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PERSISTENCE OF CORRUPTION AND SOME MACROECONOMIC IMPLICATIONS: AN EVOLUTIONARY GAME APPROACH

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For my daughter

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I would like to thank Professor Jaylson Silveira for his inspiration and guidance.

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Last but not the least, I am greatful to my parents, Nilson and Nazaré, for always supporting and encouraging me in my intelectual and personal growth.

"When you see corruption being rewarded and honesty becoming a self-sacrifice, you may know that your society is doomed."

(Ayn Rand, 1957)

RESUMO

Serão apresentados dois modelos sobre corrupção de um ponto de vista de jogos evolucionários. O primeiro trata da persistência da corrupção do ponto de vista microeconômico, como resultado de interações entre indivíduos. Nós mostramos que a dinâmica evolucionária sempre leva a economia a um estado com corrupção permanente. A depender do valor de parâmetros como tributação, probabilidade de punição e tamanho da pena, a corrupção pode se apresentar em maior ou menor grau na sociedade. O segundo modelo incorpora o nível de corrupção ao modelo de crescimento de Solow-Swan com gasto produtivo do governo, baseado em Barro (1990). Nós mostramos que o nível de corrupção depende da punição esperada e da taxa tributária. Quanto maior a proporção de indivíduos corruptos, menor a renda per capita, o que traz à luz a importância da corrupção para explicar o subdesenvolvimento.

Palavras-chave: Corrupção. Jogos Evolucionários. Dinâmica Satisficing.

ABSTRACT

We will develop two models about corruption under an evolutionary perspective. The first one shows that the pervasiveness of corruption is a result of interactions between individuals. We show that the evolutionary dynamics will always lead the economy to a situation with permanent corruption. The level of corruption depends on parameters such as the tax rate, the probability of being punished and the punishment size. The second model introduces corruption to the Solow-Swan model with productive government expenditure, based on Barro (1990). We will show that the level of corruption depends on the expected punishment and on the tax rate. The more widespread the corruption, the smaller the income per capita, what enlightens the issue of how corruption affects development.

Keywords: Corruption. Evolutionary Games. Satisficing Dynamics.

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1 INTRODUCTION

Recent international corruption scandals like the one involving Brazilian enterprises and several countries' governments around the globe brought the subject back to the spotlights. In special, the relationship between widespread corruption and fragile economical and political performances call attention to the importance of the quality of public governance for economic development.

Several empirical studies show that high levels of corruption are associated with lower investment and entrepreneurship as well as slower development (see Avnimelech, Zelekha e Sharabi (2014), Wei (2000), Swaleheen e Stansel (2007)). These relationships together with the quality of institutions in force in each society (see Treisman (2000)) enlighten the issue of why corruption tends to be so persistent and pervasive in some societies.

Some of the institutional features that have been found by empirical works as important to explaining corruption are hard (or even impossible) to change through policy. This fact together with evidence that the relationship between corruption and slower economic development is bicausal (TREISMAN, 2000) help to understand why corruption is so hard to eradicate in the short run.

There are many definitions of corruption. Here, we will define it as an illegal rent seeking activity where the public official uses the public office for private gain. It is common to mistake other illegal activities, such as money laundry for corruption. But these activities do not constitute corruption as we understand it, since they do not depend on public officials taking benefits from their public office. Corruption can occur at different spheres of the public sector, from the bureaucratic level to the political and judicial levels. We will focus on bureaucratic corruption, sometimes also called petty corruption.

We understand corruption—just as any other feature of a society—as a result of continuous interactions of individuals making decisions to obtain the maximum well being possible. Although corruption produces a worse outcome for the society as a whole, in some scenarios it might be the best response for reasonable individuals. When corruption is too widespread and poorly combated, non-corrupt people might be excluded from obtaining certain benefits offered by the public sphere, such as services or contracts.

Evolutionary games allow us to describe the dynamics of the processes that explain the evolution of cultural features of a society, such as its level of corruption. They offer a mechanism for equilibrium selection and allow us to take into account the dynamic aspect of the process we are analyzing. We will be able to investigate why corruption persists and why it may vary widely among economies.

We will develop two models. The first one will focus on the persistence and pervasiveness of corruption in a society, from a microeconomic point of view. We will develop an evolutionary game model based on individual decisions about being productive or corrupt, to explain the persistence of corruption. The model is followed by a discussion of its implications.

In the second model we introduce corruption in an augmented Solow-Swan model with productive government expenditure, again from an evolutionary perspective. This allows us to analyze some effects corruption has on the long run per capita income. From this result, we can infer why widespread corruption is commonly associated with lower levels of development.

The thesis is organized as follows. In Chapter 2 we will define corruption and briefly review the empirical and theoretical works on the subject. Next, in Chapter 3, we will present the first model, followed by the second model, in the fourth chapter. Finally, we will discuss our conclusions.

2 CORRUPTION: A REVIEW

In the past few decades a lot of effort has been made by economists to explain why income inequality between countries persists. Countries with similar economic structures will not necessarily converge to the same steady state or balanced growth path. These facts turned the attention of economists towards the importance of institutions that define the rules according to which individuals play the social-economic game. The quality of the institutions in place in a country will determine both the initial conditions and the path through which the development will happen. By institutions we mean "the humanly devised constraints that structure human interaction. They consist of formal rules (constitutions, stature and common law, and regulations), informal rules (conventions, moral rules, and social norms), and the enforcement characteristics of each" (MANTZAVINOS; NORTH; SHARIQ, 2004, p.77).

One important consequence of a country's institutions is its level of corruption. Although it is well known that the level of corruption affects the growth path of an economy, its role in economic development has not been fully understood so far. Some ways that corruption may affect income distribution are by concentrating the efforts and benefits of public policies to some particular interest. It may also distort the investment decisions by altering the returns of different activities as well as moving consumption to foreign countries, as the stolen money might be stored in foreign banks (MACRAE, 1982).

An important characteristic of corruption is that, when it happens at higher levels of the government, it can affect the quality of institutions, (e.g. the quality of the legal system and its enforcement) and lead to a vicious cycle, where corruption generates bad institutions that protect corrupt behavior, which in turn generates more bad institutions. In fact, that is what the more general definition of corruption states: to corrupt is to degenerate; to modify something, tampering with its original features. In that sense, history is also an important factor to explain how corruption evolves in a society. History provides the starting point that will lead to an equilibrium where the society will be trapped. This pervasiveness of corruption can be explained by the historical inertia of the institutions and social norms that support corrupt acts.

The definition of corruption we will assume here is less broad, but contains this same essence. We will define corruption as Jain (2001, p.73) suggests as “acts in which the power of public office is used for personal gain in a manner that contravenes the rules of the game”. Moreover, as Macrae (1982) advocates, we understand that corruption is not a behavior that opposes the profit motivated operations. It is instead “part of a rational calculus and an integral and often deeply-rooted method by which reasonable men take decisions” (MACRAE, 1982, p.678). As Bicchieri e Rovelli (1995) emphasize, corruption is not the result of extortion or violence, rather it is the result of voluntary exchanges.

We can also think of the dishonest behavior of public officials from the principal-agent problem perspective. The social benefit of public services is not aligned with the public official’s interests, much like the misalignment of the interests of a private firm and those of its employees (ANDVIG; MOENE, 1990).

The ancient records about the practice and penalties for corruption (22nd century B.C. in Babylon and 14th century Egypt) point to the fact that corruption is a common practice in different societies throughout history (MISHRA, 2006). Even in societies with anti-corruption policies, corruption has been shown to be pervasive. The ways in which corruption manifests itself vary, but we can classify it in three different categories, according to Jain (2001). The first one is “grand corruption”, where the ruling elite defines the public policies in its own self-interest to the detriment of the public interest. The second, “bureaucratic corruption” (often called petty corruption) happens when bureaucrats take bribes either to provide the service that they are already supposed to (e.g. speeding up a bureaucratic procedure), or to do something against their duties that benefits some private interest. As we will see, this is usually the type of corruption addressed in most of the economic works, since it is easier to identify its economical motivations. The third category is “legislative corruption”, when legislators take bribes that influence their decisions. For example, “legislators can be bribed by interest groups to enact legislation that can change the economic rents associated with assets” (JAIN, 2001, p.75). The fact that corruption is present in important functions of the government, which cannot be easily eliminated, is one reason that may help us to understand why corruption is so persistent (MISHRA, 2006).

For corruption to happen three necessary conditions have to be fulfilled. First, the public authority has to have discretionary power to design and administer regulations; second there must be economic rents associated with this power; and third, the legal system has to have some degree of ineffectiveness in detecting and punishing corrupt behavior (JAIN, 2001).

2.1 EMPIRICAL STUDIES

One could question the reliability of the data on corruption for its subjective nature as well as for its measurement limitations. Nonetheless, there are several empirical works that could help us refine our understanding of corruption and its implications for an economy's performance. Avnimelech, Zelekha e Sharabi (2014), using a sample of 176 countries, found empirical evidence that countries with high levels of corruption, measured by the Transparency International Corruption Perception Index (CPI), usually exhibit low levels of productive entrepreneurship. Mo (2001) and Wei (2000) using cross-country data show that there is a negative relationship between corruption and economic growth. For measuring corruption, Mo (2001) uses the CPI, for the period between 1980 and 1985, for 46 countries. For Mo (2001) the main channel through which corruption affects growth is political instability. The other channels he considers include the share of private investment and human capital. He argues that political instability is generated by income inequality, which in turn is caused by corruption. Political instability is responsible for uncertainty about protection of property rights and reduces the level of investment and productivity, negatively affecting the growth rate. Wei (2000) argues that one important transmission channel is foreign direct investment. Countries with a higher level of corruption tend to receive less direct investment, while foreign bank loans are not necessarily affected, distorting the composition of their capital inflows. This may be due to the vulnerability of direct investment to the intervention of corrupt local officials. For measuring corruption, Wei (2000) creates a composite index for 69 countries of two other indices based on surveys of business executives. These are the Global Competitiveness Report (GCR) and the World Development Report (WDR).

In contrast, Swaleheen e Stansel (2007) find that although corruption has a negative impact on the economic growth of countries with less economic freedom, in countries with higher economic freedom corruption is likely to positively affect economic growth. A reason for that could be that in some countries, bribes make public officials less likely to enforce restrictions in the private sector, allowing greater free exchange. The authors estimate a panel of 60 countries between the years 1995 and 2004. For measuring corruption they use the CPI, while for measuring economic freedom, they use Economic Freedom Index (EFI) by the Heritage Foundation. In the same vein, Méon e Weill (2010), arguing that corruption might remove bureaucratic obstacles, find evidence that corruption may function as "grease in the wheels" in countries with sluggish bureaucratic systems. They estimate a panel using 69 countries between 2000 and 2003. To assess the effect of corruption, they use two different indices, the corruption index provided by the World Bank and the CPI. To measure the institutional efficiency and excesses of bureaucracy, they use a range of indicators of the quality of governance, provided by Kaufmann et al (1999a, 1999b).

Another possible effect of corruption is on fiscal policy. Using Romanian data from 2000 to 2011, Apergis, Dănuleţiu et al. (2013) find that corruption, as well as other institutional variables, have a bi-causal relationship with the public deficit. They show that an improvement in freedom and corruption lowers the public deficit in Romania.

To address the issue of the importance of the size and scope of government on the incidence of corruption, Goel e Nelson (2010) estimate cross-country random effects models, covering the period between 1995 and 2003. The authors also analyze the effect of historical and geographical aspects on the level of corruption. They show that both the size of the government as well as the level of centralization are important variables to explain corruption, using the CPI index. They measure the government intervention level calculating two different indices, one emphasizing fiscal and the other monetary aspects of government intervention. Their results point to the fact that although greater governments are associated with more corruption, governments that engage in more intervention tend to reduce corruption, maybe due to a higher vigilance against corruption. They also find that the more decentralized the government, the lower the corruption practices. That does not rule out the hypothesis that bigger intervention on the regulatory area might promote corruption. Their study also shows that the level of corruption responds to the prevailing commercial code. Relative to historical and geographical aspects, Goel e Nelson (2010) find that the population distribution over the country and the influence of past institutions and norms are important in explaining the corruption level.

Some of these studies help to support our results that the higher the government participation—that we measure as the tax rate—the higher the corruption level. When corruption is more widespread, the costs of corruption are smaller and the incentives for being corrupt are stronger. In this scenario, the level of entrepreneurship and the output produced are lower, because fewer individuals are going to choose to engage in productive activities when it is possible to get a higher return on corrupt activities.

2.2 CORRUPTION PERSISTENCY

It was not until the late 1960s and early 1970s that much attention was paid to the effects of bribery and rent seeking on economic performance. In this section we will present a summary of some of the most relevant research that has been done about the subject. The vast majority of the economic works on this subject adopts Becker (1968) assertion that crimes are committed as a response to incentives.

In her seminal paper, Krueger (1974) shows that the welfare costs of rent seeking exceed those of government intervention by analyzing the competition for import licenses. It is important to differentiate rent seeking from bribery and corruption. Not all rent seeking activities are illegal or depend on bribery. For example, when competing for an import license, the allocation of which is proportional to the firm's physical plant, some firms may expand their productive capacity even when that would not be efficient in a scenario without import licenses. In this case, the rent seeker is adapting its strategy to the new rules defined by a government intervention. It chooses a new strategy that will yield the highest payoff given that the other firms will also change their behaviors. A special case of rent seeking happens when bribery is a possible strategy to influence the government decision about the allocation of contracts. For corruption to occur, the bribing firm has to meet a bureaucrat that is willing to accept the bribe to influence the allocation of the contracts.

Following this premise Rose-Ackerman (1975) proposes three models wherein firms compete for a contract while bureaucrats decide if they take bribes to influence the results of the competition. Rose-Ackerman (1975) designs her utility maximization models considering two possible market structures, a competitive market and a bilateral monopoly. She finds that there is an optimum value of the bribe that equates the firm's impatience to the bureaucrat's lack of urgency. Furthermore, her results point to the fact that the type of market structure affects the corrupt relationships and the incentives for bribery.

Introducing a game theory approach, Macrae (1982) contributes to the debate of the fundamentals of corruption by developing a prisoner's dilemma game. By doing that, he attempts to take into account the bargaining dynamics and risks of this sort of situation that a mere utility maximization approach cannot capture. His objective was to explain why corruption is omnipresent in different societies focusing on the rationality of the decisions to be corrupt. He finds that being corrupt is an equilibrium. He also argues that the legal solutions are ineffective in controlling the level of corruption. That argument is justifiable if we consider that in reality corruption tends to spread to different levels of a society, also tampering with the legal system.

The decay of moral values and social norms or the lack of some sort of legal or moral punishment may be appealing arguments to explain the pervasiveness of corruption. However, they do not seem to be sufficient to explain the differences in corruption within countries (e.g. northern and southern Italy) or how corruption could be eradicated in some countries in a short period of time (e.g. Singapore). One possible explanation for this phenomena is the existence of multiple equilibria. That arises from the fact that the decision to be corrupt depends on the level of corruption. Moreover, it depends on how individuals perceive how much corruption takes place around them. If a person sees that there is a great deal of corruption occurring, she is more likely to act in the same manner because she does not fear disapproval or moral coercion. In addition, when corruption is more widespread, the costs of finding a corrupt bureaucrat are lower. The level of corruption perceived affects both the expected benefits of corrupt acts and the perception of the probability of being punished. These together compose the expected return on corruption. The level of corruption affects the expected benefits by reducing the income available, since more people engage in and waste time on corrupt activities instead of being productive. It also reduces the gains from corruption. When a bureaucrat is bribed by everyone that is competing for a contract, the chance of a given briber getting the contract decreases. On the side of the bureaucrat, if there is more corruption, bribers will probably be willing to pay less, since they are going to have to bribe more often. When corruption is more widespread, people might perceive more impunity and think the probability of being punished is lower (BARDHAN, 2006).

It is worth noting the distinction between being caught and being punished. In a country with widespread corruption, the chances of being caught might be high, but it might be easy to pay a different bribe to convince the authorities to look the other way. Being punished, in contrast, means facing the negative consequences. One should also pay attention to the distinction between the perceived probability and the actual probability of being punished. The actual probability depends on the legal system. If corruption has spread to the legal system, the enforcement of the law will be negatively affected. The perceived probability, on the other hand, depends on peoples expectations. When there is no uncertainty about what the effectiveness of the law is, then they are equivalent.

Considering the dynamic nature of corruption, Andvig e Moene (1990) develop a model with multiple equilibria where they seek to explain why different levels of corruption can arise in societies with similar socioeconomic structures. Although this is not formally an evolutionary game, it presents some important features that resemble an evolutionary approach. They argue that the return on corrupt acts is associated with its relative frequency among bureaucrats. That is, the more corrupt bureaucrats there are, the higher the payoff of being corrupt. Their model focuses on the bureaucrat's decision to take bribes, given that there is a demand for corrupt acts. One interesting feature of their model is that the probability of being punished is associated with the probability of other bureaucrats being corrupt. That leads to the somewhat counter-intuitive conclusion that the value of the bribes increase when corruption is more widespread. This conclusion is counter-intuitive because one would expect that the bribe size rises with the probability of being detected, as shown by Basu, Basu e Cordella (2014), which in the Andvig e Moene (1990) model is assumed to fall with the number of corrupt officials.

In their paper, Bicchieri e Rovelli (1995) design infinitely repeated prisoner's dilemma, where individuals again chose between being corrupt or honest. A small number of the individuals are assumed not to change their behavior throughout time, while all the others choose their strategy based on what the other individuals did in the past period. They let the payoffs decrease over time to account for the negative effect that corruption has in the society, in a manner that the payoffs ordination is maintained. They find that corruption is a dominant strategy and the society will converge to an equilibrium where many people choose to be corrupt. That happens until a certain critical period, when the honest strategy starts to offer a higher payoff and becomes dominant, due to the erosion effect of corruption. Their model shares some characteristics with an evolutionary game, in the sense that it allows individuals to adapt their strategies over time after observing the results of that period. To do so, though, they have to assume that the payoffs and played strategies of each player are public knowledge after each period, which is not quite realistic.

One important conclusion that runs through many of the presented works so far is that the wages of the public officials are one of the determinants of the level of corruption at the equilibrium (see Rose-Ackerman (1975) and Andvig e Moene (1990)). Therefore, many economists would argue that increasing the wage in the public sector could be a good policy to push the country towards the low-corruption equilibrium. This argument seems to have some problems. First of all, it encourages corrupt politicians and corrupt bureaucrats to make their extraction official, by giving themselves raises. Second, in practice that is not what we observe. In countries like Brazil where, on average, public officials' revenues are higher than those of the private sector (according to Souza e Medeiros (2013), public employees are paid on average 20% more than private employees in equivalent functions), corruption is still perceived to be high (according to Transparency International, in 2015, Brazil scored 38 out of 100 in the Corruption Perception Index, being in position 76 in a sample of 168 countries). In fact, Porta et al. (1999) argue that in countries where bureaucrats have relatively more power, they collect both higher wages and higher bribes.

Multiple equilibria models played an important role in unfolding some features of corruption. However, these models cannot explain why different equilibria arise in different situations as well as how equilibria change over time. In the same manner, a pure institutional approach is insufficient to explain the differences in corruption across comparable countries or regions within the same country. Knowing this, Sah (2007) attempts to analyze corruption focusing on its inertial nature. Individuals observe their environment and perceive corruption at a certain level, and then make their decisions based on their perceptions. Sah (2007) develops a model of overlapping generations where citizens choose whether or not to cheat, and bureaucrats choose whether or not to be corrupt. Individuals construct and revise their beliefs about what is the better strategy at each period based on what they observe in the past period. The decision about being corrupt is made ex post,after the non-corrupt individual meets a corrupt one.

He concludes that even people who live in the same environment could have different perceptions of corruption, which would affect their behavior. Also, he finds that a greater prevalence of cheating or corruption in the past leads to a greater prevalence of cheating and corruption in the future. This could explain differences between countries or regions with similar institutions. A disturbance at some point in time in one region or country could lead to a greater level of corruption that would be carried for the future. In his model, there will always be some level of corruption. Sah (2007) does not examine the relationships between corruption and growth, efficiency and welfare. Lastly, he derives a simple evolutionary dynamic, from which he concludes that the level of corruption in the economy will persist in the future.

Following Sah's steps, our argument here is that an evolutionary game theory approach is a more suitable way to understand the persistence and pervasiveness of corruption. With such a theory it is possible to make formally explicit the microeconomic motivations that determine each agent's choices in a strategic environment. The macroeconomic setting, in turn, is affected by the individuals collectively and affects their choices, in a way that the adopted strategies and the macroeconomic environment co-evolve.

Evolutionary games provide a mechanism for equilibrium selection that is suitable for our analysis, since we are analyzing a dynamic process in which individuals from a big population make decisions throughout several periods of time. This allows us to analyze the path to the long run equilibrium and the parameters that determine its shape. Contrary to a Nash equilibrium, evolutionary games do not rely on the requirement that players have correct beliefs about other players rationality. They allow us to formulate a model where the players have a bounded rationality, in the sense that they do not necessarily have the whole game formulated in their minds before they make decisions and take actions. Instead, they do not know or do not care about the effects of their decisions on the rest of the society (SAMUELSON, 1998).

Not every Nash equilibrium is an evolutionary stable equilibrium, but every evolutionary stable strategy is at least a weak best response to itself, and is hence a Nash equilibrium (SAMUELSON, 2002). It is worth noting that, as evolutionary stable strategies are also Nash equilibria, there is some level of rationality on the decisions players make (VEGA-REDONDO, 1996).

I will now highlight two other main works that treat corruption from an evolutionary standpoint: Mishra (2006), and Verma e Sengupta (2015). In Mishra (2006) model, the corrupt strategy is a "mutation" that invades a society with originally only honest citizens. Mishra assumes that individuals change their strategies by an imitation process. The corrupt behavior is shown to be an evolutionarily stable strategy, while the honest behavior is not.

His model ignores some important features of corruption and leads to quite an unrealistic result. In a society it is not likely that we have an equilibrium situation where everyone is corrupt. Instead, what we observe is some combination of individuals being corrupt and some being honest. Moreover, someone who is dishonest in one situation is not necessarily dishonest in another. It is probable that individuals are going to adapt their behavior depending on the situation they are facing, based on a learning process. This implies that each individual has in its set of strategies the possible strategies ex ante. Furthermore, the size of the corrupt population is likely to affect the payoffs by decreasing the total income available in this society.

Verma e Sengupta (2015), considering such characteristics of corruption, design an asymmetric evolutionary game where there are two kinds of players: citizens and public officials. The citizens choose between three possible strategies: paying bribes silently, paying and complaining to the legal authorities and not paying; while the public officials choose between demanding bribes or not. The citizen's payoffs depend on the benefits of the public service, the bribe size, the costs of complaining, the probability that a complaint will lead to a prosecution, the refund for the citizen in case the bribe is found out and the penalties, which they allow to be different for citizens and public officials. They use replicator dynamics, assuming that in every period the least-fit player is replaced by a better-fit one, to explain each subpopulation's arrangements. They find that allowing for asymmetric penalties and considering the ease of denouncing as well as if the bribe is fully refunded, corruption can be extinguished from a society. If the rate of apathetic citizens who pay the bribe silently is bigger than a threshold, all the honest public officials are eliminated.

Then, they consider the scenario where public officials also demand public services. In this model they again use an imitation process to explain how the subpopulations evolve. Their conclusions regarding penalties, refunds and ease of denouncing remain the same. This new model allows consideration of the effect of empathy on bribery. Public officials who pay bribes to other public officials are less likely to demand bribes from citizens in the future.

Our proposal for the first model is to take into account the effect of each subpopulation's relative frequency on each strategy's expected payoff, and consequently on the players choices. We base our first model in the Lotka-Volterra model developed by Griebeler e Hillbrecht (2015), where this hypothesis is considered. Griebeler argues that in some economies, the productive sector is unable to develop due to the presence of a "parasite" sector, a sector that survives and multiplies at the expense of the productive sector. The size of each population depends on institutional variables, like law enforcement and guarantee of property rights. Furthermore, his model points to the possibility of a poverty trap. Considering a developing economy, where the initial income is low, it is not possible for the economy to grow consistently in the presence of parasites. Both populations will fluctuate around the low income equilibrium and there will not be endogenous forces able to push it towards the high income equilibrium.

2.3 CORRUPTION AND ECONOMIC GROWTH

As it has already been argued in the previous sections, there is empirical evidence of the negative relationship between corruption and economic growth, investment and entrepreneurship. The effects of corruption in growth models have been explored by some theoretical works. We are going to highlight the main ones that relate to the model we will develop in Chapter 4.

Extending the Solow model to include corruption as a determinant of government expenditure, private investment and foreign aid, Farida, Ahmadi-Esfahani et al. (2008) conclude that corruption reduces the effectiveness of physical and human capital and the output per worker. By reducing the output per worker, corruption indirectly reduces investment which in turn has a negative effect on the growth of output per worker. Next, they test these results empirically for Lebanon, between 1985 and 2005. They find evidence that corruption decreases Lebanon's standards of living, investment and human capital productivity.

In the same vein, Ellis e Fender (2006) devise an augmented Ramsey model to account for corruption and how it affects the growth rate and the level of output. He shows that higher levels of corruption are associated with lower level of output growth, yet this relationship is not causal. In their model, corruption is determined by "deep economic parameters" including the degree of transparency of the fiscal system. It does not make clear the direct relationship between corruption and economic growth. The model was not able to address the negative relationship between corruption and private capital accumulation.

Blackburn, Bose e Haque (2006) develop a dynamic general equilibrium model of growth in which households bribe bureaucrats for tax evasion. The costs of keeping this activity secret reduce the available resources for productive investments. In their model, there is a two-way causality between corruption and economic development. The interaction between bureaucrats and economic activity gives rise to an endogenous threshold effect and the possibility of multiple equilibria. The existence of multiple equilibria can explain why countries with similar structural characteristics may present very different levels of development and quality of governance.

Our proposal for the second model will be to introduce corruption in the Solow-Swan growth model, following Barro (1990) and Farida, Ahmadi-Esfahani et al. (2008)'s approach. Unlike the first model, the gains of corruption now derive from the difference between the government's tax revenue and its productive expenditure. In this second model we will be able to shed some light on how corruption affects the capital stock and the income per capita.

3 THE PESRISTENCE OF CORRUPTION AS AN EVOLUTIONARY EQUILIBRIUM

To study the pervasiveness of corruption we now develop an evolutionary game, in which each individual can choose among two strategies, namely, to be productive or to be corrupt. Each individual in this society is free to change her strategy at each period, whenever she thinks the other strategy will be more beneficial to herself.

The evolutionary game approach allows us to consider that the decision that each individual makes rationally in each period of time may lead them to a non-optimum situation in the long run. That happens because the corrupt individuals do not take into account the agregate negative impact of their own acts on the macroeconomic outcome that, in turn, affects their gain. In that sense we could say the individuals have a bounded rationality. They might act like that because they think their impact is small enough to not to be considered or because they do not aknowledge the consequences of their actions. Furthermore, as evolutionary games analyze dynamic process where individuals grope for the best strategy by revising their strategies recurrently in a bounded rationality environment, it is possible to consider the contagious effect of corruption and how it evolves over time.

3.1 THE SHORT RUN: COURNOT EQUILIBRIUM

We will assume a sufficiently large society, with n individuals, where each individual's behavior has a small impact on the whole. Let n_1 be the number of corrupt individuals at a given period, so that their proportion in the society is given by $x = n_1/n$. The number of productive individuals is n_2, so their proportion in the society is given by $1 - x = n_2/n$.

We will denote the corrupt individuals by the subscript c and the non-corrupt by the subscript n. Each productive individual behaves as a firm, maximizing its profit. Drawing on Griebeler e Hillbrecht (2015), we will assume that there is one homogeneous good produced by all the producers and the corrupt individuals expropriate the taxes paid by the productive ones through bribes. We can interpret this expropriation as the situation where public officials divert public resources to their own or their cohort benefit. In the present model, we will not make assumptions about how the taxes are expropriated from the public domain. One explanation, though, could be through bribes on overpriced contracts that benefit specific groups in the society.

The short run is defined as the time frame during which the proportion of individuals playing the corrupt strategy, x, is predetermined and quantities and prices adjust to bring about equilibrium in the market, which will be taken as a Cournot oligopoly with $n_2 = (1-x)n$ producers.

The producers face an inverse linear market demand of the form

$$p(Q) = a - bQ, \tag{3.1}$$

where a and b are strictly positive constants, $p(Q)$ is the market price, $Q = \sum_{i=1}^{n_2} q_i$ is the total demanded quantity in the market and q_i is producer i's demanded quantity.

We will assume that there are no fixed costs and the total cost function of the ith firm is linear, that is:

$$C(q_i) = cq_i, \tag{3.2}$$

where $c \in (0, a) \subset \mathbb{R}$ is the constant marginal cost of firm i.

Taxes are collected from profits. Thus, considering (3.1), firm i's objective function is given by:

$$\pi_n = (1-\tau)(a - bQ - c)q_i, \tag{3.3}$$

where $\tau \in (0, 1) \subset \mathbb{R}$ is the tax rate.

At each period, quantities and prices are decided as in a Cournot oligopoly game. Firms maximize their profit choosing the quantity they produce while taking into account their expectations on what other producer's chosen quantity will be. The firm i's best-response function is, thus,

$$q_i^* = \frac{a - bQ_{-i} - c}{2b}, \tag{3.4}$$

where Q_{-i} is the sum of all producers' quantities except from firm i. As all producers have the same cost function and expect the same behavior from their rivals they will all supply the same quantity, $q_i^* = q_j^* \equiv q^*$, so that the total quantity in the short-run Cournot equilibrium is simply:

$$Q^* = \sum_{i=1}^{n_2} q_i = n_2 q^*. \tag{3.5}$$

Substituting (3.5) in (3.4) and recalling that $n_2 = (1-x)n$, after rearranging the terms in (3.4) we find the quantity suplied by each firm in the short-run Cournot equilibrium, as follows:

$$q^* = \frac{a-c}{b[(1-x)n+1]}. \tag{3.6}$$

The short-run Cournot equilibrium market price is easily found by substituting (3.6) in (3.1):

$$p^* = p^*(x, q^*) = \frac{a+(1-x)nc}{(1-x)n+1}. \tag{3.7}$$

Note that as corruption falls to zero, the number of producers increases aproaching n and the market becomes more competitive. Since n is taken as suficiently large, when n_2 approaches n, the price becomes closer to the marginal cost.

From now on, without loss of generality, we will normalize $n = 1$. The impact of x on both price and individual producers quantities are shown below:

$$\frac{\partial q^*}{\partial x} = \frac{a-c}{b(2-x)^2} > 0 \tag{3.8}$$

and

$$\frac{\partial p^*}{\partial x} = \frac{a-c}{(2-x)^2} > 0. \tag{3.9}$$

When the number of corrupt individuals increases, the number of producers falls. That leads to a rise in the quantities produced by each producer as well as a rise in the market price.

The producer's expected profit after taxes depends on the proportion of producers in the market. Substituting (3.6) and (3.7) in equation (3.3), the expected profit each firm earns after taxes is given by

$$\pi_n^* = \frac{(1-\tau)}{b}\left(\frac{a-c}{2-x}\right)^2. \tag{3.10}$$

From (3.10) it follows that:

$$\frac{\partial \pi_n^*}{\partial x} = \frac{2(1-\tau)}{b}\frac{(a-c)^2}{(2-x)^3} > 0 \tag{3.11}$$

for all $x \in [0,1] \subset \mathbb{R}$. In other words, when the proportion of corrupt individuals is higher, the profit of the productive individuals increase. This happens because there is less competition in the market and both individual produced quantity and market price rise, as shown in (3.8) and (3.9).

Now let us consider the problem of those individuals who see in the amount of taxes collected by the government an opportunity for rent seeking, more specifically, an opportunity for diversion of money. We will not discuss here how these individuals perform this diversion of money. We will also not consider the role of the bureaucrats or politicians responsible for the allocation of the government budget. Instead, we will assume that they do not demand bribes to enable the diversion of money. We will assume that the corrupt individual faces a probability $\rho \in (0,1) \subset \mathbb{R}$ of getting caught and punished with an exogenous cost ε and a probability of $(1-\rho)$ of succeeding in the corrupt activity without being noticed. The cost ε can be interpreted as the cost of spending time in prison as a punishment. Note that we are not discussing the ability of the judicial system to enforce the law once corruption has been detected. Instead, ρ should be understood as the ability of the judicial system on detecting corruption. Another issue we will not consider here, is the case where law enforcement is affected by the level of corruption. In that scenario, our parameter ρ would be somehow affected by the level of corruption. We will leave that for future research.

We can write each corrupt individual's share of the government total revenue as

$$T = \frac{\tau}{b}\left(\frac{a-c}{2-x}\right)^2 \frac{n_2}{n_1} = \frac{\tau}{b}\left(\frac{a-c}{2-x}\right)^2 \frac{(1-x)}{x}. \tag{3.12}$$

Based on (3.12)

$$\frac{\partial T}{\partial x} = T\frac{(-2x^2+3x-2)}{(2-x)x^2} < 0 \tag{3.13}$$

for all $x \in [0,1]$. Since the number of corrupt individuals who share the government revenue increases when corruption rises, the number of producers falls and the individual producer's profit rises as shown in (3.11). This alone would have a positive effect on the government revenue. However, the fall in the number of producers and the rise in the number of corrupt individuals reduce both the government revenue and the share for each corrupt individual. This effect, as shown in (3.12), surpasses the positive effect and T falls. In short, as the number of corrupt individuals increases, each corrupt individual's share decreases.

The expected return of the corrupt individuals is the weighted average between T and the cost of being punished, ε, as follows:

$$\pi_c = (1-\rho)T + \rho(-\varepsilon) \tag{3.14}$$

Substituting (3.12) in (3.14), we find the the expected payoff of the corrupt individuals in the short-run Cournot equilibrium:

$$\pi_c^* = (1-\rho)\frac{\tau}{b}\left(\frac{a-c}{2-x}\right)^2 \frac{(1-x)}{x} - \rho\varepsilon. \tag{3.15}$$

Considering (3.12), from (3.15) we can conclude that the effect on the corrupt individuals payoff is negative:

$$\frac{\partial \pi_c^*}{\partial x} = (1-\rho)\frac{\partial T}{\partial x} < 0. \tag{3.16}$$

When the proportion of corrupt individuals in the population rises, the price increases, generating an extraordinary profit for the individuals that remain productive and produce,individually, a greater quantity, making this activity relatively more attractive. Although the rise in the profit of producers has a positive effect on π_c^*, the negative effect of a decrease in the relative frequency of producers is bigger. This shrinks the available rent for corruption per corrupt individual.

3.2 THE LONG RUN: EVOLUTIONARY EQUILIBRIUM

In Evolutionary Game Theory, a usual way of describing the process for changes in strategies in a population of agents is using an imitation principle as a microfundation to get an evolutionary dynamics. In that case, we have a social learning process behind the evoltionary dynamics, where each agent learns by comparing her payoff with other agents' payoffs. As corruption gains are often kept secret just as most firm's profits, we will assume that the dynamics here follows the satisficing principle, which is an individual learning process.

3.2.1 Transition between short runs as an evolutionary dynamics

Each individual j of type $k = c, n$ sets a target return μ_k, that she considers to be the minimum acceptable. At each period, each individual compares her current observed payoff, π_k^*, with μ_k and decide if she switches her strategy or not. She mantains the current strategy if $\pi_k^* \geq \mu_j$. Otherwise, she changes her strategy.

Let us assume that the acceptable returns are randomly and independently determined accross agents and over time, with a monotonically increasing cumulative distribution function $F : \mathbb{R}_+ \to [0, 1]$. Let F be continuous and differentiable. Thus, the individual j of type $k = c, n$ will be dissatisfied with her current strategy with probability $Prob(\mu_j > \pi_k^*) = 1 - Prob(\mu_j \leq \pi_k^*) = 1 - F(\pi_k^*)$. Therefore, a corrupt individual will be dissatisfied with her current payoff with probability $1 - F(\pi_c^*)$. Following Vega-Redondo (1996, p.91), we assume that the probability with which she will choose the alternative strategy depends on the relative frequency with which this strategy is played. In other words, the dissatisfied individual measures the successfulness of the other strategy by its relative frequency. The relative frequency, therefore, is the probability that the dissatisfied individual will in fact switch her strategy. In that manner, supposing these events are statistically independent, the outflow from the corrupt population is given by $x[1 - F(\pi_1^*)](1 - x)$. Analogously, the outflow from the productive population—which corresponds to the inflow to the corrupt population—is given by $(1 - x)[1 - F(\pi_2^*)]x$. Combining the outflow with the inflow, we have the net flow for the corrupt subpopulation:

$$\dot{x} = x(1 - x)\varphi(x), \tag{3.17}$$

where

$$\varphi(x) = F(\pi_c^*) - F(\pi_n^*). \tag{3.18}$$

The satisficing evolutionary dynamics in (3.17) has as its space state the interval $(0, 1] \subset \mathbb{R}$ and reflects an appropriate idea of selection. When the payoff of the corrupt strategy is greater (less) than the payoff of the alternative strategy, the proportion of individuals playing such a strategy increases (decreases).

3.2.2 Long-run equilibria

Two different equilibria arise from the evolutionary dynamics in (3.17). The monomorphic equilibrium is reached at $x = 1$, that is, when the whole population is corrupt. The polymorphic equilibrium is reached when $\varphi = 0$, that is, when the two types of strategy coexist. The latter is the most relevant one, as in most societies we have a combination of corrupt and honest individuals. We will now prove that the polymorphic equilibrium exists and is unique, using the Intermediate Value Theorem.

Based on (3.15) and (3.10), we know that:

$$\lim_{x \to 0^+} \pi_c^*(x) - \pi_n^*(x) = \infty, \tag{3.19}$$

and

$$\pi_c^*(1) - \pi_n^*(1) = -\frac{1-\tau}{b}(a-c)^2 < 0. \tag{3.20}$$

As F is strictly increasing, from (3.19) and (3.20), it follows that:

$$\lim_{x \to 0^+} \varphi(x) = 1 - F(\pi_n^*(0)) > 0, \tag{3.21}$$

and

$$\varphi(1) = F(\pi_c^*(1)) - F(\pi_n^*(1)) < 0. \tag{3.22}$$

As $\varphi(x)$ is continuous, considering (3.21) and (3.22), by the Intermediate Value Theorem, there exists an $x = x^*$ such that $\varphi(x^*) = 0$.

Furthermore, since F is strictly positive, from (3.11) and (3.16) we know that:

$$\varphi'(x) = F'(\pi_c^*)\frac{\partial \pi_c^*}{\partial x} - F'(\pi_n^*)\frac{\partial \pi_n^*}{\partial x} < 0. \tag{3.23}$$

Thus, x^* is unique.

From (3.11) and (3.16) we know that:

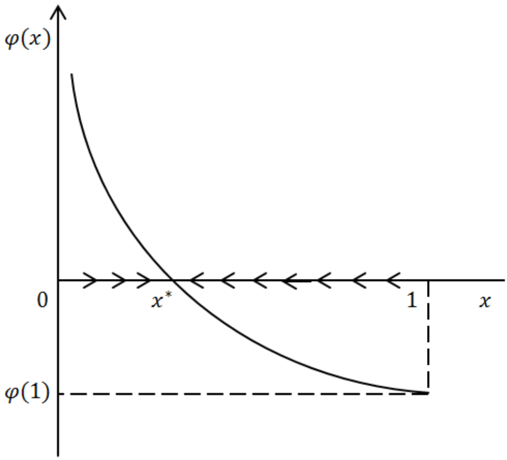
$$\frac{\partial \pi_c^*}{\partial x} - \frac{\partial \pi_n^*}{\partial x} < 0. \tag{3.24}$$

Since in the equilibrium $\pi_c^*(x^*) = \pi_n^*(x^*)$, it follows from (3.24) that $\pi_c^*(x^*) > \pi_n^*(x^*)$, for all $x \in (0, x^*) \subset \mathbb{R}$ and $\pi_c^*(x^*) < \pi_n^*(x^*)$, for all $x \in (x^*, 1) \subset \mathbb{R}$. As F is strictly increasing and $x(1-x) > 0$ for all $x \in (0,1) \subset \mathbb{R}$, it follows that for all $x \in (0, x^*) \subset \mathbb{R}$, $\varphi(x) > 0$ and therefore $\dot{x} > 0$. And for all $x \in (x^*, 1) \subset \mathbb{R}$, $\varphi(x) < 0$ and therefore $\dot{x} < 0$. Thus, we can conclude that $x = x^*$ is an asymptotically stable equibrium.

Since $\dot{x} < 0$ for values of x less than one nearby $x = 1$, this equilibrium is locally unstable. This results are summarized in the Figure 1.

In other words, when the economy begins at any $x \in (0,1) \subset \mathbb{R}$ the only equilibrium to where the population is attracted to is $x = x^*$. In other words, $x = x^*$ is evolutionarily stable.

Figure 1 – Stability of x^*



Source: Created by the author.

3.3 LONG-RUN IMPACTS ON CORRUPTION

Using the existence condition $F(\pi_c^*) - F(\pi_n^*) = 0$, we can take the implicit derivatives of x^* with respect to the parameters, and investigate their impact on the long run level of corruption.

The effect of the tax rate on x^* is given by:

$$\frac{\partial x^*}{\partial \tau} = -\frac{1}{b\varphi'(x)}\left(\frac{a-c}{2-x}\right)^2\left[(1-\rho)\left(\frac{1-x}{x}\right)+1\right] > 0 \tag{3.25}$$

The greater the tax rate, the greater the available rent for corrupt activities, therefore, the greater the long run level of corruption.

The derivative of x^* with respect to ρ, considering (3.12) is:

$$\frac{\partial x^*}{\partial \rho} = \frac{F'(\pi^*)(T+\varepsilon)}{\varphi'(x)} < 0, \tag{3.26}$$

where $\pi^* \equiv \pi_c^* = \pi_n^*$ and, therfore, $F'(\pi^*) = F'(\pi_c^*) = F'(\pi_n^*)$. As the probability of being punished increases, the long run relative frequency of corruption in the society decreases. The same is true for the size of the punishment. The derivative of x^* with respect to the punishment,

ε, is:

$$\frac{\partial x^*}{\partial \varepsilon} = \frac{F'(\pi^*)\rho(T+1)}{\varphi'(x)} < 0. \tag{3.27}$$

The greater the penalty for engaging in corrupt activities, the lower the level of corruption in the long run.

Recall that, in the equilibrium, $\pi_c^*(x^*) = \pi_n^*(x^*)$. Using (3.10) and (3.15) this equality can be expressed as follows:

$$\frac{1}{b}\left(\frac{a-c}{2-x}\right)^2 \left[(1-\rho)\tau\frac{(1-x)}{x} - (1-\tau)\right] = \rho\varepsilon. \tag{3.28}$$

Considering (3.28), the effect of the marginal cost, c, on x^* is:

$$\frac{\partial x^*}{\partial c} = \frac{2F'(\pi^*)}{\varphi'(x)(a-c)}\rho\varepsilon > 0. \tag{3.29}$$

The greater the marginal cost, the less attractive is to be a producer, and the greater is the level of corruption on the long run.

Lastly, the effect on x^* of a rise on a, considering (3.28), is given by:

$$\frac{\partial x^*}{\partial a} = -\frac{2F'(\pi^*)}{\varphi'(x)(a-c)}\rho\varepsilon < 0. \tag{3.30}$$

That is, a positive demand shock causes the level of corruption to fall on the long run.

3.4 FINAL REMARKS

Our results suggest that the evolutionary dynamics will always lead the economy to a situation with permanent corruption. The level of that corruption will depend on the parameters we are taking into account. In our model, there is also a polymorphic (mixed) equilibrium, where both corrupt and non-corrupt individuals co-exist.

When any non-zero and less than x^* percentage of the population initially chooses to be corrupt, being corrupt is a best response and the dynamics converge to the equilibrium in which x^* of the individuals choose to be corrupt. If more than x^* of the population, but not everyone, initially chooses to be corrupt, then being honest is a best response, and the evolutionary dynamics again converges to x^*. In other words, x^* is an evolutionary equilibrium. $x = 1$ is unstable, and thus is not an evolutionary equilibrium.

We can deduce some important relationships between our parameters and the level of corruption. As the chance of impunity increases (as ρ gets closer to one) the proportion of corrupt individuals in the equilibrium increases. In the same sense, as the tax rate increases, the proportion of corrupt individuals in the equilibrium also increases. That is due to two effects. On one hand, a higher tax rate discourages private initiative for legal activities. On the other hand it makes the amount of money available for corruption bigger, encouraging such a practice. These results suggest that a strong state determined to combat corruption may lower the incidence of such a practice. On the other hand, a big government in the sectors that do not affect the probability of punishment, tend to stimulate corruption. This results corroborate the empirical evidence pointed by Goel e Nelson (2010) that bigger governments are associated with higher levels of corruption, yet more intervention might increase the likelihood of being punished.

The size of the punishment, ε, as it was expected, affects corruption negatively, as it represents the direct costs of being dishonest. A higher marginal cost makes it less attractive to be a producer, raising the level of corruption in the long run. Finally, a positive demand shock causes the level of corruption to fall in the long run.

4 CORRUPTION AND ECONOMIC GROWTH: AN EVOLUTIONARY GAME APPROACH

4.1 A SOLOW-SWAN MODEL WITH PRODUCTIVE GOVERNMENT EXPENDITURE AND CORRUPTION

We will analyze the impact of corruption on economic growth from an evolutionary game perspective, where the population of corrupt individuals co-evolves with the capital stock over time. Following Barro (1990) and Farida, Ahmadi-Esfahani et al. (2007) we will introduce government expenditure and the level of corruption in a neoclassical growth model to verify how corruption affects capital accumulation in the long run. Our main hypotheses is that there is a leak in the public budget, so that government spending does not match its tax revenues. In that manner, corruption negatively affects the portion of tax revenues allocated as productive government spending, G. The more widespread the corruption, the smaller the government productive expenditure and, therefore, the smaller the output.

The model economy is closed, producing a single homogeneous good for both investment and consumption purposes. We assume that the government provides services as an input to private production, as in Barro (1990). The profit-maximing firms take as given the productive government and combine two (physically homogeneous) factors of production, capital and labor, by means of a Cobb-Douglas technology. As previously argued, in the presence of corruption, there is a leak in the government budget, so that the productive government expenditure, G, is lower than the tax revenue, T. The difference between them is the amount of tax revenue diverted for corruption. We assume that there is a fixed and large number H of households in the economy. At a given moment, there is a fraction $x = \frac{H_c}{H}$ of households that chooses to be corrupt, where H_c is the number of corrupt households, which varies over time according to an evolutionary dynamics to be shown in Section 4.2. Thus, the amount of money each corrupt household earns is

$$\frac{T - G}{H_c} = \phi(x)\frac{T}{H}, \tag{4.1}$$

where $\phi(x)$ gives the fraction of tax revenue which is diverted for corruption, formally it satisfies the following conditions: $\phi'(x) > 0$, $\phi(0) = 0$ and $\phi(1) = 1$. In sum, equation (4.1) states that the amount of tax revenue diverted for corruption depends on the relative frequency of corruption in the economy.

We will assume that the government revenue is given by a flat-rate income tax:

$$T = \tau Y, \tag{4.2}$$

where $\tau \in [0, 1] \subset \mathbb{R}$ is the income tax rate and Y is the output.

From now on, for simplicity, we will assume $\phi(x) = x$. Thus, using (4.2) we can rewrite (4.1) as follows:

$$G = (1 - x^2)\tau Y. \tag{4.3}$$

Therefore, when the degree of corruption, x, goes down, the productive government spending, G, rises. This happens because a higher number of households engaging in corruption increases the leak on the tax revenue. When corruption is absolute, $x = 1$, government expenditure is zero. When there is no corruption, $x = 0$, the tax revenue is fully used in productive government expenditure.

Dividing (4.3) by L, we have the government expenditure per capita:

$$g = (1 - x^2)\tau y, \tag{4.4}$$

where $g \equiv \frac{G}{L}$ and $y \equiv \frac{Y}{L}$ are the productive government expenditure per capita and the output per capita, respectively.

We will assume that the aggregate output is produced according to a Cobb-Douglas function that satisfies the Inada conditions, as follows:

$$Y = K^{\alpha} G^{\beta} L^{1-(\alpha+\beta)}, \tag{4.5}$$

where K and L are the aggregate quantities of capital and labor, respectively, and $\alpha \in (0, 1) \subset \mathbb{R}$ and $\beta \in (0, 1) \subset \mathbb{R}$ are parametric constants. Furthermore, $\alpha + \beta < 1$. Note that the Cobb-Douglas functional form assumed shows constant returns to scale and states that every input is equally necessary for production. In the absence of any of the inputs, the production is null.

The intensive form of the aggregate production function (4.5) is given by:

$$y = k^{\alpha} g^{\beta}, \tag{4.6}$$

where $k \equiv \frac{K}{L}$ is the capital stock per capita.

Substituting (4.4) in (4.6), we have

$$y = k^{a} [(1 - x^2)\tau]^{b}, \tag{4.7}$$

where

$$a \equiv \frac{\alpha}{1-\beta} \in (0,1) \subset \mathbb{R} \tag{4.8}$$

and

$$b \equiv \frac{\beta}{1-\beta} \in (0,1) \subset \mathbb{R}. \tag{4.9}$$

The disposable income in this economy is not only the net income after taxes, $Y - T$, but also the portion of the government revenue that was diverted by corruption, $T - G$. In short, $Y^d = Y - T + T - G = Y - G$. Thus, from (4.3) the aggregate disposable income is given by:

$$Y^d = [1 - (1 - x^2)\tau]Y. \tag{4.10}$$

As in Solow (1956), the savings are a fraction of the disposable income, $S = sY^d$, where $s \in [0,1] \subset \mathbb{R}$ is the constant savings rate. Using (4.10), the aggregate savings can be written as:

$$S = s[1 - (1 - x^2)\tau]Y. \tag{4.11}$$

Following Solow (1956), we will assume that the savings are fully converted to investment, so that $S = I$. Besides, we will suppose for simplicity, that there is no depreciation, so that the rate of change in the capital stock is equal to the aggregate investment, $\dot{K} = I$. Thus, using (4.11) we have:

$$\dot{K} = s[1 - (1 - x^2)\tau]Y. \tag{4.12}$$

Let us consider that the population, taken as equal to the labor force, L, grows at a constant rate $n > 0$, that is:

$$\frac{\dot{L}}{L} = n. \tag{4.13}$$

As $k \equiv \frac{K}{L}$, the growth rate of the capital per capita is given by:

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}. \tag{4.14}$$

Substituting (4.12) and (4.13) into (4.14), we obtain:

$$\dot{k} = s[1 - (1 - x^2)\tau]y - nk. \tag{4.15}$$

This equation states that the capital stock per capita will increase (decrease) only if the savings per capita is greater (smaller) than the investment necessary to offset the population growth.

Substituting (4.7) in (4.15) we obtain the fundamental equation of growth parameterized by the level of corruption:

$$\dot{k} = sh(x,\tau)k^a - nk, \tag{4.16}$$

where

$$h(x,\tau) \equiv [1-(1-x^2)\tau][(1-x^2)\tau]^b. \tag{4.17}$$

Note that $h(x,\tau)$ carries two different effects. The term $1-(1-x^2)\tau$ can be called *disposable income effect.* When corruption increases, the tax revenue leak will be greater, that accrues to corrupt households, which will increase their consumption and savings. However, the rise of corruption squeezes the productive government spending. This effect is represented by the term $[(1-x^2)\tau]^b$.

We can rewrite (4.16) in terms of growth rate as follows:

$$\hat{k} \equiv \frac{\dot{k}}{k} = sh(x,\tau)k^{a-1} - n. \tag{4.18}$$

In equilibrium, when the aggregate capital stock grows at the same rate as the population, that is when $\dot{k} = 0$, the steady state level of capital per capita can be seen as a function of the corruption level:

$$k^*(x) = \left[\frac{sh(x,\tau)}{n}\right]^{\frac{1}{1-a}}. \tag{4.19}$$

Furthermore, note that as k approaches zero, $\hat{k}$ goes toward infinity. As k goes to infinity, $\hat{k}$ approaches $-n$. The growth rate of the capital stock per capita is a decreasing and strictly convex function of the capital stock per capita:

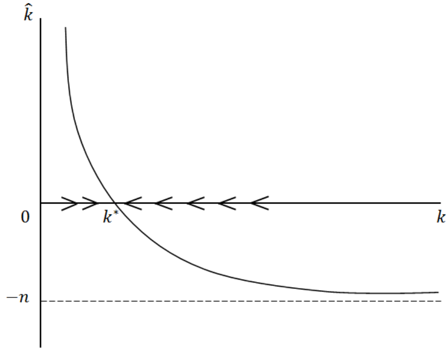
$$\frac{\partial\hat{k}}{\partial k} = (a-1)sh(x,\tau)k^{a-2} < 0. \tag{4.20}$$

Summarizing this information, we can draw a phase diagram for $\hat{k}$ for a given x, as shown in Figure 2. It is straightforward that $k^*(x)$ is a global attractor for a given x.

Before we analyze the relationship between the equilibrium capital stock per capita and the level of corruption, let us make some observations about the tax rate that will help understand such relationship.

Suppose the government's goal is to maximize the growth rate of

Figure 2 – Phase diagram for $\hat{k}$



Source: Created by the author.

the capital stock per capita in the economy choosing a tax rate. Since τ is limited in the interval $[0, 1] \subset \mathbb{R}$ and considering (4.9), then the tax rate that maximizes (4.18) is

$$\tau^* = \min\left\{\frac{\beta}{1-x^2}, 1\right\}. \tag{4.21}$$

Figure 3 shows the graph for τ^* with respect to x

Note that when corruption increases, τ^* increases until it reaches its maximum value, 1, where $x = \sqrt{1-\beta}$. In other words, when the level of corruption is higher, the leakage in the public budget is bigger and the tax rate that allows the government to achieve its optimum expenditure level is higher.

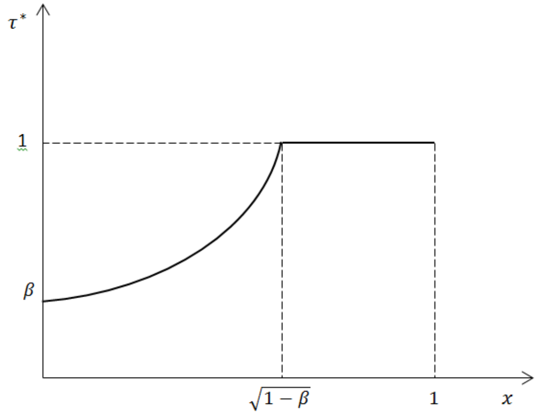
The impact on the stock of capital per capita of the level of corruption as well as the impact of the tax rate itself depend on the gap between the actual tax rate and the optimum tax rate. When the government overtaxes, it is reasonable to think that there is some inefficiency or leak in public spending that will affect the availability of resources for corruption. Formally:

$$\frac{\partial h}{\partial x} = 2x\tau b[(1-x^2)\tau]^{b-1}\left(\frac{\tau}{\tau^*} - 1\right) \tag{4.22}$$

and

$$\frac{\partial h}{\partial \tau} = b(1-x^2)[(1-x^2)\tau]^{b-1}\left(1 - \frac{\tau}{\tau^*}\right). \tag{4.23}$$

Figure 3 – Optimum tax rate as a function of the level of corruption



Source: Created by the author.

Their signs will depend on the actual tax rate practiced by the government in comparison to the optimal tax rate as sumarized below:

$$\frac{\partial h}{\partial x}\begin{cases} < 0, & \text{if } 0 \leq \tau < \tau^*, \\ = 0, & \text{if } \tau = \tau^*, \\ > 0, & \text{if } \tau^* < \tau \leq 1, \end{cases} \tag{4.24}$$

and

$$\frac{\partial h}{\partial \tau}\begin{cases} > 0, & \text{if } 0 \leq \tau < \tau^*, \\ = 0, & \text{if } \tau = \tau^*, \\ < 0, & \text{if } \tau^* < \tau \leq 1. \end{cases} \tag{4.25}$$

Thus, an increase in the tax rate can have three different impacts on the capital stock per capita. Deriving (4.19) with respect to τ we have

$$\frac{\partial k^*}{\partial \tau} = \frac{1}{1-a}\frac{sk^{*a}}{n}\frac{\partial h}{\partial \tau}. \tag{4.26}$$

Based on (4.25) and (4.24), we know that when $\tau = \tau^*$ then the capital stock per capita in the steady state is at its maximum. Thus, when the government chooses $\tau = \tau^*$, it is not only maximizing the growth rate of the capital stock per capita, it is also maximizing the stock of capital per capita and the output per capita.

The effects of the level of corruption on the capital stock per capita also depend on the ratio between the actual tax rate charged by the government and the optimum tax rate. Let us take the derivative of k^* with respect to x. This yields the expression below.

$$\frac{\partial k^*}{\partial x} = \frac{1}{1-a}\frac{sk^{*a}}{n}\frac{\partial h}{\partial x}. \tag{4.27}$$

Recall from (4.24), that, when the tax rate is below its optimum level, $\frac{\partial h}{\partial x}$ is negative. In that case, the level of corruption has a negative impact on the stock of capital per capita. On the other hand, when the tax rate is above its optimum level, $\frac{\partial h}{\partial x}$ is positive, and the level of corruption has a positive impact on k.

This result can be understood as analoguous to the "grease in the wheels" argument. When the tax rate is above the optimum level, there are more funds available for corruption, which induces corrupt behavior. Since we assumed a closed economy with a homogeneous savings rate, more corruption results in more available income. Therefore, corruption transforms inefficient government expenditure (above the optimum) into relatively more efficient private investment, leading to a higher capital stock per capita. On the other hand, when the tax rate is below the optimum, corruption would divert resources from a relatively more efficient government to the private sector, causing an adverse effect on the capital stock.

4.2 EVOLUTIONARY DYNAMICS OF CORRUPTION

To complete our analysis, let us now define how the level of corruption evolves over time.

We will assume that the society is composed of households who can engage in corrupt activities or not. The number of households is constant over time. There are H households, each one consisting of $\frac{L}{H}$ individuals. Since H is fixed, each household grows at the same rate n.

Each member of a non-corrupt household (here identified by the subscript n) has a disposable income given by:

$$y_n^d = \left[\frac{(1-\tau)Y}{H}\right]\frac{1}{\frac{L}{H}} = (1-\tau)y. \tag{4.28}$$

Because each household is concern about its $\frac{L}{H}$ members' payoffs, then the relevant payoff of an non-corrupt household is its per capita consumption. Considering (4.28), this consumption is given by:

$$u_n = (1 - s)(1 - \tau)y. \tag{4.29}$$

Substituting (4.7) into (4.29), we obtain:

$$u_n = (1 - s)(1 - \tau)[(1 - x^2)\tau]^b k^a. \tag{4.30}$$

In addition to the disposable income enjoyed by all members of society, each member of a corrupt household (identified by the subscript c), earns a share of the surplus (4.1), given by $\frac{T-G}{H_c}\frac{1}{\frac{L}{H}} = \frac{x\tau Y}{H}\frac{1}{\frac{L}{H}} = x\tau y$. Thus, their disposable income before considering the risk of punishment is given by:

$$(1 - \tau)y + x\tau y. \tag{4.31}$$

Let $\gamma > 0$ be the percentage of the income the corrupt individual will have to pay back in case of being punished. Note that γ can be greater than one. In that case, the punishment not only compensates for corruption but also makes the corrupt household forfeit an amount larger than his income. His disposable income after punishment would be

$$(1 - \gamma)[1 - (1 - x)\tau]y. \tag{4.32}$$

Let ϵ be the probability of punishment. Thus the income of each member of a corrupt household can be expressed as the following expected value:

$$y_c^d = \epsilon(1 - \gamma)[1 - (1 - x)\tau]y + (1 - \epsilon)[1 - (1 - x)\tau]y. \tag{4.33}$$

We can rewrite (4.33) simply as

$$y_c^d = (1 - \rho)[1 - (1 - x)\tau]y, \tag{4.34}$$

where $\rho \equiv \epsilon\gamma$ is the expected punishment.

The expected payoff of each member of a corrupt household is the portion of his expected disposable income reserved for consumption and can be written as:

$$u_c = (1 - s)(1 - \rho)[1 - (1 - x)\tau]y. \tag{4.35}$$

Substituting (4.7) in (4.35) and using (4.17) we have:

$$u_c = (1 - s)(1 - \rho)h(x, \tau)k^a. \tag{4.36}$$

Although in some situations the money from corruption is diverted to other illegal activities or kept in bank accounts in other countries, and therefore does not return as an investment to the domestic economy, here we assume a closed economy and the savings rate is homogeneous across households. That is, the saving rate of each household does not depend on its type (corrupt or non-corrupt).

To describe the learning process through which the evolutionary path happens, we will use the argument of the satisficing dynamics. We will assume that household j has a target return that it considers to be the minimum acceptable, denoted by μ^j. At instant t, if its current payoff $u^j = u_\ell$, with $\ell = c, n$, is higher than its target return, it will not consider changing its strategy. Otherwise, when $u^j < \mu^j$, it will become a strategy reviser. Let us assume that the target return is randomly and independently determined accross households, with a cumulative distribution function given by $F : \mathbb{R} \to [0, 1] \subset \mathbb{R}$, which is continuously differentiable and strictly increasing. That being said, the probability with which a household will find its current return unacceptable is

$$Prob(\mu^j > u^j) = 1 - Prob(\mu^j \leq u^j) = 1 - F(u^j). \tag{4.37}$$

Thus, the non-corrupt household is willing to change its strategy with probability $1 - F(u_n)$. It will do so by imitating the corrupt individuals, who exhibit the alternative strategy, at a rate given by the proportion of corrupt individuals it observes in the society, x. Hence, the inflow for the corrupt subpopulation is given by:

$$(1 - x)[1 - F(u_n)]x. \tag{4.38}$$

Analogously, the outflow from the corrupt subpopulation is given by:

$$x[1 - F(u_c)](1 - x). \tag{4.39}$$

Subtracting (4.39) from (4.38) we find the following satisficing evolutionary dynamics:

$$\dot{x} = x(1 - x)[F(u_c) - F(u_n)]. \tag{4.40}$$

As F is a strictly increasing function, an increase in the per capita consumption of non-corrupt households will cause the number of dissatisfied corrupt households to rise. Analogously, if there is a rise in the corrupt household's per capita consumption, the corrupt activity will become relatively more appealing and the number of dissatisfied non-corrupt households will rise. In sum, the evolutionary dynamic in

(4.40) reflects the behavior of a selection mechanism according to which the proportion of corrupt households varies positively with the relative fitness of such a strategy.

4.3 EQUILIBRIUM ANALYSIS

We will now show that our model has two monomorphic (pure strategy) equilibria where only one of the strategies survive and one long run polymorphic (mixed strategy) equilibrium, where both corrupt and non-corrupt strategies survive in the long run. Then, we will analyze the stability of each of these equilibria.

The state transition of the economy is determined by the ordinary differetial equations (4.16) and (4.40), which are repeated here by convenience:

$$\begin{cases} \dot{x} = x(1-x)\big[F\big(u_c(x,k(x))\big) - F\big(u_n(x,k(x))\big)\big], \\ \dot{k} = sh(x,\tau)k^a - nk. \end{cases} \tag{4.41}$$

The state space is defined by $\Theta \equiv \{(x,k) \in \mathbb{R}^2 : 0 \leq x \leq 1, k \geq 0\}$

It is straightforward that $x = 0$ and $x = 1$ both satisfy $\dot{x} = 0$. These are the two monomorphic equilibria.

When being corrupt or non-corrupt yield the same expected payoff, that is, when $u_c - u_n = 0$, the system reaches a polymorphic equilibria. In this kind of equilibrium there might still exist dissatisfied households that are willing to change their strategies, but the outflow of both subpopulations will be equal. Subtracting (4.29) from (4.35) we have

$$u_c - u_n = (1-s)y[(1-\rho)\tau x - \rho(1-\tau)]. \tag{4.42}$$

When the difference between the payoffs is equal to zero, the solution x^* for (4.42), which is the polymorphic equilibrium, is

$$x^* = \frac{1-\tau}{\tau}\frac{\rho}{1-\rho} \tag{4.43}$$

The conditions according to which $x^* \in (0,1) \subset \mathbb{R}$ are summarized in Table 1.

To show that the system is in equilibrium in each of the cases described above, we need to show for what k values the condition $\dot{k} = 0$ is satisfied. More specifically, when all the households choose to play

Table 1 – Conditions for existence of x^*

$\rho = 0$	$x^* = 0$
$0 < \rho < \tau$	$x^* \in (0,1) \subset \mathbb{R}$
$\rho = \tau$	$x^* = 1$
$\rho > \tau$	$\nexists x^* \in (0,1) \subset \mathbb{R}$

Source: Created by the author

the non-corrupt strategy, $x = 0$, the equilibrium capital stock per capita in this economy is, from (4.19):

$$k^*(0) = \left[\frac{s(1-\tau)\tau^b}{n}\right]^{\frac{1}{1-a}}. \tag{4.44}$$

When all the households choose to play the corrupt strategy, $x = 1$, the equilibrium capital stock per capita is:

$$k^*(1) = 0. \tag{4.45}$$

Finally, in the polymorphic equilibrium, when a portion of the households choose to be corrupt and the other portion does not, the equilibrium capital stock per capita is:

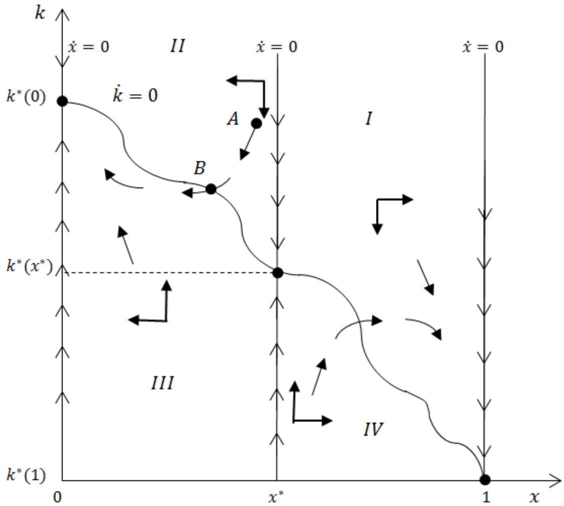
$$k^*(x^*) = \left[\frac{sh(x^*, \tau)}{n}\right]^{\frac{1}{1-a}}. \tag{4.46}$$

We know, from (4.19), that for all values of k^*, $\dot{k} = 0$. We can then draw the isocline $\dot{k} = 0$ in Figure 4. In $\dot{k} = 0$, the impact of a small change in k over $\dot{k}$ is given by:

$$\frac{\partial \dot{k}}{\partial k} = (a-1)n < 0, \tag{4.47}$$

which is negative, since by assumption a is strictly less than 1. Suppose the economy begins in a point $k_0 = k^*(x)$, that is, k_0 is on the isocline $\dot{k} = 0$. A positive shock in k, so that k now is above the isocline $\dot{k} = 0$, will cause the economy to move back towards the isocline $\dot{k} = 0$, as (4.47) states. This movement is represented by the arrows pointing down on regions I and II in Figure 4. Analogously, if there is a negative shock in k, the economy will again move back towards the isocline $\dot{k}$. This movement is represented by the arrows pointing up on regions III and IV in Figure 4.

Figure 4 – Phase diagram for k and x, when $0 < \rho < \tau$



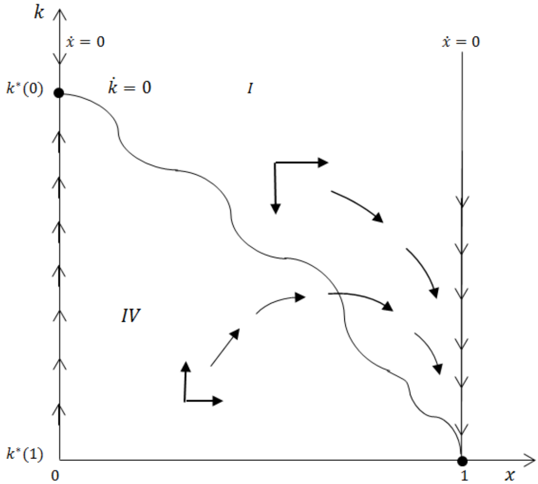
Source: Created by the author.

We can draw three vertical isoclines for when $\dot{x} = 0$. One for $x = 0$, one for $x = 1$ and a last one for $x = x^*$. We know that in the polymorphic equilibrium, when $x = x^*$, from (4.42), $u_c - u_n = 0$ and hence $\dot{x} = 0$. Thus, when $0 < x < x^*$, we know that $u_c - u_n < 0$ and therefore $\dot{x} < 0$. This is represented by the arrows pointing to the left in regions II and III in Figure 4. Analogously, when $x^* < x < 1$, $u_c - u_n > 0$ and $\dot{x} > 0$. This is represented by the arrows pointing to the right in regions I and IV in Figure 4.

Equilibrium $(x^*, k^*(x^*))$ is a saddle point, while equilibria $(0, k^*(0))$ and $(0, 1)$ are local attractors. The stable arm of x^* is the boundary line between the attraction basins of the two monomorphic equilibria. The proof for the stability of the equilibria $(0, k^*(0))$ and $(0, 1)$ are shown in Apendix A and Apendix B, respectively.

When the expected punishment, ρ, moves towards zero, the boundary line x^* moves towards $x = 0$, increasing the attraction basin of the full corruption equilibrium. Alternatively, when ρ moves towards τ, the boundary x^* moves towards $x = 1$, increasing the attraction basin of the equilibrium in absence of corruption. The extreme situations, when $\rho = 0$ and when $\rho = \tau$, are shown in Figure 5 and Figure 6, respectively.

Figure 5 – Phase diagram for k and x, when $\rho = 0$



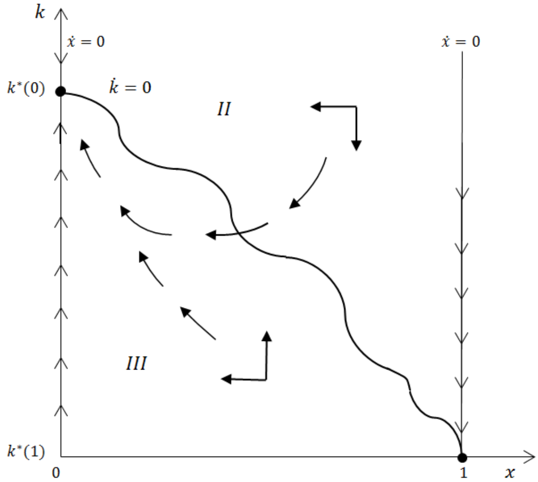
Source: Created by the author.

When the tax rate increases, for a given expected punishment, the polymorphic equilibrium level of corruption decreases, increasing the attraction basin for the full corruption equilibrium. If the tax rate decreases, x^* increases, increasing the attraction basin for the equilibrium with no corruption.

The relationship between τ and τ^* does not affect our conclusions about the stability of the equilibria, although it might affect the transition dynamics through which the equilibria are reached. When τ is greater than β, the demarcation curve $\dot{k} = 0$ can be nonmonotonic. When that happens, although k converges to the equilibria, it might not converge in a monotonic manner.

Differently from the original Solow-Swan model, in which the transition dynamics on the stationary state is monotonic, in our model the dynamics is not monotonic. If the starting point in the economy is, for example, point A in Figure 4, where k is above its stationary level, the effect of the productive government expenditure exceeds the effect of the available income, in a way that even when corruption is declining, we observe a fall in the stock of capital per capita. However, after a certain point with low enough corruption (point B), the available income effect starts to exceed the productive government expenditure effect, in a way that the agregate savings now generate gross investment

Figure 6 – Phase diagram for k and x, when $\rho \geq \tau$



Source: Created by the author.

that overcomes breakeven investment, leading to a rise on the stock of capital per capita.

4.4 FINAL REMARKS

The stability of the equilibria depends on the relationship between the expected punishment, ρ, and the tax rate, τ. Three cases arise. When the expected punishment is in between zero and the tax rate, there exists a polymorphic (mixed) equilibrium, x^*. This equilibrium is, however, a saddle point. If the dynamic starts in a level of corruption below x^*, it leads the economy towards the equilibrium where corruption is erradicated. On the other hand, if the dynamic starts in a level of corruption above x^*, it leads the economy to a collapse, where corruption is the norm and the capital stock per capita as well as the aggregate output are null.

This extreme result is due to our choice concerning the production function functional form. The Cobb-Douglas technology implies that every factor of production is essential. Therefore, when the government revenue is fully diverted as corruption, so that G is zero, the economy collapses. For future research we suggest using a functional form that allows for zero government expenditure without an economical breakdown, which is a more realistic premise.

The second case happens when the expected punishment is equal to or greater than the tax rate. In this case, when the expected punishment is sufficiently high, the dynamic leads to the equilibrium where corruption is erradicated. That is, the expected punishment is able to control outright corruption.

In the third case, when households expect no punishment for corrupt acts, we have that corruption becomes the norm in the society and again the economy collapses.

Both variables, the expected punishment and the tax rate, are under the control of the public sector. However, the former is less flexible and demands more complex institutional reforms to be changed. The tax rate, in contrast, can be adjusted through relatively less complicated decisions. If the tax rate is too high, the expected punishment sufficient to make the full corruption equilibrium unstable and eliminate the mixed equilibrium, eradicating corruption, must be greater. On the other hand, a smaller tax rate guarantees that even relatively small expected punishments could erradicate corruption.

5 CONCLUSION

We analyze some of the features of corruption as well as their implications in two different models from an evolutionary games approach. We believe that evolutionary games are a well suited tool to deal with the corruption issue in economics, since they allow us to consider a scenario where a large group of individuals makes decisions simutaneosly with a bounded understanding of the consequences of their actions.

In the first model we devised an evolutionary game with a homogeneous population where individuals may choose to behave as a producer or as a corrupt citizen who expropriates the taxes payed to the government. We show that corruption will always be present in this context and its relative frequency will depend on the probability of being punished, the size of the punishment, the tax rate and the marginal cost of the productive strategy. This result points to the importance of the public sector in fighting corruption, since many of these variables are under control of the public choice or can be affected by it.

More specifically, as the chance of impunity increases (as ρ gets closer to one) the proportion of corrupt individuals in the long run equilibrium increases. In the same sense, as the tax rate increases, the proportion of corrupt individuals in the equilibrium also increases. That is due to two effects. On one hand, a higher tax rate discourages private initiative for legal activities. On the other hand it makes the amount of money available for corruption bigger, encouraging such a practice. These results suggest that a strong government determined to combat corruption may lower the incidence of such a practice. On the other hand, a big government in the sectors that do not affect the probability of punishment, tend to stimulate corruption. These results corroborate the empirical evidence found by Goel e Nelson (2010) that bigger governments are associated with higher levels of corruption, yet more intervention might increase the likelihood of being punished.

The size of the punishment, ε, as it was expected, affects corruption negatively, as it represents the direct costs of being dishonest. A higher marginal cost makes it less attractive to be a producer, raising the level of corruption in the long run. Finally, a positive demand shock causes the level of corruption to fall in the long run.

In the second model we augmented the Solow-Swan model with productive government spending to consider for corruption. We then developed an evolutionary game with two dynamic equations. The first describes the change in the capital stock per capita and the second describes the evolution on the population of corrupt individuals over time.

Three situations arise in the model depending on the interaction between the tax rate and the expected punishment. When the expected punishment is non-zero but too low, below the tax rate, there exists a polymorphic equilibrium. This equilibrium is a saddle point, that is, if the dynamic starts in a level of corruption below x^*, it leads the economy towards the equilibrium where corruption is erradicated. On the other hand, if the dynamic starts in a level of corruption above x^*, it leads the economy to a collapse, where corruption is the norm and the capital stock per capita as well as the aggregate output are null. Intuitively, the mixed equilibrium is a watershed. If the system starts at a point below the mixed equilibrium, it will be attracted to a corruption-free situation. If it starts above the mixed equilibrium, the dynamics will lead to a situation where everyone chooses to act in a corrupt manner.

The second case happens when the expected punishment is equal to or greater than the tax rate. In this case, when the expected punishment is sufficiently high, the dynamic leads to the equilibrium where corruption is erradicated. That is, the expected punishment is able to control outright corruption.

In the third case, when households expect no punishment for corruption acts, we have that corruption becomes the norm in the society and the economy collapses.

Our models are only an attempt to help to understand the phenomenon of corruption and how it affects economic growth and try to explain why it is so persistent in societies. The results we found corroborate some of the empirical evidence in the literature. For future research we suggest developing our second model for a different production function technology, where the government expenditure has a positive but not crucial role in the output. Furthermore, we suggest the expected punishment and the tax rate to be endogenized as the government becomes a player in the game. We also suggest our results to be tested empirically.

APPENDIX A – Proof of the local asymptotic stability of the equilibrium $(0, k^*(0)) \in \Theta$

Let $(x_0, k_0) \equiv \big(x(0), k(0)\big) \in \Theta$ be the initial state of the economy. In order to show that for any initial condition $(x_0, k_0) \in \{(x, k) \in \Theta : 0 < x < x^*\}$ the economy moves asymptotically toward the equilibrium $(0, k^*(0)) \in \Theta$, we need to express the dynamic system (4.41) in terms of the deviation from the equilibrium $(0, k^*(0)) \in \Theta$ to apply the second (or direct) method of Lyapunov for stability (GANDOLFO, 1996). As the state variable x vanishes at this equilibrium, it is already the deviation of the level of corruption from the steady state $(0, k^*(0)) \in \Theta$. Let $\tilde{k} \equiv k - k^*(0)$ be the deviation of state variable k from its value at the equilibrium $(0, k^*(0)) \in \Theta$. Thus, the dynamic system (4.41) can be rewritten in terms of the variables x and $\tilde{k}$ as follows:

$$\begin{cases} \dot{x} = x(1-x)\big[F\big(u_c(x, \tilde{k} + k^*(0))\big) - F\big(u_n(x, \tilde{k} + k^*(0))\big)\big] \\ \dot{\tilde{k}} = sh(x, \tau)\big(\tilde{k} + k^*(0)\big)^a - n\big(\tilde{k} + k^*(0)\big), \end{cases} \tag{A.1}$$

where

$$u_c\big(x, \tilde{k} + k^*(0)\big) = (1-s)(1-\rho)[1-(1-x)\tau] \big(\tilde{k} + k^*(0)\big)^a [(1-x^2)\tau]^b, \tag{A.2}$$

and

$$u_n\big(x, \tilde{k} + k^*(0)\big) = (1-s)(1-\tau)\big(\tilde{k} + k^*(0)\big)^a [(1-x^2)\tau]^b. \tag{A.3}$$

If the economy begins in region II, that is, if $(x_0, k_0) \in R_{II} \equiv \{(x, k) \in \Theta : 0 < x < x^*, k > k^*(x)\}$, as t increases, the economy reaches a point at $\dot{k} = 0$ locus. At this point the economy reaches the following (bounded and closed) compact and positive invariant set:

$$\Omega_{III} = \{(x, k) \in \Theta : 0 \leq x \leq x_0 < \min\{x^*, 1\}, \min\{k_0, k^*(x_0)\} \leq k \leq k^*(x), \tag{A.4}$$

or in terms of the new coordinate $(x, \tilde{k})$:

$$\tilde{\Omega}_{III} = \{(x, \tilde{k}) \in \mathbb{R}^2 : 0 \leq x \leq x_0 < \min\{x^*, 1\}, \min\{k_0 - k^*(0), k^*(x_0) - k^*(0)\} \leq k \leq k^*(x) - k^*(0)\}. \tag{A.5}$$

If the economy begins in region III, that is, if $(x_0, k_0) \in R_{III} \equiv \{(x, k) \in \Theta : 0 < x < x^*, k \leq k^*(x)\}$, it is already in the set Ω_{III}.

Consider the following continuously differentiable function:

$$V(x,\tilde{k}) = \frac{\tilde{k}^2 + x^2}{2}. \tag{A.6}$$

V is clearly positive definite, since $V(x,\tilde{k}) > 0$ for any $(x,\tilde{k}) \neq 0$ and $V(0,0) = 0$.

As it has already been shown (cf. Figure 4), we know that $\dot{x} < 0$ and $\dot{k} > 0$ for all $(x,k) \in R_{III} \equiv \{(x,k) \in \Theta : 0 < x < x^*, k \leq k^*(x)\}$. Therefore, since $\Omega_{III} \subset R_{III}$, it follows that $\dot{x} < 0$ and $\dot{\tilde{k}} = \dot{k} > 0$ for all $(x,k) \in \Omega_{III}$ and, consequently, $\dot{x} < 0$ and $\dot{\tilde{k}} = \dot{k} > 0$ for all $(x,\tilde{k}) \in \tilde{\Omega}_{III}$. Considering all these properties, we are able to infer that:

$$\dot{V} = \tilde{k}\dot{k} + x\dot{x} < 0 \tag{A.7}$$

for all $(x,k) \in \tilde{\Omega}_{III}$. Thus, (A.6) is a Lyapunov function and the equilibrium $(0, k^*(0))$ is locally asymptotically stable. In other words, if the economy begins in regions II or III, it converges towards the equilibrium $(0, k^*(0)) \in \Theta$.

APPENDIX B – Proof of the local asymptotic stability of the equilibrium $(1, 0) \in \Theta$

Let $(\boldsymbol{x_0}, \boldsymbol{k_0}) \equiv \big(\boldsymbol{x}(\mathbf{0}), \boldsymbol{k}(\mathbf{0})\big) \in \boldsymbol{\Theta}$ be the initial state of the economy. It has already been shown that, if $(\boldsymbol{x_0}, \boldsymbol{k_0}) \in \{(\boldsymbol{x}, \boldsymbol{k}) \in \boldsymbol{\Theta} : \boldsymbol{x} = \mathbf{1}$ or $\boldsymbol{k} = \mathbf{0}\}$, the economy will converge asymptotically towards the equilibrium $(\mathbf{1}, \mathbf{0}) \in \boldsymbol{\Theta}$, as represented in the Figure 4. In order to show that for any initial condition $(\boldsymbol{x_0}, \boldsymbol{k_0}) \in \{(\boldsymbol{x}, \boldsymbol{k}) \in \boldsymbol{\Theta} : \boldsymbol{x^*} < \boldsymbol{x} < \mathbf{1}\}$ the economy moves asymptotically toward the equilibrium $(\mathbf{1}, \mathbf{0}) \in \boldsymbol{\Theta}$, we need to express the dynamic system (4.41) in terms of the deviation from the equilibrium $(\mathbf{1}, \mathbf{0}) \in \boldsymbol{\Theta}$ to apply the second (or direct) method of Lyapunov for stability (GANDOLFO, 1996). As the state variable $\boldsymbol{k}$ vanishes at this equilibrium, it is already the deviation of capital stock per capita from the steady state $(\mathbf{1}, \mathbf{0}) \in \boldsymbol{\Theta}$. Let $\tilde{\boldsymbol{x}} \equiv \boldsymbol{x} - \mathbf{1}$ be the deviation of state variable $\boldsymbol{x}$ from its value at the equilibrium $(\mathbf{1}, \mathbf{0}) \in \boldsymbol{\Theta}$. Thus, the dynamic system (4.41) can be rewritten in terms of the variables $\tilde{\boldsymbol{x}}$ and $\boldsymbol{k}$ as follows:

$$\begin{cases} \dot{\tilde{x}} = -\tilde{x}(1+\tilde{x})\big[F\big(u_c(1+\tilde{x},k)\big) - F\big(u_n(1+\tilde{x},k)\big)\big] \\ \dot{k} = sh(1+\tilde{x},\tau)k^a - nk, \end{cases} \tag{B.1}$$

where

$$u_c(1+\tilde{x},k) = (1-s)(1-\rho)[1+\tilde{x}\tau]k^a\big[(1-(1+\tilde{x})^2)\tau\big]^b, \tag{B.2}$$

$$u_n(1+\tilde{x},k) = (1-s)(1-\tau)k^a\big[(1-(1+\tilde{x})^2)\tau\big]^b \tag{B.3}$$

and

$$h(1+\tilde{x},k) = [1-(1+\tilde{x})^2\tau](-\tilde{x}\tau)^b. \tag{B.4}$$

If the economy begins in region $\boldsymbol{IV}$, that is, if $(\boldsymbol{x_0}, \boldsymbol{k_0}) \in \boldsymbol{R_{IV}} \equiv \{(\boldsymbol{x}, \boldsymbol{k}) \in \boldsymbol{\Theta} : \boldsymbol{x^*} < \boldsymbol{x} < \mathbf{1}, \mathbf{0} < \boldsymbol{k} < \boldsymbol{k^*}(\boldsymbol{x})\}$, as $\boldsymbol{t}$ increases, the economy reaches a point at $\dot{\boldsymbol{k}}$ locus. At this point the economy reaches the following (bounded and closed) compact and positive invariant set:

$$\begin{aligned} \Omega_I = \{(x,k) \in \Theta : \max\{0, x^*\} < x_0 \le x \le 1, \\ k^*(x) \le k \le \max\{k_0, k^*(x_0)\}\}, \end{aligned} \tag{B.5}$$

or in terms of the new coordinates $(\tilde{\boldsymbol{x}}, \boldsymbol{k})$:

$$\begin{aligned} \tilde{\Omega}_I = \{(\tilde{x},k) \in \mathbb{R}^2 : \max\{-1, x^*-1\} < x_0 - 1 \le \tilde{x} \le 0, \\ k^*(x) \le k \le \max\{k_0, k^*(x_0)\}\}, \end{aligned} \tag{B.6}$$

If the economy begins in region I, that is, if $(\boldsymbol{x_0}, \boldsymbol{k_0}) \in \boldsymbol{R_I} \equiv \{(\boldsymbol{x}, \boldsymbol{k}) \in \boldsymbol{\Theta} : \boldsymbol{x}^* < \boldsymbol{x} < \mathbf{1}, \boldsymbol{k} \geq \boldsymbol{k}^*(\boldsymbol{x})\}$, it is already in the set $\boldsymbol{\Omega_I}$.

Consider the following continuously differentiable function:

$$\boldsymbol{V}(\tilde{\boldsymbol{x}}, \boldsymbol{k}) = \frac{\boldsymbol{k}^{\mathbf{2}} + \tilde{\boldsymbol{x}}^{\mathbf{2}}}{\mathbf{2}}. \tag{B.7}$$

$\boldsymbol{V}$ is clearly positive definite, since $\boldsymbol{V}(\tilde{\boldsymbol{x}}, \boldsymbol{k}) > \mathbf{0}$ for any $(\tilde{\boldsymbol{x}}, \boldsymbol{k}) \neq \mathbf{0}$ and $\boldsymbol{V}(\mathbf{0}, \mathbf{0}) = \mathbf{0}$.

As it has already been shown (cf. Figure 4), we know that $\dot{\boldsymbol{x}} > \mathbf{0}$ and $\dot{\boldsymbol{k}} < \mathbf{0}$ for all $(\boldsymbol{x}, \boldsymbol{k}) \in \boldsymbol{R_I} \equiv \{(\boldsymbol{x}, \boldsymbol{k}) \in \boldsymbol{\Theta} : \boldsymbol{x}^* < \boldsymbol{x} < \mathbf{1}, \boldsymbol{k} \geq \boldsymbol{k}^*(\boldsymbol{x})\}$. Therefore, since $\boldsymbol{\Omega_I} \subset \boldsymbol{R_I}$, it follows that $\dot{\tilde{\boldsymbol{x}}} = \dot{\boldsymbol{x}} > \mathbf{0}$ and $\dot{\boldsymbol{k}} < \mathbf{0}$ for all $(\boldsymbol{x}, \boldsymbol{k}) \in \boldsymbol{\Omega_I}$ and, consequently, $\dot{\tilde{\boldsymbol{x}}} = \dot{\boldsymbol{x}} > \mathbf{0}$ and $\dot{\boldsymbol{k}} < \mathbf{0}$ for all $(\boldsymbol{x}, \boldsymbol{k}) \in \tilde{\boldsymbol{\Omega}}_{\boldsymbol{I}}$. Considering all these properties, we are able to infer that:

$$\dot{\boldsymbol{V}} = \boldsymbol{k}\dot{\boldsymbol{k}} + \boldsymbol{x}\dot{\boldsymbol{x}} < \mathbf{0} \tag{B.8}$$

for all $(\boldsymbol{x}, \boldsymbol{k}) \in \tilde{\boldsymbol{\Omega}}_{\boldsymbol{I}}$. Thus, (B.7) is a Lyapunov function and the equilibrium $(\mathbf{1}, \mathbf{0})$ is locally asymptotically stable. In other words, if the economy begins in regions $\boldsymbol{I}$ or $\boldsymbol{IV}$, it converges towards the equilibrium $(\mathbf{1}, \mathbf{0}) \in \boldsymbol{\Theta}$.

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