta^{G}, \theta^{\mu_1}, \theta^{\mu_2}, \theta^{\mu_3}, \theta^{\mu_4}, \theta^{\mu_5}, \theta^{\mu_6}],$$

is a $N_S \times 16$ matrix of coefficients.

2. Formulate an initial guess for the matrix of coefficients, Θ_X^0, and specify some convergence criterion ϵ_{tol}. We set $\epsilon_{\text{tol}} = 10^{-8}$.

3. At each iteration j, the matrix of coefficients is updated Θ_X^j by implementing the following time iteration procedure:

   - For each collocation node $s \in S$, compute the possible values of future policy functions $X(s')$ for $k = 1, \ldots, q$. That is:

$$X(s') = \Omega(s')\Theta_X^{j-1}.$$

   where q is the number of nodes in a Gauss-Hermite quadrature[21]. Note that:

$$\Omega(s') = T_{j_b}(\xi(b'))T_{j_\epsilon}(xi(\epsilon')),$$

   is a $q \times N_s$ matrix, for $j_b = 0, \ldots, n_b$ and $j_\epsilon = 0, \ldots, n_\epsilon$, with $b' = \hat{b}(s; \theta^b)$ and:

$$\ln(\epsilon') = (1-\rho_\epsilon)\ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon) + z_k\sqrt{2\sigma_\epsilon^2}.$$

   The two auxiliary functions can be calculated in a similar way:

$$\begin{aligned} M(s') &\approx \left(\hat{C}^o(s'; \theta^{c^o})\right)^{-\sigma} \hat{Y}(s'; \theta^y) \frac{\hat{\Pi}(s'; \theta^{\Pi})}{\Pi^*} \left(\frac{\hat{\Pi}(s'; \theta^{\Pi})}{\Pi^*} - 1\right), \\ L(s') &\approx \left(\hat{C}^o(s'; \theta^{c^o})\right)^{-\sigma} \left(\hat{\Pi}(s'; \theta^{\Pi})\right)^{-1} \left(1 + \frac{\rho \hat{P^M}(s'; \theta^{\tilde{p}})}{\Pi^* - \rho\beta}\right). \end{aligned}$$

   Following Leeper, Leith, and Liu (2016, 2019), in the numerical analysis we approximate the function $\tilde{P}_t^M = (\Pi^* - \rho\beta)P_t^M$ rather than P_t^M, since the former is less sensitive to variations in the maturity structure. The hat notation indicates that the policy functions are only approximated.

   - Let ω_k denote the weights of the Gauss-Hermite quadrature, the expectation terms

[21] In this work we set an 12-point Gauss-Hermite quadrature rule.

can be calculated, at each node s, as:

$$\begin{aligned}
\mathbb{E}[M(s')] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\hat{C}^o(s';\theta^{c^o})\right)^{-\sigma}\hat{Y}(s';\theta^y)\frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*}\left(\frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*}-1\right)\equiv\bar{M}(s',q),\\
\mathbb{E}[L(s')] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\hat{C}^o(s';\theta^{c^o})\right)^{-\sigma}\left(\hat{\Pi}(s';\theta^\Pi)\right)^{-1}\left(1+\frac{\rho\hat{P^M}(s';\theta^{\tilde{p}})}{\Pi^*-\rho\beta}\right)\equiv\bar{L}(s',q),\\
\mathbb{E}_t\left[\left(\frac{1+\rho P^M_{t+1}}{\Pi_{t+1}}\right)\mu_{3t+1}\right] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\frac{1+\frac{\rho\hat{P^M}(s';\theta^{\tilde{p}})}{\Pi^*-\rho\beta}}{\hat{\Pi}(s';\theta^\Pi)}\right)\hat{\mu_3}(s';\theta^{\mu_3})\equiv\Lambda(s',q).
\end{aligned}$$

Hence:

$$\mathbb{E}[Z(X(s'))]\approx\mathbb{E}[\hat{Z}(X(s'))]=\begin{bmatrix}\bar{M}(s',q)\\ \bar{L}(s',q)\\ \Lambda(s',q)\end{bmatrix}.$$

- To calculate the partial derivatives under expectations, $\mathbb{E}[Z_b(X(s'))]$, we obtain the following terms:

$$\begin{aligned}
\frac{\partial C^o_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{c^o}_{j_b j_\epsilon}}{b_{\max}-b_{\min}}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon'))\equiv\hat{C}^o_b(s'),\\
\frac{\partial Y_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{y}_{j_b j_\epsilon}}{b_{\max}-b_{\min}}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon'))\equiv\hat{Y}_b(s'),\\
\frac{\partial \Pi_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{\Pi}_{j_b j_\epsilon}}{b_{\max}-b_{\min}}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon'))\equiv\hat{\Pi}_b(s'),\\
\frac{\partial P^M_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{\tilde{p}}_{j_b j_\epsilon}}{(b_{\max}-b_{\min})(\Pi^*-\rho\beta)}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon'))\equiv\hat{P^M_b}(s').
\end{aligned}$$

Hence, the partial derivatives under expectations can be approximated as:

$$\begin{gathered}
\frac{\partial\mathbb{E}[M(s')]}{\partial b}\\
\approx\tfrac{1}{\sqrt{\pi}}\textstyle\sum_{k=1}^{q}\omega_k\begin{bmatrix}-\sigma\left(\hat{C}^o(s';\theta^{c^o})\right)^{-\sigma-1}\hat{Y}(s';\theta^y)\frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*}\left(\frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*}-1\right)\hat{C}^o_b(s')\\
+\left(\hat{C}^o(s';\theta^{c^o})\right)^{-\sigma}\frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*}\left(\frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*}-1\right)\hat{Y}_b(s')\\
+\left(\hat{C}^o(s';\theta^{c^o})\right)^{-\sigma}\frac{\hat{Y}(s';\theta^y)}{\Pi^*}\left(\frac{2\hat{\Pi}(s';\theta^\Pi)}{\Pi^*}-1\right)\hat{\Pi}_b(s')\end{bmatrix}\\
\equiv\hat{M}_b(s',q),\\
\frac{\partial\mathbb{E}[L(s')]}{\partial b}\\
\approx\tfrac{1}{\sqrt{\pi}}\textstyle\sum_{k=1}^{q}\omega_k\begin{bmatrix}-\sigma\left(\hat{C}^o(s';\theta^{c^o})\right)^{-\sigma-1}\left(\hat{\Pi}(s';\theta^\Pi)\right)^{-1}\left(1+\frac{\rho\hat{P^M}(s';\theta^{\tilde{p}})}{\Pi^*-\rho\beta}\right)\hat{C}^o_b(s')\\
-\left(\hat{C}^o(s';\theta^{c^o})\right)^{-\sigma}\left(\hat{\Pi}(s';\theta^\Pi)\right)^{-2}\left(1+\frac{\rho\hat{P^M}(s';\theta^{\tilde{p}})}{\Pi^*-\rho\beta}\right)\hat{\Pi}_b(s')\\
+\rho\left(\hat{C}^o(s';\theta^{c^o})\right)^{-\sigma}\left(\hat{\Pi}(s';\theta^\Pi)\right)^{-1}\hat{P}^M_b(s')\end{bmatrix}\\
\equiv\hat{L}_b(s',q).
\end{gathered}$$

That is,

$$\mathbb{E}[Z_b(X(s'))]\approx\mathbb{E}[\hat{Z}_b(X(s'))]=\begin{bmatrix}\hat{M}_b(s',q)\\ \hat{L}_b(s',q)\end{bmatrix}.$$

4. At each collocation node s, solve the following functional for $X(s)$:

$$\Gamma(s, X(s), \mathbb{E}[\hat{Z}(X(s'))], \mathbb{E}[\hat{Z}_b(X(s'))]) = 0,$$

using some routine to solve systems of nonlinear equations. Once $X(s)$ is obtained, the matrix of coefficients can be calculated as follows:

$$\hat{\Theta}_X^j = \left(\Omega(S)^T\Omega(S)\right)^{-1}\Omega(S)^T X(S).$$

5. Update the approximating coefficients, $\Theta_X^j = \eta\hat{\Theta}_X^j + (1-\eta)\Theta_X^{j-1}$, where $0 \leq \eta \leq 1$ is a dampening parameter used for improving convergence.

6. If $||\Theta_X^j - \Theta_X^{j-1}|| < \epsilon_{\text{tol}}$, the algorithm converged. Otherwise, restart the procedure from Step 3.

1.C EULER EQUATION ERRORS

To assess the accuracy of our numerical solutions, we follow Judd (1992) in performing Euler equation errors (EEE) analyses to address the difference between the exact and the approximated solutions. As it is convention in the literature, we calculate the $\log_{10}|EEE(b_{t-1}, \epsilon_t)|$, for a discussion see Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016). Table 1.C.1 summarizes some basic statistics about the Euler equation errors for the numerical results presented in Subsection 1.6.1, the most common reported statistics for the EEE are the mean of the Euler equation errors, in our case a simple average[22], and the maximum of the EEE. The reported statistics were calculated using an evenly-spaced grid consisting of 40 points for the stock of debt, b_t, and 40 points for the elasticity of substitution between intermediate goods, $\ln(\epsilon_t)$. The results are similar on a finer grid[23].

Table 1.C.1 – Euler equation errors (EEE).

	Benchmark calibration			$\phi = 50$			$\frac{\bar{\epsilon}-1}{\bar{\epsilon}} = 6\%$		
	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$
$\log_{10}\max\lvert\text{EEE}\rvert$	-6.41	-6.65	-6.50	-6.56	-6.53	-5.61	-6.16	-6.36	-2.26
$\log_{10}\text{mean}\lvert\text{EEE}\rvert$	-7.78	-7.81	-7.79	-7.84	-7.74	-7.51	-7.70	-7.75	-5.64
$\log_{10}\text{median}\lvert\text{EEE}\rvert$	-11.06	-10.28	-11.3	-11.6	-9.30	-8.73	-9.67	-9.47	-7.93
std(EEE)	5.20e-08	4.86e-08	-7.27	-7.38	-7.28	-7.02	-7.22	-7.27	-4.27

A visual representation of the accuracy of the approximated solutions is presented in Figure 1.C.1. In this figure, the model is solved under the benchmark calibration for $\lambda = 0.3$.

[22] Alternatively, some researchers use some estimate of the ergodic distribution of state variables.

[23] For all the other models considered, with different maturities and degrees of myopia, the accuracy of the approximations is similar.

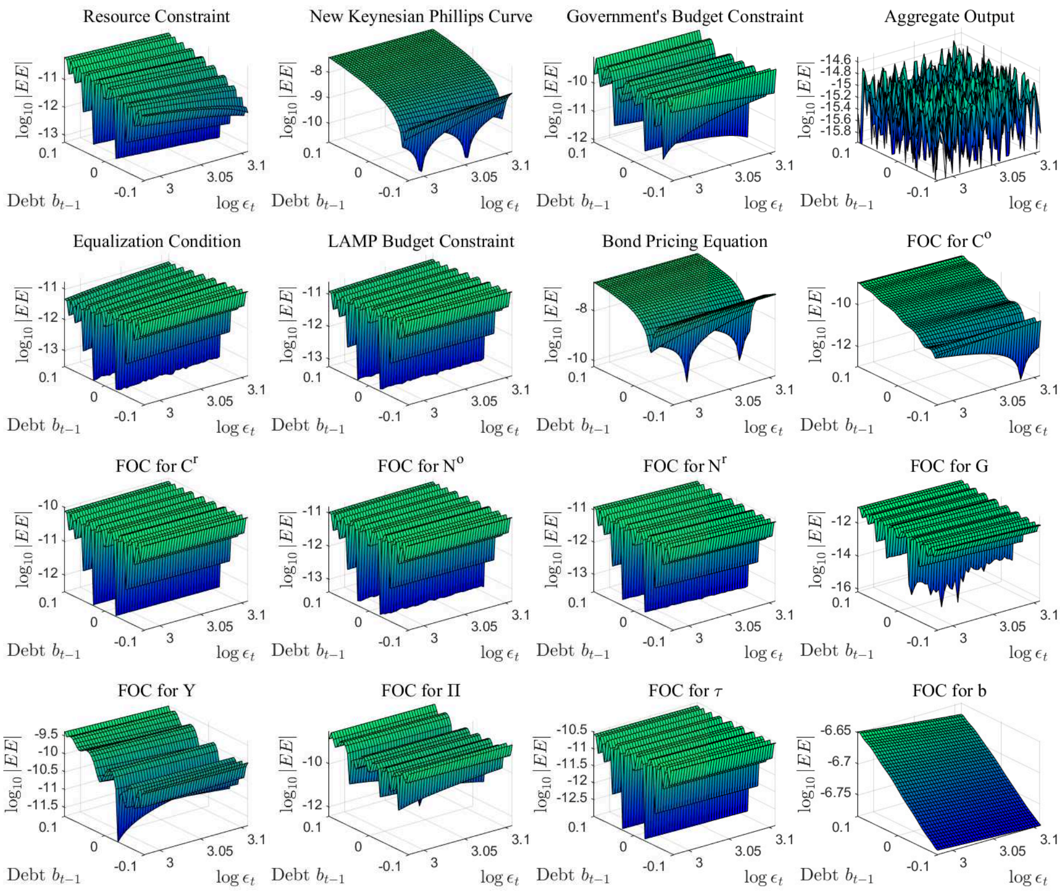


Figure 1.C.1 – Euler Equation Errors (EEE): benchmark calibration with $\lambda = 0.3$.

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Chapter 2

Strategic interactions, inflation conservatism and the level of government debt: A nonlinear analysis

Abstract

This essay addresses the state dependencies in the strategic interactions between an inflation conservative central bank and a benevolent fiscal authority. Building on a standard New Keynesian model extended to include fiscal policy and nominal government debt, we consider the effects of independent, discretionary and possibly non-cooperative policymakers, rather than joint optimal policies. The main contribution of this work is to solve for the discretionary equilibrium of the policy games using nonlinear global solution techniques. We found that under a fiscal leadership policy game, delegating monetary policy to a conservative authority can function as a device for fiscal discipline and reduces both the stabilization and level inflationary biases. The consequences in terms of welfare are mostly harmless in this case. Nonetheless, a simultaneous move policy game not only increases the gap between actual inflation and the target rate, in comparison with the cooperative setup, but is also associated with higher welfare losses. The loss in terms of welfare is an increasing function of the degree of monetary conservatism.
Keywords: Monetary policy; Fiscal policy; Strategic interactions; Time-consistency; Government debt; Inflation conservatism.

2.1 INTRODUCTION

Conventional macroeconomic models show that the inability of a monetary policymaker to commit to an optimal plan can give rise to an inefficiently high equilibrium rate of inflation - a 'level bias' - despite the absence of a long-run trade-off between output and inflation (KYDLAND; PRESCOTT, 1977; BARRO; GORDON, 1983). This level bias stems from the public's knowledge that the policymaker is constrained to act under discretion and, thus, has an incentive to induce surprise inflation to push output temporarily above its normal level.

To avoid, or at least mitigate, the distortions caused by the lack of monetary commitment, the most popular approach is to delegate the control of monetary policy to a central bank that is instrument independent, i.e. has full control over the instruments of monetary policy, and inflation conservative - places a higher relative weight on inflation than society as a whole[1]. As shown by Rogoff (1985), when the monetary policy is conducted by an independent central banker who cares strongly about inflation - an inflation conservative monetary authority - the desire to pursue an expansionary policy is lowered and private expectations are formed accordingly, hence, reducing the inflationary bias.

Although the delegation proposal reduces the inflationary bias, it can still be associated with a larger volatility of welfare-relevant economic variables than it is optimal. Svensson (1997) calls this higher variability a 'stabilization bias' and demonstrates that an inflation-target conservative central bank, rather than the Rogoff's weight-conservative, could still lead to an improved outcome. Furthermore, Gertler, Gali, and Clarida (1999) show that discretionary monetary conservatism can achieve the same level of welfare as the optimal precommitment-to-rules policy.

Overall, it seems that the optimal delegation to a conservative central bank proposal is sound, as it deals with both the level and stabilization bias. Nonetheless, all the aforementioned works abstract from fiscal policy considerations, assuming it to be exogenously given or passive. On the other hand, there is also a literature focusing on optimal and time-consistent fiscal policy in dynamic models without money (CHARI; KEHOE, 1990; KLEIN; KRUSELL; RÍOS-RULL, 2008; PICHLER; PEREIRA; ORTIGUEIRA, 2012). However, interactions between independent fiscal and monetary authorities can lead to macroeconomic outcomes that are very different from those implied by policy analyses in isolation. More specifically, it is not reasonable to assume that an independent and conservative monetary authority eliminates potential influences of fiscal policy. Fiscal objectives can deviate from what is socially optimal. In addition to that, even if it remains entirely benevolent, the fiscal authority may still behave strategically, affecting the resulting equilibrium in the model economy. Hence, the presence of a fiscal policymaker could lead both to an inflationary and a stabilization bias.

Dixit and Lambertini (2003) explores the monetary and fiscal policy interactions when the monetary authority is more inflation-conservative than the fiscal and society. Using a static Barro-Gordon model, extended to include fiscal policy, they show that a conflict of interests between authorities leads to suboptimal outcomes. In the steady state equilibrium, levels of inflation and unemployment are both higher than is socially optimal, meaning that an inflationary bias still arises.

A dynamic stochastic sticky price economy with monopolistic distortions and monetary-fiscal interactions is studied by Adam and Billi (2008). Their results show that inflation conservatism remains desirable. Nevertheless, they mostly focus on implications for the steady state equilibrium rather than dynamics, and restrict their analysis of stabilization bias to the case

[1] Another potential improvements over discretion are: (i) incentive contracts along the lines of Persson and Tabellini (1993) and Walsh (1995), in which the central banker is penalized for inflation; (ii) reputation, as developed in Backus and Driffill (1985) and Barro (1986); and (iii) punishment (BARRO; GORDON, 1983).

of fiscal leadership and flexible prices. But, most importantly, the assumption of lump-sum taxation implies that public debt plays no role in determining equilibrium outcomes.

The literature that looks at strategic interactions between policy authorities into optimal policy analysis often relies on simplifying assumptions to obtain tractable results. Dixit and Lambertini (2003) and Adam and Billi (2008), by abstracting the existence of government debt, rule out the restrictions on stabilization abilities of fiscal and monetary authorities that debt stabilization can impose (LEEPER, 1991).

Blake and Kirsanova (2011) explore the strategic interactions between policymakers in a standard dynamic New Keynesian model extended to include both fiscal policy and nominal government debt. In this framework, they allow the possibility of strategic and non-cooperative behavior of both authorities in order to achieve their objectives. Three forms of strategic interactions are considered: either monetary or fiscal leadership or a simultaneous Nash game between authorities. Besides examining different leadership regimes, their work: (i) studies the stabilization rather than inflationary bias, and focus on dynamics instead steady state implications; (ii) incorporates the effects of potential fiscal insolvency; and (iii) shows that different levels of steady state government debt have profound implications for the equilibrium outcomes resulting from each type of strategic game played.

The striking result of Blake and Kirsanova (2011) is that the delegation proposal to an inflation conservative and independent central bank, when the fiscal authority remains entirely benevolent but acts strategically, usually increases the stabilization bias leading to a reduction in welfare. As they put, "*what works well in an economy with a single policymaker may not work at all in an economy with two strategic policymakers*" (BLAKE; KIRSANOVA, 2011, p. 45). Despite being robust to model specification and the choice of instruments by fiscal policy, the quantitative results are dependent on the level of government debt around which the economy is linearized. This suggests that their model could be incorporated in a nonlinear framework to account for the state-dependencies that the presence of government debt imposes onto the model.

Given the aforementioned background, this paper addresses the state dependencies in the strategic interactions between an inflation conservative policymaker and a benevolent fiscal authority. Following Leeper, Leith, and Liu (2016, 2019), we build on a standard dynamic New Keynesian model extended to include fiscal policy and nominal government debt but, rather, consider the effects of independent and possibly non-cooperative policymakers, rather than joint optimal policies. The assumption of discretionary policies yield the optimal behavior of policymakers characterized by generalized Euler equations, which involves derivatives of some equilibrium decision rules, and, thus, linearization around a deterministic steady state becomes a complicate matter as now this steady state is endogenous and a priori unknown. A key contribution of this work is to solve for the discretionary equilibrium of the dynamic rational expectations model, where the interplay between independent and potential strategic authorities is explicitly considered, using non-linear projection methods, rather in a linear quadratic rational expectations framework as in Blake and Kirsanova (2011).

Our main results can be summarized as follows:

1. The degree of monetary conservatism have monotonic effects over the steady state values of macroeconomic variables, regardless the structure of the policy game. In a simultaneous move game, the fiscal authority fails to fully internalize the behavior of monetary policy, by taking the monetary instrument as given. As a consequence of that, in the long-run equilibrium, inflation and nominal interest rate are higher, while fiscal policy is tightened as the degree of conservatism increases. When the fiscal authority can act as a Stackelber leader, by its turn, an independent and inflation conservative central bank can function as a fiscal discipline device. Given that, the higher the degree of conservatism, the lower the incentive to induce inflation surprises in the economy. This almost offset the level inflationary bias and the inflation rate converges to the target. Furthermore, the fiscal authority's task to stabilize debt resorts more on distortionary taxation and public consumption.

2. The optimal dynamics in response to a shock to the firms' markup are sensitive to the structure of the game considered. While under a fiscal leadership policy game, a conservative central bank imposes a strong debt stabilization bias on the fiscal authority and also reduces the state-dependent inflationary bias problem, reducing the volatility of welfare relevant macroeconomic variables in comparison to the cooperative setup. If the independent authorities play a simultaneous move game, delegating monetary policy to an inflation conservative central bank increases the stabilization bias.

3. Welfare analysis show that, while inflation conservatism is relatively harmless when the fiscal authority acts as a Stackelberg leader, the welfare losses in relation to the cooperative solution are an increasing function of the degree of monetary conservatism.

The paper is structured in the following way. Section 2.2 describes the model economy, and the first-best allocation is discussed on Section 2.3. The sequential discretionary policy-making is presented in Section 2.4. The solution method and the model parameterization are discussed in Section 2.E, numerical results are presented in Section 2.6. Section 2.7 concludes.

2.2 THE MODEL

Building on a standard New Keynesian model, the model economy is augmented to incorporate the government's budget constraint, where the optimally chosen government expenditures are financed by distortionary taxation and/or issuing single-period debt.

2.2.1 Households

A representative infinitely-lived household derives utility from private consumption, C_t, from the provision of public goods, G_t, and disutility from the supply of labor, N_t. Its objective is to maximize the expected separable utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{2.1}$$

where $\beta \in (0,1)$ is the household's discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution between private consumption, $\sigma_g > 0$ has the analogous interpretation for public consumption, $\chi > 0$ is a scaling parameter, and $\varphi > 0$ is the inverse of the Frisch labor supply elasticity.

Private consumption is a consumption index à la Dixit and Stiglitz (1977) defined by:

$$C_t \equiv \left(\int_0^1 C_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}},$$

where $C_t(j)$ represents the quantity of a variety j consumed by the household in period t, and ϵ_t is a time-varying elasticity of substitution between goods that evolves according to the following AR(1) stochastic process:

$$\ln \epsilon_t = (1 - \rho_\epsilon) \ln \bar{\epsilon} + \rho_\epsilon \ln \epsilon_{t-1} + \sigma_\epsilon \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0,1). \tag{2.2}$$

As in Beetsma and Jensen (2004), the assumption of a stochastic elasticity of substitution is a device for introducing mark-up shocks to the model.

Optimal allocation of consumption among the different varieties, for a given level of expenditure, yields the following demand equation for good j:

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_t} C_t,$$

where $P_t(j)$ is the price of variety j and $P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon_t} dj \right)^{\frac{1}{1-\epsilon_t}}$ is an aggregate price index.

The sequence of budget constraints can, then, be written as:

$$P_t C_t + \mathbb{E}_t \left[Q_{t,t+1} B_t \right] \leq \Xi_t + B_{t-1} + (1 - \tau_t) W_t N_t, \tag{2.3}$$

where $P_t C_t = \int_0^1 P_t(j) C_t(j) dj$, Ξ_t denotes the representative household's share of profits in the imperfectly competitive firms, W_t is the nominal wage, τ_t is a wage income tax rate[2], B_t represents the acquired quantity of one-period bonds in period t with maturity in $t+1$ at the price of $Q_{t,t+1}$.

The representative household maximizes (2.1) subject to (2.3) to obtain the following optimality conditions:

$$\beta \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{1}{\Pi_{t+1}} \right\} = \mathbb{E}_t [Q_{t,t+1}] = R_t^{-1}, \tag{2.4}$$

$$(1 - \tau_t) \left(\frac{W_t}{P_t} \right) = N_t^{\varphi} C_t^{\sigma}, \tag{2.5}$$

[2] It is important to emphasize that, since fiscal policy is a crucial element in this work, the fiscal authority cannot levy on any kind of lump-sum tax to offset the distortionary effects of monopolistic competition. As

where R_t is the gross short-term nominal interest rate, which is also the central bank's policy instrument, and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the gross rate of inflation.

Equation (2.4) is a standard consumption Euler equation that shows the representative households' optimal allocation of private consumption across time, based on the pricing of one period bonds, that is inversely linked to the nominal interest rate. The latter first-order condition (2.5) is a labor supply decision and implies that the marginal rate of substitution between consumption and leisure must be equal to the after-tax real wage rate.

Necessary and sufficient conditions for optimality also require a solvency constraint that prevents Ponzi-type schemes and that the households' budget constraint holds with equality. The no-Ponzi-game condition can be written as:

$$\lim_{T\to\infty} \mathbb{E}_t \left[\frac{1}{R_{t,T}} \frac{B_{T-1}}{P_T} \right] \geq 0, \tag{2.6}$$

where $R_{t,T} \equiv \Pi_{s=t}^{T-1} \left(R_s \frac{P_s}{P_{s+1}} \right)$, for $T \geq 1$ and $R_{t,t} = 1$.

2.2.2 Firms

There is a continuum of monopolistically competitive firms of unit mass, indexed by $j \in [0,1]$, producing differentiated intermediate goods. A firm j in the intermediate good sector produces its good subject to a linear production function:

$$Y_t(j) = N_t(j). \tag{2.7}$$

Each firm faces a demand schedule for their product given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_t} Y_t, \tag{2.8}$$

which is the sum of private and public demand, where $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$.

As in Rotemberg (1982), firms face a resource cost for adjusting nominal prices that is quadratic in the change prices for its product and proportional to nominal GDP. This quadratic cost is defined for a monopolistic firm j as[3]:

$$\eta_t(j) \equiv \frac{\phi}{2} \left(\frac{P_t(j)}{\Pi^* P_{t-1}(j)} - 1 \right)^2 Y_t, \tag{2.9}$$

where Π^* is the chosen inflation target and $\phi \geq 0$ measures the degree of nominal price stickiness. The higher the ϕ the more sluggish are price adjustments. If $\phi = 0$, prices are flexible.

emphasized by Leeper, Leith, and Liu (2019), this is a typical, but unrealistic, assumption in linear quadratic analyses in New Keynesian models.

[3] We consider the Rotemberg (1982) pricing approach rather than Calvo (1983) pricing because this reduces the number of endogenous state variables. In the latter, price dispersion becomes an additional endogenous state variable, which complicate matters when solving the model through nonlinear methods. However, as Leith and Liu (2016) and Sims and Wolff (2017) conclude, the form of nominal inertia adopted is not

After defining the real marginal costs of production as $mc_t \equiv W_t/P_t$. The problem facing firm j is to maximize the discounted value of nominal profits:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{z=0}^{\infty} Q_{t,t+z}\Xi_{t+z}(j),$$

where nominal profits are defined as,

$$\Xi_t(j) \equiv P_t(j)Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi^* P_{t-1}(j)} - 1\right)^2 P_t Y_t,$$

subject to the linear production function (2.7), the demand curve for their product (2.8), and the quadratic adjustment costs in changing prices (2.9).

The first-order condition for a symmetric equilibrium where $P_t(j) = P_t$ implies the following nonlinear Phillips curve relation:

$$\frac{\Pi_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*} - 1\right) = \beta\mathbb{E}_t\left[\left(\frac{C_t^o}{C_{t+1}^o}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right)\right] + \phi^{-1}((1-\epsilon_t) + \epsilon_t mc_t), \tag{2.10}$$

linking current inflation to future expected inflation and to the level of activity.

2.2.3 Government

Government expenditures consist on the provision of public goods. The level of aggregate public consumption takes the same form as private consumption:

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right)^{\frac{\epsilon_t}{\epsilon_t - 1}},$$

which implies a government demand for individual goods given by:

$$G_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_t} G_t.$$

The government issues one-period nominal debt, B_t, and levies on a distortionary tax on labor income, τ_t, to pay the principle and interests on outstanding debt and to fund potential deficits. The government's sequential budget constraint, in real terms, can then be written as:

$$\frac{1}{R_t} b_t = \frac{b_{t-1}}{\Pi_t} - \tau_t w_t N_t + G_t, \tag{2.11}$$

where $b_t \equiv B_t/P_t$ denotes real debt and $w_t \equiv W_t/P_t$ are real wages. Fiscal policy instruments are tax rates, τ_t, and government consumption, G_t.

2.2.4 Market Clearing

Goods market clearing requires, for each good j,

$$Y_t(j) = C_t(j) + G_t(j) + \eta_t(j),$$

such that, in a symmetrical equilibrium,

$$Y_t\left[1 - \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*} - 1\right)^2\right] = C_t + G_t. \tag{2.12}$$

A competitive rational expectations equilibrium for the private sector is defined as follows.

Definition 2.1 (Competitive equilibrium). *A competitive rational expectations equilibrium consists of a plan $\{C_t, N_t, Y_t, \Pi_t\}_{t=0}^{\infty}$ satisfying: (i) household budget constraint (2.3); (ii) the production function $Y_t = N_t$ and optimality conditions (2.4), (2.5) and (2.10); (iii) the government's budget constraint (2.11); (iv) the market clearing condition (2.12); and (v) the no-Ponzi-game condition (2.6), given the government policies $\{R_t, G_t, \tau_t, b_t\}_{t=0}^{\infty}$, prices $\{W_t, P_t\}_{t=0}^{\infty}$, the exogenous process $\{\epsilon_t\}_{t=0}^{\infty}$ and an initial level of government debt b_{t-1}.*

2.3 FIRST-BEST ALLOCATION

This section outlines the first-best allocation associated with the model previously described. In this allocation, a social planner implements a policy abstracting from monopoly distortions and nominal rigidities. Therefore, the social planner aims to maximize the representative consumer's utility (2.1), subject to the technology constraint $Y_t = N_t$ and the aggregate resource constraint $Y_t = C_t + G_t$. Thus, the first-best allocation solves:

$$\begin{aligned} \max_{\{C_t, G_t, N_t\}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \\ \text{subject to} \quad & Y_t = C_t + G_t \\ & Y_t = N_t. \end{aligned}$$

Assuming that the optimal allocation is given by $\{C_t^*, G_t^*, N_t^*\}$, first-order conditions imply that:

$$(C_t^*)^{-\sigma} = \chi (G_t^*)^{-\sigma_g} = (N_t^*)^{\varphi} = (Y_t^*)^{\varphi},$$

showing that, given the technological and resource constraints, it is optimal to equate the marginal utility of private and public consumption to the marginal disutility of labor. The optimal public expenditure-to-output ratio is given by:

$$\frac{G_t^*}{Y_t^*} = \chi^{\frac{1}{\sigma_g}} (Y_t^*)^{-\frac{\varphi + \sigma_g}{\sigma_g}}.$$

Such that, assuming $\sigma = \sigma_g$, in a deterministic steady state this optimal ratio is:

$$\frac{G^*}{Y^*} = \left(1 + \chi^{-\frac{1}{\sigma}}\right)^{-1},$$

which implies that the first-best level of output can be stated as follows:

$$(Y^*)^{\varphi+\sigma} \left(1 - \frac{G^*}{Y^*}\right)^{\sigma} = 1. \tag{2.13}$$

Following Leeper, Leith, and Liu (2016), the optimal allocation achieved in steady state under the decentralized equilibrium, described in Appendix 2.A, is compared to its counterpart in the first-best allocation by finding the policies and prices that make the decentralized equilibrium mimics the social planner's optimal plan. The steady state level of output in the decentralized economy is given by[4]:

$$Y^{\varphi+\sigma} \left(1 - \frac{G}{Y}\right)^{\sigma} = (1-\tau) \left(\frac{\epsilon - 1}{\epsilon}\right). \tag{2.14}$$

innocuous for higher order of approximations.

[4] See equation (2.27).

Under the assumption that the steady state ratio of public expenditure-to-GDP is the same under the decentralized economy and the first-best allocation, equations (2.13) and (2.14) imply that those two expressions will be identical when the labor income tax rate is set to:

$$\tau^* = -\frac{1}{\epsilon - 1}, \tag{2.15}$$

which means that, effectively, τ^* is a labor income subsidy that offsets the monopolistic distortion in the economy. By the government's sequential budget constraint, this requires that the government has accumulated an annualized stock of assets[5] given by:

$$\frac{b^*}{4R^*Y^*} = \frac{\beta}{4(1-\beta)}\left[-\frac{1}{\epsilon} - \left(1 + \chi^{-\frac{1}{\sigma}}\right)^{-1}\right],$$

this result shows that an optimizing policymaker will have to finance all its expenditures with public consumption and labor income subsidies from a 'war chest'[6]. Given our benchmark calibration, to be described below, the implied stock of nominal assets is of 1182% of GDP.

2.4 SEQUENTIAL DISCRETIONARY POLICYMAKING

In this section the structure of the different policy games played by two independent authorities analysed throughout the paper is presented. First, we introduce the case in which policy objectives amongst fiscal and monetary authorities are not distorted, i.e. there is full cooperation between policymakers. Then, the cases of sequential discretionary policymaking between non-cooperative authorities are discussed.

2.4.1 Cooperative policy

Under a cooperative or centralised discretionary policy, a single and benevolent policymaker sets is policy instruments in order to maximize the utility of the representative household (2.1) subject to the system of equations describing the evolution of the economy[7]. By being constrained to act in a time-consistent manner, the policymaker is unable to commit to any particular future plan and, instead, reoptimizes its responses period by period. However, as already mentioned before, the presence of an endogenous state variable in the form of government debt yields the optimal discretionary policy state-dependent and, thus, decisions taken today can have effects on future expectations through the stock of debt that the policy bequeaths to the future.

Defining the following state-dependent auxiliary functions to capture those future ex-

[5] The necessity to accumulate a stock of assets to finance public expenditures is somewhat common in analyses of optimal fiscal policy, see Aiyagari et al. (2002) for a case with incomplete markets and Ramsey policy.

[6] The term comes from Hume (1777) in his observation, on 'Of Public Credit', about the ability of ancient policies to ensure a 'war chest' against disasters and the recurrent threatening of war.

[7] As Stehn and Vines (2008) pointed out, this setup can be alternatively interpreted either as a dependent central bank or as a case of full cooperation between a fiscal and a monetary authority with no distortion in their objectives.

pectations:

$$M(b_t, \epsilon_{t+1}) \equiv (C_{t+1})^{-\sigma} Y_{t+1} \frac{\Pi_{t+1}}{\Pi^*} \left(\frac{\Pi_{t+1}}{\Pi^*} - 1 \right), \tag{2.16}$$

$$F(b_t, \epsilon_{t+1}) \equiv (C_{t+1})^{-\sigma} \Pi_{t+1}^{-1}. \tag{2.17}$$

The consumption Euler equation (2.4), the New Keynesian Phillips curve (2.10) and the government's budget constraint (2.11) can then, after some algebraic manipulations, be rewritten as follows:

$$\frac{C_t^{-\sigma}}{R_t} = \beta \mathbb{E}_t [F(b_t, \epsilon_{t+1})], \tag{2.18}$$

$$\phi \frac{\Pi_t}{\Pi^*} \left(\frac{\Pi_t}{\Pi^*} - 1 \right) = \phi \beta C_t^{\sigma} Y_t^{-1} \mathbb{E}_t [M(b_t, \epsilon_{t+1})] + (1 - \epsilon_t) + \epsilon_t (1 - \tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma}, \tag{2.19}$$

$$\frac{1}{R_t} b_t = \frac{b_{t-1}}{\Pi_t} - \left(\frac{\tau_t}{1 - \tau_t} \right) Y_t^{1+\varphi} C_t^{\sigma} + G_t. \tag{2.20}$$

Formally, the cooperative policy under discretion is described by a set of decision rules $\{C_t, Y_t, \Pi_t, R_t, G_t, \tau_t, b_t\}$ that maximizes the following value function:

$$V(b_{t-1}, \epsilon_t) = \max \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{Y_t^{1+\varphi}}{1+\varphi} + \beta \mathbb{E}_t [V(b_t, \epsilon_{t+1})] \right\}, \tag{2.21}$$

subject to the aggregate resource constraint (2.12), the consumption Euler equation (2.18), the nonlinear New Keynesian Phillips curve (2.19) and the government's budget constraint (2.20). The Lagrangian formulation of the policy problem and its associated first-order conditions are presented in Appendix 2.B.

2.4.2 Non-cooperative policy regimes

In what follows the structure of the policy games played by two independent discretionary policymakers is considered. The central bank's instrument is the nominal interest rate, R_t, while the fiscal authority sets the level of government expenditures, G_t, and the distortionary labor income tax rate, τ_t. We assume a benevolent fiscal authority, which means that its objective function coincides with the representative household in the model economy. The central bank, by its turn, is assumed to be a 'weight conservative' monetary authority à la Rogoff (1985) and, thus, it can be more inflation averse than society. This distortion in the objectives of authorities yields the equilibrium outcome dependent on the timing of policy moves, i.e., on whether fiscal policy actions are taken before, after or simultaneously to monetary decisions. Since casual observation suggests that it is less likely for the central bank to exploit the fiscal policy reaction function, given that fiscal decisions are taken at a much lower frequency, we disregard the monetary leadership case for convenience. We thus focus our attention to the following cases of strategic interactions: (i) simultaneous move (Nash) policy game, and (ii) when the fiscal authority acts as a Stackelberg leader.

Nash policy game. Under a Nash policy game each independent discretionary policymaker set its instruments taking as given the current decisions of both the private sector and the other authority.

The benevolent fiscal authority policy problem is, therefore, described by a set of decision rules $\{C_t, Y_t, \Pi_t, \tau_t, G_t, b_t\}$, taking the monetary policy fiscal instrument as given, that maximizes the value function:

$$V^F(b_{t-1}, \epsilon_t) = \max\left\{\frac{C_t^{1-\sigma}}{1-\sigma} + \chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{Y_t^{1+\varphi}}{1+\varphi} + \beta\mathbb{E}_t\left[V^F(b_t, \epsilon_{t+1})\right]\right\}$$
$$\text{s.t. (2.12), (2.18), (2.19), (2.20), for all t.} \tag{2.22}$$

The implied system of first-order conditions describes the behavior of the fiscal authority and, thus, the fiscal reaction function (FRF henceforth).

The policy problem of the conservative central bank, by its turn, is to choose a policy plan $\{C_t, Y_t, \Pi_t, R_t, b_t\}$, taking fiscal authority's control variables as given, to maximize:

$$V^M(b_{t-1}, \epsilon_t) = \max\left\{(1-\alpha)\left[\frac{C_t^{1-\sigma}}{1-\sigma} + \chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{Y_t^{1+\varphi}}{1+\varphi}\right] - \frac{\alpha}{2}\left(\frac{\Pi_t}{\Pi^*} - 1\right)^2 + \beta\mathbb{E}_t\left[V^M(b_t, \epsilon_{t+1})\right]\right\}$$
$$\text{s.t. (2.12), (2.18), (2.19), (2.20), for all t,} \tag{2.23}$$

where $\alpha \in [0, 1]$ is a measure of monetary conservatism (ROGOFF, 1985). When $\alpha = 0$, the monetary objective function is identical to household's utility (2.1) and, thus, we have a benevolent central bank. For $0 < \alpha < 1$, the *partially conservative* central bank dislikes deviations of the inflation rate from its target more than society does and, in the extreme, $\alpha = 1$ means that the monetary policymaker cares only about inflation, known as a *fully conservative* monetary authority. The implied set of first-order conditions associated with the optimization problem (2.23), the monetary reaction function (MRF henceforth), alongside the reaction function of the fiscal policymaker, is presented in Appendix 2.C.

Fiscal leadership game. Under a fiscal leadership game the fiscal authority acts as a Stackelberg leader in the policy game and, thus, can exploit the reaction function of the monetary policymaker (Stackelberg follower).

While the policy problem of the monetary authority remains unchanged, given by the optimization problem in (2.23), the fiscal policymaker now optimizes for a set of decision rules $\{C_t, Y_t, \Pi_t, \tau_t, G_t, R_t, b_t\}$ the following value function:

$$V^F(b_{t-1}, \epsilon_t) = \max\left\{\frac{C_t^{1-\sigma}}{1-\sigma} + \chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{Y_t^{1+\varphi}}{1+\varphi} + \beta\mathbb{E}_t\left[V^F(b_t, \epsilon_{t+1})\right]\right\}$$
$$\text{s.t. (2.12), (2.18), (2.19), (2.20), (MRF) for all t.} \tag{2.24}$$

Note that now the fiscal authority optimizes its value function with respect to the nominal interest rate R_t, the monetary policymaker instrument, which means that fiscal policy, as a Stackelberg leader, can choose the best point in the monetary reaction function. The associated system of first-order conditions is described in Appendix 2.D.

2.5 NUMERICAL METHODS AND CALIBRATION

This section outlines the numerical method used to solve for the discretionary equilibrium and the calibration of parameters.

2.5.1 Solution method

For the model described in sections 2.2 and 2.4, the equilibrium policy functions cannot be computed analytically and, thus, numerical methods are necessary. Linearization around the steady state, a common approach in economics, is not feasible given the presence of generalized Euler equations - involving the derivatives of some equilibrium decision rules - yielding that steady state endogenous and, therefore, *a priori* unknown. Following Leeper, Leith, and Liu (2016, 2019), we resort on a global approximation method to solve for the time consistent equilibrium of the model[8]. More specifically, the policy functions are approximated by Chebyshev polynomials and the nonlinear system of equations is iterated until a set of time-invariant equilibrium of policy rules mapping the vector of state variables to the optimal decisions is reached[9]. The numerical algorithm is detailed in Appendix 2.E.

2.5.2 Calibration

The model is calibrated to a quarterly frequency. The baseline parameterization is summarized on Table 2.1, which is in line with Leeper, Leith, and Liu (2016). The discount factor for Ricardian households is assumed to be $\beta = 0.995$, implying an annual real interest rate of 2%. The intertemporal elasticity of substitution between private consumption, σ, and public consumption, σ_g, are set to one half ($\sigma = \sigma_g = 2$). The baseline value of φ is calibrated to be consistent with a Frisch elasticity of labor supply of one-third (i.e., $\varphi = 3$). The elasticity of substitution between varieties is set to $\bar{\epsilon} = 21$ to be consistent with a markup of 5%, as in Siu (2004). The scaling parameter $\chi = 0.055$ is calibrated to ensure that, in the steady state, the government spending-to-GDP ratio, G/Y, is approximately 19%. The annual inflation target, Π^*, is assumed to be 2%, a value which is in line with adopted by most inflation targeting economies. The Rotemberg price adjustment cost parameter, $\phi = 32.5$, implies that on average firms re-optimize prices approximately every six months[10] - an empirically plausible value. The parameters characterizing the cost-push exogenous process are given by $\rho_\epsilon = 0.95$ and $\sigma_\epsilon = 0.01$.

2.6 NUMERICAL RESULTS

2.6.1 Steady state

Table 2.2 shows the steady state effects, for each policy game, of different degrees of monetary conservatism. It is important to note that when the monetary authority is not infla-

[8] Time consistency problems, in general, can be treated as dynamic games. In problems of this kind, multiplicity of equilibria often arises. By using polynomial approximations, we are focusing only on continuous equilibria. See Judd (2004) for a discussion on the existence, uniqueness and alternative computational approaches to deal with problems of this kind.

[9] For textbook treatments on the numerical techniques involved see Judd (1998), Miranda and Fackler (2004) and Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016).

[10] Given the equivalence between Calvo and Rotemberg pricing for linearized models, $\phi = \frac{(\epsilon-1)\theta}{(1-\theta)(1-\beta\theta)}$, where θ is the fraction of firms that keep their prices unchanged on Calvo model, see Leith and Liu (2016) and Sims and Wolff (2017).

Parameter	Value	Description
β	0.995	Quarterly discount factor
σ	2	Relative risk aversion coefficient
σ_g	2	Relative risk aversion coefficient (government spending)
φ	3	Inverse Frisch elasticity of labor supply
χ	0.055	Scaling parameter (government spending)
Π^*	2%	Annual inflation target
ϕ	32.5	Rotemberg adjustment cost coefficient
$\bar{\epsilon}$	21	Elasticity of substitution between varieties
ρ_ϵ	0.95	AR-coefficient of cost-push shock
σ_ϵ	0.01	Standard deviation of cost-push shock

Table 2.1 – Calibration

tion conservative, $\alpha = 0$, the timing of moving becomes irrelevant and the equilibrium values of the Nash policy game and fiscal leadership are identical. This reflects the fact that, in this scenario, there are no distortions in the objectives of both policymakers and, thus, the solution of the model in the two regimes converges to the full cooperation solution.

Parameter	Steady State Values								
	Nash policy game								
α	b	$\frac{100b}{4RY}$	$100(\Pi^4-1)$	$100(R^4-1)$	Y	$\frac{G}{Y}$	$\frac{C}{Y}$	τ	V
0	-0.76	-18.20%	2.53%	4.61%	1.034	19.95%	80.05%	20.56%	-352.13
0.1	-0.33	-8.00%	2.89%	4.97%	1.033	19.78%	80.21%	20.60%	-352.15
0.2	-0.33	-7.83%	2.90%	4.98%	1.033	19.78%	80.22%	20.60%	-352.15
0.3	-0.32	-7.62%	2.91%	4.99%	1.033	19.77%	80.22%	20.60%	-352.15
0.4	-0.31	-7.38%	2.92%	5.00%	1.033	19.77%	80.22%	20.60%	-352.15
0.5	-0.30	-7.08%	2.93%	5.01%	1.033	19.77%	80.23%	20.60%	-352.16
0.6	-0.28	-6.70%	2.94%	5.03%	1.033	19.76%	80.23%	20.61%	-352.16
0.7	-0.26	-6.20%	2.96%	5.05%	1.033	19.75%	80.24%	20.61%	-352.16
0.8	-0.23	-5.49%	2.99%	5.08%	1.033	19.74%	80.25%	20.61%	-352.16
0.9	-0.18	-4.32%	3.04%	5.12%	1.033	19.72%	80.27%	20.62%	-352.17
0.95	-0.14	-3.33%	3.08%	5.16%	1.032	19.71%	80.28%	20.62%	-352.17
0.99	-0.09	-2.04%	3.13%	5.22%	1.032	19.69%	80.30%	20.63%	-352.18
1	-0.07	-1.58%	3.15%	5.24%	1.032	19.68%	80.30%	20.63%	-352.18

Parameter	Steady State Values								
	Fiscal leadership policy game								
α	b	$\frac{100b}{4RY}$	$100(\Pi^4-1)$	$100(R^4-1)$	Y	$\frac{G}{Y}$	$\frac{C}{Y}$	τ	V
0	-0.76	-18.20%	2.53%	4.61%	1.034	19.95%	80.05%	20.56%	-352.13
0.1	-0.76	-18.14%	2.53%	4.61%	1.034	19.95%	80.05%	20.56%	-352.13
0.2	-0.76	-18.07%	2.53%	4.60%	1.034	19.94%	80.05%	20.56%	-352.13
0.3	-0.75	-17.97%	2.53%	4.60%	1.034	19.94%	80.06%	20.56%	-352.13
0.4	-0.75	-17.85%	2.53%	4.60%	1.034	19.94%	80.06%	20.56%	-352.13
0.5	-0.74	-17.69%	2.52%	4.60%	1.034	19.94%	80.06%	20.56%	-352.13
0.6	-0.73	-17.44%	2.52%	4.60%	1.034	19.93%	80.07%	20.56%	-352.13
0.7	-0.71	-17.06%	2.52%	4.59%	1.034	19.92%	80.07%	20.56%	-352.13
0.8	-0.68	-16.34%	2.51%	4.58%	1.034	19.91%	80.09%	20.56%	-352.13
0.9	-0.61	-14.58%	2.48%	4.56%	1.033	19.87%	80.13%	20.55%	-352.13
0.95	-0.51	-12.17%	2.44%	4.51%	1.033	19.81%	80.19%	20.55%	-352.13
0.99	-0.27	-6.43%	2.28%	4.35%	1.032	19.65%	80.35%	20.50%	-352.13
1	-0.13	-3.02%	2.15%	4.22%	1.032	19.54%	80.46%	20.46%	-352.13

Table 2.2 – Steady state values under alternative degrees of monetary conservatism (Nash and fiscal leadership policy games).

Numerical results show that when the central bank is dependent or, alternatively, monetary and fiscal authorities are fully cooperative, the steady state level of debt is negative, implying that government holds a net stock of nominal assets, rather than liabilities, of 18.2% of output. This value lies well below of the amount of accumulated level of assets necessary to finance government consumption and eliminate the distortions in the economy, associated with the first-best allocation (1,182% of GDP).

Alongside, in the steady state, there is an overshooting of the inflation target. The long-run equilibrium value of the annualized inflation rate is 2.53%, which outweighs the target rate of 2%. The reasoning for an overshooting of the target can be understood as follows: when the policymaker is constrained to act in a time-consistent manner, it suffers from an incentive to induce surprise inflation in the economy aiming to push output closer to its efficient level, since the presence of monopolist distortions yields the equilibrium level of GDP suboptimal. In standard analyses the magnitude of this so-called *inflationary bias* is determined solely by the degree of monopolistic competition in the model economy that distorts the steady state.

However, in the presence of government debt and distortionary taxation, this inflationary bias problem becomes state-dependent, since at higher levels of debt or taxes, the inefficiency is more pronounced and, therefore, increases the policymaker's desire to induce surprise inflation. In addition to that, as observed in Leeper, Leith, and Liu (2016), by inducing unexpected inflation in the economy the policymaker, as a byproduct, reduces the real value of the government debt (since a higher inflation causes bond prices to fall) and mitigates the costs of distortionary taxation, reducing the associated future state-dependent inflationary bias - *debt stabilization bias* as Leith and Wren-Lewis (2013) dubbed it. Nevertheless, once debt levels turn negative, the optimizing discretionay policymaker faces a trade-off, as now inflation surprises causes a reduction in the real market value of the assets that it is holding. Therefore, in the long-run equilibrium, those opposing forces between an incentive to increase inflation to boost output and a desire to cause deflation to raise the value of assets have to be balanced. Since we are only considering single-period maturity debt, the endogenous inflationary bias problem dominates the debt stabilization bias causing an overshoot of inflation[11].

Next we turn to the effects of monetary conservatism and strategic interactions between the fiscal and monetary authorities over the steady state.

Nash steady state. The degree of inflation conservatism by an independent central bank has monotonic effects over the steady state values of the model in a simultaneous move game. In the long-run, the more averse to inflation the monetary authority is, the higher is the inflation and nominal interest rates. The fiscal side, by its turn, is characterized by increases in the distortionary labor income rate, while the public consumption-to-GDP ratio and the stock of assets held by the government fall. To summarize, fiscal policy is tightened as the degree of conservatism by the central banker increases. The annualized interest rate rises from 2.53% in the cooperative scenario to 3.15% under a full conservative central bank. The intuition can be understood as follows. Under a Nash game the fiscal authority takes the nominal interest rate as given and, thus, its incentive to induce a surprise inflation in order to push the suboptimal output closer to its efficient level remains unchanged in the face of a more conservative central bank. Therefore, initially, for the fiscal policymaker the inflationary bias is more relevant than stabilize its finances. Nevertheless, the monetary authority perceiving the inflationary pressures fiscally created responds strongly raising the nominal interest rate. As a result, the real return on the stock of assets falls and this, alongside the fact that fiscal authority is now accumulating more assets, worsen the future inefficiencies in the economy. As time proceeds, the endogenous inflationary bias gets higher but also the debt stabilization bias. Once the latter overturns the former, the economy converges to a long run equilibrium associated with a higher inflation rate (caused by the initial attempts of the fiscal authority to approximate the current output to its efficient level) and labor income tax rates, and with lower government consumption and stock of assets (since, at the end of the day, the fiscal policymaker had to ensure fiscal stabilization).

Fiscal leadership steady state. When the fiscal policymaker can act as a Stackelberg leader, its optimal decisions are taken conditional on the knowledge it has on how the

[11] Leeper, Leith, and Liu (2016) show that as the maturity of debt rises, the debt stabilization bias starts to

conservative central bank will respond. Therefore, it can exploit the best point on the monetary reaction function. An inflation averse central bank can, thus, function as a device of fiscal discipline. The higher the degree of conservatism of the monetary authority the less likely he is to induce surprise inflation to boost the suboptimal economy. Since the fiscal authority, under a fiscal leadership policy game, is aware of the lower inflationary bias that a conservative central bank has, his incentive to resort on inflationary pressures to stimulate the economy falls. As a result of that, the debt stabilization bias, through channels other than inflation, increases in importance in relation to the state-dependent inflationary bias. Numerical results show, then, that for higher values of α lower steady state values of the inflation rate, labor income tax rate, public expenditure-to-GDP ratio and net stock of nominal assets can coexist. Under a fully conservative central bank, $\alpha = 1$, the annual inflation rate almost converge to the target, reaching 2.15%. This suggests that, under a fiscal leadership game, the delegation proposal to an independent and conservative central bank can offset the level bias in inflation caused by a policy constrained to follow a time-consistent (discretionary) plan. Similar to the simultaneous move case, the effects of the degree of monetary conservatism over the steady state values of macroeconomic variables are also monotonic.

2.6.2 Optimal dynamics in response to a markup shock

Figure 2.1 displays the optimal dynamics of the key macroeconomic variables in response to a positive markup shock through impulse response functions (IRFs henceforth). All variables measured as percentage deviation from the steady state. Under the benchmark calibration, we consider the cases in which there is cooperation between the two policymakers, when there is a simultaneous move policy game between a benevolent fiscal authority and a fully conservative central bank and, lastly, when the fiscal policymaker can act as a Stackelberg leader[12].

outweigh the inflation bias problem. In this case, when the government is accumulating a stock of assets, the long-run equilibrium can be associated with a mildly deflation in relation to the target.

[12] The IRFs are qualitative the same when the central bank is only partially conservative, that is, $0 < \alpha < 1$. However, the variability of welfare-relevant macroeconomic variables, under a fiscal leadership regime, is very much alike the cooperative case for intermediate values of α. As we shall see, when the central bank is fully conservative, this variability is considerably smaller.

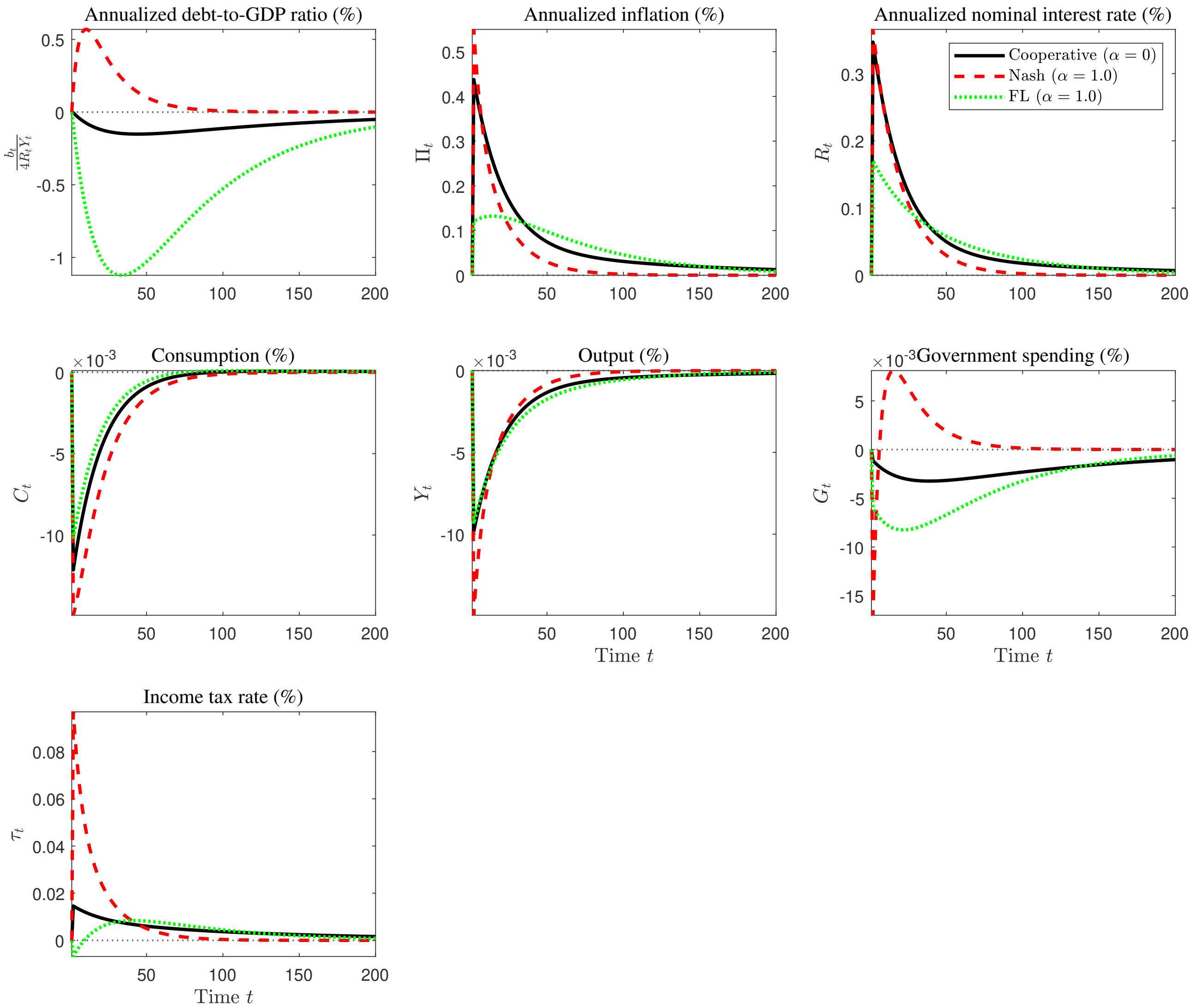

Figure 2.1 – IRFs to a positive markup shock under the benchmark calibration: cooperative solution (black solid line), Nash policy game with a fully conservative central bank (red dashed lines) and fiscal leadership policy game with a fully conservative central bank (green dotted lines). All variables measured as percentage deviation from steady state.

Before analysing the effects of the shock for each policy game, it is worth noting that, as Table 2.2 shows, for all cases considered the steady state level of public debt is mildly negative. Under this circumstance, since the government is holding a stock of assets rather than liabilities, there is a policy trade-off associated: (i) there is an incentive to induce inflation surprises in order to boost the suboptimal level of output, moving it closer to its efficient level (inflationary bias); (ii) but at the same time, there is also an incentive to create a deflation aiming to increase the real value of those assets (debt stabilization bias). As Leeper, Leith, and Liu (2016) argue, there is a tendency towards a dominant inflationary bias over the debt stabilization bias at low maturity levels. In all of the cases considered subsequently, numerical results show that, when a positive cost-push shock hits the model economy, inflation initially rises.

Cooperative dynamics. Following a negative shock of one standard-deviation, σ_ϵ, in the elasticity of substitution between varieties (which implies an increase in firms markup) the optimal response of a centralised policy or, equivalently, the full cooperative arrangement between benevolent fiscal and monetary authorities, is to accommodate only partially the inflationary pressures resulting from the cost-push shock. Therefore, the inflation rate initially rises. Nevertheless, this increase is smaller than it would be if output remained unchanged. The surge in inflation is followed by a tightening in monetary policy through a jump in the nominal interest rates. As a result of that, the stock of nominal assets held by the government looses its real value, leading to a decrease in the amount of assets-held by the government, through running fiscal deficits, in order to be consistent with its budget constraint. Note, however, that despite the fact of being incurring in a fiscal deficit, fiscal instruments were also tightened to restrain aggregate demand - public expenditures fall and distortionary taxes rise - and, thus, mitigating the inflationary consequences of the shock.

Nash dynamics. When the benevolent fiscal authority and a fully conservative central bank play a simultaneous move game the optimal responses to the shock can be strikingly different. Unlike the previously case of full cooperation, under a Nash game the fiscal authority takes the nominal interest rate as given and, in doing so, does not anticipate that higher inflation bears with it lower returns on the assets that it is holding. In other words, fiscal decisions are taken as if there was no trade-off associated with inducing inflation surprises. Taken that into consideration, the initial jump in the inflation rate exceeds the increase in the cooperative setup, as now the inflationary bias is more pronounced. The reaction of the fully conservative central bank is, thus, to raise the nominal interest rate, yielding a devaluation of government assets. Notice that the fiscal policymaker, in an attempt to alleviate the effects of the shock, accumulated a larger stock of assets, since the rise in nominal interest rates was not predicted. It follows that this policy plan worsen future inefficiencies, as now private agents anticipate that to bring back the real market value of the assets to its steady state value, the optimizing discretionary government will have to generate fiscal deficits by managing its instruments. This is indeed what happens after a couple of periods after the shock, when there is an overshooting of government spending and the stock of assets begin to return to its long-run equilibrium value.

Fiscal leadership dynamics. Lastly, when the fiscal authority can act as a Stackelberg leader, fully inflation conservatism implies fiscal discipline. As now the benevolent fiscal authority anticipates that the central bank is strongly averse to variations in inflation, its optimal response to the shock is to impose a substantial decrease in the amount of assets that is holding. Moreover, the state-dependent inflationary bias is much lower and, as a result, there is a small but more persistent increase in inflation following the positive markup shock. In summary, under a fiscal leadership policy game, the response to a cost-push shock is mainly driven by the debt stabilization bias, and the inflationary consequences of the shock are smoothed over time. This suggest that, under a fiscal leadership policy game, the inflationary stabilization bias can be reduced by delegating monetary policy to an independent and conservative central banker as, now, the volatility of welfare relevant economic variables is lower than it is under the cooperative regime.

Overall, numerical results show that the optimal dynamics in response to a markup shock are sensitive to the type of the discretionary game considered. While in a fiscal leadership game a fully inflation conservative central bank can act as a device for fiscal discipline, imposing a strong debt stabilization bias on the fiscal authority and, thus, allowing a smaller variance of inflation rate, by reducing the state-dependent inflationary bias. In a simultaneous move game, the discretionary fiscal policymaker, by taking the monetary policy instrument as given, incurs initially in a larger inflationary bias and, at the same time, accumulating more assets. The unanticipated rise on nominal interest rate, nevertheless, reduces the real market value of government-held assets worsen future inefficiencies in the model economy. Finally, the government has to generate a fiscal deficit to return the stock of assets to its steady state level.

2.6.3 Welfare analysis

In this subsection we consider the welfare effects that an inflation conservative central bank impose on the model economy for each particular game structure. We follow Adam and Billi (2008) and Albonico and Rossi (2017) in adopting a measure for the consumption equivalent steady state welfare losses associated with different degrees of monetary conservatism, $\alpha \in [0,1]$, relative to the benchmark cooperative steady state[13].

Let V^C denotes the deterministic steady state of the value function for the full-cooperative case, given by:

$$V^C = \frac{1}{1-\beta} u(C, G, N), \tag{2.25}$$

where $u(C, G, N)$ is the period utility defined in equation (2.1) evaluated at the cooperative steady state. If V^A is the value function of an alternative game structure, then, the permanent reduction in private consumption, μ^A, that would imply the cooperative long-run equilibrium to be welfare equivalent to this alternative scenario is implicitly defined by the following expression:

$$V^A = \frac{1}{1-\beta} u(C(1+\mu^A), G, N). \tag{2.26}$$

Based on our baseline model calibration, Figure 2.2 displays the effects of inflation conservatism on welfare, measured as percentage points of consumption losses relative to the cooperative steady state, for a simultaneous move game and for the fiscal leadership structure. As shown in the figure, lack of inflation conservatism ($\alpha = 0$), as expected, implies no welfare losses for any of the games. This is due to the fact that, when there are no distortions in the objective functions of the independent authorities, the order of moving is of no importance and, hence, all models are equivalent. Nevertheless, when we consider an inflation conservative central bank, welfare results differ greatly between the regimes considered.

[13] We are aware that an instructive exercise would be to analyse how well, in terms of welfare, the game structures considered between the independent and discretionary authorities perform in comparison to the optimal Ramsey plan. Gertler, Gali, and Clarida (1999) argue that an inflation conservative central bank acting under discretion can replicate the same level of welfare as under an optimal commitment policy. However, their analysis is absent from fiscal considerations and, thus, disregard the potentially strategic interactions

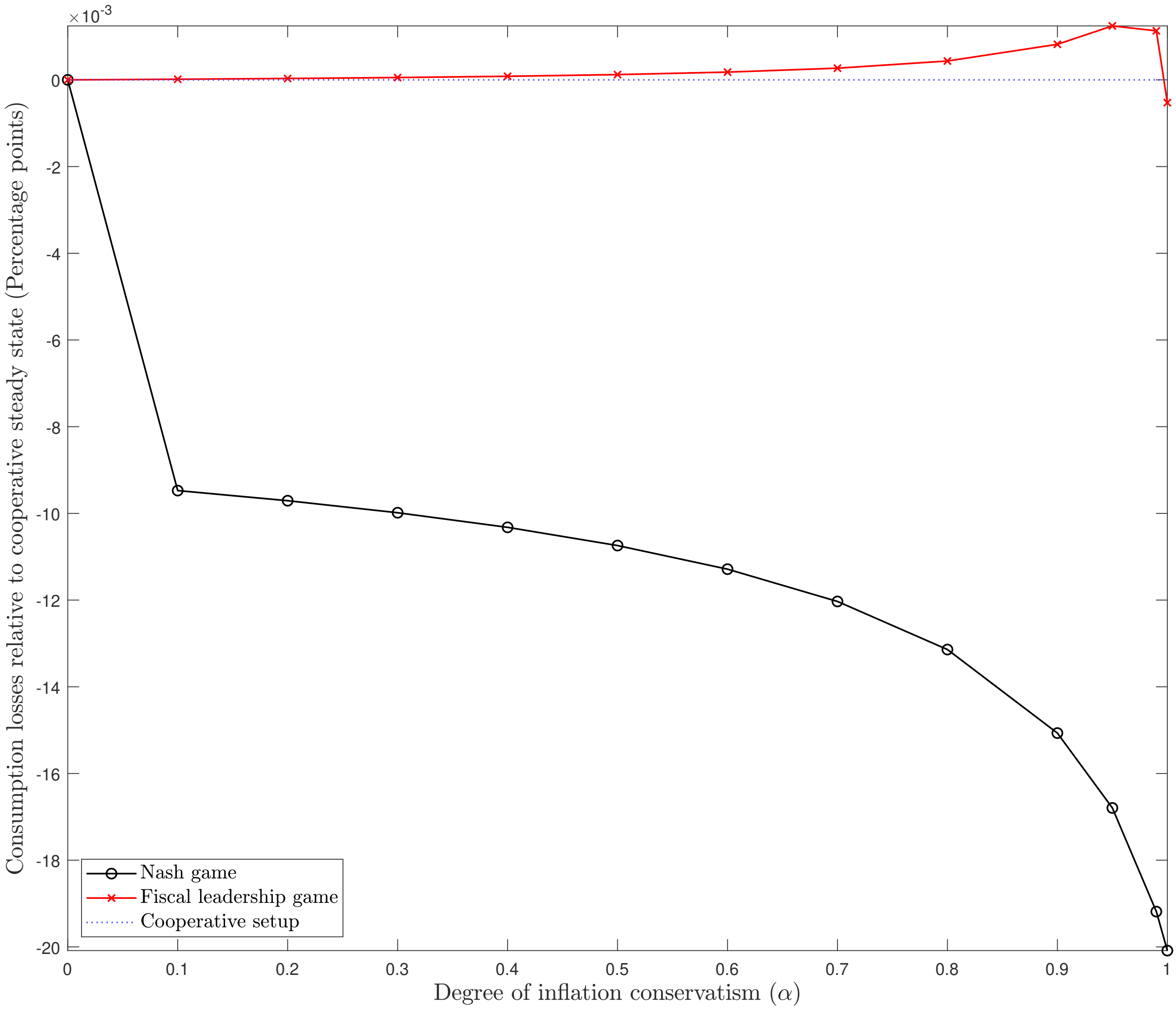

Figure 2.2 – Consumption losses in relation to the cooperative steady state as a function of the degree of inflation conservatism: simultaneous move game (black circled line) and fiscal leadership game (red crossed line).

While the effects of inflation conservatism is monotonic in the simultaneous move game, generating higher welfare losses as the degree of conservatism rises. When the fiscal authority can act as a Stackelberg leader, the implications of a more averse to inflation central bank are not monotonic. When the degree of conservatism assumes intermediate values on the open interval $\alpha \in (0, 1)$, there are small welfare gains of a fiscal leadership game in comparison to the cooperative setup. Nevertheless, when α breaches a threshold this upward trend in terms of consumption gains is reversed. In the extreme, when the monetary authority is fully conservative, there is an undershooting on welfare gains in comparison to the case where objectives are not distorted. Note, however, that the scale of the changes in welfare is very small.

The previous results call for caution when considering the policy prescription to delegate conduction of monetary policy to an independent and inflation conservative central bank as

between authorities. The Ramsey policy, nevertheless, may require a different numerical algorithm do solve the model, see Leeper and Leith (2016). We leave that for future research.

suggested by Rogoff (1985). The optimal delegation proposal asserts that a higher level of social welfare, in comparison to the case where the authority is benevolent, can be achieved by distorting the objectives of a discretionary policymaker. Nevertheless, in most analyses of this kind, fiscal policy plays a very limited role and the potential strategic interactions between two independent, and possibly conflicting, policymakers is largely absent. The numerical results described above indicate that knowledge about the timing of moving between the authorities, when these can act strategically, are crucial to good policy prescriptions. In our model economy, although increases in the degree of monetary conservatism is harmless in the fiscal leadership structure game, they are associated with greater losses in welfare when the policymakers move simultaneously, suggesting that under these circumstances an arrangement with benevolent authorities is welfare improving. Those findings are in line with the work of Blake and Kirsanova (2011) who made even further qualifications regarding the optimal delegation proposal, they found that when government debt levels are higher, welfare losses can be substantial, rather than almost negligible as we found.

2.7 CONCLUSION

This essay addresses the state dependencies in the strategic interactions between an inflation conservative central bank and a benevolent fiscal authority. We consider a standard New Keynesian model augmented to include the government's budget constraint and study the effects of independent, time-consistent and non-cooperative policymakers, rather than joint optimal policies.

We found that if the fiscal policymaker is benevolent but acts strategically, under a fiscal leadership policy game, delegating monetary policy to an inflation conservative central bank can function as a device for fiscal discipline and, thus, reduces both the stabilization and level inflationary biases. Under these circumstances, the consequences in terms of welfare are mostly harmless. Nonetheless, a simultaneous move policy game not only increases the gap between actual inflation and the target rate, in comparison to the cooperative setup, but is also associated with higher welfare losses. These losses are an increasing function of the degree of monetary conservatism.

In all the cases considered in this essay, the economy converges to a steady state with negative level of government debt. This means that the government is holding a net stock of nominal assets, rather than liabilities. This inability of the model to allow for debt accumulation can be overturned by allowing the issuing of longer-term debt and introducing a myopic fiscal authority, as in Leeper, Leith, and Liu (2019). We leave that for future research.

Appendices

2.A DETERMINISTIC STEADY STATE

Given the nonlinear system of equations describing the model economy, the corresponding deterministic steady state can be written as follows:

$$
\begin{aligned}
\frac{\beta R}{\Pi} &= 1, \\
(1-\tau)w &= N^{\varphi}C^{\sigma}, \\
(1-\epsilon)+\epsilon mc &= \phi(1-\beta)\left[\frac{\Pi}{\Pi^*}\left(\frac{\Pi}{\Pi^*}-1\right)\right], \\
Y\left[1-\frac{\phi}{2}\left(\frac{\Pi}{\Pi^*}-1\right)^2\right] &= C+G, \\
\frac{1}{R}b &= \frac{b}{\Pi}-\left(\frac{\tau}{1-\tau}\right)Y^{1+\varphi}C^{\sigma}+G \\
mc = w &= (1-\tau)^{-1}N^{\varphi}C^{\sigma}, \\
Y &= N.
\end{aligned}
$$

Hence, when the rate of inflation in the steady state is equal to the inflation target, $\Pi = \Pi^*$, we have:

$$
\begin{aligned}
R &= \frac{\Pi^*}{\beta}, \\
mc = w &= \frac{\epsilon-1}{\epsilon}, \\
\frac{C}{Y} &= \left[(1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right)\right]^{\frac{1}{\sigma}}Y^{-\frac{\varphi+\sigma}{\sigma}}, \\
\frac{G}{Y} = 1-\frac{C}{Y} &= 1-\left[(1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right)\right]^{\frac{1}{\sigma}}Y^{-\frac{\varphi+\sigma}{\sigma}}, \\
\frac{1}{R}b &= \frac{\beta}{1-\beta}\left[\tau\left(\frac{\epsilon-1}{\epsilon}\right)-\frac{G}{Y}\right]Y.
\end{aligned}
$$

Note that,

$$
Y^{\varphi+\sigma}\left(1-\frac{G}{Y}\right)^{\sigma} = (1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right), \tag{2.27}
$$

that will be used to contrast with the first-best allocation.

2.B THE COOPERATIVE PROBLEM

The Lagrangian for the cooperative policy problem is:

$$\begin{aligned}
\mathcal{L} &= \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{Y_t^{1+\varphi}}{1+\varphi} + \beta \mathbb{E}_t[V(b_t, \epsilon_{t+1})] \right\} \\
&+ \lambda_{1t} \left[\frac{C_t^{-\sigma}}{R_t} - \beta \mathbb{E}_t[F(b_t, \epsilon_{t+1})] \right] \\
&+ \lambda_{2t} \left[(1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} + \phi \beta C_t^{\sigma} Y_t^{-1} \mathbb{E}_t[M(b_t, \epsilon_{t+1})] - \phi \frac{\Pi_t}{\Pi^*} \left(\frac{\Pi_t}{\Pi^*} - 1 \right) \right] \\
&+ \lambda_{3t} \left[Y_t \left(1 - \frac{\phi}{2} \left(\frac{\Pi_t}{\Pi^*} - 1 \right)^2 \right) - C_t - G_t \right] \\
&+ \lambda_{4t} \left[\frac{1}{R_t} b_t - \frac{b_{t-1}}{\Pi_t} + \left(\frac{\tau_t}{1-\tau_t} \right) Y_t^{1+\varphi} C_t^{\sigma} - G_t \right]
\end{aligned} \tag{2.28}$$

The implied set of first-order conditions with respect to $\{C_t, Y_t, \Pi_t, R_t, \tau_t, G_t, b_t\}$ is:

$$\begin{array}{ll}
C_t & C_t^{-\sigma} - \sigma \lambda_{1t} \frac{C_t^{-\sigma-1}}{R_t} + \sigma \lambda_{2t} \left[\epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma-1} + \phi \beta C_t^{\sigma-1} Y_t^{-1} \mathbb{E}_t[M(b_t, \epsilon_{t+1})] \right] \\
& -\lambda_{3t} + \sigma \lambda_{4t} \left(\frac{\tau_t}{1-\tau_t} \right) Y_t^{1+\varphi} C_t^{\sigma-1} = 0 \\
Y_t & -Y_t^{\varphi} + \lambda_{2t} [\varphi \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi-1} C_t^{\sigma} - \phi \beta C_t^{\sigma} Y_t^{-2} \mathbb{E}_t[M(b_t, \epsilon_{t+1})]] + \lambda_{3t} \left(1 - \frac{\phi}{2} \left(\frac{\Pi_t}{\Pi^*} - 1 \right)^2 \right) \\
& +(1+\varphi) \lambda_{4t} \left(\frac{\tau_t}{1-\tau_t} \right) Y_t^{\varphi} C_t^{\sigma} = 0 \\
\Pi_t & \lambda_{4t} \frac{b_{t-1}}{\Pi_t^2} - \lambda_{2t} \frac{\phi}{\Pi^*} \left(2 \frac{\Pi_t}{\Pi^*} - 1 \right) - \phi \lambda_{3t} \frac{Y_t}{\Pi^*} \left(\frac{\Pi_t}{\Pi^*} - 1 \right) = 0 \\
R_t & \lambda_{1t} C_t^{-\sigma} + \lambda_{4t} b_t = 0 \\
\tau_t & \lambda_{2t} \epsilon_t + \lambda_{4t} Y_t = 0 \\
G_t & \chi G_t^{-\sigma_g} - \lambda_{3t} - \lambda_{4t} = 0 \\
b_t & -\beta \mathbb{E}_t \left[\lambda_{4t+1} \frac{1}{\Pi_{t+1}} \right] - \beta \lambda_{1t} \mathbb{E}_t[F_b(b_t, \epsilon_{t+1})] + \phi \beta \lambda_{2t} C_t^{\sigma} Y_t^{-1} \mathbb{E}_t[M_b(b_t, \epsilon_{t+1})] + \frac{\lambda_{4t}}{R_t} = 0,
\end{array}$$

where $X_b(b_t, \epsilon_{t+1}) \equiv \partial X(b_t, \epsilon_{t+1}) / \partial b_t$ for the functions $X \in \{M, F\}$, and we have used the envelope theorem on the first-order condition for government debt to obtain:

$$\frac{\partial V(b_{t-1}, \epsilon_t)}{\partial b_{t-1}} = -\lambda_{4t} \frac{1}{\Pi_t}.$$

2.C NASH POLICY GAME

2.C.1 Fiscal policy problem

The Lagrangian for the fiscal policy problem can be written as:

$$\begin{aligned}
\mathcal{L} &= \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{Y_t^{1+\varphi}}{1+\varphi} + \beta \mathbb{E}_t[V(b_t, \epsilon_{t+1})] \right\} \\
&+ \lambda_{1t}^F \left[\frac{C_t^{-\sigma}}{R_t} - \beta \mathbb{E}_t[F(b_t, \epsilon_{t+1})] \right] \\
&+ \lambda_{2t}^F \left[(1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} + \phi \beta C_t^{\sigma} Y_t^{-1} \mathbb{E}_t[M(b_t, \epsilon_{t+1})] - \phi \frac{\Pi_t}{\Pi^*} \left(\frac{\Pi_t}{\Pi^*} - 1 \right) \right] \\
&+ \lambda_{3t}^F \left[Y_t \left(1 - \frac{\phi}{2} \left(\frac{\Pi_t}{\Pi^*} - 1 \right)^2 \right) - C_t - G_t \right] \\
&+ \lambda_{4t}^F \left[\frac{1}{R_t} b_t - \frac{b_{t-1}}{\Pi_t} + \left(\frac{\tau_t}{1-\tau_t} \right) Y_t^{1+\varphi} C_t^{\sigma} - G_t \right]
\end{aligned} \tag{2.29}$$

The implied set of first-order conditions with respect to $\{C_t, Y_t, \Pi_t, \tau_t, G_t, b_t\}$ is:

$$\begin{aligned}
C_t \quad & C_t^{-\sigma} - \sigma\lambda_{1t}^F \frac{C_t^{-\sigma-1}}{R_t} + \sigma\lambda_{2t}^F \left[\epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma-1} + \phi\beta C_t^{\sigma-1} Y_t^{-1} \mathbb{E}_t[M(b_t, \epsilon_{t+1})]\right] \\
& -\lambda_{3t}^F + \sigma\lambda_{4t}^F \left(\frac{\tau_t}{1-\tau_t}\right) Y_t^{1+\varphi} C_t^{\sigma-1} = 0 \qquad (2.30)\\
Y_t \quad & -Y_t^{\varphi} + \lambda_{2t}^F[\varphi\epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi-1} C_t^{\sigma} - \phi\beta C_t^{\sigma} Y_t^{-2} \mathbb{E}_t[M(b_t, \epsilon_{t+1})]] + \lambda_{3t}^F \left(1 - \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*} - 1\right)^2\right) \\
& +(1+\varphi)\lambda_{4t}^F \left(\frac{\tau_t}{1-\tau_t}\right) Y_t^{\varphi} C_t^{\sigma} = 0 \qquad (2.31)\\
\Pi_t \quad & \lambda_{4t}^F \frac{b_{t-1}}{\Pi_t^2} - \lambda_{2t}^F \frac{\phi}{\Pi^*}\left(2\frac{\Pi_t}{\Pi^*} - 1\right) - \phi\lambda_{3t}^F \frac{Y_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*} - 1\right) = 0 \qquad (2.32)\\
G_t \quad & \chi G_t^{-\sigma_g} - \lambda_{3t}^F - \lambda_{4t}^F = 0 \qquad (2.33)\\
\tau_t \quad & \lambda_{2t}^F \epsilon_t + \lambda_{4t}^F Y_t = 0 \qquad (2.34)\\
b_t \quad & -\beta\mathbb{E}_t\left[\lambda_{4t+1}^F \frac{1}{\Pi_{t+1}}\right] - \beta\lambda_{1t}^F \mathbb{E}_t[F_b(b_t, \epsilon_{t+1})] + \phi\beta\lambda_{2t}^F C_t^{\sigma} Y_t^{-1} \mathbb{E}_t[M_b(b_t, \epsilon_{t+1})] + \frac{\lambda_{4t}^F}{R_t} = 0 \qquad (2.35)
\end{aligned}$$

The system of nonlinear first-order conditions (2.30)-(2.35) constitutes the fiscal reaction function (FRF).

2.C.2 Monetary policy problem

The Lagrangian for the monetary policy problem can be written as:

$$\begin{aligned}
\mathcal{L} \;=\; & \left\{\frac{C_t^{1-\sigma}}{1-\sigma} + \chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{Y_t^{1+\varphi}}{1+\varphi} + \beta\mathbb{E}_t[V(b_t, \epsilon_{t+1})]\right\} \\
+ \; & \lambda_{1t}^M \left[\frac{C_t^{-\sigma}}{R_t} - \beta\mathbb{E}_t[F(b_t, \epsilon_{t+1})]\right] \\
+ \; & \lambda_{2t}^M \left[(1-\epsilon_t) + \epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} + \phi\beta C_t^{\sigma} Y_t^{-1}\mathbb{E}_t[M(b_t, \epsilon_{t+1})] - \phi\frac{\Pi_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*} - 1\right)\right] \qquad (2.36)\\
+ \; & \lambda_{3t}^M \left[Y_t\left(1 - \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*} - 1\right)^2\right) - C_t - G_t\right] \\
+ \; & \lambda_{4t}^M \left[\frac{1}{R_t} b_t - \frac{b_{t-1}}{\Pi_t} + \left(\frac{\tau_t}{1-\tau_t}\right) Y_t^{1+\varphi} C_t^{\sigma} - G_t\right]
\end{aligned}$$

The implied set of first-order conditions with respect to $\{C_t, Y_t, \Pi_t, R_t, b_t\}$ is:

$$\begin{aligned}
C_t \quad & (1-\alpha)C_t^{-\sigma} - \sigma\lambda_{1t}^M \frac{C_t^{-\sigma-1}}{R_t} + \sigma\lambda_{2t}^M \left[\epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma-1} + \phi\beta C_t^{\sigma-1} Y_t^{-1} \mathbb{E}_t[M(b_t, \epsilon_{t+1})]\right] \\
& -\lambda_{3t}^M + \sigma\lambda_{4t}^M \left(\frac{\tau_t}{1-\tau_t}\right) Y_t^{1+\varphi} C_t^{\sigma-1} = 0 \qquad (2.37)\\
Y_t \quad & (\alpha-1)Y_t^{\varphi} + \lambda_{2t}^M[\varphi\epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi-1} C_t^{\sigma} - \phi\beta C_t^{\sigma} Y_t^{-2} \mathbb{E}_t[M(b_t, \epsilon_{t+1})]] \\
& +\lambda_{3t}^M \left(1 - \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*} - 1\right)^2\right) + (1+\varphi)\lambda_{4t}^M \left(\frac{\tau_t}{1-\tau_t}\right) Y_t^{\varphi} C_t^{\sigma} = 0 \qquad (2.38)\\
\Pi_t \quad & -\frac{\alpha}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*} - 1\right) - \lambda_{2t}^M \frac{\phi}{\Pi^*}\left(2\frac{\Pi_t}{\Pi^*} - 1\right) - \phi\lambda_{3t}^M \frac{Y_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*} - 1\right) + \lambda_{4t}^M \frac{b_{t-1}}{\Pi_t^2} = 0 \qquad (2.39)\\
R_t \quad & \lambda_{1t}^M C_t^{-\sigma} + \lambda_{4t}^M b_t = 0 \qquad (2.40)\\
b_t \quad & -\beta\mathbb{E}_t\left[\lambda_{4t+1}^M \frac{1}{\Pi_{t+1}}\right] - \beta\lambda_{1t}^M \mathbb{E}_t[F_b(b_t, \epsilon_{t+1})] + \phi\beta\lambda_{2t}^M C_t^{\sigma} Y_t^{-1} \mathbb{E}_t[M_b(b_t, \epsilon_{t+1})] + \frac{\lambda_{4t}^M}{R_t} = 0 \qquad (2.41)
\end{aligned}$$

The system of nonlinear first-order conditions (2.37)-(2.41) constitutes the monetary reaction function (MRF).

2.D FISCAL LEADERSHIP POLICY GAME

While the monetary policy problem remains unchanged, described by (2.36), the fiscal authority policy problem now optimizes its value function subject to the equations describing the evolution of the economy and the monetary reaction function. The Lagrangian for the fiscal policy game can, then, be written as:

$$
\begin{aligned}
\mathcal{L} \;=\;& \left\{\frac{C_t^{1-\sigma}}{1-\sigma} + \chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{Y_t^{1+\varphi}}{1+\varphi} + \beta\mathbb{E}_t[V^F(b_t,\epsilon_{t+1})]\right\} \\
+\;& \lambda_{1t}^F\left[\frac{C_t^{-\sigma}}{R_t} - \beta\mathbb{E}_t[F(b_t,\epsilon_{t+1})]\right] \\
+\;& \lambda_{2t}^F\left[(1-\epsilon_t) + \epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma} + \phi\beta C_t^{\sigma}Y_t^{-1}\mathbb{E}_t[M(b_t,\epsilon_{t+1})] - \phi\frac{\Pi_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*}-1\right)\right] \\
+\;& \lambda_{3t}^F\left[Y_t\left(1-\frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*}-1\right)^2\right) - C_t - G_t\right] \\
+\;& \lambda_{4t}^F\left[\frac{1}{R_t}b_t - \frac{b_{t-1}}{\Pi_t} + \left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{1+\varphi}C_t^{\sigma} - G_t\right] \\
+\;& \lambda_{5t}^F\left[(1-\alpha)C_t^{-\sigma} - \sigma\lambda_{1t}^M\frac{C_t^{-\sigma-1}}{R_t} + \sigma\lambda_{2t}^M\left[\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma-1} + \phi\beta C_t^{\sigma-1}Y_t^{-1}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]\right]\right. \\
&\left.\quad -\lambda_{3t}^M + \sigma\lambda_{4t}^M\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{1+\varphi}C_t^{\sigma-1}\right] \\
+\;& \lambda_{6t}^F\left[(\alpha-1)Y_t^{\varphi} + \lambda_{2t}^M[\varphi\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma} - \phi\beta C_t^{\sigma}Y_t^{-2}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]]\right. \\
&\left.\quad +\lambda_{3t}^M\left(1-\frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*}-1\right)^2\right) + (1+\varphi)\lambda_{4t}^M\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{\varphi}C_t^{\sigma}\right] \\
+\;& \lambda_{7t}^F\left[-\frac{\alpha}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*}-1\right) - \lambda_{2t}^M\frac{\phi}{\Pi^*}\left(2\frac{\Pi_t}{\Pi^*}-1\right) - \phi\lambda_{3t}^M\frac{Y_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*}-1\right) + \lambda_{4t}^M\frac{b_{t-1}}{\Pi_t^2}\right] \\
+\;& \lambda_{8t}^F\left[\lambda_{1t}^M C_t^{-\sigma} + \lambda_{4t}^M b_t\right] \\
+\;& \lambda_{9t}^F\left[-\beta\mathbb{E}_t\left[\lambda_{4t+1}^M\frac{1}{\Pi_{t+1}}\right] - \beta\lambda_{1t}^M\mathbb{E}_t[F_b(b_t,\epsilon_{t+1})] + \phi\beta\lambda_{2t}^M C_t^{\sigma}Y_t^{-1}\mathbb{E}_t[M_b(b_t,\epsilon_{t+1})] + \frac{\lambda_{4t}^M}{R_t}\right]
\end{aligned}
$$

The implied set of first-order conditions with respect to $\{C_t, Y_t, \Pi_t, R_t, \tau_t, G_t, b_t, \lambda_{1t}^M, \lambda_{2t}^M, \lambda_{3t}^M, \lambda_{4t}^M\}$ is:

$$
\begin{aligned}
C_t \quad & C_t^{-\sigma} - \sigma\lambda^F_{1t}\frac{C_t^{-\sigma-1}}{R_t} + \sigma\lambda^F_{2t}\Big[\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma-1} + \phi\beta C_t^{\sigma-1}Y_t^{-1}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]\Big] \\
& -\lambda^F_{3t} + \sigma\lambda^F_{4t}\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{1+\varphi}C_t^{\sigma-1} + \lambda^F_{5t}\bigg[\sigma(\alpha-1)C_t^{-\sigma-1} + \sigma(1+\sigma)\lambda^M_{1t}\frac{C_t^{-\sigma-2}}{R_t} \\
& +\sigma(\sigma-1)\lambda^M_{2t}[\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma-2} + \phi\beta C_t^{\sigma-2}Y_t^{-1}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]] + \sigma(\sigma-1)\lambda^M_{4t}\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{1+\varphi}C_t^{\sigma-2}\bigg] \\
& +\lambda^F_{6t}\bigg[\sigma\lambda^M_{2t}[\varphi\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma-1} - \phi\beta C_t^{\sigma-1}Y_t^{-2}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]] + \sigma(1+\varphi)\lambda^M_{4t}\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{\varphi}C_t^{\sigma-1}\bigg] \\
& -\sigma\lambda^F_{8t}\lambda^M_{1t}C_t^{-\sigma-1} + \phi\beta\sigma\lambda^F_{9t}\lambda^M_{2t}C_t^{\sigma-1}Y_t^{-1}\mathbb{E}_t[M(b_t,\epsilon_{t+1})] = 0 \qquad (2.42)
\end{aligned}
$$

$$
\begin{aligned}
Y_t \quad & -Y_t^{\varphi} + \lambda^F_{2t}[\varphi\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma} - \phi\beta C_t^{\sigma}Y_t^{-2}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]] + \lambda^F_{3t}\left(1-\frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*}-1\right)^2\right) \\
& +(1+\varphi)\lambda^F_{4t}\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{\varphi}C_t^{\sigma} - \phi\lambda^F_{7t}\frac{\lambda^M_{3t}}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*}-1\right) - \phi\beta\lambda^F_{9t}\lambda^M_{2t}C_t^{\sigma}Y_t^{-2}\mathbb{E}_t[M_b(b_t,\epsilon_{t+1})] \\
& +\lambda^F_{5t}\bigg[\sigma\lambda^M_{2t}[\varphi\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma-1} - \phi\beta C_t^{\sigma-1}Y_t^{-2}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]] + \sigma(1+\varphi)\lambda^M_{4t}\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{\varphi}C_t^{\sigma-1}\bigg] \\
& +\lambda^F_{6t}\Big[\varphi(\alpha-1)Y_t^{\varphi-1} + \lambda^M_{2t}[\varphi(\varphi-1)\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi-2}C_t^{\sigma} + 2\phi\beta C_t^{\sigma}Y_t^{-3}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]] \\
& +\varphi(1+\varphi)\lambda^M_{4t}\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{\varphi-1}C_t^{\sigma}\Big] = 0 \qquad (2.43)
\end{aligned}
$$

$$
\begin{aligned}
\Pi_t \quad & \lambda^F_{4t}\frac{b_{t-1}}{\Pi_t^2} - \lambda^F_{2t}\frac{\phi}{\Pi^*}\left(2\frac{\Pi_t}{\Pi^*}-1\right) - \phi\lambda^F_{3t}\frac{Y_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*}-1\right) - \lambda^F_{6t}\lambda^M_{3t}\frac{\phi}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*}-1\right) \\
& +\lambda^F_{7t}\left[-\frac{\alpha}{(\Pi^*)^2} - 2\lambda^M_{2t}\frac{\phi}{(\Pi^*)^2} - \phi\lambda^M_{3t}\frac{Y_t}{(\Pi^*)^2} - 2\lambda^M_{4t}\frac{b_{t-1}}{\Pi_t^3}\right] = 0 \qquad (2.44)
\end{aligned}
$$

$$
R_t \quad -\lambda^F_{1t}\frac{C_t^{-\sigma}}{R_t^2} - \lambda^F_{4t}\frac{b_t}{R_t^2} + \sigma\lambda^F_{5t}\lambda^M_{1t}\frac{C_t^{-\sigma-1}}{R_t^2} - \lambda^F_{9t}\frac{\lambda^M_{4t}}{R_t^2} = 0 \qquad (2.45)
$$

$$
G_t \quad \chi G_t^{-\sigma_g} - \lambda^F_{3t} - \lambda^F_{4t} = 0 \qquad (2.46)
$$

$$
\tau_t \quad \lambda^F_{2t}\epsilon_t + \lambda^F_{4t}Y_t + \lambda^F_{5t}[\sigma\lambda^M_{2t}\epsilon_t C_t^{-1} + \sigma\lambda^M_{4t}Y_tC_t^{-1}] + \lambda^F_{6t}[\varphi\lambda^M_{2t}\epsilon_t Y_t^{-1} + (1+\varphi)\lambda^M_{4t}] = 0 \qquad (2.47)
$$

$$
\begin{aligned}
b_t \quad & -\beta\mathbb{E}_t\left[\lambda^F_{4t+1}\frac{1}{\Pi_{t+1}} - \lambda^F_{7t+1}\lambda^M_{4t+1}\frac{1}{\Pi^2_{t+1}}\right] - \beta\lambda^F_{1t}\mathbb{E}_t[F_b(b_t,\epsilon_{t+1})] + \phi\beta\lambda^F_{2t}C_t^{\sigma}Y_t^{-1}\mathbb{E}_t[M_b(b_t,\epsilon_{t+1})] + \frac{\lambda^F_{4t}}{R_t} \\
& +\phi\beta\sigma\lambda^F_{5t}\lambda^M_{2t}C_t^{\sigma-1}Y_t^{-1}\mathbb{E}_t[M_b(b_t,\epsilon_{t+1})] - \phi\beta\lambda^F_{6t}\lambda^M_{2t}C_t^{\sigma}Y_t^{-2}\mathbb{E}_t[M_b(b_t,\epsilon_{t+1})] + \lambda^F_{8t}\lambda^M_{4t} \\
& +\lambda^F_{9t}\Big[\phi\beta\lambda^M_{2t}C_t^{\sigma}Y_t^{-1}\mathbb{E}_t[M_{bb}(b_t,\epsilon_{t+1})] - \beta\mathbb{E}_t[H_b(b_t,\epsilon_{t+1})] - \beta\lambda^M_{1t}\mathbb{E}_t[F_{bb}(b_t,\epsilon_{t+1})]\Big] = 0 \qquad (2.48)
\end{aligned}
$$

$$
\lambda^M_{1t} \quad -\sigma\lambda^F_{5t}\frac{C_t^{-\sigma-1}}{R_t} + \lambda^F_{8t}C_t^{-\sigma} - \beta\lambda^F_{9t}\mathbb{E}_t[F_b(b_t,\epsilon_{t+1})] = 0 \qquad (2.49)
$$

$$
\begin{aligned}
\lambda^M_{2t} \quad & \sigma\lambda^F_{5t}[\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma-1} + \phi\beta C_t^{\sigma-1}Y_t^{-1}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]] - \frac{\phi}{\Pi^*}\lambda^F_{7t}\left(2\frac{\Pi_t}{\Pi^*}-1\right) + \phi\beta\lambda^F_{9t}C_t^{\sigma}Y_t^{-1}\mathbb{E}_t[M_b(b_t,\epsilon_{t+1})] \\
& +\lambda^F_{6t}[\varphi\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma} - \phi\beta C_t^{\sigma}Y_t^{-2}\mathbb{E}_t[M(b_t,\epsilon_{t+1})]] = 0 \qquad (2.50)
\end{aligned}
$$

$$
\lambda^M_{3t} \quad -\lambda^F_{5t} + \lambda^F_{6t}\left(1-\frac{\phi}{2}\left(\frac{\Pi_t}{\Pi^*}-1\right)^2\right) - \phi\lambda^F_{7t}\frac{Y_t}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*}-1\right) = 0 \qquad (2.51)
$$

$$
\lambda^M_{4t} \quad \sigma\lambda^F_{5t}\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{1+\varphi}C_t^{\sigma-1} + (1+\varphi)\lambda^F_{6t}\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{\varphi}C_t^{\sigma} + \lambda^F_{7t}\frac{b_{t-1}}{\Pi_t^2} + \frac{\lambda^F_{9t}}{R_t} + \lambda^F_{8t}b_t = 0, \qquad (2.52)
$$

where $M_{bb}(b_t,\epsilon_{t+1}) \equiv \partial^2 M(b_t,\epsilon_{t+1})/\partial b_t^2$, and we have defined the following auxiliary function:

$$
H(b_t,\epsilon_{t+1}) \equiv \lambda^M_{4t+1}\frac{1}{\Pi_{t+1}}.
$$

2.E NUMERICAL ALGORITHM

In this appendix we outline the Chebyshev collocation method with time iteration employed to solve the model described in the paper. This method can be framed under a more general approach, namely, projection methods. Projection methods handle DSGE models by building a basis function, indexed by fixed coefficients, that approximates a policy function in order find the vector of coefficients which minimizes some given residual function. For textbook treatments of the numerical methods, see Miranda and Fackler (2004), Judd (1998) and

Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016). Our exposition heavily relies on the description of the techniques found in Leeper, Leith, and Liu (2019).

Define the vector of states $s_t = (b_{t-1}, \epsilon_t)$, where b_{t-1} is the real stock of debt, which is endogenous, and ϵ_t is the elasticity of substitution between varieties, assumed to be an exogenously given process. The law of motion for each state variable is given by:

$$\begin{aligned}\frac{1}{R_t} b_t &= \frac{b_{t-1}}{\Pi_t} - \tau_t w_t N_t + G_t,\\ \ln(\epsilon_t) &= (1-\rho_\epsilon)\ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon_{t-1}) + \sigma_\epsilon \varepsilon_t, \quad \varepsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1),\end{aligned}$$

where $0 \leq \rho_\epsilon < 1$.

The following subsections present the algorithm for the simultaneous move and the fiscal leadership policy games.

2.E.1 Nash policy game

We approximate the following 15 functional equations associated with 7 endogenous variables and 8 Lagrange multipliers, $\Big\{C_t(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), G_t(s_t), R_t(s_t), \lambda^F_{1t}(s_t), \lambda^F_{2t}(s_t),$ $\lambda^F_{3t}(s_t), \lambda^F_{4t}(s_t), \lambda^M_{1t}(s_t), \lambda^M_{2t}(s_t), \lambda^M_{3t}(s_t), \lambda^M_{4t}(s_t)\Big\}$. In order to collect the policy functions of endogenous variables, we define a function $X : \mathbb{R}^2 \to \mathbb{R}^{15}$, where:

$$\begin{aligned}X(s_t) &= \Big(C_t(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), G_t(s_t), \lambda^F_{1t}(s_t), \lambda^F_{2t}(s_t), \lambda^F_{3t}(s_t), \lambda^F_{4t}(s_t),\\ &\quad \lambda^M_{1t}(s_t), \lambda^M_{2t}(s_t), \lambda^M_{3t}(s_t), \lambda^M_{4t}(s_t)\Big).\end{aligned}$$

In doing so, the equilibrium conditions of the model can be rewritten compactly as:

$$\Gamma\left(s_t, X(s_t), \mathbb{E}_t[Z(X(s_{t+1}))], \mathbb{E}_t[Z_b(X(s_{t+1}))]\right) = 0,$$

where $\Gamma : \mathbb{R}^{2+15+4+4} \to \mathbb{R}^{15}$, and:

$$Z(X(s_{t+1})) = \begin{bmatrix} Z_1(X(s_{t+1})) \\ Z_2(X(s_{t+1})) \\ Z_3(X(s_{t+1})) \\ Z_4(X(s_{t+1})) \end{bmatrix} \equiv \begin{bmatrix} M(b_t, \epsilon_{t+1}) \\ F(b_t, \epsilon_{t+1}) \\ (\Pi_{t+1})^{-1}\lambda^F_{4t+1} \\ (\Pi_{t+1})^{-1}\lambda^M_{4t+1} \end{bmatrix},$$

with:

$$\begin{aligned}M(b_t, \epsilon_{t+1}) &= (C_{t+1})^{-\sigma} Y_{t+1} \frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right),\\ F(b_t, \epsilon_{t+1}) &= (C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1},\end{aligned}$$

and:

$$Z_b(X(s_{t+1})) = \begin{bmatrix} \frac{\partial Z_1(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_2(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_3(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_4(X(s_{t+1}))}{\partial b_t} \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial M(b_t, \epsilon_{t+1})}{\partial b_t} \\ \frac{\partial F(b_t, \epsilon_{t+1})}{\partial b_t} \\ \frac{\partial [(\Pi_{t+1})^{-1}\lambda^F_{4t+1}]}{\partial b_t} \\ \frac{\partial [(\Pi_{t+1})^{-1}\lambda^M_{4t+1}]}{\partial b_t} \end{bmatrix}.$$

Taking the derivatives yields:

$$\begin{aligned} M_b(b_t,\epsilon_{t+1}) &\equiv \frac{\partial M(b_t,\epsilon_{t+1})}{\partial b_t} = -\sigma(C_{t+1})^{-\sigma-1}Y_{t+1}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*}-1\right)\frac{\partial C_{t+1}}{\partial b_t} \\ &+ (C_{t+1})^{-\sigma}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*}-1\right)\frac{\partial Y_{t+1}}{\partial b_t} + (C_{t+1})^{-\sigma}\frac{Y_{t+1}}{\Pi^*}\left(\frac{2\Pi_{t+1}}{\Pi^*}-1\right)\frac{\partial \Pi_{t+1}}{\partial b_t}, \end{aligned}$$

and

$$F_b(b_t,\epsilon_{t+1}) \equiv \frac{\partial F(b_t,\epsilon_{t+1})}{\partial b_t} = -\sigma(C_{t+1})^{-\sigma-1}(\Pi_{t+1})^{-1}\frac{\partial C_{t+1}}{\partial b_t} - (C_{t+1})^{-\sigma}(\Pi_{t+1})^{-2}\frac{\partial \Pi_{t+1}}{\partial b_t}.$$

As in Leeper, Leith, and Liu (2019), we are relying on the Interchange of Integration and Differentiation Theorem and assuming that $\mathbb{E}_t[Z_b(X(s_{t+1}))] = \partial\mathbb{E}_t[Z(X(s_{t+1}))]/\partial b_t$. To solve the model, we then use projection methods to find a vector-valued function X that Γ maps to some "approximately" zero function.

In order to easy notation, we follow the convention in the literature by using $s(b,\epsilon)$ to denote the current state of the economy, and s' to represent the next period state.

The Chebyshev collocation algorithm to solve the nonlinear system describing the model economy can be described as follows:

1. Define the collocation nodes and the space of linearly independent basis functions to approximate the policy functions:

   - Choose an order of approximation[14] (i.e., the polynomial degrees) n_b and n_ϵ for each dimension of the state space $s=(b,\epsilon)$, then there are $N_s=(n_b+1)\times(n_\epsilon+1)$ nodes. Let $S=(S_1,S_2,\ldots,S_{N_s})$ denote the set of collocation nodes.
   - Compute the n_b+1 and $n_\epsilon+1$ zeros of the Chebyshev polynomials of order n_b+1 and $n_\epsilon+1$ as:

   $$\begin{aligned} z_b^i &= \cos\left(\frac{(2i-1)\pi}{2(n_b+1)}\right), \quad i=1,2,\ldots,n_b+1. \\ z_\epsilon^i &= \cos\left(\frac{(2i-1)\pi}{2(n_\epsilon+1)}\right), \quad i=1,2,\ldots,n_\epsilon+1. \end{aligned}$$

   Chebyshev polynomials have the convenient property that they are smooth and bounded between $[-1,1]$. Besides, their roots are quadratically clustered toward ± 1.
   - Given that the domain of Chebyshev polynomials is $[-1,1]$ and that the state variables of our DSGE model are different, we should use some form of linear translation. In order to do that, we compute the collocation points ϵ_i as:

   $$\epsilon_i = \frac{\epsilon_{\max}+\epsilon_{\min}}{2} + \frac{\epsilon_{\max}-\epsilon_{\min}}{2}z_\epsilon^i = \frac{\epsilon_{\max}-\epsilon_{\min}}{2}(z_\epsilon^i+1)+\epsilon_{\min}, \quad i=1,2,\ldots,n_\epsilon+1,$$

   which map $[-1,1]$ onto $[\epsilon_{\min},\epsilon_{\max}]$. Similarly, the collocation points b_t are:

   $$b_i = \frac{b_{\max}+b_{\min}}{2} + \frac{b_{\max}-b_{\min}}{2}z_b^i = \frac{b_{\max}-b_{\min}}{2}(z_b^i+1)+b_{\min}, \quad i=1,2,\ldots,n_b+1,$$

[14] In this work we have used polynomials of 11^{th} degree for each state variable.

mapping $[-1,1]$ onto $[b_{\min}, b_{\max}]$. Note that:

$$S = \{(b_i, \epsilon_j)|i = 1, 2, \ldots, n_b + 1, j = 1, 2, \ldots, n_\epsilon + 1\},$$

are the tensor grids, with $S_1 = (b_1, \epsilon_1), S_2 = (b_1, \epsilon_2), \ldots, S_{N_s} = (b_{n_b+1}, \epsilon_{n_\epsilon+1})$.

- The space of the approximating functions, denoted as Ω, is a matrix of two-dimensional Chebyshev polynomials. More specifically,

$$\Omega(S) = \begin{bmatrix} \Omega(S_1) \\ \Omega(S_2) \\ \vdots \\ \Omega(S_{n_\epsilon+1}) \\ \vdots \\ \Omega(S_{N_s}) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_1))) & T_0(\xi(b_1)T_2(\xi(\epsilon_1))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_1))) \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_2))) & T_0(\xi(b_1)T_2(\xi(\epsilon_2))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_2))) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_{n_\epsilon+1}))) & T_0(\xi(b_1)T_2(\xi(\epsilon_{n_\epsilon+1}))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1}))) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & T_0(\xi(b_{n_b+1})T_1(\xi(\epsilon_{n_\epsilon+1}))) & T_0(\xi(b_{n_b+1})T_2(\xi(\epsilon_{n_\epsilon+1}))) & \cdots & T_{n_b}(\xi(b_{n_b+1})T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1}))) \end{bmatrix}_{N_S \times N_S}$$

where $\xi(x) = 2\frac{x - x_{\min}}{x_{\max} - x_{\min}} - 1$ is a linear translation mapping the domain of $x \in [x_{\min}, x_{\max}]$ onto $[-1,1]$, and T defines the Chebyshev polyomials recursively, with $T_0(x) = 1, T_1(x) = x$, and the general $n+1$-th order polynomial is given by:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x).$$

- Then, at each node $s \in S$, the policy functions $X(s)$ are approximated by $X(s) = \Omega(s)\Theta_X$, where

$$\Theta_X = [\theta^c, \theta^y, \theta^\Pi, \theta^b, \theta^\tau, \theta^G, \theta^R, \theta^{\lambda_1^F}, \theta^{\lambda_2^F}, \theta^{\lambda_3^F}, \theta^{\lambda_4^F}, \theta^{\lambda_1^M}, \theta^{\lambda_2^M}, \theta^{\lambda_3^M}, \theta^{\lambda_4^M}],$$

is a $N_S \times 15$ matrix of coefficients.

2. Formulate an initial guess for the matrix of coefficients, Θ_X^0, and specify some convergence criterion ϵ_{tol}. We set $\epsilon_{\text{tol}} = 10^{-8}$.

3. At each iteration j, the matrix of coefficients is updated Θ_X^j by implementing the following time iteration procedure:

   - For each collocation node $s \in S$, compute the possible values of future policy functions $X(s')$ for $k = 1, \ldots, q$. That is:

$$X(s') = \Omega(s')\Theta_X^{j-1}.$$

where q is the number of nodes in a Gauss-Hermite quadrature[15]. Note that:

$$\Omega(s') = T_{j_b}(\xi(b'))T_{j_\epsilon}(xi(\epsilon')),$$

[15] In this work we set an 12-point Gauss-Hermite quadrature rule.

is a $q \times N_s$ matrix, for $j_b = 0, \ldots, n_b$ and $j_\epsilon = 0, \ldots, n_\epsilon$, with $b' = \hat{b}(s; \theta^b)$ and:

$$\ln(\epsilon') = (1 - \rho_\epsilon)\ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon) + z_k \sqrt{2\sigma_\epsilon^2}.$$

The two auxiliary functions can be calculated in a similar way:

$$\begin{aligned} M(s') &\approx \left(\hat{C}(s';\theta^c)\right)^{-\sigma} \hat{Y}(s';\theta^y) \frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*}\left(\frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*} - 1\right), \\ F(s') &\approx \left(\hat{C}(s';\theta^c)\right)^{-\sigma}\left(\hat{\Pi}(s';\theta^\Pi)\right)^{-1}. \end{aligned}$$

The hat notation indicates that the policy functions are only approximated.

- Let ω_k denote the weights of the Gauss-Hermite quadrature, the expectation terms can be calculated, at each node s, as:

$$\begin{aligned} \mathbb{E}[M(s')] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q} \omega_k \left(\hat{C}(s';\theta^c)\right)^{-\sigma} \hat{Y}(s';\theta^y)\frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*}\left(\frac{\hat{\Pi}(s';\theta^\Pi)}{\Pi^*} - 1\right) \equiv \bar{M}(s', q), \\ \mathbb{E}[F(s')] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q} \omega_k \left(\hat{C}(s';\theta^c)\right)^{-\sigma}\left(\hat{\Pi}(s';\theta^\Pi)\right)^{-1} \equiv \bar{L}(s', q), \\ \mathbb{E}_t\left[\left(\frac{1}{\Pi_{t+1}}\right)\lambda^F_{4t+1}\right] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q} \omega_k \left(\frac{1}{\hat{\Pi}(s';\theta^\Pi)}\right)\hat{\lambda}^F_4(s';\theta^{\lambda^F_4}) \equiv \Lambda^F(s', q), \\ \mathbb{E}_t\left[\left(\frac{1}{\Pi_{t+1}}\right)\lambda^M_{4t+1}\right] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q} \omega_k \left(\frac{1}{\hat{\Pi}(s';\theta^\Pi)}\right)\hat{\lambda}^M_4(s';\theta^{\lambda^M_4}) \equiv \Lambda^M(s', q), \end{aligned}$$

  Hence:

$$\mathbb{E}[Z(X(s'))] \approx \mathbb{E}[\hat{Z}(X(s'))] = \begin{bmatrix} \bar{M}(s', q) \\ \bar{L}(s', q) \\ \Lambda^F(s', q) \\ \Lambda^M(s', q) \end{bmatrix}.$$

- To calculate the partial derivatives under expectations, $\mathbb{E}[Z_b(X(s'))]$, we obtain the following terms:

$$\begin{aligned} \frac{\partial C_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon} \frac{2\theta^c_{j_b j_\epsilon}}{b_{\max} - b_{\min}} T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{C}_b(s'), \\ \frac{\partial Y_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon} \frac{2\theta^y_{j_b j_\epsilon}}{b_{\max} - b_{\min}} T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{Y}_b(s'), \\ \frac{\partial \Pi_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon} \frac{2\theta^\Pi_{j_b j_\epsilon}}{b_{\max} - b_{\min}} T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{\Pi}_b(s'). \end{aligned}$$

Hence, the partial derivatives under expectations can be approximated as:

$$\begin{aligned}
&\frac{\partial \mathbb{E}[M(s')]}{\partial b} \\
\approx \frac{1}{\sqrt{\pi}} \sum_{k=1}^{q} \omega_k &\begin{bmatrix} -\sigma \left(\hat{C}(s';\theta^c)\right)^{-\sigma-1} \hat{Y}(s';\theta^y) \frac{\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*} \left(\frac{\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*} - 1\right) \hat{C}_b(s') \\ + \left(\hat{C}(s';\theta^c)\right)^{-\sigma} \frac{\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*} \left(\frac{\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*} - 1\right) \hat{Y}_b(s') \\ + \left(\hat{C}(s';\theta^c)\right)^{-\sigma} \frac{\hat{Y}(s';\theta^y)}{\Pi^*} \left(\frac{2\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*} - 1\right) \hat{\Pi}_b(s') \end{bmatrix} \\
&\equiv \hat{M}_b(s',q), \\
&\frac{\partial \mathbb{E}[F(s')]}{\partial b} \\
\approx \frac{1}{\sqrt{\pi}} \sum_{k=1}^{q} \omega_k &\begin{bmatrix} -\sigma \left(\hat{C}(s';\theta^c)\right)^{-\sigma-1} \left(\hat{\Pi}(s';\theta^{\Pi})\right)^{-1} \hat{C}_b(s') \\ - \left(\hat{C}(s';\theta^c)\right)^{-\sigma} \left(\hat{\Pi}(s';\theta^{\Pi})\right)^{-2} \hat{\Pi}_b(s') \end{bmatrix} \\
&\equiv \hat{F}_b(s',q).
\end{aligned}$$

That is,

$$\mathbb{E}[Z_b(X(s'))] \approx \mathbb{E}[\hat{Z}_b(X(s'))] = \begin{bmatrix} \hat{M}_b(s',q) \\ \hat{F}_b(s',q) \end{bmatrix}.$$

4. At each collocation node s, solve the following functional for $X(s)$:

$$\Gamma(s, X(s), \mathbb{E}[\hat{Z}(X(s'))], \mathbb{E}[\hat{Z}_b(X(s'))]) = 0,$$

using some routine to solve systems of nonlinear equations. Once $X(s)$ is obtained, the matrix of coefficients can be calculated as follows:

$$\hat{\Theta}_X^j = \left(\Omega(S)^T \Omega(S)\right)^{-1} \Omega(S)^T X(S).$$

5. Update the approximating coefficients, $\Theta_X^j = \eta \hat{\Theta}_X^j + (1-\eta)\Theta_X^{j-1}$, where $0 \leq \eta \leq 1$ is a dampening parameter used for improving convergence.

6. If $||\Theta_X^j - \Theta_X^{j-1}|| < \epsilon_{\text{tol}}$, the algorithm converged. Otherwise, restart the procedure from Step 3.

2.E.2 Fiscal leadership policy game

We approximate the following 20 functional equations associated with 7 endogenous variables and 13 Lagrange multipliers, $\Big\{C_t(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), G_t(s_t), R_t(s_t), \lambda_{1t}^F(s_t), \lambda_{2t}^F(s_t), \lambda_{3t}^F(s_t), \lambda_{4t}^F(s_t), \lambda_{5t}^F(s_t), \lambda_{6t}^F(s_t), \lambda_{7t}^F(s_t), \lambda_{8t}^F(s_t), \lambda_{9t}^F(s_t), \lambda_{1t}^M(s_t), \lambda_{2t}^M(s_t), \lambda_{3t}^M(s_t), \lambda_{4t}^M(s_t)\Big\}$. In order to collect the policy functions of endogenous variables, we define a function $X : \mathbb{R}^2 \to \mathbb{R}^{20}$, where:

$$\begin{aligned}
X(s_t) \;=\; &\Big(C_t(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), G_t(s_t), \lambda_{1t}^F(s_t), \lambda_{2t}^F(s_t), \lambda_{3t}^F(s_t), \lambda_{4t}^F(s_t), \lambda_{5t}^F(s_t), \\
&\lambda_{6t}^F(s_t), \lambda_{7t}^F(s_t), \lambda_{8t}^F(s_t), \lambda_{9t}^F(s_t), \lambda_{1t}^M(s_t), \lambda_{2t}^M(s_t), \lambda_{3t}^M(s_t), \lambda_{4t}^M(s_t)\Big).
\end{aligned}$$

In doing so, the equilibrium conditions of the model can be rewritten compactly as:

$$\Gamma\left(s_t, X(s_t), \mathbb{E}_t[Z(X(s_{t+1}))], \mathbb{E}_t[Z_b(X(s_{t+1}))], \mathbb{E}_t[Z_{bb}(X(s_{t+1}))]\right) = 0,$$

where $\Gamma : \mathbb{R}^{2+15+5+5} \to \mathbb{R}^{20}$, and:

$$Z(X(s_{t+1})) = \begin{bmatrix} Z_1(X(s_{t+1})) \\ Z_2(X(s_{t+1})) \\ Z_3(X(s_{t+1})) \\ Z_4(X(s_{t+1})) \\ Z_5(X(s_{t+1})) \end{bmatrix} \equiv \begin{bmatrix} M(b_t, \epsilon_{t+1}) \\ F(b_t, \epsilon_{t+1}) \\ (\Pi_{t+1})^{-1}\lambda^F_{4t+1} \\ (\Pi_{t+1})^{-1}\lambda^M_{4t+1} \\ (\Pi_{t+1})^{-2}\lambda^M_{4t+1}\lambda^F_{7t+1} \end{bmatrix},$$

with:

$$\begin{aligned} M(b_t, \epsilon_{t+1}) &= (C_{t+1})^{-\sigma} Y_{t+1} \frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right), \\ F(b_t, \epsilon_{t+1}) &= (C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}, \end{aligned}$$

and:

$$Z_b(X(s_{t+1})) = \begin{bmatrix} \frac{\partial Z_1(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_2(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_3(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_4(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_5(X(s_{t+1}))}{\partial b_t} \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial M(b_t, \epsilon_{t+1})}{\partial b_t} \\ \frac{\partial F(b_t, \epsilon_{t+1})}{\partial b_t} \\ \frac{\partial[(\Pi_{t+1})^{-1}\lambda^F_{4t+1}]}{\partial b_t} \\ \frac{\partial[(\Pi_{t+1})^{-1}\lambda^M_{4t+1}]}{\partial b_t} \\ \frac{\partial[(\Pi_{t+1})^{-2}\lambda^M_{4t+1}\lambda^F_{7t+1}]}{\partial b_t} \end{bmatrix}.$$

Taking the derivatives yields:

$$\begin{aligned} M_b(b_t, \epsilon_{t+1}) &\equiv \frac{\partial M(b_t, \epsilon_{t+1})}{\partial b_t} = -\sigma(C_{t+1})^{-\sigma-1} Y_{t+1}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right)\frac{\partial C_{t+1}}{\partial b_t} \\ &+ (C_{t+1})^{-\sigma}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*} - 1\right)\frac{\partial Y_{t+1}}{\partial b_t} + (C_{t+1})^{-\sigma}\frac{Y_{t+1}}{\Pi^*}\left(\frac{2\Pi_{t+1}}{\Pi^*} - 1\right)\frac{\partial \Pi_{t+1}}{\partial b_t}, \end{aligned}$$

and

$$F_b(b_t, \epsilon_{t+1}) \equiv \frac{\partial F(b_t, \epsilon_{t+1})}{\partial b_t} = -\sigma(C_{t+1})^{-\sigma-1}(\Pi_{t+1})^{-1}\frac{\partial C_{t+1}}{\partial b_t} - (C_{t+1})^{-\sigma}(\Pi_{t+1})^{-2}\frac{\partial \Pi_{t+1}}{\partial b_t}.$$

As in Leeper, Leith, and Liu (2019), we are relying on the Interchange of Integration and Differentiation Theorem and assuming that $\mathbb{E}_t[Z_b(X(s_{t+1}))] = \partial \mathbb{E}_t[Z(X(s_{t+1}))]/\partial b_t$. To solve the model, we then use projection methods to find a vector-valued function X that Γ maps to some "approximately" zero function.

Lastly, in the fiscal leadership policy games, we need to take second derivatives of auxiliary functions:

$$Z_{bb}(X(s_{t+1})) = \begin{bmatrix} \frac{\partial^2 Z_1(X(s_{t+1}))}{\partial b_t^2} \\ \frac{\partial^2 Z_2(X(s_{t+1}))}{\partial b_t^2} \\ \frac{\partial^2 Z_3(X(s_{t+1}))}{\partial b_t^2} \\ \frac{\partial^2 Z_4(X(s_{t+1}))}{\partial b_t^2} \\ \frac{\partial^2 Z_5(X(s_{t+1}))}{\partial b_t^2} \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial^2 M(b_t, \epsilon_{t+1})}{\partial b_t^2} \\ \frac{\partial^2 F(b_t, \epsilon_{t+1})}{\partial b_t^2} \\ \frac{\partial^2[(\Pi_{t+1})^{-1}\lambda^F_{4t+1}]}{\partial b_t^2} \\ \frac{\partial^2[(\Pi_{t+1})^{-1}\lambda^M_{4t+1}]}{\partial b_t^2} \\ \frac{\partial^2[(\Pi_{t+1})^{-2}\lambda^M_{4t+1}\lambda^F_{7t+1}]}{\partial b_t^2} \end{bmatrix}.$$

Taking the derivatives yields:

$$
\begin{aligned}
M_{bb}(b_t,\epsilon_{t+1}) \equiv & \frac{\partial^2 M(b_t,\epsilon_{t+1})}{\partial b_t^2} = \sigma(1+\sigma)(C_{t+1})^{-\sigma-2}Y_{t+1}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*}-1\right)\left(\frac{\partial C_{t+1}}{\partial b_t}\right)^2 \\
- & 2\sigma(C_{t+1})^{-\sigma-1}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*}-1\right)\frac{\partial Y_{t+1}}{\partial b_t}\frac{\partial C_{t+1}}{\partial b_t} \\
- & 2\sigma(C_{t+1})^{-\sigma-1}\frac{Y_{t+1}}{\Pi^*}\left(\frac{2\Pi_{t+1}}{\Pi^*}-1\right)\frac{\partial \Pi_{t+1}}{\partial b_t}\frac{\partial C_{t+1}}{\partial b_t} \\
- & \sigma(C_{t+1})^{-\sigma-1}Y_{t+1}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*}-1\right)\frac{\partial^2 C_{t+1}}{\partial b_t^2} \\
+ & 2\frac{(C_{t+1})^{-\sigma}}{\Pi^*}\left(\frac{2\Pi_{t+1}}{\Pi^*}-1\right)\frac{\partial \Pi_{t+1}}{\partial b_t}\frac{\partial Y_{t+1}}{\partial b_t} + 2(C_{t+1})^{-\sigma}\frac{Y_{t+1}}{(\Pi^*)^2}\left(\frac{\partial \Pi_{t+1}}{\partial b_t}\right)^2 \\
+ & (C_{t+1})^{-\sigma}\frac{\Pi_{t+1}}{\Pi^*}\left(\frac{\Pi_{t+1}}{\Pi^*}-1\right)\frac{\partial^2 Y_{t+1}}{\partial b_t^2} + (C_{t+1})^{-\sigma}\frac{Y_{t+1}}{\Pi^*}\left(2\frac{\Pi_{t+1}}{\Pi^*}-1\right)\frac{\partial^2 \Pi_{t+1}}{\partial b_t^2},
\end{aligned}
$$

and:

$$
\begin{aligned}
F_{bb}(b_t,\epsilon_{t+1}) \equiv & \frac{\partial^2 F(b_t,\epsilon_{t+1})}{\partial b_t^2} = \sigma(1+\sigma)(C_{t+1})^{-\sigma-2}(\Pi_{t+1})^{-1}\left(\frac{\partial C_{t+1}}{\partial b_t}\right)^2 \\
+ & 2(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-3}\left(\frac{\partial \Pi_{t+1}}{\partial b_t}\right)^2 + 2\sigma(C_{t+1})^{-\sigma-1}(\Pi_{t+1})^{-2}\frac{\partial \Pi_{t+1}}{\partial b_t}\frac{\partial C_{t+1}}{\partial b_t} \\
- & (C_{t+1})^{-\sigma}(\Pi_{t+1})^{-2}\frac{\partial^2 \Pi_{t+1}}{\partial b_t^2} - \sigma(C_{t+1})^{-\sigma-1}(\Pi_{t+1})^{-1}\frac{\partial^2 C_{t+1}}{\partial b_t^2}.
\end{aligned}
$$

In order to easy notation, we follow the convention in the literature by using $s(b,\epsilon)$ to denote the current state of the economy, and s' to represent the next period state.

The Chebyshev collocation algorithm to solve the nonlinear system describing the model economy can be described as follows:

1. Define the collocation nodes and the space of linearly independent basis functions to approximate the policy functions:

   - Choose an order of approximation[16] (i.e., the polynomial degrees) n_b and n_ϵ for each dimension of the state space $s=(b,\epsilon)$, then there are $N_s=(n_b+1)\times(n_\epsilon+1)$ nodes. Let $S=(S_1,S_2,\ldots,S_{N_s})$ denote the set of collocation nodes.
   - Compute the n_b+1 and $n_\epsilon+1$ zeros of the Chebyshev polynomials of order n_b+1 and $n_\epsilon+1$ as:

$$
\begin{aligned}
z_b^i &= \cos\left(\frac{(2i-1)\pi}{2(n_b+1)}\right), \quad i=1,2,\ldots,n_b+1. \\
z_\epsilon^i &= \cos\left(\frac{(2i-1)\pi}{2(n_\epsilon+1)}\right), \quad i=1,2,\ldots,n_\epsilon+1.
\end{aligned}
$$

   Chebyshev polynomials have the convenient property that they are smooth and bounded between $[-1,1]$. Besides, their roots are quadratically clustered toward ±1.

[16] In this work we have used polynomials of 11^{th} degree for each state variable.

- Given that the domain of Chebyshev polynomials is $[-1, 1]$ and that the state variables of our DSGE model are different, we should use some form of linear translation. In order to do that, we compute the collocation points ϵ_i as:

$$\epsilon_i = \frac{\epsilon_{\max} + \epsilon_{\min}}{2} + \frac{\epsilon_{\max} - \epsilon_{\min}}{2} z_\epsilon^i = \frac{\epsilon_{\max} - \epsilon_{\min}}{2}(z_\epsilon^i + 1) + \epsilon_{\min}, \quad i = 1, 2, \ldots, n_\epsilon + 1,$$

  which map $[-1, 1]$ onto $[\epsilon_{\min}, \epsilon_{\max}]$. Similarly, the collocation points b_t are:

$$b_i = \frac{b_{\max} + b_{\min}}{2} + \frac{b_{\max} - b_{\min}}{2} z_b^i = \frac{b_{\max} - b_{\min}}{2}(z_b^i + 1) + b_{\min}, \quad i = 1, 2, \ldots, n_b + 1,$$

  mapping $[-1, 1]$ onto $[b_{\min}, b_{\max}]$. Note that:

$$S = \{(b_i, \epsilon_j) | i = 1, 2, \ldots, n_b + 1, j = 1, 2, \ldots, n_\epsilon + 1\},$$

  are the tensor grids, with $S_1 = (b_1, \epsilon_1), S_2 = (b_1, \epsilon_2), \ldots, S_{N_s} = (b_{n_b+1}, \epsilon_{n_\epsilon+1})$.
- The space of the approximating functions, denoted as Ω, is a matrix of two-dimensional Chebyshev polynomials. More specifically,

$$\Omega(S) = \begin{bmatrix} \Omega(S_1) \\ \Omega(S_2) \\ \vdots \\ \Omega(S_{n_\epsilon+1}) \\ \vdots \\ \Omega(S_{N_s}) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_1))) & T_0(\xi(b_1)T_2(\xi(\epsilon_1))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_1))) \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_2))) & T_0(\xi(b_1)T_2(\xi(\epsilon_2))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_2))) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_{n_\epsilon+1}))) & T_0(\xi(b_1)T_2(\xi(\epsilon_{n_\epsilon+1}))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1}))) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & T_0(\xi(b_{n_b+1})T_1(\xi(\epsilon_{n_\epsilon+1}))) & T_0(\xi(b_{n_b+1})T_2(\xi(\epsilon_{n_\epsilon+1}))) & \cdots & T_{n_b}(\xi(b_{n_b+1})T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1}))) \end{bmatrix}_{N_S \times N_S}$$

  where $\xi(x) = 2\frac{x - x_{\min}}{x_{\max} - x_{\min}} - 1$ is a linear translation mapping the domain of $x \in [x_{\min}, x_{\max}]$ onto $[-1, 1]$, and T defines the Chebyshev polyomials recursively, with $T_0(x) = 1, T_1(x) = x$, and the general $n + 1$-th order polynomial is given by:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x).$$

- Then, at each node $s \in S$, the policy functions $X(s)$ are approximated by $X(s) = \Omega(s)\Theta_X$, where

$$\begin{aligned} \Theta_X \quad = \quad & [\theta^c, \theta^y, \theta^\Pi, \theta^b, \theta^\tau, \theta^G, \theta^R, \theta^{\lambda_1^F}, \theta^{\lambda_2^F}, \theta^{\lambda_3^F}, \theta^{\lambda_4^F}, \theta^{\lambda_5^F}, \theta^{\lambda_6^F}, \theta^{\lambda_7^F}, \theta^{\lambda_8^F}, \theta^{\lambda_9^F}, \\ & \theta^{\lambda_1^M}, \theta^{\lambda_2^M}, \theta^{\lambda_3^M}, \theta^{\lambda_4^M}], \end{aligned}$$

  is a $N_S \times 20$ matrix of coefficients.

2. Formulate an initial guess for the matrix of coefficients, Θ_X^0, and specify some convergence criterion ϵ_{tol}. We set $\epsilon_{\text{tol}} = 10^{-8}$.

3. At each iteration j, the matrix of coefficients is updated Θ_X^j by implementing the following time iteration procedure:

- For each collocation node $s \in S$, compute the possible values of future policy functions $X(s')$ for $k = 1, \ldots, q$. That is:

$$X(s') = \Omega(s')\Theta_X^{j-1}.$$

where q is the number of nodes in a Gauss-Hermite quadrature[17]. Note that:

$$\Omega(s') = T_{j_b}(\xi(b'))T_{j_\epsilon}(xi(\epsilon')),$$

is a $q \times N_s$ matrix, for $j_b = 0, \ldots, n_b$ and $j_\epsilon = 0, \ldots, n_\epsilon$, with $b' = \hat{b}(s; \theta^b)$ and:

$$\ln(\epsilon') = (1 - \rho_\epsilon)\ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon) + z_k\sqrt{2\sigma_\epsilon^2}.$$

The two auxiliary functions can be calculated in a similar way:

$$\begin{aligned} M(s') &\approx \left(\hat{C}(s'; \theta^c)\right)^{-\sigma} \hat{Y}(s'; \theta^y)\frac{\hat{\Pi}(s'; \theta^{\Pi})}{\Pi^*}\left(\frac{\hat{\Pi}(s'; \theta^{\Pi})}{\Pi^*} - 1\right), \\ F(s') &\approx \left(\hat{C}(s'; \theta^c)\right)^{-\sigma}\left(\hat{\Pi}(s'; \theta^{\Pi})\right)^{-1}. \end{aligned}$$

The hat notation indicates that the policy functions are only approximated.

- Let ω_k denote the weights of the Gauss-Hermite quadrature, the expectation terms can be calculated, at each node s, as:

$$\begin{aligned} \mathbb{E}[M(s')] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\hat{C}(s'; \theta^c)\right)^{-\sigma}\hat{Y}(s'; \theta^y)\frac{\hat{\Pi}(s'; \theta^{\Pi})}{\Pi^*}\left(\frac{\hat{\Pi}(s'; \theta^{\Pi})}{\Pi^*} - 1\right) \equiv \bar{M}(s', q), \\ \mathbb{E}[F(s')] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\hat{C}(s'; \theta^c)\right)^{-\sigma}\left(\hat{\Pi}(s'; \theta^{\Pi})\right)^{-1} \equiv \bar{L}(s', q), \\ \mathbb{E}_t\left[\left(\frac{1}{\Pi_{t+1}}\right)\lambda_{4t+1}^F\right] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\frac{1}{\hat{\Pi}(s'; \theta^{\Pi})}\right)\hat{\lambda_4^F}(s'; \theta^{\lambda_4^F}) \equiv \Lambda^F(s', q), \\ \mathbb{E}_t\left[\left(\frac{1}{\Pi_{t+1}}\right)\lambda_{4t+1}^M\right] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\frac{1}{\hat{\Pi}(s'; \theta^{\Pi})}\right)\hat{\lambda_4^M}(s'; \theta^{\lambda_4^M}) \equiv \Lambda^M(s', q), \\ \mathbb{E}_t\left[\left(\frac{1}{\Pi_{t+1}^2}\right)\lambda_{4t+1}^M\lambda_{7t+1}^F\right] &\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\frac{1}{\hat{\Pi}^2(s'; \theta^{\Pi})}\right)\hat{\lambda_4^M}(s'; \theta^{\lambda_4^M})\hat{\lambda_7^F}(s'; \theta^{\lambda_7^F}) \equiv \Upsilon(s', q). \end{aligned}$$

Hence:

$$\mathbb{E}[Z(X(s'))] \approx \mathbb{E}[\hat{Z}(X(s'))] = \begin{bmatrix} \bar{M}(s', q) \\ \bar{L}(s', q) \\ \Lambda^F(s', q) \\ \Lambda^M(s', q) \\ \Upsilon(s', q) \end{bmatrix}.$$

- To calculate the partial derivatives under expectations, $\mathbb{E}[Z_b(X(s'))]$, we obtain the following terms:

$$\begin{aligned} \frac{\partial C_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta_{j_b j_\epsilon}^c}{b_{\max} - b_{\min}}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{C}_b(s'), \\ \frac{\partial Y_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta_{j_b j_\epsilon}^y}{b_{\max} - b_{\min}}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{Y}_b(s'), \\ \frac{\partial \Pi_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta_{j_b j_\epsilon}^{\Pi}}{b_{\max} - b_{\min}}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{\Pi}_b(s'). \end{aligned}$$

[17] In this work we set an 12-point Gauss-Hermite quadrature rule.

Hence, the partial derivatives under expectations can be approximated as:

$$\frac{\partial \mathbb{E}[M(s')]}{\partial b}$$

$$\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k \begin{bmatrix} -\sigma\left(\hat{C}(s';\theta^c)\right)^{-\sigma-1}\hat{Y}(s';\theta^y)\frac{\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*}\left(\frac{\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*}-1\right)\hat{C}_b(s') \\ +\left(\hat{C}(s';\theta^c)\right)^{-\sigma}\frac{\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*}\left(\frac{\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*}-1\right)\hat{Y}_b(s') \\ +\left(\hat{C}(s';\theta^c)\right)^{-\sigma}\frac{\hat{Y}(s';\theta^y)}{\Pi^*}\left(\frac{2\hat{\Pi}(s';\theta^{\Pi})}{\Pi^*}-1\right)\hat{\Pi}_b(s') \end{bmatrix}$$

$$\equiv \hat{M}_b(s',q),$$

$$\frac{\partial \mathbb{E}[F(s')]}{\partial b}$$

$$\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k \begin{bmatrix} -\sigma\left(\hat{C}(s';\theta^c)\right)^{-\sigma-1}\left(\hat{\Pi}(s';\theta^{\Pi})\right)^{-1}\hat{C}_b(s') \\ -\left(\hat{C}(s';\theta^c)\right)^{-\sigma}\left(\hat{\Pi}(s';\theta^{\Pi})\right)^{-2}\hat{\Pi}_b(s') \end{bmatrix}$$

$$\equiv \hat{F}_b(s',q).$$

That is,

$$\mathbb{E}[Z_b(X(s'))] \approx \mathbb{E}[\hat{Z}_b(X(s'))] = \begin{bmatrix} \hat{M}_b(s',q) \\ \hat{F}_b(s',q) \end{bmatrix}.$$

- The procedure to obtain the approximated second order partial derivatives under expectations is the same as the one used for first order derivatives. Therefore, we obtain:

$$\mathbb{E}[Z_{bb}(X(s'))] \approx \mathbb{E}[\hat{Z}_{bb}(X(s'))] = \begin{bmatrix} \hat{M}_{bb}(s',q) \\ \hat{F}_{bb}(s',q) \end{bmatrix}.$$

4. At each collocation node s, solve the following functional for $X(s)$:

$$\Gamma(s, X(s), \mathbb{E}[\hat{Z}(X(s'))], \mathbb{E}[\hat{Z}_b(X(s'))], \mathbb{E}[\hat{Z}_{bb}(X(s'))]) = 0,$$

using some routine to solve systems of nonlinear equations. Once $X(s)$ is obtained, the matrix of coefficients can be calculated as follows:

$$\hat{\Theta}_X^j = \left(\Omega(S)^T\Omega(S)\right)^{-1}\Omega(S)^T X(S).$$

5. Update the approximating coefficients, $\Theta_X^j = \eta\hat{\Theta}_X^j + (1-\eta)\Theta_X^{j-1}$, where $0 \leq \eta \leq 1$ is a dampening parameter used for improving convergence.

6. If $||\Theta_X^j - \Theta_X^{j-1}|| < \epsilon_{\text{tol}}$, the algorithm converged. Otherwise, restart the procedure from Step 3.

2.F EULER EQUATION ERRORS

To assess the accuracy of our numerical solutions, we follow Judd (1992) in performing Euler equation errors (EEE) analyses to address the difference between the exact and the approximated solutions. As it is convention in the literature, we calculate the $\log_{10}|EEE(b_{t-1}, \epsilon_t)|$, for a discussion see Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016). Table 2.F.1 summarizes some basic statistics about the Euler equation errors for the numerical results presented in Subsection 1.6.1, the most common reported statistics for the EEE are the mean of the Euler equation errors, in our case a simple average[18], and the maximum of the EEE. The reported statistics were calculated using an evenly-spaced grid consisting of 40 points for the stock of debt, b_t, and 40 points for the elasticity of substitution between intermediate goods, $\ln(\epsilon_t)$. The results are similar on a finer grid.

Parameter	Statistics of Euler equation errors							
	Nash policy game				Fiscal leadership policy game			
α	$\log_{10}$ max |EEE|	$\log_{10}$ mean|EEE|	std(EEE)	$\log_{10}$ median|EEE|	$\log_{10}$ max |EEE|	$\log_{10}$ mean|EEE|	std(EEE)	$\log_{10}$ median|EEE|
0	-8.01	-8.92	2.44e-09	-10.89	-8.00	-9.04	2.18e-09	-11.16
0.1	-7.93	-9.01	2.21e-09	-10.58	-7.94	-9.08	2.57e-09	-11.22
0.2	-7.92	-9.03	2.11e-09	-10.52	-7.90	-9.06	2.63e-09	-11.22
0.3	-7.92	-9.04	2.03e-09	-10.44	-7.85	-9.04	2.72e-09	-11.24
0.4	-7.92	-9.04	1.96e-09	-10.34	-7.79	-9.01	2.85e-09	-11.28
0.5	-7.91	-9.04	1.92e-09	-10.20	-7.71	-8.98	3.06e-09	-11.31
0.6	-7.90	-9.02	1.96e-09	-10.04	-7.62	-8.93	3.42e-09	-11.29
0.7	-7.77	-8.97	2.22e-09	-9.86	-7.51	-8.87	4.08e-09	-11.30
0.8	-7.41	-8.84	3.42e-09	-9.66	-7.35	-8.76	5.45e-09	-11.17
0.9	-6.93	-8.52	9.26e-09	-9.16	-7.20	-8.58	8.95e-09	-13.07
0.95	-6.84	-8.23	1.56e-08	-9.06	-7.43	-8.74	5.77e-09	-12.95
0.99	-5.26	-6.86	4.07e-07	-7.75	-7.71	-9.21	2.42e-09	-13.20
1	-7.91	-9.04	2.52e-09	-12.07	-8.12	-9.56	9.82e-10	-13.56

Table 2.F.1 – Euler equation errors (EEE).

[18] Alternatively, some researchers use some estimate of the ergodic distribution of state variables.

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Chapter 3

Strategic fiscal and monetary interactions in the Brazilian economy

Abstract

This essay identifies the leadership structure of the game played by monetary and fiscal authorities in the Brazilian economy after the inflation targeting regime in 1999. A stylized small-scale New Keynesian model augmented with fiscal policy is estimated using Bayesian methods. We assume that monetary and fiscal authorities can act strategically under discretion in a non-cooperative setup and compare three different forms of games: (i) simultaneous move; (ii) fiscal leadership; and (iii) monetary leadership. We find strong empirical support for the hypothesis that the Brazilian fiscal authority acts as a Stackelberg leader. The results obtained can shed some light to the improvement of policy design in the Brazilian economy.
Keywords: Monetary and fiscal policies; Strategic interactions; Policy games; Joint stabilization policies.

3.1 INTRODUCTION

The Brazilian economy, in the aftermath of the global financial crisis of 2008, presents a compelling case to study the interactions between fiscal and monetary policies. In a period of only three years, from 2014 to 2016, the country experienced a quick fiscal deterioration, reversing half of the decrease in the public debt obtained previously, positioning itself amongst the most indebted economies in the world (ORAIR; GOBETTI, 2017). As stressed out by Sims (2013) and Bai, Kirsanova, and Leith (2017), the fiscal environment plays a key role in determining the effectiveness of monetary policies. Both level and structure of maturity of debt influence inflation determination. Hence, this recent lack of fiscal discipline combined with a single-minded inflation targeting may have created inflationary pressures, forcing the CPI (consumer price index) to breach double digits by the end of 2015 (LEEPER; LEITH, 2016). Nevertheless, the majority of the literature typically abstracts from the behavior of the fiscal authority, implicitly assuming that the only concern of fiscal policy is debt stabilization.

The empirical literature on monetary-fiscal interactions suggests that fiscal policy does more than just allow automatic stabilizers to operate (AUERBACH, 2002; FAVERO; MONACELLI, 2005). Since the works of Sargent and Wallace (1981) and Leeper (1991), joint stabilization problems had received more attention from macroeconomists. Most of the theoretical literature developed since then studies these matters assuming that policymakers operate with simple rules or in an optimizing framework in a cooperative setup with well-defined social objectives[1].

The adoption of simple rules requires that authorities are able to pre-commit to these rules (CURRIE; LEVINE, 1993) and do not have any clear links to policy objectives. However, Svensson (2003) argues that what we observe as rules are the equilibrium outcome of an optimization problem solved by the authority in charge. The assumption of complete cooperation in an optimizing framework, in its turn, is seldom realistic. It is more likely that authorities act in a strategic manner, since they do not necessarily cooperate on all targets (FRAGETTA; KIRSANOVA, 2010). For example, fiscal policy can assign a higher weight on output stabilization than the monetary authority, or might be concerned with debt stabilization. Monetary policy can be delegated to an inflation conservative Central Bank who puts a heavier weight on inflation than does the fiscal authority and society.

When both authorities are allowed to interact strategically, each policymaker has its own policy objective function and chooses its instrument to minimize the losses. Since authorities can assign different weights to their objectives or pursuit different targets, this non-cooperation can lead to a conflict between monetary and fiscal policymakers. The result of this fight very much depends on how policy is conducted (under commitment or discretion), the choice of policy instruments and, specially, the sequencing of the game played between the two authorities. For example, Dixit and Lambertini (2003) and Blake and Kirsanova (2011) show that delegation of monetary policy to an inflation conservative central bank can make the equilibrium outcome suboptimal when both authorities play a simultaneous move (Nash) game. Furthermore, Kirsanova, Stehn, and Vines (2005) show that the solution of a game where the fiscal authority acts as a Stackelberg leader Pareto dominates the Nash game when there is an excessive weight on output stabilization in fiscal objectives and/or the fiscal authority has a myopic behavior (discounts the future too much).

Ergo, what could be considered as a good policy design for one country may lead to welfare losses in another if the structure of the game played by both authorities is different. To address questions of good policy design, and similar issues, it is necessary to know the way the authorities actually interact with each other.

Motivated by these considerations, the main aim of this essay is to study empirically the strategic interactions between monetary and fiscal policies and identify the leadership regime that prevails in the game played by the two authorities in the Brazilian economy after the adoption of inflation targeting in 1999. We are unaware of any previous empirical research that aims to identify the leadership structure of monetary and fiscal interactions for the Brazilian

[1] In this case, fiscal and monetary policies are both driven by a unique authority. See, e.g., Schmitt-Grohe and Uribe (2004a,b, 2007).

case[2].

In order to do so, we use a small stylized standard dynamic New Keynesian model with monopolistic competition and sticky prices in the goods market, extended to include fiscal policy and nominal government debt, as proposed by Blake and Kirsanova (2011).

Some crucial assumptions about the nature of interactions between monetary and fiscal authorities are made, specifically: (i) both policymakers behave in a non-cooperative strategic manner, with non-identical objectives; and (ii) policy for both authorities is conducted under discretion. Regarding the degree of pre-commitment of authorities, we assume that policymakers act under discretion since the empirical literature shows that commitment policies are strongly dominated by discretion, for both monetary and monetary-fiscal regimes, (BAI; KIRSANOVA, 2015)[3].

As in Fragetta and Kirsanova (2010), the model is estimated using Bayesian methods. Different assumptions about the sequencing of the game lead to different methods of solution and linear feedback rules, which allow us to identify the leadership regime that best describes the behavior of the Brazilian economy in the analyzed period. Three different models are estimated and compared based on the Bayes factor: (i) a simultaneous move (Nash) game; (ii) a game where the fiscal authority acts as a Stackelberg leader; and (iii) a monetary leadership game[4].

Our empirical findings suggest that there is strong evidence in favor of a fiscal leadership in the Brazilian economy after the implementation of the inflation targeting regime. This result is in line with our *prior* belief since fiscal policy is made on a much lower frequency than monetary decisions, besides, under an inflation targeting regime, monetary policy is expected to be credible and clear which allows the fiscal authority to exploit the reaction function of the monetary policymaker. This means that, the Brazilian monetary policymaker, by acting as a follower, can discipline the fiscal policy (LIBICH; STEHLÍK, 2012). Moreover, estimation of policy objectives show that there are no evidences of neither output nor inflation conservatism by part of monetary or fiscal authorities. The major concern of the monetary policy is, as expected, inflation stabilization, while for the fiscal authority is the smoothing of its policy instrument.

This essay is structured in the following way. Section 3.2 outlines the model economy. In Section 3.3 we first discuss the choice of policy and instruments, the microfounded welfare

[2] The empirical literature on monetary and fiscal interactions for the Brazilian economy mainly focuses on identifying the prevailing regime of dominance between policies, see e.g. Fialho and Portugal (2005), Moreira, Souza, and Almeida (2007), Ornellas and Portugal (2011) and Lima, Maka, and Pumar (2012). Most closely related to the present essay, in terms of topics, is the work of Saulo, Rêgo, and Divino (2013). They study strategic interactions between fiscal and monetary policies in a model calibrated to the Brazilian economy after the implementation of the Real Plan. Nonetheless, they do not attempt to estimate the model and identify the leadership structure of the game played by policymakers.

[3] See, e.g., Le Roux and Kirsanova (2013) and Chen, Kirsanova, and Leith (2014, 2017) for, respectively, the UK, Euro area and US. And Palma and Portugal (2011) for monetary policy in Brazil.

[4] Unlike Fragetta and Kirsanova (2010), we do not disregard the monetary leadership regime *a priori* as implausible.

metrics and all policy scenarios of interest. Section 3.4 discusses the econometric methodology, tests the leadership hypotheses, presents the empirical results and, finally, the impulse responses analysis. Section 3.5 concludes.

3.2 THE MODEL ECONOMY

We consider a standard dynamic New Keynesian model with monopolistic competition and sticky prices in the goods market, similar to those presented by Woodford (2003) and Galí (2008), extended to include fiscal policy and nominal government debt as proposed by Blake and Kirsanova (2011). There are two policymakers, a fiscal authority (the government) and a monetary authority (the central bank).

3.2.1 Households

The economy is populated by a representative infinitely-lived household who seeks to maximize the expected utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \zeta \frac{G_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{3.1}$$

subject to a standard sequence of flow budget constraints. $\beta \in (0,1)$ is household's discount factor, σ the inverse of intertemporal elasticity of substitution in consumption, φ is the inverse labour supply elasticity and ζ a relative weight on consumption of public goods. The aggregate variables C_t, G_t and N_t are, respectively, private consumption, government spending and labour supplied.

Maximization of (3.1) is subject to a conventional period budget constraint of the form:

$$P_t C_t + Q_t A_t \leq A_{t-1} + (1-\tau) W_t N_t + T, \tag{3.2}$$

where W_t is the nominal wage, A_t is the nominal portfolio of one-period bonds at a price Q_t, T is a constant lump-sum tax or subsidy and τ is an exogenous income tax rate. Following Dixit and Stiglitz (1977), $C_t = \left[\int_0^1 C_t(i)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} di \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$ is a consumption index with an elasticity of substitution between goods that varies over time according to some stationary stochastic process $\{\varepsilon_t\}$[5]. Finally, $P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon_t} di \right]^{\frac{1}{1-\varepsilon_t}}$ is an aggregate price index.

Log-linearization around the deterministic steady state with zero inflation of first-order conditions and the national income identity allow us to obtain a dynamic IS equation[6,7]:

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma} (i_t - \mathbb{E}_t[\pi_{t+1}]) - \mathbb{E}_t[\Delta \tilde{g}_{t+1}], \tag{3.3}$$

where the endogenous variables are aggregate output y_t, government spending $\tilde{g}_t \equiv (G/C)(g_t - y_t)$, nominal interest rate i_t and inflation rate π_t.

[5] By adopting a stochastic elasticity of substitution we allow for variations in desired price markups, which makes possible to generate shocks to the markups of firms, as in Beetsma and Jensen (2004).

[6] Lowercase letters denote log deviations of a variable from its steady-state value, $x_t = \log X_t - \log X$.

[7] In all derivations we follow Woodford (2003) and Galí (2008) who study closely related models.

3.2.2 Firms and price-setting

There is a continuum of identical monopolistically competitive firms, each of which produces a differentiated good using a production function given by:

$$Y_t(i) = A_t N_t(i)^{1-\alpha},$$

where A_t is an exogenous time-varying level of technology, common to all firms, and $1-\alpha$ is the labour-share.

We assume an AR(1) process for $\{a_t\}$:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \tag{3.4}$$

where $\rho_a \in [0,1]$ and $\{\varepsilon_t^a\}$ is a zero mean white noise process with constant variance σ_a^2.

Price-setting is based on Calvo (1983) contracts where, at each period, only a fraction $1-\theta$ of firms may optimally reset its prices. Hence, a fraction θ of firms keep their prices unchanged. Aggregation across prices yields a New Keynesian Phillips curve[8]:

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa(y_t - y_t^e) - \lambda\sigma\tilde{g}_t + \eta_t^\pi, \tag{3.5}$$

where $\kappa = \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$, $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$ and $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$. η_t^π is a cost-push shock, which reflects variations in desired price markups (BEETSMA; JENSEN, 2004) or any other disturbance to marginal costs. The variable y_t^e denotes output in the efficient allocation (in the absence of nominal rigidities and distortionary cost-push shocks) and is given by:

$$y_t^e = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t.$$

For the cost-push shock, it is assumed that it follows an exogenous AR(1) process:

$$\eta_t^\pi = \rho_\eta \eta_{t-1}^\pi + \varepsilon_t^\pi, \tag{3.6}$$

where $\rho_\eta \in [0,1)$ and $\{\varepsilon_t^\pi\}$ is a white noise process with constant variance σ_π^2.

3.2.3 The government solvency constraint

Following Blake and Kirsanova (2011) and Fragetta and Kirsanova (2010), the government issues one-period nominal debt B_t in order to pay the principle and interests on the existing debt and to fund discrepancies between spending and tax revenues. The log-linearized government solvency constraint, or evolution of debt, can be written as:

$$\tilde{b}_t = \chi i_t + \frac{1}{\beta}\left[\tilde{b}_{t-1} - \chi\pi_t + \frac{\bar{C}}{\bar{Y}}\tilde{g}_t + \left(1 - \frac{\bar{C}}{\bar{Y}} - \tau\right) y_t\right], \tag{3.7}$$

[8] Following Fragetta and Kirsanova (2010), we assume that, in the efficient equilibrium, there are no solvency problems. This yields that, under the assumption of ζ constant, government spending in the efficient allocation is zero, $\tilde{g}_t^e = 0$, see Galí and Monacelli (2005).

where $\tilde{b}_t = \chi \mathcal{B}_t / P_{t-1}$[9], $\mathcal{B}_t$ is nominal debt stock, χ is the steady state debt to GDP ratio, $\bar{C}/\bar{Y}$ is steady state consumption to GDP ratio. A model-consistent value of τ can be obtained, given the steady state debt to GDP and consumption to GDP ratios, by $\tau = \chi(1-\beta) + (1 - \bar{C}/\bar{Y})$.

A private sector rational expectations equilibrium consists of plan $\{y_t, \pi_t, \tilde{b}_t\}$ satisfying the dynamic IS equation, the New Keynesian Phillips curve and the evolution of debt equation, given the policies $\{i_t, \tilde{g}_t\}$, the exogenous processes $\{\eta_t^\pi, a_t\}$, and initial conditions $\tilde{b}_0$.

3.3 POLICY MAKING

3.3.1 Choice of Policy and Instruments

Following current convention, we assume that the monetary policymaker uses the short-term nominal interest rate, i_t, as its instrument of policy. This is, indeed, the case for the Brazilian inflation targeting regime that uses the Selic (Special System of Clearance and Custody) as the primary instrument of monetary policy. The choice of fiscal instrument is more arbitrary, since there is no well-established form of fiscal policy rule (BLAKE; KIRSANOVA, 2011). For the Brazilian case, Castro et al. (2011) argue that changes in government spending take place more often than variations in tax rate, given that a large part of taxes are not allowed to move during the fiscal year. This mainly motivates our choice of government spending, $\tilde{g}_t$, as the fiscal authority's control variable.

In what regards the way agents' expectations are dealt with in the optimization problems, we assume that both policymakers adopt discretionary policies. Under discretion, policymakers can, and are expected to, reoptimize in each period, thus it is a time-consistent and credible policy (FRAGETTA; KIRSANOVA, 2010). The assumption of an optimal discretionary policy seems to be in line with the empirical evidence about the Brazilian monetary authority's preferences. In a paper by Palma and Portugal (2011), the authors show that the behaviour of the Brazilian economy, after the implementation of an inflation targeting regime, is better described by an authority acting under discretion when compared to the commitment optimal plan[10].

When considering fiscal policy, however, there are no obvious reasons to expect that it has been conducted optimally (CHEN; LEEPER; LEITH, 2015). As pointed out by Fragetta and Kirsanova (2010), fiscal authorities are likely to be able to precommit to rules. Hence, by restricting ourselves to discretionary policies we are ruling out such possibility, as well as the time-inconsistent Ramsey policy, where the policymaker is able to credibly commit to a policy plan, and cases where policies are formulated in terms of simple rules[11].

[9] This definition allows us to work with the same model even if $\chi = 0$, see Blake and Kirsanova (2011).

[10] Similar results were found to the US and Euro-area economies by Chen, Kirsanova, and Leith (2017, 2014) respectively, they found that monetary policy is best described as optimal and time-consistent, i.e. discretionary, rather than operating under commitment.

[11] Rules-based policies are time-inconsistent and requires that policymakers are able to commit to the coefficients of rules. Besides, they are also non-strategic policies Fragetta and Kirsanova (2010).

3.3.2 Social Welfare

Following Rotemberg and Woodford (1998), Woodford (2003) and Blake and Kirsanova (2011), we assume that both authorities set their instruments to maximize a quadratic approximation (a second-order Taylor expansion) to the expected aggregate utility function of the representative household given by (3.1). The adoption of a quadratic loss function is quite attractive since that, given a system of linear restrictions, under this linear-quadratic framework we obtain linear policy rules. We show in Appendix 3.B that this approximation implies that a benevolent policymaker minimizes the discounted sum of all future losses:

$$\mathbb{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t W_t^s, \tag{3.8}$$

with an intra-period loss function, W_t^s, given by:

$$W_t^s = \pi_t^2 + \tilde{\Phi}_Y (y_t - y_t^e)^2 + \tilde{\Phi}_G \tilde{g}_t^2 + \mathcal{O}(||\xi||^3), \tag{3.9}$$

where the weights $\tilde{\Phi}$ are functions of the structural parameters of the model[12] and are rescaled in order to normalize the coefficient on inflation to one, $\mathcal{O}(||\xi||^3)$ collects terms of higher order and terms independent of policy.

The expression (3.9) contains a quadratic term in $\tilde{g}$, this is due to the fact that the representative household derives utility from the consumption of public goods (BLAKE; KIRSANOVA, 2011).

3.3.3 Non-cooperative policies under discretion

If both fiscal and monetary authorities are benevolent, they use the same intra-period loss function (3.9) as their objective function to minimize the welfare loss (3.8) subject to the system (3.3)-(3.7).

The micro-founded coefficients of the intra-period loss function derived in the previous subsection place very tight cross-equation restrictions on the model, this can make the estimation problematic, but also, are thought to be implausible (CHEN; LEEPER; LEITH, 2015). Therefore, following Fragetta and Kirsanova (2010) and Chen, Leeper, and Leith (2015), we allow the weights on the objective functions of both policymakers to be freely estimate. In doing so, we assume that both authorities are not benevolent and can act non-cooperatively. By this we mean that there can be distortions between their targets and the social optimal values, such as an inflation conservatism of the monetary authority, or they can pursuit additional policy objectives that are not present in the social optimal objectives, such as instrument smoothing or a debt stabilization target in the fiscal authority objectives.

We assume that the monetary authority objective function is of the form:

$$W_t^M = \pi_t^2 + \Phi_{MY}(y_t - y_t^e)^2 + \Phi_{MG}\tilde{g}_t^2 + \Phi_{\Delta I}(i_t - i_{t-1})^2, \tag{3.10}$$

where the weights attached to the output gap and government spending can be different from the social optimal. The reason for this can be policy delegation to a conservative monetary

[12] See Appendix 3.B.

authority, or simply because the Central Bank cannot compute the social optimal. Besides, there is an additional interest rate smoothing target. This is motivated by the results of Woodford (2003) who shows that, under discretion, it is possible to reduce the 'stabilization bias' when the authority chooses to smooth movements in its instrument.

For the fiscal authority, our preferred specification, following Fragetta and Kirsanova (2010), is given by:

$$W_t^F = \pi_t^2 + \Phi_{FY}(y_t - y_t^e)^2 + \Phi_{FG}\tilde{g}_t^2 + \Phi_{\Delta G}(\tilde{g}_t - \tilde{g}_{t-1})^2 + \Phi_{FB}\tilde{b}_t^2, \tag{3.11}$$

where, as in the monetary objective, weights on output gap and government spending can deviate from the social optimal. Since fiscal policy is relatively inflexible - current period spending decisions are often based on past period allocations - the fiscal authority also have an additional government spending smoothing target (FRAGETTA; KIRSANOVA, 2010). Finally, we assume that the fiscal authority pursues a target for the stabilization of the public sector debt-to-GDP ratio, $\tilde{b}_t$, which is in accordance to the fiscal regime in place in Brazil since 1999 (CASTRO et al., 2011).

3.3.4 Strategic interactions

We allow the monetary and fiscal authorities to play strategic games with each other[13]. Specifically, we assume that the optimal problem outlined in the previous subsection is solved for two policymakers under three different structures of strategic interactions: (i) when the fiscal authority acts as a Stackelberg leader (fiscal leadership) in the policy game and chooses the best point in the monetary policymaker's reaction function, (ii) the other way around, where the monetary authority acts as the leader (monetary leadership), or (ii) a simultaneous moves regime (Nash game).

For both the simultaneous move and the leader-follower case, we can describe the evolution of the economy given by equations (3.3)-(3.7) by the following linear system:

$$A_0 z_t = A_1 z_{t-1} + A_2 \mathbb{E}_t z_{t+1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_4 \mathbb{E}_t x_{t+1} + \tilde{A}_t \mathbb{E}_{t+1} \tilde{x}_{t+1} + A_5 v_t, \tag{3.12}$$

where z_t is a vector of endogenous variables, x_t and $\tilde{x}_t$ are vectors of policy instruments for each policymaker and v_t a vector of stochastic disturbances.

The quadratic loss functions (3.10) and (3.11) can be rewritten as:

$$\mathbb{W}_1 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(y_t' W y_t + x_t' Q_1 x_t + \tilde{x}_t' Q_2 \tilde{x}_t\right), \tag{3.13}$$

$$\mathbb{W}_2 = \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left(y_t' \tilde{W} y_t + x_t' \tilde{Q}_1 x_t + \tilde{x}_t' \tilde{Q}_2 \tilde{x}_t\right). \tag{3.14}$$

In the Appendix 3.A we outline the solution procedure for optimal discretionary policies in a linear-quadratic rational expectations framework like (3.12)-(3.14), for both simultaneous moves and leader-follower cases. It is important to note that different cases of strategic interaction require different solution procedures and lead to different solutions for the linear-quadratic

[13] But not with their future selves.

optimization problem described. Since the linear policy reaction for each policymaker is different between the Nash and the Stackelberg games[14], we can identify the structure of the game played by the monetary and fiscal authorities in the Brazilian economy by performing a model comparison based on the marginal data density for each model.

3.4 ESTIMATION

Following An and Schorfheide (2007) and Fragetta and Kirsanova (2010), the model is estimated using a system-based Bayesian approach[15]. The Bayesian estimation allows the use of additional information in the estimation process in the form of prior distributions. This prior information can add curvature to a likelihood function that can be flat along the dimension of weakly identifiable parameters. Moreover, as stated in Castro et al. (2011), the estimation of DSGE models for the Brazilian economy by classical full information methods can be even more difficult due to short span of the data sample.

3.4.1 Data Description

In order to identify the structure of the game played by monetary and fiscal authorities in the Brazilian economy, we estimated the model described on the previous subsections using four Brazilian data series as observable variables spanning from 1999.Q3 to 2019.Q1: real GDP, nominal interest rate, inflation and government spending to GDP ratio (see Table 3.1).

Table 3.1 – Description of the data series used in estimation

Variable	Description	Source
Y_t	Gross Domestic Product	IBGE
G_t	Final consumption - Government	IBGE
i_t	Nominal interest rate - Selic (% per quarter)	BCB
π_t	CPI inflation: IPCA (% per quarter)	IBGE

Acronyms: IBGE - Brazilian Institute of Geography and Statistics;
BCB - Central Bank of Brazil.

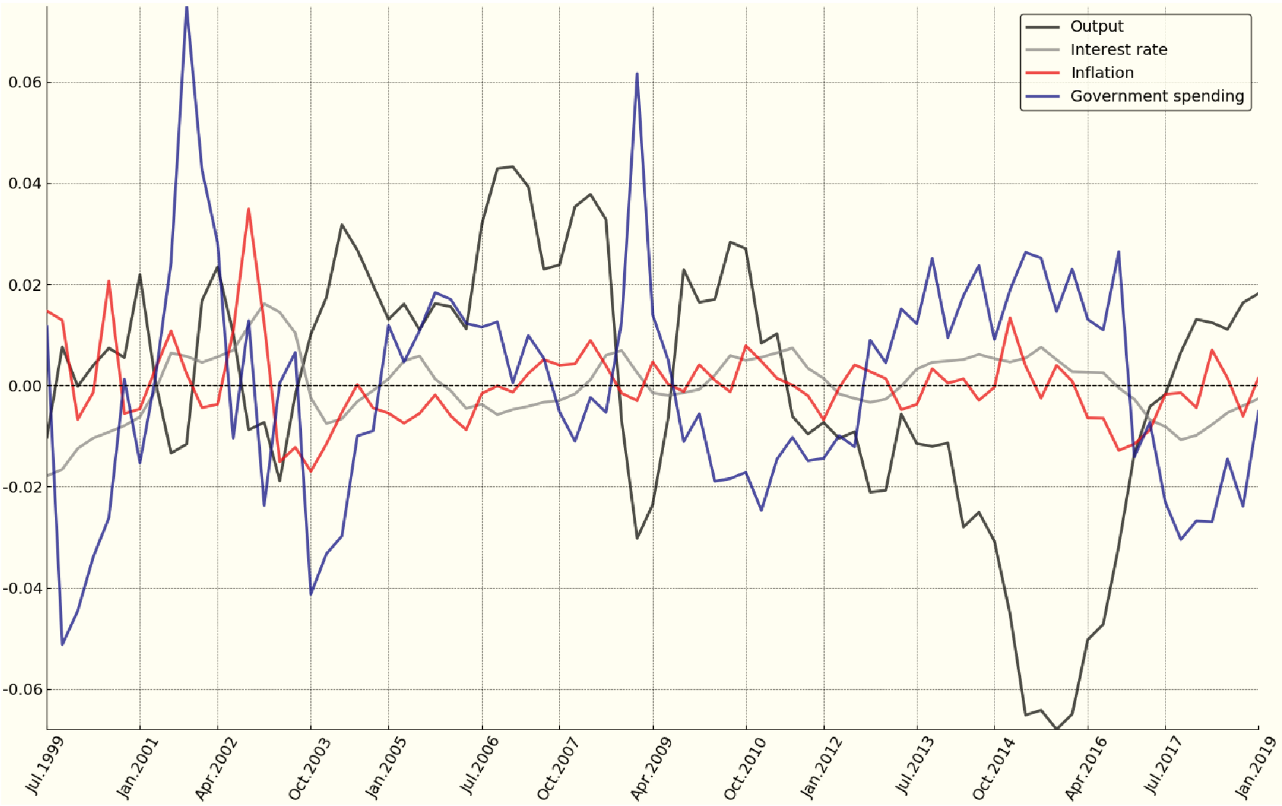
We have chosen to use data for the period after the adoption of the inflation targeting regime in Brazil, which was formally adopted on June, 1999. All data are at quarterly frequencies, seasonally adjusted, detrended and demeaned prior to estimation. Following Stock and Watson (1999), data were detrended using a one-sided version of the Hodrick-Prescott (1997) filter[16]. Figure 3.1 depicts the resulting data series for the period analyzed.

[14] As shown in Appendix 3.A, under a Stackelberg game the linear feedback function of the follower depends on the instrument of the leader. This is not true for the simultaneous moves game, where the reaction functions for both policymakers depend on the same set of variables.

[15] For Bayesian analysis of DSGE models see, for instance, An and Schorfheide (2007), Herbst and Schorfheide (2016) and Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016).

[16] The smoothing parameter λ was set to 1600. To obtain the filtered series, we used data from 1996.Q1 to

Figure 3.1 – Time Series.



3.4.2 Calibrated Parameters

To conduct our empirical analysis, we calibrate eight parameters. Table 3.2 presents the list of the calibrated parameters and their corresponding values to be used in the estimation of the model.

Based on the sample average calculated from the National Accounts, we set the government spending to GDP ratio in the steady state, $\bar{G}$, to 0.1919. The same procedure was used to obtain the debt to GDP ratio, χ, whose value matches the sample average of the total net public sector debt, obtained from the Brazilian Institute of Geography and Statistics (IBGE).

We calibrate three parameter values following the work of Castro et al. (2011) for the Brazilian economy: (i) the discount factor, β, is set to 0.989, which implies an annual steady state interest rate of approximately 4.4%; (ii) the mean of the stochastic elasticity of substitution, ε, is 11 which implies a 10% price markup; and (iii) the parameter α related to the labor-share in the production function is fixed in 0.448.

The autoregressive coefficient of the markup shock, ρ_η, is set to zero based on Fragetta and Kirsanova (2010).

Finally, from the set of calibrated parameters defined previously, we can calculate the model consistent values for the consumption to GDP ratio in the steady state, $\bar{C}$, and the constant income tax rate, τ.

2019.Q1 and discarded the observations from 1996.Q1 to 1999.Q2.

Table 3.2 – Calibrated parameters

Parameter	Value	Description	Source
β	0.989	Discount factor	Castro et al. (2011)
$\bar{G}$	0.1919	Spending/GDP in steady-state	Sample average
$\bar{C}$	0.8081	Consumption/GDP in steady-state	Implied by the model
χ	0.3776	Steady-state debt-to-GDP ratio	Sample average
τ	0.1961	Constant income tax rate	Implied by the model
ρ_η	0.00	AR(1) coefficient of markup shock	Fragetta and Kirsanova (2010)
α	0.448	Related to the labor-share	Castro et al. (2011)
ϵ	11.00	Mean of markup shock	Castro et al. (2011)

3.4.3 Prior Distributions

The parameters on which we will perform the estimation can be divided in three major groups: (i) structural parameters; (ii) policy objectives coefficients - for both monetary and fiscal authorities; and (iii) shocks-related parameters - persistence and standard deviation of innovations. Table 3.3 presents the prior distribution for each parameter for both Stackelberg and simultaneous move cases. Whenever possible, we choose priors that are widely used in the literature on estimation of New Keynesian models and avoided using tight priors, since prior information is associated with a large degree of uncertainty.

Prior distributions of structural parameters are set following the work of Smets and Wouters (2007). Given its compact support, a Beta distribution with mean 0.5 and standard deviation of 0.1 is chosen for the Calvo parameter, θ. Normal distributions with, respectively, means of 1.50 and 2.00 are set for the two preference parameters estimated, the inverse of the intertemporal elasticity of substitution, σ, and the labout disutility parameter, φ.

In our preferred specification of policy objectives (3.10)-(3.11) there are two types of coefficients, the ones associated with the social optimal objective function and additional targets that each authority can pursuit. For the former, the coefficients related to output and government spending stabilization, we set the means of the prior distributions to match the implied theoretical values (given the priors for the structural parameters). For these coefficients we chose to use Beta distributions with loose priors that, given its compact support, allow us to exclude negative and unrealistically high values of weights. For the latter, we set a Beta distribution with mean 0.50 and standard deviation of 0.20 for the debt stabilization target for the fiscal authority and Gamma distributions with mean 0.70 for the smoothing instruments of both authorities[17].

Finally, we follow Smets and Wouters (2007) to set the priors for the parameters related to the innovation processes. We assume a Beta distribution with mean 0.50 and standard deviation of 0.20 for the autoregressive coefficient of the technology process, and Inverse Gamma

[17] While Fragetta and Kirsanova (2010) assume Beta distributions for the smoothing coefficients, we chose Gamma distributions, given that those parameters have only a lower bound.

distributions with mean 0.10 and 2 degrees of freedom for the standard deviations of shocks.

Table 3.3 – Prior distribution

	Domain	**Density**	**Mean**	**Std. Dev.**
θ	$[0,1]$	Beta	0.50	0.10
σ	$\mathbb{R}_+$	Normal	1.50	0.37
φ	$\mathbb{R}_+$	Normal	2.00	0.50
Φ_{MY}	$[0,1]$	Beta	0.0317	0.020
Φ_{MG}	$[0,1]$	Beta	0.0292	0.015
$\Phi_{\Delta I}$	$\mathbb{R}_+$	Gamma	0.70	0.35
Φ_{FY}	$[0,1]$	Beta	0.0317	0.020
Φ_{FG}	$[0,1]$	Beta	0.0292	0.015
$\Phi_{\Delta G}$	$\mathbb{R}_+$	Gamma	0.70	0.35
Φ_{FB}	$[0,1]$	Beta	0.50	0.20
ρ_a	$[0,1]$	Beta	0.50	0.20
σ_a	$\mathbb{R}_+$	InvGamma	0.10	2
σ_η	$\mathbb{R}_+$	InvGamma	0.10	2
σ_r	$\mathbb{R}_+$	InvGamma	0.10	2
σ_g	$\mathbb{R}_+$	InvGamma	0.10	2

3.4.4 Model Comparison

Table 3.4 presents the posterior odds for cases of simultaneous moves, fiscal leadership or monetary leadership regimes. We treat each regime as equally probable by setting prior probabilities to one. Based on the Bayes factor, we found strong evidence[18] in the estimation results that a fiscal leadership regime is dominant in the Brazilian economy for the period considered.

The likelihood that the data were generated under a simultaneous move regime is 0.0356 when compared to the case where the fiscal authority acts as a Stackelberg leader. For the monetary leadership regime, the Bayes factor is even smaller, which reflects the fact that monetary decisions take place in a much higher frequency than fiscal decisions (FRAGETTA; KIRSANOVA, 2010), which make it less likely for the Central Bank to exploit the reaction function of the fiscal policymaker.

Table 3.4 – Model comparison

Model	**Log marginal data density**	**Bayes Factor**
Fiscal Leadership	712.261	1.0
Simultaneous Move	708.926	0.0356
Monetary Leadership	707.807	0.0116

[18] See Raftery (1995) for grades of evidence in Bayesian model selection.

3.4.5 Posterior Estimates

The posterior parameter estimates were computed by the Metropolis-Hastings sampling algorithm based on 100,000 draws[19]. According to the Geweke (1992) univariate diagnostic - see Appendix 3.C - a sample of 100,000 was sufficient to ensure convergence of the Metropolis-Hastings algorithm. Table 3.5 reports the posterior mean, standard deviation and the credible interval (5^{th} and 95^{th} percentiles) for the estimated parameters of each one of the models: simultaneous move (Nash), fiscal leadership (FL) and monetary leadership (ML). Figure 3.2 depicts the priors (dashed blue lines) and posteriors (shaded purple areas) of structural and policy objectives parameters for the dominant regime of fiscal leadership in the Brazilian economy[20].

Table 3.5 – Empirical posterior estimates

Parameter	Prior Distribution		Posterior Distribution												
			Simultaneous Move				Fiscal Leadership				Monetary Leadership				
	Mean	Std.	Mean	Std.	5%	95%	Mean	Std.	5%	95%	Mean	Std.	5%	95%
θ	0.5	0.1	0.6348	0.0722	0.5090	0.7464	0.6382	0.0662	0.5217	0.7400	0.6178	0.0737	0.4926	0.7353
σ	1.5	0.37	1.7087	0.3161	1.2102	2.2505	1.5878	0.2948	1.1238	2.0987	1.6650	0.3347	1.1518	2.2607
φ	2.0	0.5	1.6189	0.5613	0.6832	2.5217	1.7203	0.5284	0.8335	2.5730	1.6366	0.5543	0.7097	2.5384
Φ_{MY}	0.0317	0.02	0.0104	0.0058	0.0028	0.0213	0.0086	0.0046	0.0025	0.0171	0.0106	0.0062	0.0027	0.0226
Φ_{MG}	0.0292	0.015	0.0251	0.0129	0.0082	0.0493	0.0242	0.0123	0.0080	0.0476	0.0188	0.0096	0.0060	0.0371
$\Phi_{\Delta I}$	0.7	0.35	0.3638	0.1578	0.1589	0.6699	0.3641	0.1570	0.1670	0.6625	0.3839	0.1749	0.1682	0.6988
Φ_{FY}	0.0317	0.02	0.0311	0.0190	0.0075	0.0672	0.0332	0.0201	0.0075	0.0717	0.0315	0.0191	0.0074	0.0672
Φ_{FG}	0.0292	0.015	0.0285	0.0149	0.0093	0.0569	0.0270	0.0139	0.0088	0.0536	0.0279	0.0143	0.0092	0.0551
$\Phi_{\Delta G}$	0.7	0.35	1.1058	0.4248	0.5141	1.8824	1.2493	0.4525	0.6116	2.0779	1.0924	0.4203	0.5201	1.8768
Φ_{FB}	0.5	0.2	0.0173	0.0146	0.0015	0.0456	0.0106	0.0083	0.0015	0.0277	0.0111	0.0095	0.0009	0.0326
ρ_a	0.5	0.2	0.8591	0.0384	0.7929	0.9202	0.8450	0.0400	0.7780	0.9079	0.8659	0.0397	0.7979	0.9288
σ_a	0.1	2	0.0357	0.0067	0.0272	0.0477	0.0347	0.0058	0.0268	0.0455	0.0337	0.0057	0.0262	0.0445
σ_η	0.1	2	0.0194	0.0017	0.0169	0.0224	0.0196	0.0017	0.0170	0.0225	0.0195	0.0017	0.0169	0.0225
σ_r	0.1	2	0.0168	0.0014	0.0148	0.0192	0.0168	0.0013	0.0147	0.0192	0.0167	0.0014	0.0147	0.0191
σ_g	0.1	2	0.0172	0.0014	0.0151	0.0196	0.0172	0.0014	0.0151	0.0196	0.0171	0.0013	0.0150	0.0194

Overall, there are no considerable differences between the estimated parameters in each one of the models. For large part of the parameters, observed data was informative in the estimation process[21]. Aside from the output stabilization target in the fiscal authority objective, the posterior distributions are more concentrated than the priors or are shifted to different points on the support (Figure 3.2).

Estimation of deep parameters of the models fall within plausible ranges. The estimated means of Calvo parameter (a measure of price stickiness), θ, implies that prices remain fixed, on average, for approximately three quarters indicating that price changes are as frequent as in most developed countries[22]. Moreover, its posterior distribution is tighter and shifted to the right when compared to the prior, which reflects the fact that observed data is informative along this dimension. The intertemporal elasticity of substitution in consumption, σ, obtained in the estimation procedure does not contradict the findings of Castro et al. (2011) for the Brazilian

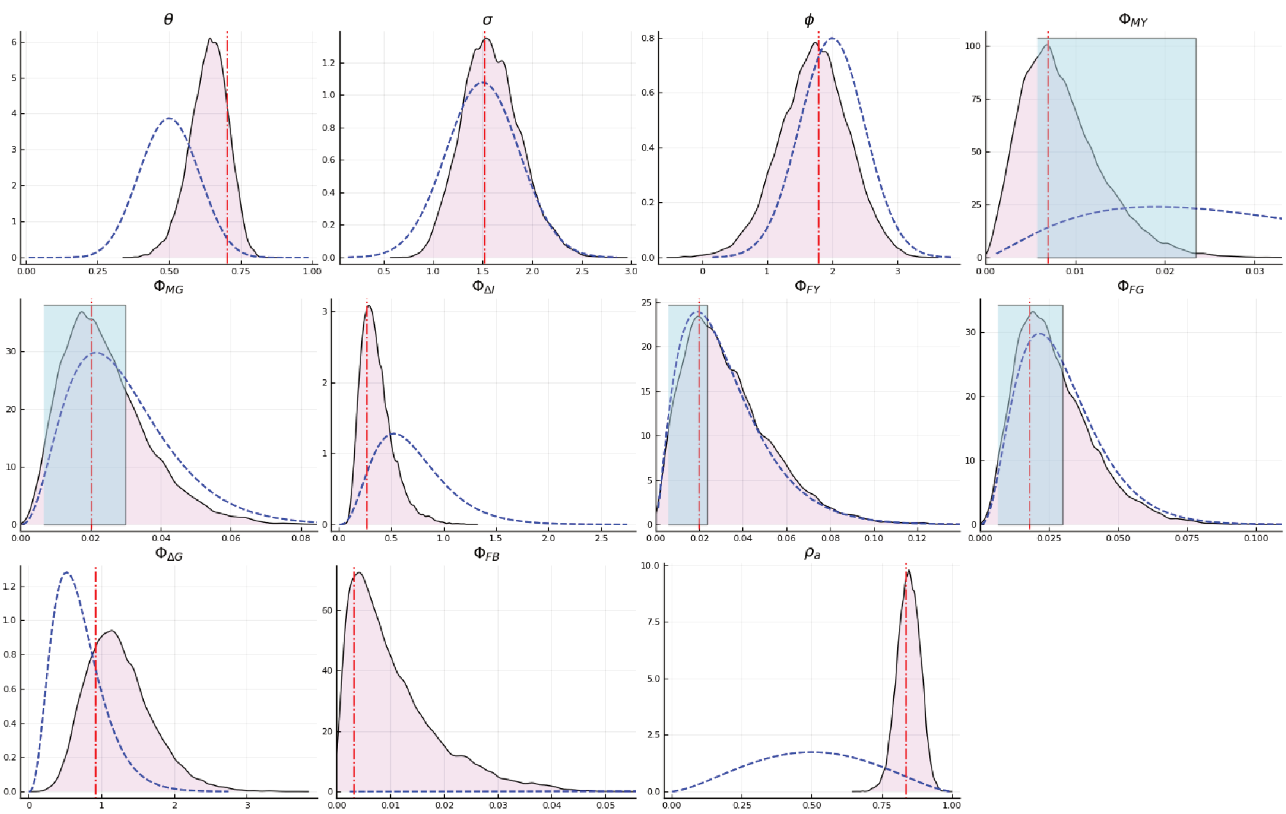
[19] The acceptance rate was approximately 30% on average for each one of the estimated models.

[20] Dash-dotted red lines depict the estimated posterior mode, obtained by the direct maximization of the log of the posterior distribution with respect to the parameters.

[21] Çebi (2012) argues that, in this kind of DSGE models, it is a common finding that the means of prior and posterior distributions are similar.

[22] Estimating an equally stylized model, Fragetta and Kirsanova (2010) found that prices are kept constant for between 3 quarters to one year for the US, UK and Sweden. Similarly, Smets and Wouters (2007) found an

Figure 3.2 – Prior and posterior distributions



economy, given that their results for this parameter fall within the 90% credible interval we obtain. Furthermore, estimates of φ are in line with the results of Fragetta and Kirsanova (2010) for the UK, US and Sweden, and imply an intermediate value of elasticity of labour supply.

Concerning the autoregressive process, there is a high degree of persistence on the technology shock, although the value of ρ_a is smaller than the one obtained by Castro et al. (2011).

Estimates of policy objective parameters suggest, in the period analyzed, that the preferences of both the Brazilian fiscal and monetary authorities are stable between the three regimes considered. As one would expect, the Central Bank of Brazil is more concerned with inflation stabilization than with other targets, once it puts a heavier weight on this objective. The other targets that the monetary authority pursuit in our specification are, in order of importance to the monetary authority, interest rate smoothing, government spending and output stabilization. The fiscal authority, in its turn, gives more attention to the smoothing of the fiscal policy instrument, followed by, respectively, inflation, output, government spending and debt stabilization. This reflects the fact that current decisions of fiscal policy are, indeed, based on past period allocations.

A closer inspection of Figure 3.2 makes evident that there is no evidence indicating neither inflation nor output conservatism by the monetary authority. In this figure, shaded rectangular areas show the theoretical distributions for the optimal social weights of policy parameters[23] that are based on the estimates obtained for the structural parameters of the

average duration of about 3 quarters of price contracts for the US economy.

[23] Namely, parameters associated with output and government spending stabilization for both authorities.

model (with a 90% credible interval). Given that the posterior distributions of Φ_{MY} and Φ_{MG} overlap with the respective theoretical distributions, it is not possible to assert that those coefficients are different from the social optimal. Moreover, we do not find a substantial degree of interest rate smoothing by part of the Central Bank of Brazil[24]. Palma and Portugal (2011) found that the weights on output stabilization, Φ_{MY}, and interest rate smoothing, $\Phi_{\Delta I}$, when the Central Bank of Brazil acts under discretion are, respectively, 0.01 and 0.2, which does not contradict our findings (see Table 3.5).

Just as for the monetary authority, estimates of fiscal policy preference parameters indicate that the values for Φ_{FY} and Φ_{FG} are not different from the social optimal weights. Although it is important to note that the marginal likelihood for output stabilization in the fiscal objectives, Φ_{FY}, is flat so that its posterior distribution agrees with the prior, while the posterior for Φ_{FG} is slightly more concentrated than the prior. Furthermore, we found a substantial weight on the government spending smoothing target, $\Phi_{\Delta G}$, being the major concern of the Brazilian fiscal authority. While the posterior of $\Phi_{\Delta G}$ is less concentrated than its respective prior, the mean of the distribution is shifted to the right on the support. Finally, there is little evidence of debt stabilization given that the estimated value of the posterior mean is small and the distribution is shifted to the lower end of the support, being very close to the origin.

Summarizing our results, we found that the model which best describes the behavior of the Brazilian economy spanning from 1999.Q3 to 2019.Q1 is one of a fiscal leadership regime, where the fiscal authority acts as a Stackelber leader and chooses the best point on the reaction function of the central bank. There are no evidences indicating neither output nor inflation conservatism by part of monetary or fiscal authorities. As expected, the major concern of the monetary authority is inflation stabilization, while for the fiscal authority is the smoothing of the fiscal instrument. And the estimated weight of debt stabilization on the fiscal objectives is small, with the posterior distribution being shifted towards the origin.

3.4.6 Impulse Response Analysis

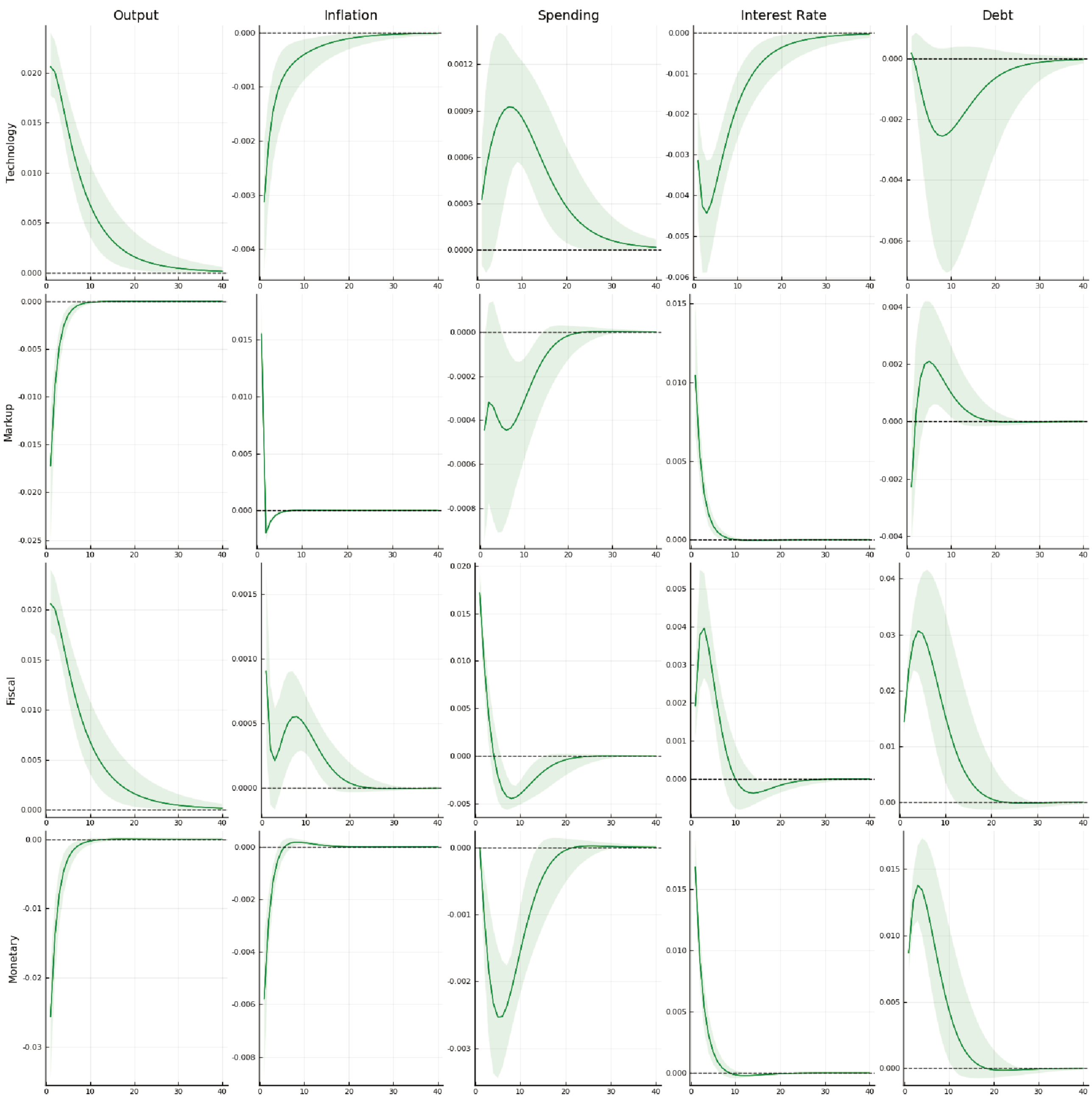
The dynamic properties of the model can be investigated by an impulse response analysis. Figure 3.3 displays the impulse response functions for the fiscal leadership regime[25] in terms of mean responses of the observable variables along with the unobservable debt accumulation and a 90% confidence interval.

Following a positive productivity shock, the efficient level of output increases and the efficient interest rate decreases. A higher efficient level of output reduces the marginal costs of the firms leading to a fall in inflation. At the time of the impact, monetary policy does not respond and the real interest rate initially rises. Afterwards, the reduction of the nominal interest rate causes the real interest rate to fall. This stimulates the economy, raising the output.

[24] Fragetta and Kirsanova (2010) found that the posterior means of this coefficient of 1.5, 0.8 and 0.9 for the UK, Sweden and US, respectively.

[25] Impulse responses for the other regimes are very much alike.

Figure 3.3 – Impulse response analysis



Moreover, debt reduces following a higher output and a decrease on nominal interest rate. The fall of inflation and debt stock make the government responds with a small expansionary fiscal policy.

A positive markup shock raises inflation. In order to stabilize inflation, the monetary authority increases the nominal interest rate, which reduces output. The effect of a higher inflation on debt offsets the effect of a higher interest rate, which leads to an initial fall of the debt stock. Since the incentive to increase government spending in order to stabilize output practically offsets the opposite incentive to keep debt under control, fiscal policy does not change much (the fall of the fiscal instrument is very small).

An expansionary fiscal policy raises government spending and, consequently, output. Although an increase in government spending has a negative impact over inflation, the positive effect of a higher output prevails, increasing inflation. In order to keep inflation under control, the central bank raises the nominal interest rate. The increases in government spending and nominal interest rate raises the debt stock. Following this expansionary fiscal policy, the government subsequently reduces the government spending in order to stabilize debt. This contractionary policy is kept for long enough to bring output and inflation to the steady state level.

Finally, a positive shock to nominal interest rate lowers inflation and output. This higher interest rate leads to an increase on debt accumulation. In order to stabilize the debt stock, the fiscal authority reduces government spending. Hence, a tight monetary policy is followed by a tight fiscal policy.

3.5 CONCLUSION

This essay addresses empirically some questions of joint stabilization problems. A stylized small-scale New Keynesian model, extended to include fiscal policy and nominal government debt, is specified and estimated through Bayesian methods in order to identify the leadership structure of the game played by monetary and fiscal authorities in the Brazilian economy after the implementation of inflation targeting regime in 1999. Under the assumption that monetary and fiscal authorities can act non-cooperatively under discretion, Bayesian model comparison provides a strong empirical support for the hypothesis that the Brazilian monetary policymaker disciplines the fiscal authority. Put differently, a regime of fiscal leadership fits the data better than a simultaneous move game or the monetary leadership case.

Empirical estimates of policy objectives show that there are no evidence of inflation or output conservatism in the objective functions of both monetary and fiscal authorities. Moreover, as expected, we find that the greater concern of the Brazilian Central Bank is inflation stabilization and that it assigns a very small weight to output stabilization. The fiscal authority, in its turn, operates with considerable smoothing of its instrument and shows little concern in stabilizing debt.

The analyses presented in this essay can shed some light to the improvement of policy design. The identification of the structure of the game played by the Brazilian monetary and fiscal authorities is important since it can help to mitigate the welfare losses caused by a potentially strategic interaction between them.

The behavior of the Brazilian economy over the last decades exhibit some events of possible conflicts between the monetary and fiscal authorities, as the hyperinflationary period studied by Loyo (1999) and the uprising inflation at the end of 2015. This suggests that the monetary policy not always works as a fiscal disciplinary tool. Thus, a fixed regime model, as considered in this essay, fails to capture these changes between conflict and cooperation in the conduct of policies. A possible extension to this work is to consider a model where optimal fiscal and monetary policies can switch over time, as done by Chen, Leeper, and Leith (2015).

Further developments of this work include extensions of the model considered here to capture some characteristics of the Brazilian economy as the presence of administered prices, financially constrained households and an open economy.

Appendices

3.A MODEL SOLUTION PROCEDURE

In this appendix we outline the solution procedure for optimal discretionary policies in a linear-quadratic (LQ) rational expectations framework. The methods employed here describe the solution for both the simultaneous move (Nash game) and the leader-follower (Stackelberg game) cases, and are taken from Dennis and Ilbas (2016). Unlike the framework developed by Blake and Kirsanova (2011), in Dennis and Ilbas (2016) the LQ optimal policy problem is put in a generalized structural form. By avoiding the matrix partitioning required by state-space methods, this procedure can be easily applied to larger models.

For both the simultaneous move and the leader-follower cases, the evolution of the economy can be described by the following linear system:

$$A_0 y_t = A_1 y_{t-1} + A_2 \mathbb{E}_t y_{t+1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_4 \mathbb{E}_t x_{t+1} + \tilde{A}_4 \mathbb{E}_t \tilde{x}_{t+1} + A_5 v_t, \tag{3.15}$$

where y_t is a vector of endogenous variables, x_t is the vector of policy instruments for one policymaker, $\tilde{x}_t$ is the vector of policy instruments of the other policymaker, $v_t \sim i.i.d.[0, \Omega]$ is a vector of stochastic disturbances, and matrices $A_0 - A_5$ contain the model's parameters[26].

The quadratic loss functions for the two policymakers are assumed to be given by:

$$\mathbb{W}_1 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(y_t' W y_t + x_t' Q_1 x_t + \tilde{x}_t' Q_2 \tilde{x}_t \right), \tag{3.16}$$

$$\mathbb{W}_2 = \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left(y_t' \tilde{W} y_t + x_t' \tilde{Q}_1 x_t + \tilde{x}_t' \tilde{Q}_2 \tilde{x}_t \right), \tag{3.17}$$

where matrices W and Q contain the policy preferences of the policymakers and are symmetric and positive semi-definite. Both authorities are said to be in a cooperative setup if the following conditions are satisfied: $\beta = \tilde{\beta}$, $W = \tilde{W}$, $Q_1 = \tilde{Q}_1$ and $Q_2 = \tilde{Q}_2$. Equations (3.16) and (3.17) make clear that the objective function for each authority is allowed to depend on both policymaker's policy instruments, not just their own.

3.A.1 Simultaneous move

As discussed in Dennis (2007), quadratic objective functions like (3.16) and (3.17), with linear constraints, lead to linear decision rules. We assume that a stationary solution to the

26 A_0 is assumed to be a non-singular matrix.

optimization problems exists and is given by:

$$\begin{aligned} y_t &= H_1 y_{t-1} + H_2 v_t, & (3.18)\\ x_t &= F_1 y_{t-1} + F_2 v_t, & (3.19)\\ \tilde{x}_t &= \tilde{F}_1 y_{t-1} + \tilde{F}_2 v_t. & (3.20) \end{aligned}$$

Substituting this set of equations into the linear constraints, we can rewrite equation (3.15) as:

$$D y_t = A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t, \tag{3.21}$$

where

$$D \equiv A_0 - A_2 H_1 - A_4 F_1 - \tilde{A}_t \tilde{F}_1. \tag{3.22}$$

Using the properties of convergent geometric series[27], we can rewrite the loss functions as follows[28]:

$$\begin{aligned} \mathbb{W}_1 &= (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t)' D'^{-1} P D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t)\\ &\quad + x_t' Q_1 x_t + \tilde{x}_t' Q_2 \tilde{x}_t + \frac{\beta}{1-\beta} tr[(F_2' Q_1 F_2 + \tilde{F}_2' Q_2 \tilde{F}_2 + H_2 P H_2)\Omega], & (3.23)\\ \mathbb{W}_2 &= (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t)' D'^{-1} \tilde{P} D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t)\\ &\quad + x_t' \tilde{Q}_1 x_t + \tilde{x}_t' \tilde{Q}_2 \tilde{x}_t + \frac{\tilde{\beta}}{1-\tilde{\beta}} tr[(F_2' \tilde{Q}_1 F_2 + \tilde{F}_2' \tilde{Q}_2 \tilde{F}_2 + H_2 \tilde{P} H_2)\Omega], & (3.24) \end{aligned}$$

where

$$\begin{aligned} P &= W + \beta(F_1' Q_1 F_1 + \tilde{F}_1' Q_2 \tilde{F}_1 + H_1' P H_1), & (3.25)\\ \tilde{P} &= \tilde{W} + \tilde{\beta}(F_1' \tilde{Q}_1 F_1 + \tilde{F}_1' \tilde{Q}_2 \tilde{F}_1 + H_1' \tilde{P} H_1). & (3.26) \end{aligned}$$

Differentiating the objective functions with respect to the vector of instrument variables for each policymaker gives us the following set of first order conditions:

$$\begin{aligned} \frac{\partial \mathbb{W}_1}{\partial x_t} &= A_3' D'^{-1} P D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t) + Q_1 x_t = 0,\\ \frac{\partial \mathbb{W}_2}{\partial \tilde{x}_t} &= \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t) + \tilde{Q}_2 \tilde{x}_t = 0. \end{aligned}$$

Since D, P and $\tilde{P}$ are implicit functions of the matrices in (3.18)-(3.20), the simultaneous move solution can be obtained as a fixed point in the following numerical procedure:

1. Initialize $H_1, H_2, F_1, F_2, \tilde{F}_1$ and $\tilde{F}_2$.

2. Compute D using equation (3.22), P using equation (3.25) and $\tilde{P}$ using equation (3.26).

[27] For further details, see the Appendix in Dennis (2007).

[28] This transformation requires D to have full rank, which is satisfied since A_0 is a non-singular matrix.

3. Update $H_1, H_2, F_1, F_2, \tilde{F}_1$ and $\tilde{F}_2$ according to

$$\begin{aligned}
F_1 &= -(Q_1 + A_3' D'^{-1} P D^{-1} A_3)^{-1} A_3' D'^{-1} P D^{-1} (A_1 + \tilde{A}_3 \tilde{F}_1), \\
F_2 &= -(Q_1 + A_3' D'^{-1} P D^{-1} A_3)^{-1} A_3' D'^{-1} P D^{-1} (A_5 + \tilde{A}_3 \tilde{F}_2), \\
\tilde{F}_1 &= -(\tilde{Q}_2 + \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} \tilde{A}_3)^{-1} \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} (A_1 + A_3 F_1), \\
\tilde{F}_2 &= -(\tilde{Q}_2 + \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} \tilde{A}_3)^{-1} \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} (A_5 + A_3 F_2), \\
H_1 &= D^{-1}(A_1 + A_3 F_1 + \tilde{A}_3 \tilde{F}_1), \\
H_2 &= D^{-1}(A_5 + A_3 F_2 + \tilde{A}_3 \tilde{F}_2).
\end{aligned}$$

4. Iterate over steps 2-4 until convergence.

3.A.2 Leader-follower

Without loss of generality, let us designate policymaker 1 as the leader and policymaker 2 as the follower. Besides, we assume that at each time t, the policymaker who acts as a follower observes the current decision rule x_t of the leader. Hence, given this assumption, the conjectured reaction function for the follower takes the form of a linear feedback function:

$$\tilde{x}_t = \tilde{F}_1 y_{t-1} + \tilde{F}_2 v_t + L x_t, \tag{3.27}$$

while the conjectured solutions for the private sector and the leader continue to be given by equations (3.18) and (3.19). The reaction function (3.27) implies that the follower takes into account the behavior of the leader when formulating its policy.

The solution procedure for the leader-follower case is similar to the one described for the simultaneous move case. Substituting the conjectured solutions into the linear constraints we obtain the same equation (3.21), but now D is given by:

$$D = A_0 - A_2 H_1 - A_4 F_1 - \tilde{A}_4 \tilde{F}_1 - \tilde{A}_4 L F_1. \tag{3.28}$$

The loss functions for the two policymakers are, then, given by:

$$\begin{aligned}
\mathbb{W}_1 &= y_t' P y_t + x_t' Q_1 x_t + \tilde{x}_t' Q_2 \tilde{x}_t \\
&\quad + \frac{\beta}{1-\beta} tr[(F_2' Q_1 F_2 + (\tilde{F}_2 + L F_2)' Q_2 (\tilde{F}_2 + L F_2) + H_2 P H_2)\Omega], \\
\mathbb{W}_2 &= y_t' \tilde{P} y_t + x_t' \tilde{Q}_1 x_t + \tilde{x}_t' \tilde{Q}_2 \tilde{x}_t \\
&\quad + \frac{\tilde{\beta}}{1-\tilde{\beta}} tr[(F_2' \tilde{Q}_1 F_2 + (\tilde{F}_2 + L F_2)' \tilde{Q}_2 (\tilde{F}_2 + L F_2) + H_2 \tilde{P} H_2)\Omega],
\end{aligned}$$

where

$$P = W + \beta(F_1' Q_1 F_1 + \tilde{F}_1' Q_2 \tilde{F}_1 + H_1' P H_1), \tag{3.29}$$

$$\tilde{P} = \tilde{W} + \beta(F_1' \tilde{Q}_1 F_1 + (\tilde{F}_1 + L F_1)' \tilde{Q}_2 (\tilde{F}_1 + L F_1) + H_1' \tilde{P} H_1). \tag{3.30}$$

After some algebraic manipulations and differentiating the two loss functions with respect to x_t and $\tilde{x}_t$, respectively, we obtain the following set of first order conditions:

$$\begin{aligned}
\frac{\partial \mathbb{W}_1}{\partial x_t} &= (A_3 + \tilde{A}_3 L)' D'^{-1} P D^{-1} [(A_1 + \tilde{A}_3 \tilde{F}_1) y_{t-1} + (A_3 + \tilde{A}_3 L) x_t + (A_5 + \tilde{A}_3 \tilde{F}_2) v_t] \\
&\quad + Q_1 x_t + L' Q_2 (\tilde{F}_1 y_{t-1} + \tilde{F}_2 v_t + L x_t) = 0, \qquad (3.31) \\
\frac{\partial \mathbb{W}_2}{\partial \tilde{x}_t} &= \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t) + \tilde{Q}_2 x_t = 0. \qquad (3.32)
\end{aligned}$$

The leader-follower solution can now be obtained as a fixed point of the following iterative procedure:

1. Initialize $H_1, H_2, F_1, F_2, \tilde{F}_1, \tilde{F}_2$ and L.

2. Compute D using equation (3.28), P using equation (3.29) and $\tilde{P}$ using equation (3.30).

3. Update $H_1, H_2, F_1, F_2, \tilde{F}_1, \tilde{F}_2$ and L according to

$$\begin{aligned}
F_1 &= -[Q_1 + L'Q_2L + (A_3 + \tilde{A}_3L)'D'^{-1}PD^{-1}(A_3 + \tilde{A}_3L)]^{-1} \\
&\quad \times(A_3 + \tilde{A}_3L)'D'^{-1}PD^{-1}(A_1 + \tilde{A}_3\tilde{F}_1 + L'Q_2\tilde{F}_1), \\
F_2 &= -[Q_1 + L'Q_2L + (A_3 + \tilde{A}_3L)'D'^{-1}PD^{-1}(A_3 + \tilde{A}_3L)]^{-1} \\
&\quad \times(A_3 + \tilde{A}_3L)'D'^{-1}PD^{-1}(A_5 + \tilde{A}_3\tilde{F}_2 + L'Q_2\tilde{F}_2), \\
\tilde{F}_1 &= -(\tilde{Q}_2 + \tilde{A}_3'D'^{-1}\tilde{P}D^{-1}\tilde{A}_3)^{-1}\tilde{A}_3'D'^{-1}\tilde{P}D^{-1}(A_1 + A_3F_1), \\
\tilde{F}_2 &= -(\tilde{Q}_2 + \tilde{A}_3'D'^{-1}\tilde{P}D^{-1}\tilde{A}_3)^{-1}\tilde{A}_3'D'^{-1}\tilde{P}D^{-1}(A_5 + A_3F_2), \\
H_1 &= D^{-1}(A_1 + A_3F_1 + \tilde{A}_3\tilde{F}_1 + \tilde{A}_3LF_1), \\
H_2 &= D^{-1}(A_5 + A_3F_2 + \tilde{A}_3\tilde{F}_2 + \tilde{A}_3LF_2), \\
L &= -(\tilde{Q}_2 + \tilde{A}_3'D'^{-1}\tilde{P}D^{-1}\tilde{A}_3)^{-1}\tilde{A}_3'D'^{-1}\tilde{P}D^{-1}A_3.
\end{aligned}$$

4. Iterate over steps 2-4 until convergence.

3.B SOCIAL WELFARE

We assume that the intra-temporal utility function is given by:

$$U(C_t, \zeta G_t, N_t),$$

and is separable in consumption, government spendings and hours (i.e., $U_{cn} = U_{cg} = U_{gn} = 0$).

A second-order Taylor expansion of U_t around the steady state allocation (C, G, N) yields:

$$\begin{aligned} U_t - U &\simeq U_c C \left(\frac{C_t - C}{C}\right) + \zeta U_g G \left(\frac{G_t - G}{G}\right) + U_n N \left(\frac{N_t - N}{N}\right) \\ &+ \frac{1}{2} U_{cc} C^2 \left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2} \zeta U_{gg} G^2 \left(\frac{G_t - G}{G}\right)^2 + \frac{1}{2} U_{nn} N^2 \left(\frac{N_t - N}{N}\right)^2. \end{aligned}$$

In terms of log-deviations,

$$U_t - U \simeq U_c C \left(c_t + \frac{1-\sigma}{2} c_t^2\right) + \zeta U_g G \left(g_t + \frac{1-\sigma}{2} g_t^2\right) + U_n N \left(n_t + \frac{1+\varphi}{2} n_t^2\right),$$

where $\sigma = -\frac{U_{cc}}{U_c} C = -\frac{U_{gg}}{U_g} G$ and $\varphi = \frac{U_{nn}}{U_n} N$.

From the definition, we know that:

$$g_t = \frac{C}{G} \tilde{g}_t + y_t,$$

which implies

$$g_t^2 = \left(\frac{C}{G}\right)^2 \tilde{g}_t^2 + 2\frac{C}{G} \tilde{g}_t y_t + y_t^2.$$

A second-order Taylor expansion of the national income identity yields:

$$c_t + \frac{1}{2} c_t^2 \simeq y_t + \frac{1}{2} y_t^2 - \frac{G}{C}(g_t - y_t) - \frac{1}{2}\frac{G}{C}(g_t^2 - y_t^2),$$

which can be rewritten in terms of $\tilde{g}_t$ as:

$$c_t + \frac{1}{2} c_t^2 \simeq y_t + \frac{1}{2} y_t^2 - \tilde{g}_t - \frac{1}{2}\frac{C}{G} \tilde{g}_t^2 - \tilde{g}_t y_t.$$

A first-order expansion of the national identity yields:

$$c_t = y_t - \tilde{g}_t,$$

then:

$$c_t^2 = y_t^2 - 2\tilde{g}_t y_t + \tilde{g}_t^2.$$

Hence, we have that:

$$c_t + \frac{1-\sigma}{2} c_t^2 \simeq y_t + \frac{1-\sigma}{2} y_t^2 - \tilde{g}_t - (1-\sigma)\tilde{g}_t y_t - \frac{1}{2}\left(\frac{C}{G} + \sigma\right) \tilde{g}_t^2. \tag{3.33}$$

For the government spending, we can write:

$$g_t + \frac{1-\sigma}{2} g_t^2 = \frac{C}{G} \tilde{g}_t + y_t + \left(\frac{1-\sigma}{2}\right)\left(\frac{C}{G}\right)^2 \tilde{g}_t^2 + (1-\sigma)\frac{C}{G} \tilde{g}_t y_t + \frac{1-\sigma}{2} y_t^2. \tag{3.34}$$

To rewrite n_t in terms of output, we have:

$$(1-\alpha)n_t = y_t - a_t + \frac{\varepsilon}{2\Theta}var_i\{p_t(i)\},$$

which implies that[29]:

$$n_t + \frac{1+\varphi}{2}n_t^2 = \frac{1}{1-\alpha}\left(y_t + \frac{\varepsilon}{2\Theta}var_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)}(y_t-a_t)^2\right). \tag{3.35}$$

Substituting equations (3.33)-(3.35) into the expansion of utility, we obtain:

$$\begin{aligned}
\frac{U_t-U}{U_cC} &\simeq y_t + \frac{1-\sigma}{2}y_t^2 - \tilde{g}_t - (1-\sigma)\tilde{g}_t y_t - \frac{1}{2}\left(\frac{C}{G}+\sigma\right)\tilde{g}_t^2 \\
&+ \frac{\zeta U_g G}{U_c C}\left(\frac{C}{G}\tilde{g}_t + y_t + \left(\frac{1-\sigma}{2}\right)\left(\frac{C}{G}\right)^2\tilde{g}_t^2 + (1-\sigma)\frac{C}{G}\tilde{g}_t y_t + \frac{1-\sigma}{2}y_t^2\right) \\
&+ \frac{UnN}{(1-\alpha)U_cC}\left(y_t + \frac{\varepsilon}{2\Theta}var_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)}(y_t-a_t)^2\right).
\end{aligned}$$

Then, collecting terms:

$$\begin{aligned}
\frac{U_t-U}{U_cC} &\simeq \left[1+\frac{\zeta U_g G}{U_c C} + \frac{UnN}{(1-\alpha)U_cC}\right]y_t + \left[\frac{\zeta U_g}{U_c}-1\right]\tilde{g}_t \\
&+ \frac{1}{2}\left[1-\sigma+\frac{\zeta U_g G}{U_c C}(1-\sigma) + \frac{UnN}{(1-\alpha)U_cC}\frac{1+\varphi}{(1-\alpha)}\right]y_t^2 \\
&+ \left[\frac{\zeta U_g}{U_c}(1-\sigma)-(1-\sigma)\right]\tilde{g}_t y_t + \frac{1}{2}\left[\frac{\zeta U_g C}{U_c G}(1-\sigma) - \left(\frac{C}{G}+\sigma\right)\right]\tilde{g}_t^2 \\
&- 2\frac{UnN}{(1-\alpha)U_cC}\frac{1+\varphi}{2(1-\alpha)}y_t a_t + \frac{UnN}{(1-\alpha)U_cC}\frac{\varepsilon}{2\Theta}var_i\{p_t(i)\}.
\end{aligned}$$

Now, if we assume that ζ is such that, in the steady state, $\zeta\frac{U_G}{U_C}=1$, and $-\frac{U_n}{U_c} = MPN = (1-\alpha)\frac{Y}{N}$, we can eliminate the linear terms and obtain:

$$\begin{aligned}
\frac{U_t-U}{U_cC} &\simeq -\frac{1}{2}\frac{Y}{C}\left[\sigma+\frac{\alpha+\varphi}{1-\alpha}\right]y_t^2 - \frac{1}{2}\frac{Y}{C}\left[\sigma\frac{C}{G}\right]\tilde{g}_t^2 \\
&+ \frac{Y}{C}\frac{1+\varphi}{1-\alpha}y_t a_t - \frac{1}{2}\frac{Y}{C}\left(\frac{\varepsilon}{\Theta}\right)var_i\{p_t(i)\}.
\end{aligned}$$

Since the efficient level of output, in log-deviations from the steady state, is given by $y_t^e = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}a_t$, then, we can rewrite:

$$\frac{U_t-U}{U_cC} \simeq -\frac{1}{2}\frac{Y}{C}\left[\frac{\varepsilon}{\Theta}var_i\{p_t(i)\} + \left(\sigma+\frac{\alpha+\varphi}{1-\alpha}\right)(y_t-y_t^e)^2 + \sigma\frac{C}{G}\tilde{g}_t^2\right].$$

Finally, making use of a result found in Woodford (2003), we can express the terms involving the price dispersion as a function of inflation:

$$\sum_{t=0}^{\infty}\beta^t var_i\{p_t(i)\} = \frac{\theta}{(1-\beta\theta)(1-\theta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2.$$

[29] Excluding terms that are independent of policy.

Then, the welfare losses can be expressed as:

$$\mathbb{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{Y}{C}\frac{\varepsilon}{\lambda}\pi_t^2 + \frac{Y}{C}\left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right)(y_t - y_t^e)^2 + \sigma\frac{Y}{G}\tilde{g}_t^2 \right],$$

where $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}\Theta$.

Or, equivalently:

$$\mathbb{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \frac{\lambda}{\varepsilon}\left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right)(y_t - y_t^e)^2 + \sigma\frac{\lambda}{\varepsilon}\frac{C}{G}\tilde{g}_t^2 \right]. \tag{3.36}$$

3.C CONVERGENCE DIAGNOSTIC

This appendix presents the convergence diagnostics of Geweke (1992) for the estimated models. This diagnostic computes a normal-based test statistic comparing the sample means in two windows containing the initial 10% and the last 50% iterations. A non-significant p-value or, equivalently, a Z-score smaller than 1.96 in absolute value indicates convergence.

Table 3.C.1 shows that there are evidences to rule out the possibility of non-convergence for all the three models estimated, given that for all the parameters we found non-significant p-values.

Table 3.C.1 – Geweke diagnostic

	Nash		**FL**		**ML**	
	Z-score	**p-value**	**Z-score**	**p-value**	**Z-score**	**p-value**
θ	-0.896	0.3702	1.830	0.0672	-1.354	0.1758
σ	0.516	0.6060	-0.028	0.9775	0.892	0.3722
φ	-0.607	0.5437	-0.596	0.5511	-0.952	0.3411
Φ_{MY}	1.157	0.2471	-0.958	0.3378	0.532	0.5945
Φ_{MG}	0.154	0.8776	1.112	0.2660	1.491	0.1358
$\Phi_{\Delta I}$	0.276	0.7829	0.038	0.9700	0.550	0.5822
Φ_{FY}	0.551	0.5814	-1.102	0.2704	-0.838	0.4019
Φ_{FG}	1.084	0.2785	-1.509	0.1313	-0.338	0.7356
$\Phi_{\Delta G}$	-0.447	0.6547	-0.396	0.6924	-0.008	0.9937
Φ_{FB}	-1.474	0.1404	-1.765	0.0776	1.894	0.0583
ρ_a	0.596	0.5513	-0.473	0.6359	1.248	0.2119
σ_a	-1.415	0.1570	0.976	0.3293	1.492	0.1356
σ_η	-0.476	0.6339	0.268	0.7887	-1.100	0.2714
σ_r	-1.807	0.0707	-0.424	0.6714	0.117	0.9066
σ_g	0.727	0.4674	-1.740	0.0819	1.359	0.1740

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