

Colégio Imaculada

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nº 30 3º Exatas

Complementos de Matemática (2ª fase)

1) A área de um círculo inscrito em um hexágono regular é $81\pi \text{ cm}^2$. Calcule a área deste hexágono



$$S_0 = 81\pi \text{ cm}^2$$

S_0

$$S_0 = \pi r^2$$

$$81\pi = \pi r^2$$

$$r^2 = 81$$

$$r = 9$$

$$r = a$$

$$a = 9$$

$$S = p \cdot a$$

$$\left(\frac{1}{2}\right)^2 + a^2 = l^2$$

$$\frac{l^2}{4} + a^2 = l^2$$

$$\frac{l^2}{4} + 81 = l^2$$

$$81 = l^2 - \frac{l^2}{4}$$

$$81 = \frac{3l^2}{4} \Rightarrow 3l^2 = 324$$

$$l^2 = 108$$

$$l = \sqrt{108}$$

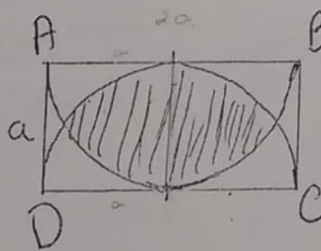
$$2p = 6 \cdot 6\sqrt{3}$$

$$2p = 36\sqrt{3} \Rightarrow p = 18\sqrt{3} \quad l = 6\sqrt{3}$$

$$S = 18\sqrt{3} \cdot 9$$

$$S = 162\sqrt{3} \text{ cm}^2$$

2) Na figura ABCD é um retângulo de lados a e $2a$. Calcule a área da região sombreada



Sol:

$$360^\circ - \pi r^2$$

$$60^\circ - S$$

$$360S = 60\pi \cdot a^2$$

$$S = \frac{60\pi a^2}{360} = \frac{\pi a^2}{6}$$

$$S_{\text{total}} = S_{\Delta} + S_p$$

$$\frac{\pi a^2}{6} - \frac{a^2 \sqrt{3}}{4} = \frac{2\pi a^2 - 3a^2 \sqrt{3}}{12}$$

$$S_p = \frac{a^2(2\pi - 3\sqrt{3})}{12}$$

$$S_{\Delta} + 2 \cdot S_p =$$

$$\frac{a^2 \sqrt{3}}{4} + 2 \cdot \frac{a^2(2\pi - 3\sqrt{3})}{12}$$

$$\frac{3a^2 \sqrt{3} + 2a^2(2\pi - 3\sqrt{3})}{12}$$

$$\frac{3a^2 \sqrt{3} + 4a^2 \pi - 6a^2 \sqrt{3}}{12}$$

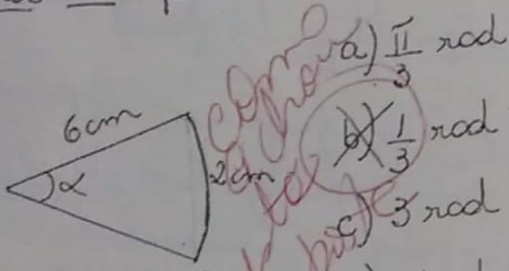
$$\frac{-3a^2 \sqrt{3} + 4a^2 \pi}{12}$$

$$\frac{a^2(4\pi - 3\sqrt{3})}{12}$$

Sol final

$$2 \times \frac{a^2(4\pi - 3\sqrt{3})}{12} = \frac{a^2(4\pi - 3\sqrt{3})}{6}$$

3) Considere a figura abaixo e calcule α (não use fórmulas dadas)



$360^\circ - \pi \cdot 6^2$
 $2\pi r - S$

$2\pi \cdot 36 = 360S$

$\alpha = \frac{360S}{\pi \cdot 36}$

$\alpha = \frac{10S}{\pi}$

a) $\frac{\pi}{3}$ rad

b) $\frac{1}{3}$ rad

c) 3 rad

d) 3π rad

e) n.d.a

$C = 2\pi r$

$C = 2\pi \cdot 6$

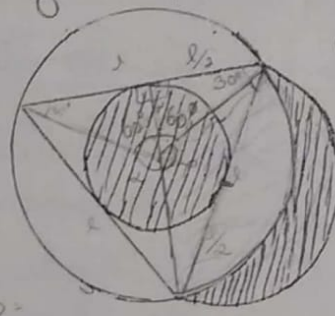
$C = 12\pi$

$12\pi = 2\pi r$

$12\pi S = 4\pi \cdot r$

$S = 24\pi$

4) Calcule a área da região sombreada



Considere o Δ equilátero de lado l

$r = a$

$\tan 30^\circ = \frac{a}{\frac{l}{2}}$

$\frac{\sqrt{3}}{3} = a \cdot \frac{2}{l}$

$\frac{\sqrt{3}}{3} = \frac{2a}{l}$

$\sqrt{3}l = 6a$

$a = \frac{\sqrt{3}l}{6}$

$S_0 = \pi r^2$

$S_0 = \pi \cdot a^2$

$S_0 = \pi \cdot \left(\frac{\sqrt{3}l}{6}\right)^2$

$S = \pi \cdot \frac{3l^2}{36}$

$S_0 = \frac{\pi l^2}{12}$

$S_{\text{shaded}} = S_0 - S$

$\frac{\pi r^2}{2} = ?$

$360^\circ - \pi r^2$

$120^\circ - S$

$360S = 120\pi \cdot \left(\frac{\sqrt{3}l}{6}\right)^2$

$360S = 120\pi \cdot \frac{3l^2}{36}$

$\frac{360S}{120} = \pi \cdot \frac{l^2}{3}$

$3S = \pi \cdot \frac{l^2}{3}$

$S = \frac{\pi \cdot l^2}{9} \cdot \frac{\pi \cdot l^2}{3} \cdot \frac{1}{3} = \frac{\pi l^2}{9}$

$S_{\text{shaded}} = S_{\Delta} - S = \frac{\sqrt{3}l}{3} \cdot l \cdot \frac{\sin 60^\circ}{2}$

$\frac{\sqrt{3}l^2}{3} \cdot \frac{1}{2} = \frac{\sqrt{3}l^2}{6} = \frac{\sqrt{3}l^2}{12}$

$\frac{\pi l^2}{9} - \frac{\sqrt{3}l^2}{12} = \frac{4\pi l^2 - 3\sqrt{3}l^2}{36}$

$S_0 = \frac{l^2(4\pi - 3\sqrt{3})}{36}$

4 exercícios:

$\frac{\pi r^2}{2} - \frac{l^2(4\pi - 3\sqrt{3})}{36} = \frac{18\pi \cdot \left(\frac{l}{3}\right)^2 - 4\pi l^2 + 3\sqrt{3}l^2}{36}$

$\frac{18\pi \cdot \frac{l^2}{9} - 4\pi l^2 + 3\sqrt{3}l^2}{36} = \frac{2\pi l^2 - 4\pi l^2 + 3\sqrt{3}l^2}{36}$

$\frac{2\pi l^2 - 4\pi l^2 + 3\sqrt{3}l^2}{36} = \frac{\pi l^2 - 4\pi l^2 + 3\sqrt{3}l^2}{36}$

$\frac{3\pi l^2 - 8\pi l^2 + 6\sqrt{3}l^2}{36} = \frac{\pi l^2 - 4\pi l^2 + 3\sqrt{3}l^2}{36}$

$\frac{-5\pi l^2 + 6\sqrt{3}l^2}{36} = \frac{-3\pi l^2 + 3\sqrt{3}l^2}{36}$

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