Caderno Escolar N.º 237

SABATINA

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Res Dna Inocência 78 1º andar

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LIVRARIA SELBACH de Selbach & Cia.

FABRICA: Rua Dr. Timóteo, 416 — PORTO ALEGRE

Indústria Brasileira

"MATEMÁTICA"

10-3-61

Irracionais = $\sqrt{\ }$

Êles não possuem uma representação decimal exata: Ex: $\sqrt{2} = 1,4142\ldots$

Racionais } reais
Irracionais }

$a x^2 + b x + c = 0$

⊕ Equação do 2º grau com uma incógnita é quando o maior expoente

a, b, c = nºs reais = +, −, 0: coeficientes

x = incógnita

1) { b = 0
   { $a x^2 + c = 0$

2) { c = 0
   { $a x^2 + b x = 0$

3) b = 0, c = 0
   $a x^2 = 0$

} incompletas

Resolver equação é achar o valor para a incógnita x.

$a x^2 = -c$ $\qquad x = \pm\sqrt{-\frac{c}{a}}$

$x^2 = -\frac{c}{a}$ $\qquad x' = +\sqrt{-\frac{c}{a}}$

$x'' = -\sqrt{-\frac{c}{a}}$

$a\varphi^2 + b\varphi \,\cancel{+c} = 0$

$\varphi(a\varphi + b) = 0$

$\varphi = 0$

$a\varphi + b = 0$

$a\varphi = -b$

$\varphi = -\frac{b}{a}$

$(b = c = 0)$

$a\varphi^2 = 0$

$\varphi^2 = \frac{0}{a}$ $a \div 0$ impossivel

$\varphi^2 = 0$

$\varphi \cdot \varphi = 0$

$\varphi' = 0$

$\varphi'' = 0$

$\varphi^2 - 1 = 0$

$\varphi^2 = 0 + 1$

$\varphi^2 = 1$

$\varphi = \pm\sqrt{1}$

$\varphi' = +\sqrt{1} = 1$

$\varphi'' = -\sqrt{1} = -1$

$5\varphi^2 = 0$

$\varphi^2 = \frac{0}{5}$

$\varphi^2 = 0$

$\varphi \times \varphi = 0$

$\varphi' = 0$

$\varphi'' = 0$

$4\varphi^2 - 9 = 0$

$4\varphi^2 = 9$

$\varphi = \pm\sqrt{\frac{9}{4}}$

$\varphi' = \frac{3}{2}$

$\varphi'' = -\frac{3}{2}$

$-7\varphi^2 = 0$

$\varphi^2 = \frac{0}{-7}$

$\varphi^2 = 0$

$\varphi \times \varphi = 0$

$\varphi' = 0$

$\varphi'' = 0$

$1 + 8\varphi^2 = 1$ $\varphi' = 0$ $\varphi'' = 0$

$-3\varphi^2 + 27 = 0$

$-3\varphi^2 = -27$

$\varphi^2 = \frac{-27}{-3}$

$\varphi^2 = 9$

$\varphi = \pm\sqrt{9}$

$\varphi' = 3$

$\varphi'' = -3$

$\frac{\varphi^2}{4} = 0$ $\varphi^2 = \frac{0}{4}$ $0 \times 4 = 0$ $\varphi^2 = 0$

$\varphi' = 0$ $\varphi'' = 0$

$4\varphi^2 - 5 = 5\varphi^2 - 6$

$4\varphi^2 - 5\varphi^2 = 5 - 6$

$-\varphi^2 = -1 \; (-1)$

$\varphi^2 = 1$

$\varphi = \pm\sqrt{1}$

$\varphi' = +1$

$\varphi'' = -1$

$\frac{9\varphi^2}{5} + 3 = \frac{1}{2} + \varphi^2 \;\therefore\; \frac{18\varphi^2 + 30}{10} = \frac{5 + 10\varphi^2}{10}$

$18\varphi^2 + 30 = 5 + 10\varphi^2 \;\therefore\; 18\varphi^2 - 10\varphi^2 = 5 - 30$

$8\varphi^2 = -25$

$\varphi^2 = \frac{-25}{8}$ impossivel

$$\frac{5\varphi^2-1}{3}-\frac{3\varphi^2+5}{2}=\frac{1}{6}$$

$$10\varphi^2-2-6\varphi^2-15=1$$
$$10\varphi^2-6\varphi^2=2-15+1$$
$$4\varphi^2=-12$$
$$\varphi^2=\frac{-12}{4}$$

$$2(5\varphi^2-1)-3(2\varphi^2+5)=1$$
$$10\varphi^2-2-6\varphi^2-15=1$$

15-3-6 1

$$5\varphi^2-\frac{3}{4}=3\varphi^2+\frac{1}{8}$$
$$8(5\varphi^2)-2\times3=8(3\varphi^2)+1$$
$$40\varphi^2-24\varphi^2=6+1$$
$$16\varphi^2=7$$
$$\varphi^2=\frac{7}{16}$$
$$\varphi=\pm\sqrt{\frac{7}{16}} \qquad \varphi'=+\frac{\sqrt{7}}{4}$$
$$\varphi''=-\frac{\sqrt{7}}{4}$$

$$(2\varphi+1)^2-1=0$$
$$4\varphi^2+4\varphi+1-1=0$$
$$4\varphi^2+4\varphi=0$$
$$\varphi(4\varphi+4)=0$$
$$\varphi=0 \rightarrow \varphi'=0$$
$$4\varphi+4=0$$
$$4\varphi=-4$$
$$\varphi=-\frac{4}{4}$$
$$\varphi''=-1$$

$a\varphi^2+b\varphi+c=0$ completa

$c=0:\ a\varphi^2+b\varphi=0$
$b=0:\ a\varphi^2+c=0$
$b=c=0:\ a\varphi^2=0$
} incompletas

$$a\varphi^2+b\varphi=0$$
$$\varphi(a\varphi+b)=0$$
$$\varphi=0 \rightarrow \therefore \varphi'=0$$
$$a\varphi+b=0$$
$$a\varphi=-b$$
$$\varphi=-\frac{b}{a}$$
$$\varphi''=-\frac{b}{a}$$

17-3-61

$ax^2+bx+c=0$ $\quad a, b, c \neq 0$

$ax^2+bx=-c$ $\quad (4a)$

$4a^2x^2+4abx=-4ac$ $\quad (+b^2)$

$4a^2x^2+b^2+4abx+b^2=b^2-4ac$

$(2ax+b)^2=b^2-4ac$

$b^2-4ac=\Delta$

$\Delta>0$

$2ax+b=\pm\sqrt{b^2-4ac}$

$2ax=-b\pm\sqrt{b^2-4ac}$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x'=\frac{-b+\sqrt{\Delta}}{2a} \qquad x''=\frac{-b-\sqrt{\Delta}}{2a}$$ 1º caso

$\Delta=0$

$$x'=\frac{-b}{2a} \qquad x''=\frac{-b}{2a}$$ 2º caso

$\Delta<0$ imaginário

$a=2$
$b=9$
$c=5$

$ax-bx-c=0$

$2x-9x-5=0$

$x=-b\pm\sqrt{b^2-4ac}$

$$x=\frac{9\pm\sqrt{81-4.2(-5)}}{2.2}$$

$$x=\frac{9\pm\sqrt{81+40}}{4}$$

$$x=\frac{9\pm\sqrt{121}}{4}$$

$$x=\frac{9\pm11}{4}$$

$$x'=\frac{9+11}{4}$$

$$x'=\frac{20}{4}=5$$

$$x''=\frac{9-11}{4}=\frac{-2}{4}=\frac{-1}{2}$$

$y^2+8y+15=0$

$-x^2+10x-25=0 \quad (-1)$

$x^2-10x+25=0$

$$x=\frac{10\pm\sqrt{100-100}}{2}$$

$$x=\frac{10}{2}=5$$

$x'=5 \qquad x''=5$

21/3/61

$Z^2 + \frac{3}{32} = \frac{7Z}{8} \therefore 32Z^2 + 3 = 28Z$

$32Z^2 - 28Z + 3 = 0$

$Z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$Z = \frac{28 \pm \sqrt{28^2 - 4 \cdot 32 \cdot 3}}{2 \cdot 32}$

$Z = \frac{28 \pm \sqrt{784 - 12 \times 32}}{64}$

$Z = \frac{28 \pm 20}{64}$

$Z' = \frac{28+20}{64} = \frac{48}{64} = \frac{3}{4}$

$Z'' = \frac{28-20}{64} = \frac{8}{64} = \frac{1}{8}$

$(y-8)^2 + (y-1)^2 = (y+1)^2$

$y^2 - 16y + 64 + y^2 - 2y + 1 = y^2 + 2y + 1$

$y^2 - 16y + 64 + y^2 - 2y + 1 - y^2 - 2y - 1 = 0$

$y^2 - 20y + 64 = 0$ $a = 1$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $b = -20$

$c = 64$

$y = \frac{+20 \pm \sqrt{400 - 256}}{2} \therefore y = \frac{+20 \sqrt{144}}{2}$

[illegible]6

[illegible]6

$\frac{15}{y} - \frac{72 - 6y}{2y^2} = 4y^2 \therefore \frac{30y - 72 - 6y}{2y^2} = 4y^2$

$-4y^2 + 36y - 72 = 0 \; (-1)$

$4y^2 - 36y + 72 = 0$

$y = \frac{36 \pm \sqrt{1296 - 4 \cdot 4 \cdot 72}}{8}$

$y = \frac{36 \pm \sqrt{1296 - 1152}}{8}$

$y' = \frac{36 + \sqrt{144}}{8}$

$y' = 6$

$y'' = 3$

22-3-61

$ay^2 - by + c = 0$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta = b^2 - 4ac$

$\Delta > 0 \rightarrow$ reais $\neq$ s

$\Delta = 0 \; \frac{-b}{2a}$ reais = s

$\Delta < 0$ raízes imaginárias

$3y^2 - 10y + 3 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (-10)^2 - 4.3.3$

$\Delta = 100 - 36$

$\Delta = 64$ as raizes são reais e diferentes

$y^2 - 8y + 16 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (-8)^2 - 4.1.16$

$\Delta = 64 - 64 = 0$ raizes reais iguais

$-2y^2 + y - 5 = 0 \ (-1)$

$2y^2 - y + 5 = 0$

$\Delta = (-1)^2 - 4.a.c$

$\Delta = 1 - 40 = -39$ raizes imaginárias

Para que valôres de M a equação $y^2 - 4y + M = 0$

1) admite raizes reais desiguais

2) admite raizes reais iguais

3) não admite raiz

$\Delta = (-4)^2 - 4.1.M$

[illegible] $- 4M > 0$

[illegible] > -16

$M < 4$ reais desiguais

$16 - 4m = 0$

$-4m = -16$

$m = 4$ raizes reais iguais

$16 - 4m < 0$

$-4m < -16$

$m > 4$ não admite raiz

24-31-61

$y^2 - 8y - n = 0$

$\Delta = b^2 - 4ac$

$\Delta = (8)^2 - 4 \cdot 1 \cdot n > 0$

$\Delta = 64 - 4(-n)$

$64 + 4n > 0$

$4n > -64$

$\boxed{n > -16}$

$\Delta = 64 + 4n = 0$

$4n = -64$

$\boxed{n = -16}$

$\Delta < 0$

$64 + 4n < 0$

$4n < -64$

$\boxed{n < -16}$

$3y^2 - 10y + 3 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (-10)^2 - 4.3.3$

$\Delta = 100 - 36$

$\Delta = 64$ as raizes são reais e diferentes

$y^2 - 8y + 16 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (-8)^2 - 4.1.16$

$\Delta = 64 - 64 = 0$ raizes reais iguais

$-2y^2 + y - 5 = 0 \ (-1)$

$2y^2 - y + 5 = 0$

$\Delta = (-1)^2 - 4.a.c$

$\Delta = 1 - 40 = -39$ raizes imaginárias

Para que valôres de M a equação $y^2 - 4y + M = 0$

1) admite raizes reais desiguais

2) admite raizes reais iguais

3) não admite raiz

$\Delta = (-4)^2 - 4.1.M$

$16 - 4M > 0$

$-4M > -16$

$M < 4$ reais desiguais

$16 - 4m = 0$

$-4m = -16$

$m = 4$ raizes reais iguais

$16 - 4m < 0$

$-4m < -16$

$m > 4$ não admite raiz

24-31-61

$y^2 - 8y - n = 0$

$\Delta = b^2 - 4ac$

$\Delta = (8)^2 - 4.1.n > 0$

$\Delta = 64 - 4(-n)$

$64 + 4n > 0$

$4n > -64$

$\boxed{n > -16}$

$\Delta = 64 + 4n = 0$

$4n = -64$

$\boxed{n = -16}$

$\Delta < 0$

$64 + 4n < 0$

$4n < -64$

$\boxed{n < -16}$

$(m-1)x^2+2(1-m)x+3m=0$

$\Delta=b^2-4ac$

$2-2m^2-4(m-1)3m=0$

$4-8m+4m^2-(12m^2-12m)=0$

$4-8m+4m^2-12m^2+12m=0$

$-8m^2+4m+4=0$

$-2m^2+m+1=0\ (-1)$

$2m^2-m-1=0$

$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$m=\frac{1\pm\sqrt{1-4.2(-1)}}{4}$

$m=\frac{1\pm\sqrt{1+8}}{4}$

$m=\frac{1\pm3}{4}$

$m'=\frac{4}{4}=1$

$m''=\frac{1-3}{4}=-\frac{2}{4}=-\frac{1}{2}$

$x^2+2kx+7k^2=0$

$\Delta=b^2-4ac$

$b^2-4ac>0$

$4k^2-28k^2>0$

$-24k^2>0\ (-1)$

$24k^2<0$ impossível de resolver

$x^2+(k-1)^2x+k-2=0$

$\Delta=0$

$\Delta=b^2-4ac$

$k^2-2k+1-[4.1.(k-2)]=0$

$k^2-2k+1-4k+8=0$

$k^2-6k+9=0$

$k=\frac{+6\pm\sqrt{36-36}}{2a}$

$k=\frac{6}{2}$

$k'=3$

$x^2-6x+m=0$	$36-(4m)>0\ (-1)$
$m=\Delta>0$	$4m<36$
$\Delta<0$	$m<9$
$\Delta=0$	
$\Delta=b^2-4ac$	

$5>2\ (-2)$

$-10<-4$

$8>4\ (-2)$

$-4<-2$

4-4-61

Relação entre coeficiente eas raizes

$a\varphi^2+b\varphi+c=0$	$\varphi=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$\varphi'+\varphi''=\frac{-b}{a}$	$\varphi'=\frac{-b+\sqrt{b^2-4ac}}{2a}$
$\varphi'.\varphi''=\frac{c}{a}$	$\varphi''=\frac{-b-\sqrt{b^2-4ac}}{2a}$

$$\varphi'+\varphi''=\frac{-b+\sqrt{b^2-4ac}}{2}+\frac{-b-\sqrt{b^2-4ac}}{2a}=\frac{-2b}{2a}=\frac{-b}{a}$$

$$\varphi'.\varphi''=\frac{-b+\sqrt{b^2-4ac}}{2a}\times\frac{-b-\sqrt{b^2-4ac}}{2a}=$$

$$=\frac{b^2-(b^2-4ac)}{4a^2}=\frac{b^2-b^2+4ac}{4a^2}=\frac{4ac}{4a^2}=\frac{c}{a}$$

$\varphi^2+b\varphi+c=0$ $\quad a=1$

$\varphi'+\varphi''=\frac{-b}{a}$

$\varphi'.\varphi''=c$

$\boxed{\varphi^2-S\varphi+P=0}$

$\varphi'=6$ $\quad\varphi^2-13\varphi+42=0$

$\varphi''=7$

$\left.\begin{matrix}\varphi'=2\\ \varphi''=5\end{matrix}\right\}\ \varphi^2-7\varphi+10=0$

$\left.\begin{matrix}\varphi'=-1\\ \varphi''=1\end{matrix}\right\}\ \varphi^2-1=0$

$\left.\begin{matrix}\varphi'=-8\\ \varphi''=-7\end{matrix}\right\}\ \varphi^2+15\varphi+56=0$

5-4-61.

$\varphi^2-7\varphi+12=0$ $\quad\varphi'=4$ $\quad\varphi''=3$

$\varphi^2-\varphi-56=0$ $\quad\varphi'=-7$ $\quad\varphi''=+8$

$\varphi^2+8\varphi+12=0$ $\quad\varphi'=-6$ $\quad\varphi''=-2$

$\varphi^2-8\varphi+16=0$ $\quad\varphi'=4$ $\quad\varphi''=4$

$\varphi^2-18\varphi+45=0$ $\quad\varphi'=15$ $\quad\varphi''=3$

$a+b=4$ $\quad\varphi'+\varphi''=4$

$a\times b=12$ $\quad\varphi'.\varphi''=12$

$\varphi^2-4\varphi-12=0$

$$\varphi=\frac{4\pm\sqrt{4^2-4.1.-12}}{2}$$

$a+b=-10$

$a\ b=16$ $\quad -8-2=-10$

$\varphi^2+10\varphi+16=0$ $\quad -8\times-2=16$

$\frac{2}{3}$ e $\frac{3}{4} = \frac{8+9}{12} = \frac{17}{12}$ $\qquad \frac{6}{12}$

$x^2 - \frac{17}{12}x + \frac{1}{2} = 0 \therefore$

$12x^2 - 17x + 6 = 0$

$x' = 3+\sqrt{2}$

$x'' = 3-\sqrt{2}$

$x'+x'' = 3+\sqrt{2}+3-\sqrt{2} = 6$

$x' \times x'' = (3+\sqrt{2})(3-\sqrt{2}) = 9-2 = 7$

$\frac{2}{5} - \frac{3}{4} = \frac{8-15}{20} = -\frac{7}{20}$ $\qquad \frac{3}{10}$

$x^2 + \frac{7}{20}x - \frac{3}{10} = 0 \therefore 20x^2 + 7x - 6 = 0$

$\frac{-4-11}{2} = \frac{-8-11}{2} = -\frac{19}{2}$

$\frac{-4 \times -11}{2} = \frac{44}{2} = 22$

$x^2 + \frac{19}{2}x + 22 = 0 \therefore 2x^2 + 19x + 22 = 0$

$\sqrt{2}+1$ e $\sqrt{2}-1$

$x'+x'' = \sqrt{2}+1+\sqrt{2}-1 = 2\sqrt{2}$

$x' \times x'' = (\sqrt{2}+1)(\sqrt{2}-1) = 2-1 = 1$

$x^2 - 2\sqrt{2}x + 1 = 0$

12 - 4 - 61.

$3x(x+2) = (x-3)2x$

$3x^2 + 6x = 2x^2 - 6x$

$3x^2 - 2x^2 + 6x + 6x = 0$

$x^2 + 12x = 0$

$x(x+12) = 0$

$x' = 0$

$x+12 = 0$

$x'' = -12$

$\frac{3x^2-5}{8} = \frac{2x-1}{4} + \frac{1}{2} = \frac{3x^2-5}{8} = 4x-2+4$

$3x^2 - 4x - 7 = 0 \qquad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{4 \pm \sqrt{16+84}}{6} \qquad x = \frac{4 \pm \sqrt{100}}{6}$

$x' = \frac{4+10}{6} = \frac{14}{6} = \frac{7}{3}$

$x'' = \frac{4-10}{6} = -\frac{6}{6} = -1$

$\frac{2}{3}+\frac{3}{4}=\frac{8+9}{12}=\frac{17}{12}$

$\frac{2}{3}\times\frac{3}{4}=\frac{3}{6}$

$\varphi^2-\frac{17}{12}\varphi+\frac{3}{6}=0 \therefore$ ~~$12\varphi^2$~~

$12\varphi^2-17\varphi+6=0$

$\varphi^2-(K-1)\varphi+K-2=0.$

$\Delta=0 \quad \Delta=b^2-4ac$

$(K-1)^2-4(K-2)=0$

$K^2-2K+1-4K+8=0$

$K^2-6K+9=0$

$K=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$K=\frac{6\pm\sqrt{36-36}}{2}$

$K=3$

$\frac{9\varphi^2}{5}+3=\frac{1}{2}+\varphi^2 \therefore 18\varphi^2+30=5+10\varphi^2$

$8\varphi^2+25=0$

$8\varphi^2=-25$

$\varphi^2=\frac{-25}{8}$

$-4 \qquad \frac{2}{9}$

$\varphi'+\varphi''=-4+\frac{2}{9}=\frac{-36+2}{9}=-\frac{34}{9}$

$9\varphi^2+34\varphi-8=0$

18-4-6 1

$\varphi^2-8\varphi+m=0$

1) $=\frac{4}{5}=\varphi'$	$\varphi'+\varphi''=-b$
2) nula	$\varphi'\cdot\varphi''=c$
3) inversa $\varphi'=\frac{1}{\varphi''}$	$\cdot\varphi'=\frac{4}{5}$
4) $\varphi'^2+\varphi''^2=40$	$\frac{4}{5}+\varphi''=8 \therefore 4+5\varphi''=40$

$5\varphi''=40-4=36$

$\varphi''=\frac{36}{5}$

$\frac{4}{5}\times\frac{36}{5}=\frac{144}{25}=m$

$\varphi^2-m\varphi+36=0$

1) $\varphi'=4$	$\varphi'=4 \quad \varphi'+\varphi''=m$
2) $\varphi'=\varphi''$	$\varphi'\cdot\varphi''=36$
3) $\varphi'=-\varphi''$	$4\cdot\varphi''=36$
4) $\varphi'=4\sqrt{3}$	$\varphi''=9$

$m=4+9=13$

$m=13.$

19-4-61

$\varphi' = 4$	$\varphi^2 - m\varphi + 36 = 0$
$\varphi' = \varphi''$	$\varphi' = \varphi''$
$\varphi' = -\varphi''$	$\varphi' + \varphi'' = m$
$\varphi' = 4\sqrt{3}$	$\varphi' \cdot \varphi'' = 36$ $\varphi'' \cdot \varphi'' = 36$

$\varphi''^2 = 36$ $\varphi'' = \sqrt{36}$ $\varphi'' = 6$

$\varphi' + \varphi'' = 12$ $m = 6 + 6 = 12$ $m = 12$

$\varphi' = -\varphi''$ $\varphi' + \varphi'' = 12$

$\varphi' \cdot \varphi'' = 36$

$-\varphi'' \cdot \varphi'' = 36$

$-\varphi''^2 = 36 \ (-1)$

$\varphi''^2 = -36$ impossível

$\varphi' = 4\sqrt{3}$

$\varphi' + \varphi'' = m$

$\varphi' \cdot \varphi'' = 36$

$4\sqrt{3} \cdot \varphi'' = 36$

$\varphi'' = \frac{36 \ 9}{4\sqrt{3}}$ $\varphi'' = \frac{9}{\sqrt{3}}$

$\varphi'' = \frac{9\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{9\sqrt{3}}{\sqrt{9}} = \frac{\overset{3}{9}\sqrt{3}}{3_1} = 3\sqrt{3}$

$= 4\sqrt{3} + 3\sqrt{3} = 7\sqrt{3}$ $m = 7\sqrt{3}$

$3\varphi^2 - (2m + \frac{1}{5})\varphi + 1 = 0$

$\varphi' + \varphi'' = \frac{1}{3}$	$5 = 10m + 1$
$\varphi' + \varphi'' = \frac{2m + \frac{1}{5}}{3}$	$10m + 1 - 5 = 0$
$\varphi' \cdot \varphi'' = \frac{1}{3}$	$10m - 4 = 0$
$\frac{1}{3} = \frac{2m + \frac{1}{5}}{3}$	$10m = 4$
$1 = 2m + \frac{1}{5}$	$m = \frac{4}{10}$ $m = \frac{2}{5}$

$\varphi^2 - 8\varphi + P = 0$

$\varphi' = 3\varphi''$

$\varphi' + \varphi'' = 8$

$\varphi' \cdot \varphi'' = P$

$3\varphi'' + \varphi'' = 8$

$4\varphi'' = 8$

$\varphi'' = \frac{8}{4}$

$\varphi'' = 2$

$\varphi' = 3\varphi''$

$\varphi' = 3 \times 2 = 6$

$P = 2 \times 6 = 12$

Soma $a + b$

Prod. ab

$\varphi' + \varphi'' =$

$\varphi^2 - (a+b)\varphi + ab = 0$

$\varphi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\varphi = \frac{(a+b) \pm \sqrt{(a+b)^2 - 4ac}}{2}$

22-4-61 recuperação

$y^2-(a+b)y+ab=0$

$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \therefore y=\frac{-b\pm\sqrt{(a+b)^2-4ac}}{2a}$

$y=\frac{a+b\pm\sqrt{a^2+2ab+b^2-4ab}}{2}$

$y=\frac{a+b\pm\sqrt{a^2-2ab+b^2}}{2}$

$y=\frac{a+b\pm\sqrt{(a-b)^2}}{2}$

$y=\frac{a+b+a-b}{2} \therefore y'=\frac{a+b+a-b}{2}=\frac{2a}{2}=a$

$y''=\frac{a+b-a+b}{2}=\frac{2b}{2}=b \qquad y''=b$

Soma = 2a   Produto a^2-b^2

$y^2-2ay+a^2-b^2=0$

$y=\frac{b\pm\sqrt{b^2-4ac}}{2a} \therefore y=\frac{b\pm\sqrt{4a^2-4(a^2-b^2)}}{2a}$

$y=2a\pm\sqrt{4a^2-4a^2+4b^2} \therefore y=2a\pm\sqrt{4b^2}$

$y=\frac{2a+2b}{2} \quad 2 \quad y'=\frac{2a+2b}{2}=\frac{2(a+b)}{2}=a+b$

$y''=\frac{2a-2b}{2}=\frac{2(a-b)}{2}=a-b$

Soma = 2a   produto a^2-b

$y^2-2ay+a^2-b=0$

$y=\frac{b\pm\sqrt{b^2-4ac}}{2}$

$y=2a\pm\sqrt{(2a)^2-4(a^2-b)}$

$y=\frac{2a\pm\sqrt{4a^2-4a^2+4b}}{2}$

$y=\frac{2a\pm\sqrt{4b}}{2}$

$y=\frac{2a\pm2\sqrt{b}}{2}$

$y'=\frac{2(a+\sqrt{b})}{2} \therefore y'=a+\sqrt{b}$

$y''=\frac{2a-2\sqrt{b}}{2} \therefore y''=\frac{2(a-\sqrt{b})}{2} \quad y''=a-\sqrt{b}$

$\frac{1+\sqrt{21}}{2} \qquad \frac{1-\sqrt{21}}{2} \qquad \frac{1+1+\sqrt{21}-\sqrt{21}}{2}$

$\frac{2}{2}=1 \qquad \frac{(1+\sqrt{21})(1-\sqrt{21})}{4}=\frac{1-21}{4}$

$\frac{-20}{4}=5$

$y^2-y-5=0$

$\frac{a}{2}-\frac{b}{3} \therefore 3a-2b \qquad y^2-(3a+2b)y-ab=0$

$\frac{1}{a+b} \quad \frac{1}{a-b} \quad \frac{a-b+a+b}{a^2-b^2} \quad \frac{2a}{a^2-b^2}$

$\frac{1}{a^2-b^2} \qquad y^2-\frac{2a}{a^2-b^2}y+\frac{1}{a^2-b^2} \therefore$

$(a^2-b^2)y^2-2ay+1=0$

$u^2 - 7u + 12 = 0$

$+3 + 4 = +7$

$+3 \times +4 = +12$

$u^2 - 2u - 120 = 0$

$-10 + 12 = +2$

$-10 \times +12 = -120$

$u^2 - 50u - 51 = 0.$

$51 - 1 = +50$

$51 \times -1 = -51$

$u^2 - 8u + 16 = 0$

$+4 + 4 = 8$

$+4 \times +4 = 16$

$u^2 + 8u + 12 = 0$

$-6 - 2 = -8$

$-6 \times -2 = +12$

$(5u^2 - 12u - 50)$

26-4-61

Trinômio do 2º = $au^2 + bu + c = 0$

Trinômio do 2º é um polinômio cujo a variável tem um expoente máximo 2; a, b e c são coeficientes da variável

$y = 5u^2 - 6u + 7$ $u = 2$

$y = 5(2^2) - 6(2) + 7$

$y = 5(4) - 6(2) + 7$

$y = 20 - 12 + 7$

$y = 15$

a: não pode ser nulo

a pode ser negativo ou positivo

$y = 5u^2 - 6u + 7$ $u = 4$

$y = 5.(16) - 6(4) + 7$

$y = 80 - 24 + 7 = 63$ $y = 63$

$y = 5.\frac{1}{4} - 6.\frac{1}{2} + 7$ $u = \frac{1}{2}$

$y = \frac{5}{4} - \frac{6}{2} + 7$ $\frac{5 - 12 + 28}{4} = \frac{21}{4}$

$y = \frac{21}{4}$

Raizes do trinômio

$au^2 + bu + c = 0$

raizes do trinômio são os valores das variável que anula o trinômio

$5u^2 - 6u + 7$

$u = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$u = \frac{6 \pm \sqrt{-104}}{2}$

$2x^2 - 9x - 5$ $\qquad x = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$x' = 5$

$x'' = -\dfrac{1}{2}$ $\qquad x = \dfrac{9 \pm \sqrt{81 + 40}}{4}$

$x'' = \dfrac{+9 - 11}{4} = -\dfrac{2}{4} = -\dfrac{1}{2}$

$x' = \dfrac{+9 + 11}{4} = \dfrac{20}{4} = 5$

$-x^2 + 10x - 25 \ (-1)$ $\qquad x = \dfrac{10 \pm \sqrt{100 - 100}}{2}$

$x^2 - 10x + 25 = 0$

$x' = 5$ $\qquad x' = \dfrac{10}{2} = 5$

$x'' = 5$ $\qquad x'' = \dfrac{10}{2} = 5$

28-4-61

$ax^2 + bx + c$

Decomposição do trinômio em fatôres do 1º grau.

$ax^2 + bx + c \therefore a\left(x^2 + \dfrac{bx}{a} + \dfrac{c}{a}\right) =$

$= a\left[x^2 - (x' + x'')x + x'x''\right] =$

$= a\left[x^2 - x'x - x''x + x'x''\right] =$

$= a\left[x(x - x') - x''(x - x')\right] =$

$= a(x - x')(x - x'')$

$ax^2 + bx + c = a(x - x')(x - x'')$

Decompor em fatôres do 1º e seg. trinômio

$y = 3x^2 - 21x + 36$ $\qquad x' + x'' = -\dfrac{b}{a}$

$\Delta = b^2 - 4ac$ $\qquad x' + x'' = \dfrac{21}{3}$

$\Delta = (21)^2 - 4 \cdot 3 \cdot 36$ $\qquad x' \cdot x'' = \dfrac{36}{3}$

$\Delta = 441 - 432 = 9 > 0$

$3x^2 - 21x + 36 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \therefore x = \dfrac{21 \pm \sqrt{441 - 432}}{6}$

$\dfrac{21 + 3}{6} = 4$ $\qquad x' = 4$

$\dfrac{21 - 3}{6} = 3$ $\qquad x'' = 3$

$3x^2 - 21x + 36 = 3(x - 4)(x - 3)$

raízes iguais $= a(x - x')(x - x')$

$= a(a - x)^2$

$-x^2 + 9x - 18 = 0$

$\Delta = -b^2 - 4a$

$\Delta = 81 - 72 = 9 > 0$

$x = \dfrac{-b \pm \sqrt{81 - 72}}{2}$ $\qquad x' = \dfrac{-9 + 3}{2} = -3$

$x'' = \dfrac{-9 - 3}{2} = -\dfrac{12}{2} = -6$

$2x^2-9x-5$ $\qquad x=\frac{b\pm\sqrt{b^2-4ac}}{2a}$

$x'=5$

$x''=-\frac{1}{2}$ $\qquad x=\frac{9\pm\sqrt{81+40}}{4}$

$x''=\frac{+9-11}{4}=-\frac{2}{4}=-\frac{1}{2}$

$x'=\frac{+9+11}{4}=\frac{20}{4}=5$

$-x^2+10x-25\ (-1)$ $\qquad x=\frac{10\pm\sqrt{100-100}}{2}$

$x^2-10x+25=0$

$x'=5$ $\qquad x'=\frac{10}{2}=5$

$x''=5$ $\qquad x''=\frac{10}{2}=5$

28-4-61.

ax^2+bx+c

Decomposição do trinômio em fatôres do 1º grau.

$ax^2+bx+c \therefore a(x^2+\frac{bx}{a}+\frac{c}{a})=$

$=a[x^2-(x'+x'')x+x'x'']=$

$=a[x^2-x'x-x''x+x'x'']=$

$=a[x(x-x'')-x'(x-x'')]=$

$=a(x-x')(x-x'')$

$ax^2+bx+c=a(x-x')(x-x'')$

Decompor em fatôres do 1º e seg. trinômio

$y=3x^2-21x+36$ $\qquad x'+x''=-\frac{b}{a}$

$\Delta=b^2-4ac$ $\qquad x'+x''=\frac{21}{3}$

$\Delta=(21)^2-4\cdot3\cdot36$ $\qquad x'\cdot x''=\frac{36}{3}$

$\Delta=441-432=9>0$

$3x^2-21x+36=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \therefore x=\frac{21\pm\sqrt{441-432}}{6}$

$\frac{21+3}{6}=4$ $\qquad x'=4$

$\frac{21-3}{6}=3$ $\qquad x''=3$

$3x^2-21x+36=3(x-4)(x-3)$

raizes iguais $=a(x-x')(x-x')$

$=a(a-x)^2$

$-x^2+9x-18=0$

$\Delta=-b^2-4a$

$\Delta=81-72=9>0$

$x=\frac{-b\pm\sqrt{81-72}}{2}$ $\qquad x'=\frac{-9+3}{2}=-3$

$x''=\frac{-9-3}{2}=-\frac{12}{2}=-6$

2-5-61

Decompor em fatôres do 1º grau

$y = 2x^2 - 12x + 18$

$2x^2 - 12x + 18 = 0$

$\Delta = b^2 - 4ac$

$\Delta = 144 - 144 = 0 \quad \Delta = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad x = \frac{b}{2a}$

$x = \frac{12}{4} = 3 \quad x' = x'' = 3$

$2x^2 - 12x + 18 = a(x - x')^2$

$= 2(x - x')^2$

$z = 2t^2 + 3t - 2 \quad 2t^2 + 3t - 2 = 0$

$\Delta = b^2 - 4ac \quad \Delta = 9 + 16 = 25$

$\Delta > 0 \quad t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-3 \pm \sqrt{25}}{4}$

$t = \frac{-3 + 5}{4} = \frac{2}{4} = \frac{1}{2} \quad t = \frac{1}{2}$

$t'' = \frac{-3 - 5}{4} = \frac{-8}{4} = -2$

$2t^2 + 3t - 2 = a(t - t')(t - t'')$

$2t^2 + 3t - 2 = a(t - \frac{1}{2})(t + 2)$

$2(\frac{2t - 1}{2})(t + 2) = (2t - 1)(t + 2)$

$y = x^2 - 3x - 10$

$x^2 - 3x - 10 = 0$

$\Delta = b^2 - 4ac \quad \Delta = 9 - (4 \times -10) \quad \Delta = 49$

$\Delta > 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad x = \frac{3 \pm \sqrt{49}}{2}$

$x = \frac{3 + 7}{2} = 5 \quad x' = 5$

$x'' = \frac{3 - 7}{2} = \frac{-4}{2} = -2 \quad x'' = -2$

$a(x - x')(x - x'')$

$x^2 - 3x - 10 = (x - x')(x - x'') = (x - 5)(x + 2)$

$y = x^2 - (m + n)x + mn$

$x^2 - (m + n)x + mn = 0$

$\Delta = b^2 - 4ac \quad \Delta = (m + n)^2 - 4mn.$

$m^2 + 2mn + n^2 - 4mn =$ (corrected: $-2mn$)

$\Delta = m^2 - 2mn + n^2 = (m - n)^2 > 0$

$x' = m \quad x^2 - (m + n)x + mn =$

$x'' = n \quad = (x - m)(x - n)$

3-5-61

Variação do sinal do trinômio

$y = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) =$

$\frac{b^2}{4a^2}$

$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right)$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \left(x + \frac{b}{2a}\right)^2 =$

$= x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} =$

$= x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} =$

$= \left(x + \frac{b}{2a}\right)^2 =$

$= a = \left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right] =$

$= -\frac{b^2}{4a^2} + \frac{c}{a} = \frac{-b^2 + 4ac}{4a^2} =$

$= -\frac{b^2 - 4ac}{4a^2}$

$ax^2 + bx + c = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right]$

v +

$\Delta < 0$ ∴ $y = a$ Sinais: $\pm y : \pm a$, $\mp y = \mp a$

$\Delta = 0$

$y = ax^2 + bx + c$

$y = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right]$

$\Delta > 0$ $y = a(x - x')(x - x'')$

$x > x'$, $x > x''$ $-\infty$ x' x' $+\infty$

$x > x'$, $x < x''$

$x < x'$, $x < x''$ ∴ $y = a(x - x')(x - x'')$ $-$ $-$

5-5-61

$\Delta > 0$ $y = a(x - x')(x - x'')$

x x x — $-\infty$ x' x'' $+\infty$

$y = a(x - x')(x - x'')$ $-$ $-$ } $x < x'$, $x < x''$

$+ = +$ $+$

$- = -$ $+$

$x > x'$, $x > x''$ } $y = a(x - x')(x - x'')$ $+$ $+$

$+ = +$ $+$

$- = -$ $+$

$x > x'$, $x < x''$ } $y = a(x - x')(x - x'')$ $+$ $-$

$- = +$ $-$

$+ = -$ $-$

$y \neq a$

$+ \neq -$

$- \neq +$

$y = x^2 - 5x + 6$

$x^2 - 5x + 6 = 0$

$\Delta = b^2 - 4ac \therefore \Delta = 25 - 24 = 1 \therefore \Delta = 1 \therefore \Delta > 0$

$x' = 2$

$x'' = 3$

$x \quad 2 \quad 3 \quad x$

$a = + \left\{ \begin{array}{l} x \text{ menor do que } 2 \\ x \text{ maior do que } 3 \end{array} \right\} y \text{ é positivo}$

$\left\{ \begin{array}{l} x \text{ maior do que } 2 \\ x \text{ menor do que } 3 \end{array} \right\} y \text{ é negativo}$

$y = -3x^2 - x + 2$

$\Delta = b^2 - 4ac \therefore \Delta = 1 - (-24) = 25 \therefore \Delta > 0$

$-3x^2 - x + 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{1 \pm 5}{-6}$

$x = -1$

$x'' = \frac{2}{3}$

$-1 \quad \frac{2}{3}$

$\left. \begin{array}{l} \text{para } x < -1 \\ \text{para } x > \frac{2}{3} \end{array} \right\} y \text{ será negativo}$

$\left. \begin{array}{l} \text{para } x > -1 \\ \text{para } x < \frac{2}{3} \end{array} \right\} y \text{ será positivo}$

9-5-61.

$y = 4x^2 + 4x + 1$

$\Delta = b^2 - 4ac \therefore \Delta = 16 - 16 = 0 \quad \Delta = 0$

Y é positivo

$y = -2x^2 + x - 4$

$\Delta = b^2 - 4ac \therefore \Delta = 1 - 32 = -31 \quad \Delta = -31$

Y é negativo

$y = x^2 - 25 \qquad x^2 = 25 \qquad x \pm \sqrt{25} \quad x \pm 5$

$x' = -5$

mesmo s. de a | s. contrario de a | mesmo s/a

$-5 \quad +5$

$x'' = 5$

$x < -5 \text{ e } x > 5 \} \; y \text{ é positivo}$

$-5 < x < 5 \} \; y \text{ é negativo}$

$y = -3z^2 + 12z \qquad -3z^2 + 12z = 0$

$z(-3z + 12) = 0$

$-3z + 12 = 0 \qquad (-3z = -12)(-1) = 3z = 12$

$z = \frac{12}{3} \quad z' = 0 \quad z'' = 4$

$- \quad + \quad -$

$0 \quad 4$

$z < 0 \text{ e } z > 4 \} \; y \text{ é neg}$

$0 < z < 4 \} \; y \text{ é positivo}$

$y = 15x^2 - x - 6$

$\Delta = b^2 - 4ac \quad \Delta = 16 - 4(15 \times 6)$

$\Delta = 16 + 360 \quad \Delta = 361 \quad \Delta > 0$

y é positivo

12-5-61

Determinar os valores de x

$y = x^2 - 6x + 8$

$\Delta = b^2 - 4ac \therefore \Delta = 36 - 32 = 4$ — 1º positivo

$\Delta = 4$ raizes reais desiguais — 2º negativo

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2} \quad x = \frac{6 \pm \sqrt{4}}{2}$ — 3º nulo.

$x = \frac{6 \pm 2}{2} \quad x' = \frac{6-2}{2} = \frac{4}{2} = 2$

$x' = 2$

$x'' = \frac{6+2}{2} = \frac{8}{2} = 4 \quad x'' = 4$

+ (2) − (4) +

y é positivo para $x < 2$ $x > 4$

y é negativo: $x > 2$ e $x < 4$

y é nulo: $x = 2$ e $x = 4$

$y = -3x^2 + x - 6$ — 1º pos.

$\Delta = b^2 - 4ac$ — 2º neg.

$\Delta = 1 - 72 = -71 \quad \Delta < 0$ — 3º nulo.

y é sempre negativo

$y = -2x^2 + 6 \quad -2x^2 = -6$

$2x^2 = 6 \quad x^2 = \frac{6}{2} \quad x^2 = 3$

$x = \pm\sqrt{3} \quad x' = -\sqrt{3} \quad x'' = +\sqrt{3}$

− ($-\sqrt{3}$) + ($+\sqrt{3}$) −

1) y é positivo $x > -\sqrt{3}$ e $x < \sqrt{3}$

2) y é negativo $x < -\sqrt{3}$ e $x > \sqrt{3}$

3) y é nulo $x = -\sqrt{3}$ e $x = \sqrt{3}$

$y = x^2 - 8x + 16$

$\Delta = b^2 - 4ac \therefore \Delta = 64 - 64 = 0 \quad \Delta = 0$

$x = \frac{8}{2} \quad x' = 4 \quad x'' = 4$

y é positivo para qualquer valor $\neq 4$. y nunca é neg.

y é nulo para $x = 4$.

$y = 3x^2 - 9x \quad 3x^2 - 9x = 0$

$x(3x - 9) = 0 \quad x = 0 \quad 3x - 9 = 0$

$3x = 9 \quad x = \frac{9}{3} = 3 \quad x' = 0 \quad x'' = 3$

+ (0) − (3) +

y é pos. para $x < 0$ e $x > 3$

y é neg para $x > 0$ e $x < 3$

y é nulo para $x = 0$ e $x = 3$

16-5-61

ψ' $a\psi^4 + b\psi^2 + c = 0$

ψ'' $\psi^2 = y$ $\quad \psi^4 = y^2$

ψ''' $a y^2 + b y + c = 0$

ψ'''' $y = -b \pm \sqrt{b^2-4ac}$

$\psi^2 = -b \pm \sqrt{b^2-4ac}$

$\psi = \pm\sqrt{\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

$\psi^4 - 5\psi^2 + 4 = 0$

$\psi = \pm\sqrt{\frac{5 \pm \sqrt{25-16}}{2}}$

$\psi = \pm\sqrt{\frac{5 \pm 3}{2}}$

$\psi' = +\sqrt{\frac{5+3}{2}} = \sqrt{4} = 2$

$\psi'' = -\sqrt{\frac{5+3}{2}} = -\sqrt{4} = -2$

$\psi''' = +\sqrt{\frac{5-3}{2}} = +\sqrt{\frac{2}{2}} = 1$

$\psi'''' = -\sqrt{\frac{5-3}{2}} = -\sqrt{\frac{2}{2}} = -1$

$\psi' = +\sqrt{\frac{-b+\sqrt{b^2-4ac}}{2a}}$

$\psi'' = -\sqrt{\frac{-b+\sqrt{b^2-4ac}}{2a}}$

$\psi''' = +\sqrt{\frac{-b-\sqrt{b^2-4ac}}{2a}}$

$\psi'''' = -\sqrt{\frac{-b-\sqrt{b^2-4ac}}{2a}}$

$\psi' = 2 \quad \psi'' = -2 \quad \psi''' = 1 \quad \psi'''' = -1$

$3\psi^4 - 25\psi^2 - 18 = 0$

$\psi = \pm\sqrt{\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

$\psi = \pm\sqrt{\frac{25 \pm \sqrt{625+216}}{2a}} \quad \psi = \pm\sqrt{\frac{25 \pm \sqrt{841}}{6}}$

$\psi' = +\sqrt{\frac{25+29}{6}} = \sqrt{\frac{54}{6}} = \sqrt{9} = 3 \quad \psi' = 3$

$\psi'' = -\sqrt{\frac{25+29}{6}} = -\sqrt{\frac{54}{6}} = -\sqrt{9} = -3 \quad \psi'' = -3$

$\psi''' = +\sqrt{\frac{25-29}{6}} = +\sqrt{\frac{-4}{6}} = \quad \psi''' = \sqrt{-\frac{2}{3}}$

$\psi'''' = -\sqrt{\frac{25-29}{6}} = -\sqrt{\frac{-4}{6}} \quad \psi'''' = -\sqrt{-\frac{2}{3}}$

17-5-61

$(4\psi^2-1)(\psi^2+1) = 26$

$4\psi^4 + 3\psi^2 - 1 = 26$

$4\psi^4 + 3\psi^2 - 1 - 26 = 0$

$\psi = \pm\sqrt{\frac{-b \pm \sqrt{b^2-4ac}}{2a}} \quad \psi = \pm\sqrt{\frac{-3 \pm \sqrt{9+432}}{8}}$

$\psi' = +\sqrt{\frac{-3+\sqrt{441}}{8}} = \sqrt{\frac{-3+21}{8}} = \sqrt{\frac{18}{8}} = \psi' = \frac{3}{2}$

$\psi'' = -\sqrt{\frac{-3+\sqrt{441}}{8}} = -\sqrt{\frac{-3+21}{8}} = -\sqrt{\frac{18}{8}} = \psi'' = -\frac{3}{2}$

$\psi''' = +\sqrt{\frac{-3-\sqrt{441}}{8}} = +\sqrt{\frac{-3-21}{8}} = +\sqrt{\frac{-24}{8}} \quad \psi''' = \sqrt{-3}$

$\psi'''' = -\sqrt{\frac{-3-\sqrt{441}}{8}} = -\sqrt{\frac{-3-21}{8}} = -\sqrt{\frac{-24}{8}} \quad \psi'''' = -\sqrt{-3}$

$(y^2-1)^2+(y^2-3)^2=20$

$y^4-2y^2+1+y^4-6y^2+9-20=0$

$2y^4-8y^2-10=0 \therefore y^4-4y-5=0$

$y=\pm\sqrt{\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$ $y=\pm\sqrt{\frac{8\pm\sqrt{64+240}}{2a}}$

$y=\pm\sqrt{\frac{4\pm\sqrt{16+20}}{2}}$

$y'=+\sqrt{\frac{4+\sqrt{36}}{2}}=\sqrt{\frac{4+6}{2}}$ $y'=\sqrt{5}$

$y''=-\sqrt{\frac{4+\sqrt{36}}{2}}=-\sqrt{\frac{4+6}{2}}$ $y''=-\sqrt{5}$

$y'''=+\sqrt{\frac{4-\sqrt{36}}{2}}=\sqrt{\frac{4-6}{2}}$ $y'''=\sqrt{-1}$

$y''''=-\sqrt{\frac{4-\sqrt{36}}{2}}=-\sqrt{\frac{4-6}{2}}$ $y''''=-\sqrt{-1}$

23-5-61

Equações irracionais: é a que contém a incognita submetida à subtração de raiz ou expoente fracionário

EX: $\sqrt{2y-1}=3y \therefore y^{\frac{4}{3}}+3=5$

$\sqrt{2y+12}=y-6 \therefore 2y+12=(y-6)^2$

$2y+12=y^2-12y+36$

$y^2-12y+36-2y-12=0$

$y^2-14y+24=0$

$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $y=\frac{14\pm\sqrt{196-96}}{2}$

$y=\frac{14\pm\sqrt{100}}{2}$ $y=\frac{14\pm 10}{2}$

$y'=\frac{14+10}{2}=12$ $y'=12$

$y''=\frac{14-10}{2}=2$ $y''=2$

$(\sqrt{4+12}=2-6)$ não serve

$\sqrt{24+12}=12-6$

$6=6$ esta serve.

$y-2\sqrt{y}=15 \therefore -2\sqrt{y}=15-y$

$4y=(15-y)^2$

$4y=y^2-30y+225$

$y^2-34y+225=0$

$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $y=\frac{34\pm\sqrt{1156-900}}{2}$

$y=\frac{34\pm\sqrt{256}}{2}$ $y=\frac{34\pm 16}{2}$

$y'=\frac{34+16}{2}=25$ $y'=25$

$y''=\frac{34-16}{2}=9$ $y''=9$

24-5-61

$\sqrt{x} = 21 - 2x \therefore x = (21-2x)^2$

$x = 441 - 84x + 4x^2$

$4x^2 - 85x + 441 = 0$

$x = \frac{85 \pm \sqrt{7225-7056}}{8} \therefore x = \frac{85 \pm \sqrt{169}}{8} \quad x = \frac{85 \pm 13}{8}$

$x' = \frac{85+13}{8} = \frac{98}{8} = 12\frac{1}{4} \qquad x' = 12\frac{1}{4}$

$x'' = \frac{85-13}{8} = \frac{72}{8} = 9 \qquad x'' = 9$

$\sqrt{x} = 21 - 2x \therefore \sqrt{9} = 21 - 2(9)$

$3 = 3$

$3 = 3$

$9 = x$ satisfaz

$5\sqrt{x} = 2(x+1) \therefore 25x = [2(x+)]^2$

$(25x = 4(x^2+1) \therefore 25x = 4x^2 +)$

$25x = 4(x^2+2x+1) \therefore 25x = 4x^2 + 8x + 4$

$4x^2 - 17x + 4 = 0$

$x = \frac{17 \pm \sqrt{289-64}}{8} \therefore x = \frac{17 \pm \sqrt{225}}{8} \therefore x = \frac{17 \pm 15}{8}$

$x' = \frac{17+15}{8} = \frac{32}{8} = 4 \qquad x' = 4$

$x'' = \frac{17-15}{8} = \frac{2}{8} = \frac{1}{4} \qquad x'' = \frac{1}{4}$

$5\sqrt{x} = 2(x+1) \therefore 5\sqrt{4} = 2(4+1) \quad 5 \times 2 = 10$

$10 = 10$

$x = 4$ satisfaz a equação

$\sqrt{x+\sqrt{x-1}} = \sqrt{2x-3} \therefore x + \sqrt{x-1} = 2x - 3$

$\sqrt{x-1} = 2x - 3 - x \therefore x - 1 = (x-3)^2$

$x - 1 = x^2 - 6x + 9 \therefore x^2 - 6x + 9 - x + 1 = 0$

$x^2 + 7x + 10 = 0$

$x = \frac{7 \pm \sqrt{49-40}}{2} \therefore x = \frac{7 \pm 3}{2}$

$x' = \frac{7+3}{2} = 5 \qquad x' = 5$

$x'' = \frac{7-3}{2} = 2 \qquad x'' = 2$

$\sqrt{x+\sqrt{x-1}} = \sqrt{2x-3}$

$\sqrt{2+\sqrt{2-1}} = \sqrt{4-3}$ } não serve

$\sqrt{2+1} = \sqrt{1}$

$\sqrt{5+\sqrt{5-1}} = \sqrt{10-3}$

$\sqrt{5+2} = \sqrt{7}$

$\sqrt{7} = \sqrt{7}$

$3\sqrt{2x} = 4x - 20 \therefore 9 \cdot 2x = (4x-20)^2$

$18x = 16x^2 - 160x + 400$

$16x^2 - 178x + 400 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad x = \frac{178 \pm \sqrt{31684-25600}}{32}$

$x = \frac{178 \pm \sqrt{6084}}{32} \qquad x = \frac{178 \pm 78}{32}$

$$ч' = \frac{178+78}{32} \therefore ч' = \frac{256}{32} = 8 \quad ч' = 8$$

$$ч'' = \frac{178-78}{32} \therefore ч'' = \frac{100}{32} = \frac{25}{8} \quad ч'' = \frac{25}{8}$$

$$3\sqrt{2ч} = 4ч - 20$$

$$3\sqrt{16} = 32 - 20$$

$$12 = 12$$

$$\sqrt{4ч^2+7ч-2} = ч+2$$

$$4ч^2+7ч-2 = ч^2+4ч+4$$

$$4ч^2+7ч-2-ч^2-4ч-4 = 0$$

$$3ч^2+3ч-6 = 0$$

$$ч = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad ч = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$ч' = \frac{-1+3}{2} = 1 \qquad ч' = 1$$

$$ч'' = \frac{-1-3}{2} = -2 \qquad ч'' = -2$$

$$\sqrt{16+14-2} = -2+2$$

$$0 = 0$$

$$\sqrt{4+7-2} = 1+2$$

$$\sqrt{9} = 3$$

$$3 = 3$$

31-5-61.

$$\sqrt{4ч+1} - \sqrt{3ч-2} = 1$$

$$\sqrt{4ч+1} = \sqrt{3ч-2} + 1$$

$$4ч+1 = 1+2\sqrt{3ч-2}+3ч-2$$

$$4ч+1-1-3ч+2 = 2\sqrt{3ч-2}$$

$$(ч+2) = (2\sqrt{3ч-2}$$

$$(ч+2)^2 = (2\sqrt{3ч-2})^2$$

$$ч^2+4ч+4 = 4(3ч-2)$$

$$ч^2+4ч+4 = 12ч-8$$

$$ч^2-8ч+12 = 0$$

$$ч' = 2$$

$$ч'' = 6$$

$$\sqrt{9} - \sqrt{4} = 1$$

$$3 - 2 = 1 \therefore 1 = 1$$

$$\sqrt{25} - \sqrt{16} = 1$$

$$5 - 4 = 1$$

$$1 = 1$$

$$\sqrt{5ч+6} = 2 + \sqrt{5ч-6}$$

$$5ч+6 = 4+4\sqrt{5ч-6}+5ч-6$$

$$5ч+6-4-5ч+6 = 4\sqrt{5ч-6}$$

$$8 = 4\sqrt{5ч-6}$$

$$64 = 16 \cdot 5ч - 6$$

$$64 = 80ч - 96$$

$$64+96 = 80ч$$

$$160 = 80ч$$

$$ч = \frac{160}{80} \qquad ч = 2$$

$\sqrt{5.2+6} = 2 + \sqrt{5.2-6}$

$\sqrt{16} = 2 + \sqrt{4}$

$4 = 2+2$

$4 = 4$

$3\sqrt{1-\frac{1}{\varphi}} = \sqrt{1-\varphi}$

$9(1-\frac{1}{\varphi}) = 1-\varphi$

~~$9\varphi - 9$~~ $9 - \frac{9}{\varphi} = 1-\varphi$

$9\varphi - 9 = \varphi - \varphi^2$

$\varphi^2 + 8\varphi - 9 = 0$

$\varphi = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\varphi = \frac{-8 \pm \sqrt{64+36}}{2}$

$\varphi = \frac{-8 \pm 10}{2}$ $\quad \varphi' = \frac{-8+10}{2}$ $\quad \varphi' = \frac{2}{2}$ $\quad \varphi' = 1$

$\varphi'' = \frac{-8-10}{2}$ $\quad \varphi'' = -\frac{18}{2}$ $\quad \varphi'' = -9$

$3\sqrt{1-1} = \sqrt{1-1}$

$3 \times 0 = 0$

$0 = 0$

2-6-61.

$3\varphi^2 + \varphi = 2$

$3\varphi^2 + \varphi - 2 = 0$

$\varphi = \frac{-1 \pm \sqrt{1+24}}{6}$

$\varphi' = \frac{-1+5}{6} = \frac{4}{6} = \frac{2}{3}$

$\varphi'' = \frac{-1-5}{6} = -1$

$3(\frac{2}{3})^2 + \frac{2}{3} = 2$

$3 \times \frac{4}{9} + \frac{2}{3} = 2$

$\frac{4}{3} + \frac{2}{3} = 2$

$\frac{6}{3} = 2$

$2 = 2$

$3(-1)^2 + (-1) = 2$

$3.1 + (-1) = 2$

$3 - 1 = 2$

$2 = 2$

$5\varphi^2 + 10 = 27\varphi$

$5\varphi^2 - 27\varphi + 10 = 0$

$\varphi = \frac{27 \pm \sqrt{729-200}}{10} \therefore \varphi = \frac{27 \pm \sqrt{529}}{10}$

$\varphi = \frac{27+23}{10} = 5$ $\quad \varphi' = 5$

$\varphi'' = \frac{27-23}{10} = \frac{4}{10}$ $\quad \varphi'' = \frac{2}{5}$

$5.25 + 10 - 135 = 0$

$125 + 10 - 135 = 0$

$135 - 135 = 0$

$0 = 0$

$5 \times \frac{4}{25} - \frac{54}{5} + 10 = 0$

$\frac{4}{5} - \frac{54}{5} + 10 = \frac{4-54+50}{5} = 0$

$0 = 0$

$\varphi^2 - 2\varphi = -1$ $\quad \varphi = \frac{2}{2}$ $\quad \varphi = 1$

$\varphi^2 - 2\varphi + 1 = 0$ $\quad \varphi' = \varphi'' = 1$

$\varphi = \frac{2 \pm \sqrt{4-4}}{2}$ $\quad 1 - 2 = -1$

$-1 = -1$

$\varphi^2+(\varphi+1)^2=25$

$\varphi^2+\varphi^2+2\varphi+1=25$

$2\varphi^2+2\varphi+1=25$

$2\varphi^2+2\varphi-24=0$

$\varphi^2+\varphi-12=0$

$\varphi'=-4$

$\varphi''=3$

$16+16-8+1=25$

$32+1-8=25$

$25=25$

7-6-61

$\varphi^2+(\varphi+1)^2+(\varphi+2)^2=194$

$x^2+x^2+2\varphi+x+x^2+4\varphi+x=194$

$3\varphi^2+6\varphi+5=194$

$3\varphi^2+6\varphi+5-194=0.$

$3\varphi^2+6\varphi-189=0$

$\varphi^2+2\varphi-63=0$

$\varphi=\frac{-2\pm\sqrt{4+252}}{2}$ $\quad \varphi=\frac{-2\pm16}{2}$

$\varphi'=\frac{-2+16}{2}=7$ $\quad \varphi'=7$

$\varphi''=\frac{-2-16}{2}=-9$ $\quad \varphi''=-9$

$\varphi^2=3\varphi+28$

$\varphi^2-3\varphi-28=0$

$\varphi'=7$

$\varphi''=-4$

$49=21+28$

$49=49$

$\left.\begin{array}{l}\varphi=7\\ \varphi=-4\end{array}\right\}$ resolvem a equação

$16=-12+28$

$16=16$

$2\varphi(2\varphi+2)=2808$

$4\varphi^2+4\varphi=2808$

$4\varphi^2+4\varphi-2808=0$

$\varphi^2+\varphi-702=0$

$\varphi=\frac{-1\pm\sqrt{1+2808}}{2}$

$\varphi=\frac{-1\pm\sqrt{2809}}{2}$ $\quad \varphi=\frac{-1\pm53}{2}$

$\varphi'=\frac{-1+53}{2}=26$ $\quad \varphi'=26$

$\varphi''=\frac{-1-53}{2}=-27$ $\quad \varphi''=-27$

$2\varphi=2\times26=52$

54

9-6-61

$(2x+1)(2x+)(2x+5)=7(2x+1+2x+3+2x+5)$

$(2x+1)(2x+3)(2x+5)=7(6x+9)$

$(2x+1)(2x+3)(2x+5)=21(2x+3)$

$(2x+1)(2x+5)=21\frac{2x+3}{2x+3}$

$4x^2+12x-16=0$

$x^2+3x-4=0$

$x'=-4$

$x''=1$

~~$(-8+1)(-8+$~~

$(2+1)(2+3)(2+5)=7\times(2+1+2+3+2+5)$

$3\times 5\times 7 = 7\times 15$

$105=105$

$6x+6+6x=5x(x+1)$

$12x+6=5x^2+5x$

$5x^2-7x-6=0$

$x=\frac{7\pm\sqrt{49+120}}{10}$ $\quad x=\frac{7\pm 13}{10}$

$x'=\frac{7+13}{10}=2$ $\quad x'=2$

$x''=\frac{7-13}{10}=\frac{-3}{5}$ $\quad x''=-\frac{3}{5}$

13-6-61

$ax^2+bx+c=0$ ∴ é completa

$ax^2+bx=0$ ∴ é incompleta ∴ $c=0$

$x(ax+b)=0$ $\quad x=0$

$ax+b=0$ $\quad x'=\frac{-b}{a}$ $\quad x''=\frac{-b}{a}$

$ax^2+c=0$ $\quad ax^2=-c$

$x^2=\frac{-c}{a}$ $\quad x=\pm\sqrt{\frac{-c}{a}}$

$ax^2=0$

$x^2=\frac{0}{a}$ $\quad 0\div a=0$ ∴ $x=0$

$x^2+4=0$ ∴ $x^2=-4$ $\quad x=\frac{\sqrt{-4}}{2}$ impossível

$5x^2=0$ $\quad x^2=\frac{0}{5}$

$x'=x''=0$

$\frac{7}{6x^2}-\frac{1}{2x^2}=6$ $\quad \frac{7-3=36x^2}{6x^2}$

$7-3=36x^2$ $\quad 4=36x^2$ $\quad 36x^2=4$ ∴ $x^2=\frac{4}{36}$

$x=\pm\sqrt{\frac{4}{36}}$ ∴ $x=\pm\frac{2}{6}=\frac{1}{3}$

$\frac{x^2}{4}+\frac{3x}{2}=0$ $\quad \frac{x^2+6x=0}{4}$

$x^2+6x=0$ ∴ $x(x+6)=0$

$x'=0$ $\quad x+6=0$ $\quad x''=-6$

$$\left(-\frac{2\psi^2}{5} + 10\psi = 0\right)(-1) \therefore \frac{2\psi^2}{5} - 50\psi = 0$$

$\psi(2\psi - 50) = 0$

$\psi = 0 \qquad 2\psi - 50 = 0 \qquad \psi = \frac{50}{2} = 25 \therefore \psi'' = 25$

$$\frac{5\psi^2 - 1}{3} - \frac{2\psi^2 + 5}{2} = \frac{1}{6} \therefore \frac{10\psi^2 - 2 - 6\psi^2 + 15}{6} = 1$$

$10\psi^2 - 2 - 6\psi^2 - 15 = 1$

$4\psi^2 - 17 = 1 \therefore 4\psi^2 = 18 \qquad \psi^2 = \frac{18}{4}$

$\psi = \pm\sqrt{\frac{18}{4}} \qquad \psi = \pm\sqrt{\frac{9}{2}}$

$\psi = \pm\frac{3}{\sqrt{2}} \qquad \psi' = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \qquad \psi' = \frac{3\sqrt{2}}{2}$

$\psi'' = -\frac{3}{\sqrt{2}} = -\frac{3\sqrt{2}}{2} \qquad \psi'' = -\frac{3\sqrt{2}}{2}$

$$5\psi^2 - \frac{3}{8} = 3\psi^2 + \frac{1}{8} \therefore \frac{40\psi^2 - 3}{8} = 24\psi^2 + 1$$

$40\psi^2 - 24\psi^2 = 1 + 3 \therefore 16\psi^2 = 4$

$\psi^2 = \frac{4}{16} \qquad \psi = \pm\sqrt{\frac{4}{16}} \qquad \psi' = \frac{1}{2},$

$\psi'' = -\frac{1}{2}$

16-6-61.

$$\frac{\psi - 1}{2} - \frac{3\psi - \psi^2}{3} = \psi + \frac{1}{3}$$

$$\frac{3\psi - 3 - 6\psi + 2\psi^2}{6} = \frac{6\psi + 2}{6} \therefore 2\psi^2 - 9\psi - 5 = 0$$

$2\psi^2 - 9\psi - 5 = 0$

$\psi = \frac{9 \pm \sqrt{81 + 40}}{4} \therefore \psi = \frac{9 \pm 11}{4}$

$\psi' = \frac{20}{4} = 5 \qquad \psi'' = \frac{-2}{4} = -\frac{1}{2}.$

$$\frac{\psi + 1}{\psi - 1} + \frac{\psi + 2}{\psi - 2} = \frac{2\psi + 13}{\psi + 1}$$

$(\psi - 2)(\psi + 1)(\psi + 1) + (\psi - 1)(\psi + 1)(\psi + 2) = (\psi - 1)($

$(\psi - 2)(2\psi + 13$

$(\psi^2 + \psi - 2\psi - 2)(\psi + 1) + (\psi^2 + \psi - \psi - 1)(\psi + 2) =$

$= (\psi^2 3 6 + 2)(2\psi + 13)$

$\psi^3 - 3\psi - 2 + \psi^3 + 2\psi^2 - \psi - 2 = 2\psi^3 + 13\psi^2 - 68\psi - 4$

$\psi^3 + \psi^3 - 2\psi^3 + 2\psi^2 - 13\psi^2 - 3\psi - \psi + 68\psi - 2 - 2 + 4 = 0$

$-5\psi^2 + 31\psi + 30 = 0 \ (-1)$

$5\psi^2 + 31\psi + 30 = 0$

$\psi' = 30$

$\psi'' = 1$

$-\frac{2\varphi^2}{5} + 10\varphi = 0$ (-1) ∴ $\frac{2\varphi^2}{5} - 50\varphi = 0$

$\varphi(2\varphi - 50) = 0$

$\varphi = 0$ $2\varphi - 50 = 0$ $\varphi = \frac{50}{2} = 25$ ∴ $\varphi'' = 25$

$\frac{5\varphi^2-1}{3} - \frac{2\varphi^2+5}{2} = \frac{1}{6}$ ∴ $\frac{10\varphi^2-2-6\varphi^2+15}{6} = 1$

$10\varphi^2-2-6\varphi^2-15=1$

$4\varphi^2-17=1$ ∴ $4\varphi^2=18$ $\varphi^2=\frac{18}{4}$

$\varphi=\pm\sqrt{\frac{18}{4}}$ $\varphi=\pm\sqrt{\frac{9}{2}}$

$\varphi=\frac{\pm3}{\sqrt2}$ $\varphi'=\frac{3}{\sqrt2}=\frac{3\sqrt2}{2}$ $\varphi'=\frac{3\sqrt2}{2}$

$\varphi''=-\frac{3}{\sqrt2}=-\frac{3\sqrt2}{2}$ $\varphi''=-\frac{3\sqrt2}{2}$

$5\varphi^2-\frac{3}{8}=3\varphi^2+\frac{1}{8}$ ∴ $\frac{40\varphi^2-3=24\varphi^2+1}{8}$

$40\varphi^2-24\varphi^2=1+3$ ∴ $16\varphi^2=4$

$\varphi^2=\frac{4}{16}$ $\varphi=\pm\sqrt{\frac{4}{16}}$ $\varphi'=\frac{1}{2}$

$\varphi''=-\frac{1}{2}$

16-6-61.

$\frac{\varphi-1}{2}-\frac{3\varphi-\varphi^2}{3}=\varphi+\frac{1}{3}$

$\frac{3\varphi-3-6\varphi+2\varphi^2=6\varphi+2}{6}$ ∴ $2\varphi^2-9\varphi-5=0$

$2\varphi^2-9\varphi-5=0$

$\varphi=\frac{9\pm\sqrt{81+40}}{4}$ ∴ $\varphi=\frac{9\pm11}{4}$

$\varphi'=\frac{20}{4}=5$ $\varphi''=\frac{-2}{4}=-\frac{1}{2}$

$\frac{\varphi+1}{\varphi-1}+\frac{\varphi+2}{\varphi-2}=\frac{2\varphi+13}{\varphi+1}$

$(\varphi-2)(\varphi+1)(\varphi+1)+(\varphi-1)(\varphi+1)(\varphi+2)=(\varphi-1)($

$(\varphi-2)(2\varphi+13)$

$(\varphi^2+\varphi-2\varphi-2)(\varphi+1)+(\varphi^2+\varphi-\varphi-1)(\varphi+2)=$

$=(\varphi^2-3\varphi+2)(2\varphi+13)$

$\varphi^3-3\varphi-2+\varphi^3+2\varphi^2-\varphi-2=2\varphi^3+13\varphi^2-68\varphi-4$…

$\varphi^3+\varphi^3-2\varphi^3+2\varphi^2-13\varphi^2-3\varphi-4+68\varphi-2-2+44$…

$-5\varphi^2+31\varphi+30=0$ (-1)

$5\varphi^2+31\varphi+30=0$

$\varphi'=30$

$\varphi''=1$

$-4m = -16 \; (-1) \quad 4m = 16$

$m = \frac{16}{4} = 4$. as raízes serão reais iguais para $m = 4$

$16 - 4m < 0$

$-4m < -16 \; (-1) \quad 4m > 16$

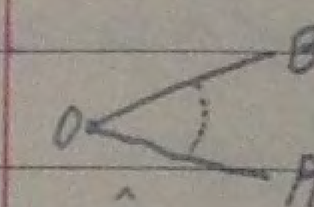
$m > \frac{16}{4} = 4$

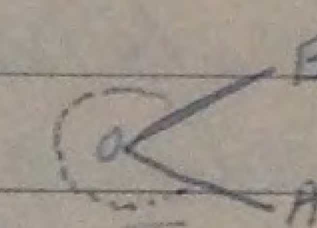
$m > 4$

$u^2 - 8u + m = 0 \qquad u = \frac{4}{5}$

4-8-61.

Ângulos


convexo

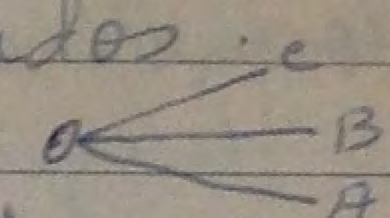

côncavo

Ângulo é a região do plano limitada por duas semi-retas; tem a mesma origem, (vértice)

Ângulo convexo não contém as semi-retas opostas aos seus lados.

Ângulo côncavo contém as semi-retas opostas à seus lados.

Ângulos consecutivos.

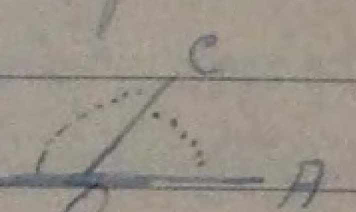


São ângulos consecutivos quando tem a mesma origem.

Os lados externos opostos OC : OA estejam em semi-planos opostos.

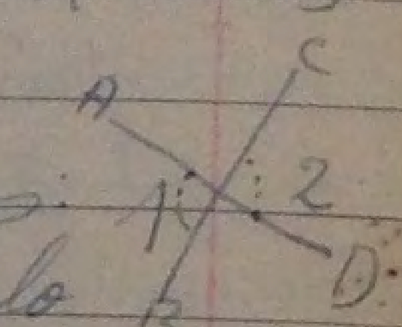
Um lado comum OB

Ângulos adjacentes:



São 2 ângulos consecutivos, cujos os lados externos são semi-retas opostas OA : OB.

Ângulos opostos pelos vértices:

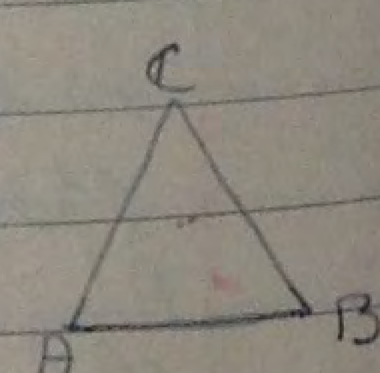


Quando os lados de um ângulo forem semi-retas opostas aos lados do outro.

Ângulo de meia volta 
Quando seus lados forem
semi-retas opostas.
Angulo de uma volta ou nulo
Quando seus lados 
coincidirem.
Angulo reto: quando tiver 90°.
Angulo agudo: quando tiver menos de 90°
Angulo obtuso: quando tiver menos
do que um ângulo raso e mais de 90°.
Angulos complementares: quando
sua soma forem 90°.
Angulos suplementares:
quando sua soma forem de 180°.
Angulos replementares: quando
sua soma forem de 360° ou 4
retos ou de uma volta.

Triângulo:

Elementos { 3 lados
principais { 3 ângulos
3 vértices, 3 alturas
3 bicetrizes, 3 medianas



Classificação quanto aos lados

Equilátero: 3 lados iguais
Isósceles: 2 lados iguais
Escaleno: 3 lados diferentes

8-8-61.

$5y^2 - \frac{3}{4} = 3y^2 + \frac{1}{8}$

$40y^2 - 24y^2 - 6 - 1 = 0$

$16y^2 - 7 = 0$

$y^2 = \frac{7}{16}$

$y = \pm\sqrt{\frac{7}{16}} \therefore y \pm \frac{\sqrt{7}}{4}$

$y' = +\frac{\sqrt{7}}{4}$

$y'' = -\frac{\sqrt{7}}{4}$

$\frac{15}{y} - \frac{72-6y}{2y^2} = 2$

$30y - 72 + 6y = 4y^2$

$4y^2 - 36y + 72 = 0$

$y = \frac{36 \pm \sqrt{36^2 - 16 \cdot 72}}{8} \therefore y = \frac{36 \pm \sqrt{1296 - 1152}}{8}$

$y = \frac{36 \pm \sqrt{144}}{8}$ $y' = \frac{36+12}{8} = 6$ $y' = 6$

$y'' = \frac{36-12}{8} = 3 \therefore y'' = 3$

$x \cdot \frac{3}{4} x = 12$

$\frac{3}{4} x^2 = 12$

$3x^2 = 12 \cdot 4$

$3x^2 = 48$

$x^2 = \frac{48}{3} = 6$

$x^2 = 6$

$x = \sqrt{16}$

$x = 4$

$y = 2x^2 + 3x - 2$ $\qquad 2x^2 + 3x - 2 = 0$

$x = \frac{-3 \pm \sqrt{9+16}}{4}$ $\qquad x = \frac{-3 \pm \sqrt{25}}{4}$

$x' = \frac{-3+5}{4} = \frac{1}{2}$ $\qquad x' = \frac{1}{2}$

$x'' = \frac{-3-5}{4} = -\frac{8}{4}$ $\qquad x'' = -2$

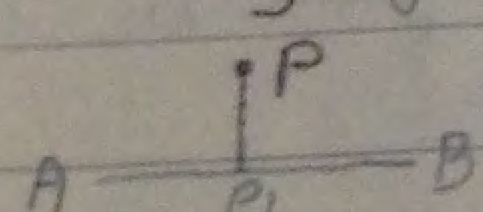
$y = 2x^2 + 3x - 2 \therefore a(x - x')(x - x'')$

$y = 2x^2 + 3x - 2 \therefore 2(x - \frac{1}{2})(x + 2) =$

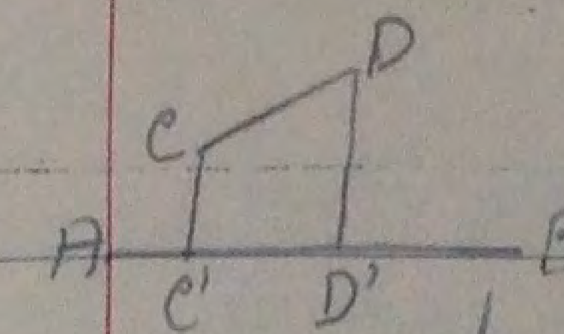
$= 2\left(\frac{2x-1}{2}\right)(x+2) =$

$= (2x-1)(x+2)$

9-8-61.

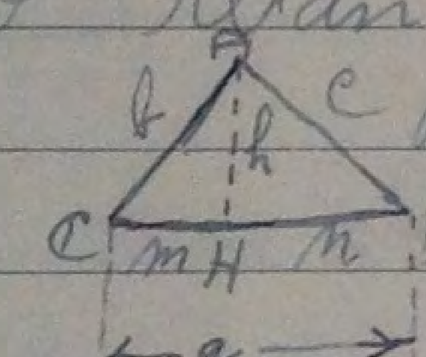


Ponto: vem a ser o pé da perpendicular traçada do ponto até a reta.



Projeção de um segmento de reta sôbre uma reta é o segmento determinado sôbre a reta pelas projeções dos extremos dos seg.

Relações métricas do triângulo retângulo.



1) Num triângulo retângulo a hipotenusa é igual a soma dos catetos. $\boxed{a = m + n}$ m = projeção do cateto b

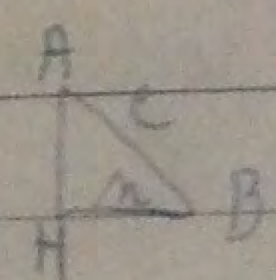
2) Sôbre a hipotenusa a; n = = projeção do cateto c sôbre a hipotenusa a.

Cada cateto é média proporcional entre a hipotenusa e sua projeção sobre ela.

$b^2 = a\,m$

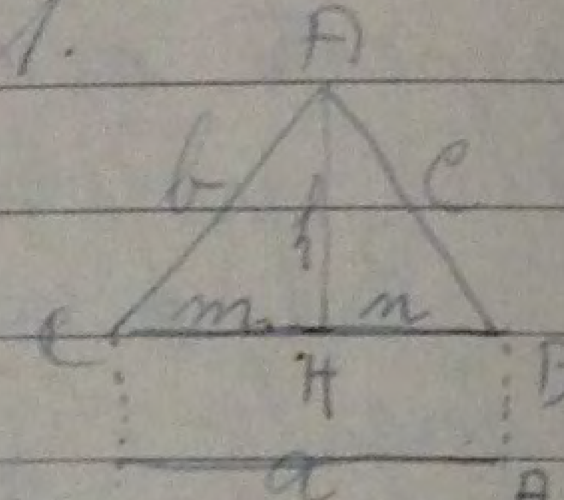
$c^2 = a\,n.$

$\left.\begin{matrix}\triangle ABC \\ \triangle AHC\end{matrix}\right\} \sim \qquad \frac{b}{a} = \frac{m}{b}$



$\frac{c}{a} = \frac{n}{c}$

11-8-61.

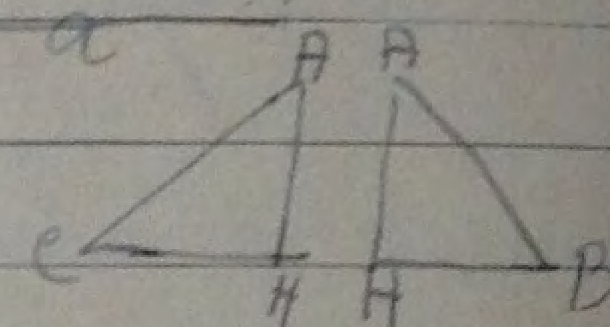


3) ABC ∾ AHC

ABC ∾ AHB

Δ AHC ∾ AHB

$h^2 = mn$



$\frac{h}{m} = \frac{n}{h}$ $h^2 = mn$

2) A altura é média proporcional entre projeções dos catetos sôbre a hipotenusa.

4) O quadrado da hipotenusa é igual a soma dos quadrados dos catetos

$b^2 = a\,m$

$c^2 = a\,n$

$b^2 + c^2 = a\,m + a\,n$

$b^2 + c^2 = a(m+n)$

$b^2 + c^2 = a \cdot a$

$b^2 + c^2 = a^2$

5º) $bc = ah$

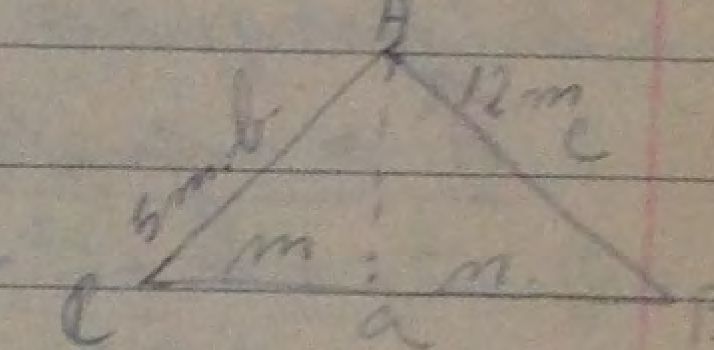
O produto dos catetos é igual ao produto da hipotenusa pela altura

$b^2c^2 = a\,m \cdot a\,n$

$b^2c^2 = a^2(m \cdot n)$

$b^2c^2 = a^2h^2$

$bc = ah$

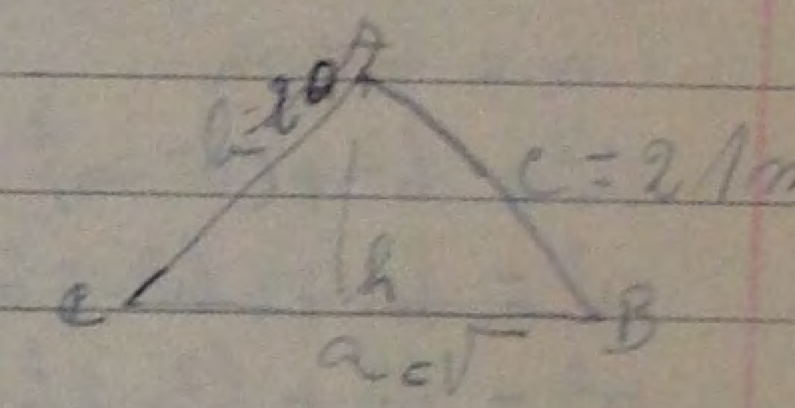


$b^2 = 25 = a\,m$

$c^2 = 144 = a\,n$

$b^2 + c^2 = 169$ $a\,m + a\,n = 169$

$a = 13$



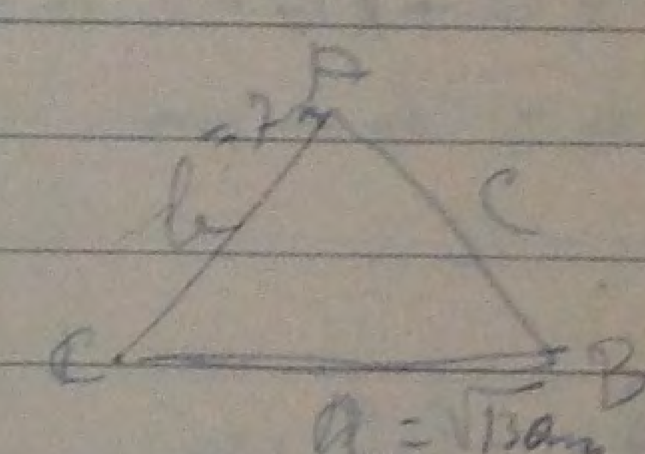
$b^2 = 400$

$c^2 = 441$

$b^2 + c^2 = a^2$

$841 = a^2$ $\sqrt{b^2+c^2} = \sqrt{a^2}$

$29 = a$ $\sqrt{841} = \sqrt{a^2}$



$a = \sqrt{130}$

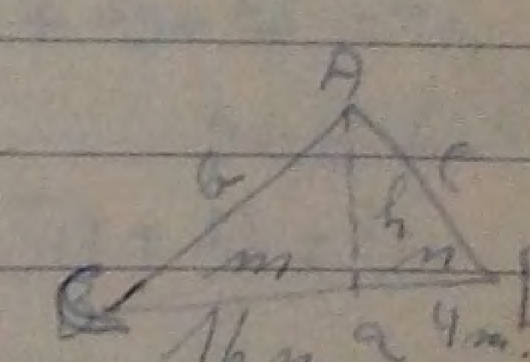
$b = 7m$

$a^2 = 130m$

$b^2 = 49m$

c^2 81

$c = 9m$



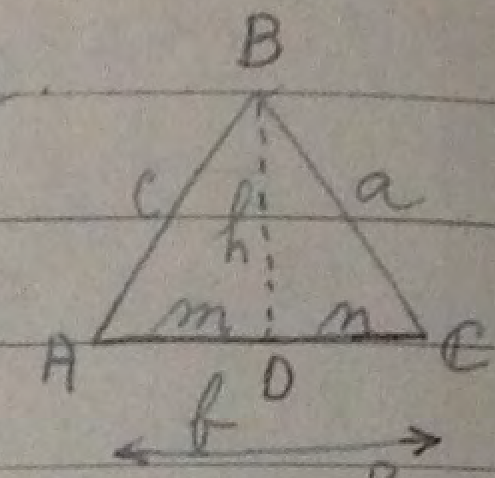
$\frac{h}{m} = \frac{n}{h}$ $h^2 = 64$

$h = 8$

$h^2 = mn$

18-8-61

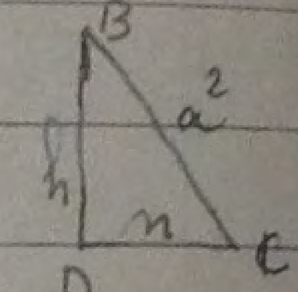
Relações métricas de um triângulo qualquer.



$\hat{A} \rightarrow$ agudo

1) $\boxed{a^2 = +b^2 + c^2 - 2bm}$

ΔBDC



$a^2 = h^2 + n^2$

$n = b - m$

$a^2 = h^2 + (b - m)^2$

$a^2 = h^2 + (b^2 - 2bm + m^2)$

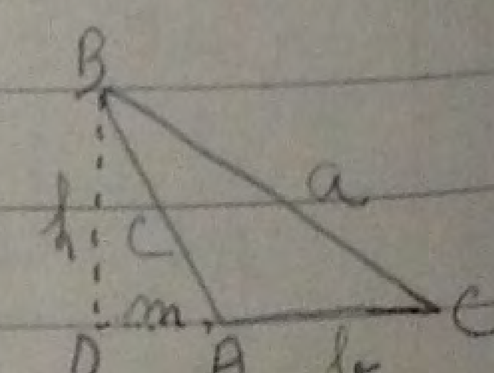
$a^2 = h^2 + b^2 - 2bm + m^2$

$h^2 = c^2 - m^2$ $\boxed{c^2 = h^2 + m^2}$

$a^2 = c^2 - ~~m^2~~ + b^2 - 2bm + ~~m^2~~$

$a^2 = c^2 + b^2 - 2bm$

$n = b - m$



$\hat{A}$ = obtuso

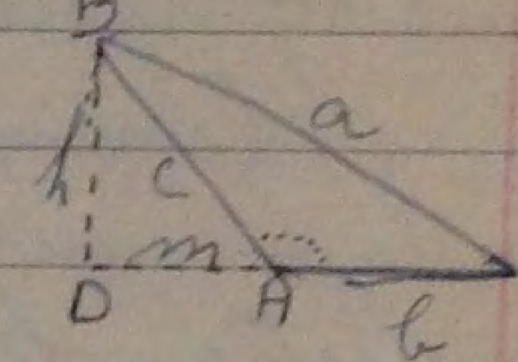
$a^2 = b^2 + c^2 + 2bm$

ΔBDC

$a^2 = h^2 + \overline{DC}^2$

22-8-61

$\hat{A} > 90°$



T{ $a^2 = b^2 + c^2 + 2bm$

ΔBDC

$a^2 = h^2 + \overline{DC}^2$

$\boxed{h^2 = c^2 - m^2}$

$a^2 = c^2 - m^2 + \overline{DC}^2$

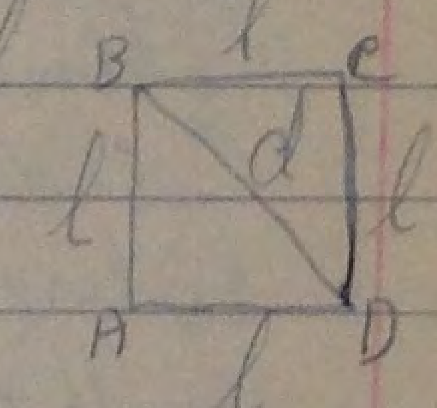
$\overline{DC} = m + b$

$a^2 = c^2 - m^2 + (m + b)^2$

$a^2 = c^2 - ~~m^2~~ + ~~m^2~~ + 2bm + b^2$

$a^2 = c^2 + 2bm + b^2$

Diagonal de um quadrado

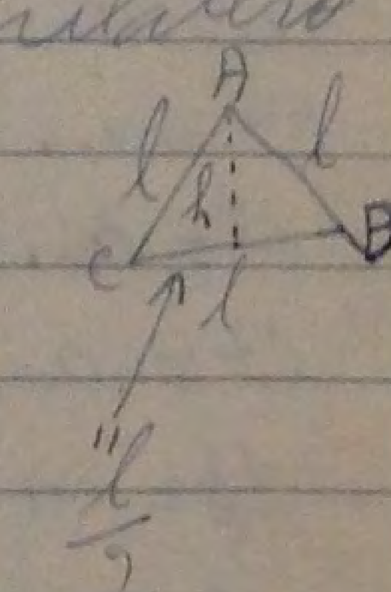


$d^2 = l^2 + l^2$

$d^2 = 2l^2$

$\boxed{d = l\sqrt{2}}$

Altura de 1Δ equilátero



$l^2 = h^2 + \left(\frac{l}{2}\right)^2$

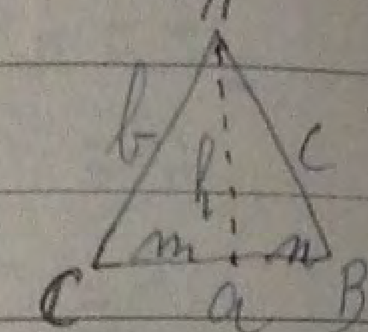
$l^2 = h^2 + \frac{l^2}{4}$

$l^2 - \frac{l^2}{4} = h^2 \therefore$

$\therefore \frac{4l^2 - l^2}{4} = h^2 \quad \frac{3l^2}{4} = h^2$

$h^2 = \frac{3l^2}{4} \therefore h = \frac{l\sqrt{3}}{2}$

A altura de um $\triangle$ retângulo determina sôbre a hipotenusa 2 segmentos; um de 32 dm e o outro de 18 dm. Calcular os catetos.



$a = m + n$

$a = 32 + 18$ $\boxed{a = 50}$

$b^2 = a.m \therefore b^2 = 50.32$

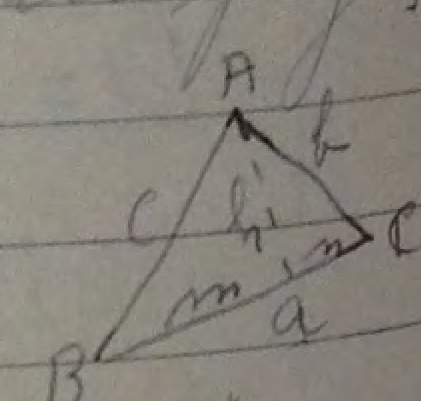
$c^2 = a.n \therefore c^2 = 50.18$

$b = \sqrt{1600} \therefore b = 40\ dm$

$c = \sqrt{900} \therefore c = 30\ dm.$

23-8-61

(A altura) Os catetos de 1 $\triangle$ retângulo medem: 3 e 4 m. Calcular o valor das projeções sôbre a hipotenusa



$a^2 = 16 + 9 \therefore a^2 = 25$

$a = \sqrt{25} \therefore \boxed{a = 5}$

$c^2 = a.m$

$b^2 = a.n$

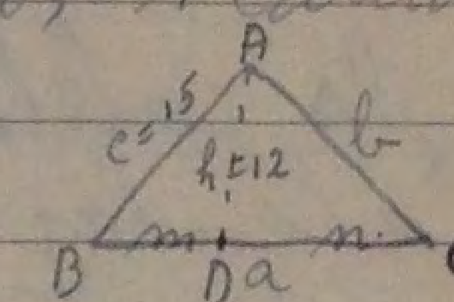
$c^2 = a\,m$ $16 = 5.m$

$\frac{16}{5} = m$ $m = 3{,}2\ m$

$b^2 = a\,n$ $9 = 5.n$

$\frac{9}{5} = n$ $\boxed{n = 1{,}8 m}$

Num $\triangle$ retângulo, 1 cateto = 15 m; altura = 12 m.



$m^2 = c^2 - h^2$

$m^2 = 225 - 144 = 81$

$\boxed{m = 9}$

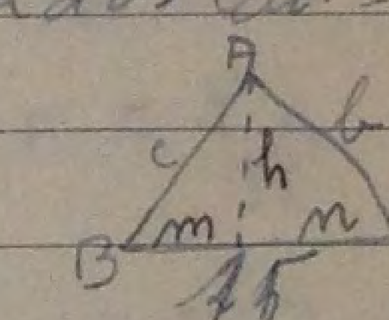
$h^2 = 9n$

$144 = 9n$ $n = \frac{144}{9}$ $\boxed{n = 16}$

$a = m + n$ $a = 9 + 16 = 25$ $\boxed{a = 25}$

$b^2 = a^2 = c^2$ $b^2 = 625 - 225$ $b^2 = 400$ $\boxed{b = 20}$

hipotenusa = 15 soma dos c.t. = 21



$b = 21 - c$

$a^2 = (21 - c)^2 + c^2$

$225 = 441 - 42c + c^2 + c^2$

$225 = 441 - 42c + 2c^2.$

$2c^2 - 42c + 441 - 225 = 0$

$2c^2 - 42c + 216 = 0$

$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $c = \frac{42 \pm \sqrt{1764 - 1728}}{4}$

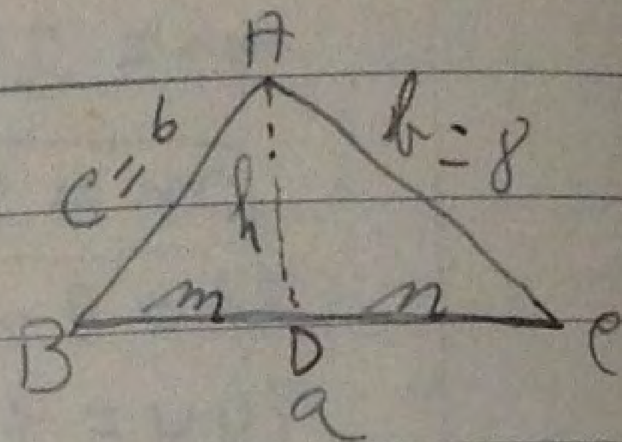
$c = \frac{42 \pm 36}{4}$ $c' = \frac{42 + 36}{4} \therefore c' = \frac{78}{4} \to 19{,}5$

$c'' = \frac{42 - 36}{4} = \frac{6}{4}$ $c'' = \frac{3}{2}$

$c = 9$ $B = 12$

25-8-61.

Os catetos medem 6 e 8 m.
Calcular a altura relativa à hipotenusa



$a^2 = b^2 + c^2$

$a^2 = 64 + 36 = 100$

$a^2 = 100 \therefore a = 10$

$h^2 = m . n$

$b^2 = a\ m$

(25-8-61)

$64 = 10\ m$

$\frac{64}{10} = m \therefore m = 6,4$ metros

$c^2 = a . n$

$36 = 10\ n$

$\frac{36}{10} = n \quad n = 3,6$ metros

$h^2 = m . n .$

$h^2 = 6,4 \times 3,6 \therefore h^2 = 23,04$

$h = \sqrt{23,04}$

$h = 4,8$ metros

A diagonal do retângulo cujas as dimensões são: base: 8 m e alt. 15.

$d^2 = b^2 + a^2$

$d^2 = 64 + 225$

$d^2 = 289$

$d = \sqrt{289}$

$d = 17$ metros.

base do retângulo = 24 metros

(comprimento / diagonal) = 25 metros

$l^2 = \frac{d^2}{b^2}$ | $l^2 = \frac{625}{576}$

$l^2 = 49 \therefore l = \sqrt{49} \therefore l = 7$ metros

Diag = 16 m   diag = 12 m.

$l^2 = \frac{D^2 + d^2}{2}$ $l^2 = 64 + 36 = 100$

$l^2 = 100 \quad l = 10$

8-9-61

Reconhecimento da natureza de um triângulo

$a^2 = b^2 + c^2$ = Δ retângulo

$a^2 = b^2 + c^2 - 2bm$ = Δ acutângulo

$a^2 < b^2 - c^2$

$a^2 = b^2 + c^2 + 2bm$ = Δ obtusângulo

$a^2 > b^3 + c^2$

9 m   12 m   15 m.

$a^2 = 225$ $b^2 = 81$ $c^2 = 144$

$a^2 = b^2 + c^2 \therefore 225 = 81 + 144.$

hip.=8   cat=11   9

a^2 64 $b^2 = 121$ $c^2 = 81$

$64 < 202 \therefore$ acutângulo

hip 14   b=8   c=7

$a^2 = 196$ $b^2 = 64$ $c^2 = 49$

$196 > 113$ = obtusângulo

hip=18   b=24   c=36

$a^2 = 324$ $b^2 = 576$ $c^2 = 1296$

a $324 < 576 + 1296 \therefore$ acutângulo

a=12m   b=8m   c=5m

Calcular o valor da projeção do lado maior sobre o menor.

$a^2 = 144$. $b^2 = 64$ $c^2 = 25$

$144 > 89$

$a^2 = b^2 + c^2 + 2cm$

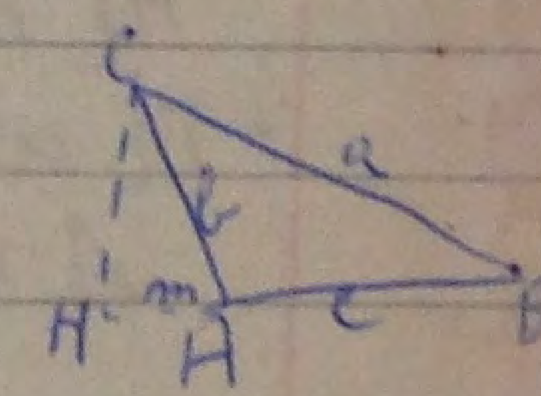
$144 = 64 + 25 + (10m$

$144 - 89 = 10m$

$55 = 10m$

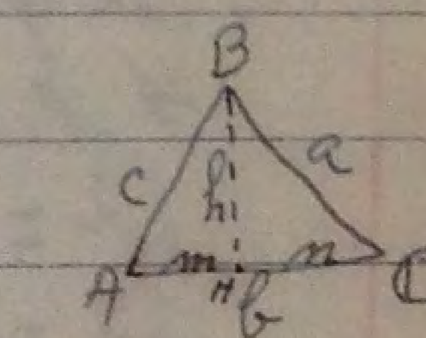
$\frac{55}{10} = m$

$5{,}5 = m.$



13-9-61

T{ $a^2 = b^2 + c^2 - 2bc.\cos \hat{A}.$



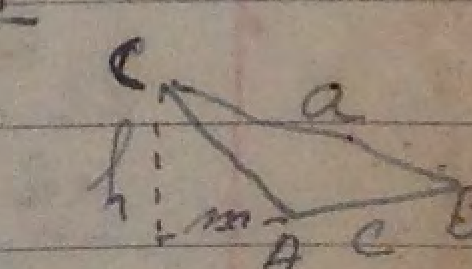
Δ ABC

H{ 1º caso: $A < 90°$ $a^2 = b^2 + c^2 - 2bm$

$m = c.\cos \hat{A}$

$a^2 = b^2 + c^2 - 2bc.\cos \hat{A}$

2º caso: $A > 90°$



$a^2 = b^2 + c^2 + 2cm$

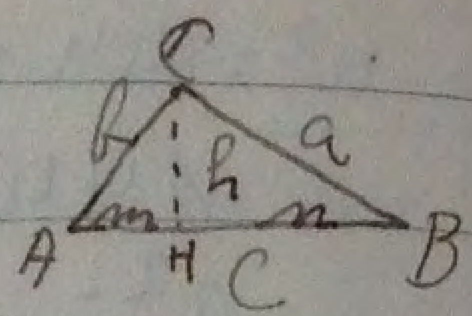
T{ $a^2 = b^2 + c^2 + 2b.c.\cos \hat{A}$

$m = b.\cos(180° - \hat{A}) = -\cos \hat{A}$

$m = -b.\cos.\hat{A}.$

$\cos 150° = -0{,}866$
$\cos 30° = 0{,}866$

$a^2 = b^2 + c^2 - 2b.c.\cos \hat{A}$



H{ $A < 90°$

T{ $a^2 = b^2 + c^2 - 2c\ m$

$\Delta\ CHB$

19-9-61

A altura determina 2 segmentos.

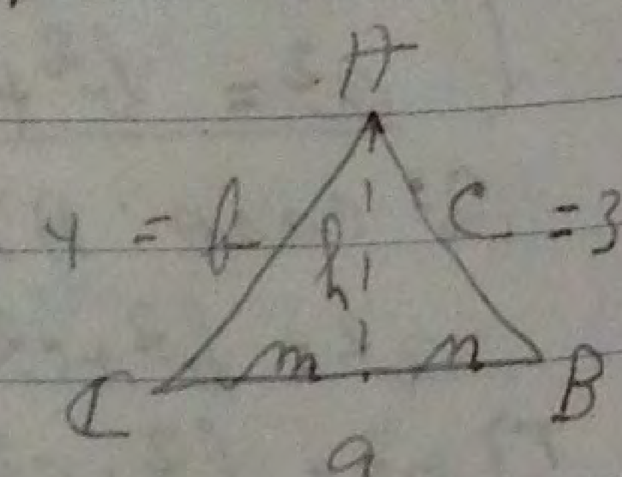
4 e 16 m.

$h^2 = m.n$

$h^2 = 4.16$

$h^2 = 64$

$h = \sqrt{64}$ $\quad h = 8$



$a\ m = b^2$

$c^2 = a\ n$

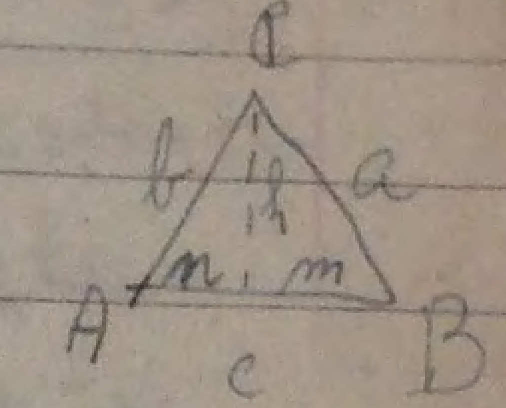
$a^2 = b^2 + c^2$

$a^2 = 16 + 9 = 25 \quad a = \sqrt{25} \quad a = 5$

$b^2 = a.m \therefore 16 = 5.m \therefore m = \frac{16}{5} \quad m = 3{,}2$

$c^2 = a\ n \therefore n = \frac{9}{5} \quad n = 1{,}8$

Sabendo-se que $\hat{A} < 90°$



T{ $a^2 = b^2 + c^2 - 2c\ n$

$\Delta\ CHB$

$a^2 = m^2 + h^2$

$b^2 = h^2 + n^2$

$h^2 = b^2 - n^2$

$a^2 = b^2 - n^2 + m^2$

$m = c - n$

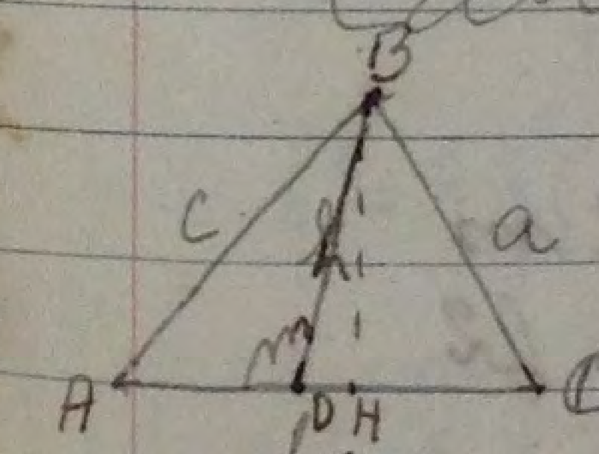
$a^2 = b^2 - n^2 + (c - n)^2$

$a^2 = b^2 - n^2 + c^2 - 2c\ n + n^2$

$a^2 = b^2 + c^2 - 2c\ n$

22-9-61

Cálculo das medianas



$AD = DC$

$\Delta\ B\hat{D}C < 90°$

$a^2 = m^2 + \overline{DC}^2 - 2\ DC.DH$

$\Delta\ BA\hat{D} > 90°$

$c^2 = m^2 + \overline{AD}^2 + 2AD.DH$

$AD = DC = \frac{b}{2}$

$$a^2 = m^2 + \overline{DC}^2 - 2DC \cdot DH$$

$$c^2 = m^2 + \overline{AD}^2 + 2DC \cdot DH$$

$$a^2 + c^2 = 2m^2 + \overline{DC}^2 + \overline{AD}^2$$

$$a^2 + c^2 = 2m^2 + \frac{b^2}{4} + \frac{b^2}{4}$$

$$a^2 + c^2 = 2m^2 + \frac{2b^2}{4}$$

$$a^2 + c^2 = 2m^2 + \frac{b^2}{2}$$

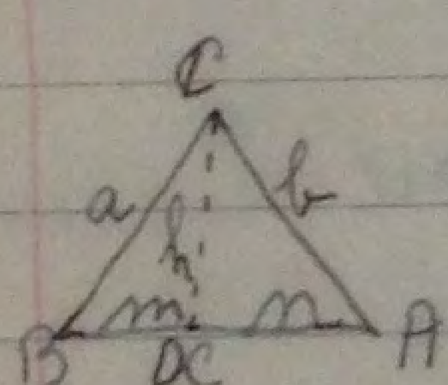
$$2m^2 = a^2 + c^2 - \frac{b^2}{2}$$

$$2m^2 = \frac{2a^2 + 2c^2 - b^2}{2}$$

$$m^2 = \frac{2a^2 + 2c^2 - b^2}{4}$$

mediana do lado b: $m = \frac{1}{2} \times \sqrt{2a^2 + 2c^2 - b^2}$

26-9-61



ΔBDC

$$h^2 = a^2 - m^2$$

$$b^2 = c^2 + a^2 - 2cm$$

$$2cm = c^2 + a^2 - b^2$$

$$m = \frac{c^2 + a^2 - b^2}{2c}$$

$$h^2 = a^2 - \frac{(c^2 + a^2 - b^2)^2}{4c^2}$$

$$h^2 = \frac{4a^2c^2 - (c^2 + a^2 - b^2)^2}{4c^2}$$

$$h^2 = \frac{(2ac + c^2 + a^2 - b^2)(2ac - c^2 - a^2 + b^2)}{4c^2}$$

$$a^2 + 2ac + c^2 = (a+c)^2$$

$$h^2 = \frac{[(a+c)^2 - b^2][b^2 - (a-c)^2]}{4c^2}$$

$$h^2 = \frac{(a+c+b)(a+c-b)(b+a-c)(b-a+c)}{4c^2}$$

$$a + b + c = 2p$$

$$a + c - b = a + b + c - 2b = 2p - 2b =$$
$$= 2(p-b)$$

$$b - a + c = b + a + c - 2a = 2p - 2a =$$
$$= 2(p-a)$$

$$h^2 = \frac{(2p)(2(p-b)(2(p-c)(2(p-a)}{4c^2}$$

$$h^2 = \frac{4p(p-b)(p-c)(p-a)}{c^2}$$

$$h^2 = \frac{4p(p-a)(p-b)(p-c)}{c^2}$$

$$h = \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}$$

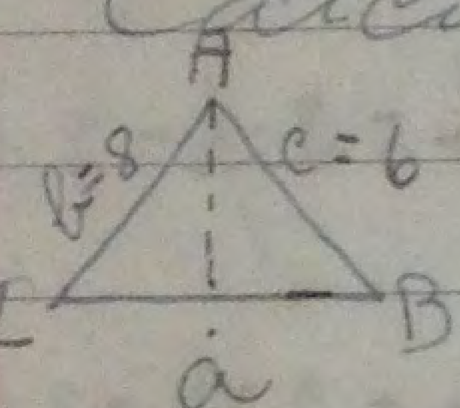
altura em relação ao lado c

altura em relação ao lado a

$h = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)}$

27-9-61.

Num triângulo, os lados valem: a = u, b = 8 c = 6 e formam entre si um ângulo de 60°. Calcular o valor de a



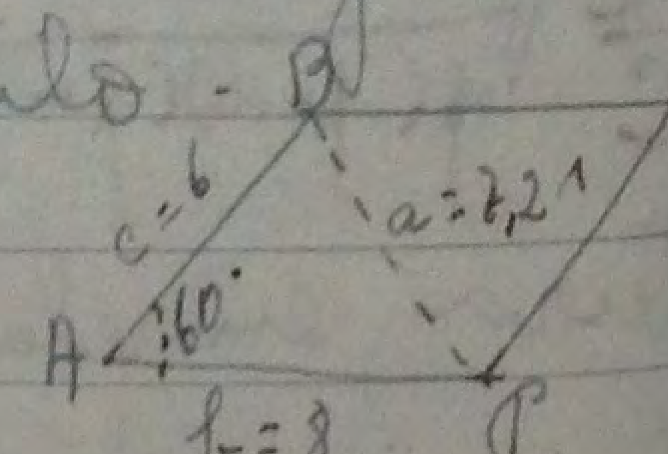
$a^2 = b^2 + c^2 - 2bc \cos 60°$ → 0,5

$a^2 = 64 + 36 - 48$

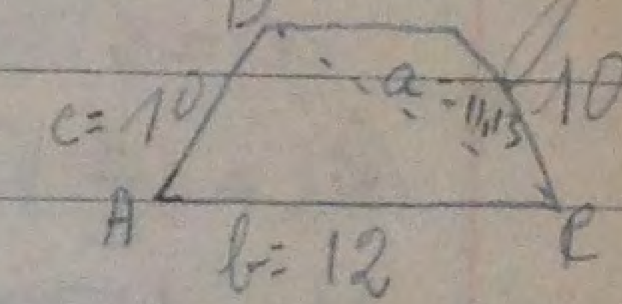
$a^2 = 52$

$a = \sqrt{52}$ $a = 7{,}21$ m.

Num paralelogramo, os lados consecutivos que medem 6 e 8 m formam entre si um ângulo de 60°. Calcular o valor da diagonal oposta a êste ângulo.



Num trapézio isósceles a parte maior mede 12 m o lado não paralelo mede 10 m, forma com a base maior um ângulo de 60°. Calcular o valor da diag. do trapézio.



$a^2 = b^2 + c^2 - 2bc \cdot \cos 60°$

$a^2 = 144 + 100 - 120$

$a^2 = 124$

$a = \sqrt{124}$

$a = 11{,}13$

Os lados de 1 triângulo medem 7, 10 e 13 m. Calcular as alturas relativas aos 3 lados.

$a = 7,\ b = 10 \quad c = 13$ m.

$h_a = \frac{2}{7}\sqrt{15(15-7)(15-10)(15-13)}$

$h_a = \frac{2}{7}\sqrt{15 \cdot 8 \cdot 5 \cdot 2}$ $h_a = \frac{2}{7}\sqrt{1200}$

$h_a = \frac{2}{7} \cdot 34{,}6$ $h_a = 9{,}8857142$ m.

29-9-61.

Cálculo das medianas.

Calcular o valor da ~~lado~~ mediana relativa ao lado b do ΔABC, cujo os lados medem.

$a=6 \quad b=9 \quad c=12$

$M_b = \frac{1}{2}\sqrt{2a^2+2c^2-b^2}$

$M_b = \frac{1}{2}\sqrt{72+288-81} \therefore M_b = \frac{1}{2}\sqrt{279}$

$M_b = \frac{1}{2} \times 16,7 \qquad M_b = 8,35 m.$

$a=4 \quad b=6 \quad c=8$

$M_a = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

$M_a = \frac{1}{2}\sqrt{72+128-16}$

$M_a = \frac{1}{2}\sqrt{184}$

$M_a = \frac{1}{2} \cdot 13,599$

$M_a = \frac{13,599}{2} \therefore M_a = 6,799.$

$a=12 \quad b=8 \quad c=6.$

Calcular M_a

$M_a = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

$M_a = \frac{1}{2}\sqrt{128+72-144}$

$M_a = \frac{1}{2}\sqrt{56}$

$M_a = \frac{1}{2} \cdot 7,48 \quad \therefore M_a = 3,74$

$A_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)}$

$A_b = \frac{2}{8}\sqrt{26(26-12)(26-8)(26-6)}$

$A_b = \frac{2}{8}\sqrt{26 \cdot 14 \cdot 18 \cdot 20}$

$A_b = \frac{1}{8}\sqrt{131040}$

$A_b = \frac{1}{8_4} 361,99$

$A_b = \frac{361,99}{8} \quad \therefore A_b$

$A_b = \frac{2}{8}\sqrt{13(13-12)(13-8)(13-6)}$

$A_b = \frac{2}{8}\sqrt{13 \cdot 1 \cdot 5 \cdot 7}$

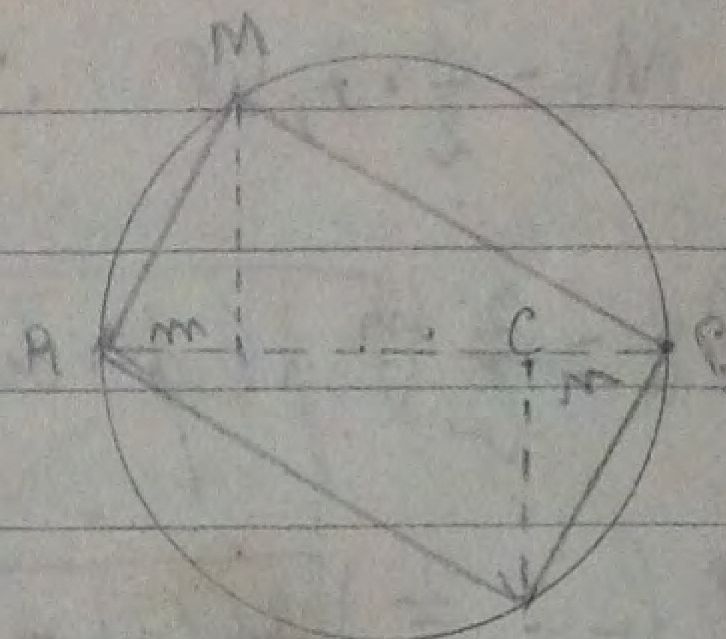
$A_b = \frac{2}{8}\sqrt{455} \quad \therefore A_b = \frac{2}{8} \cdot 21,3307$

$A_b = \frac{21,3307}{4} \quad \therefore A_b = 5,332675$

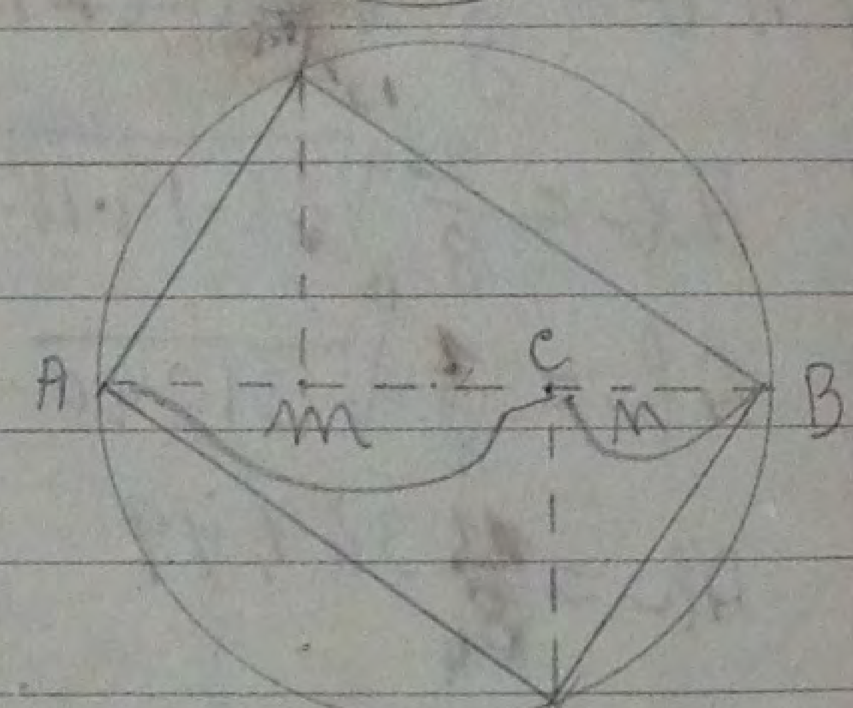
4-10-61

Relações metricas do circulo

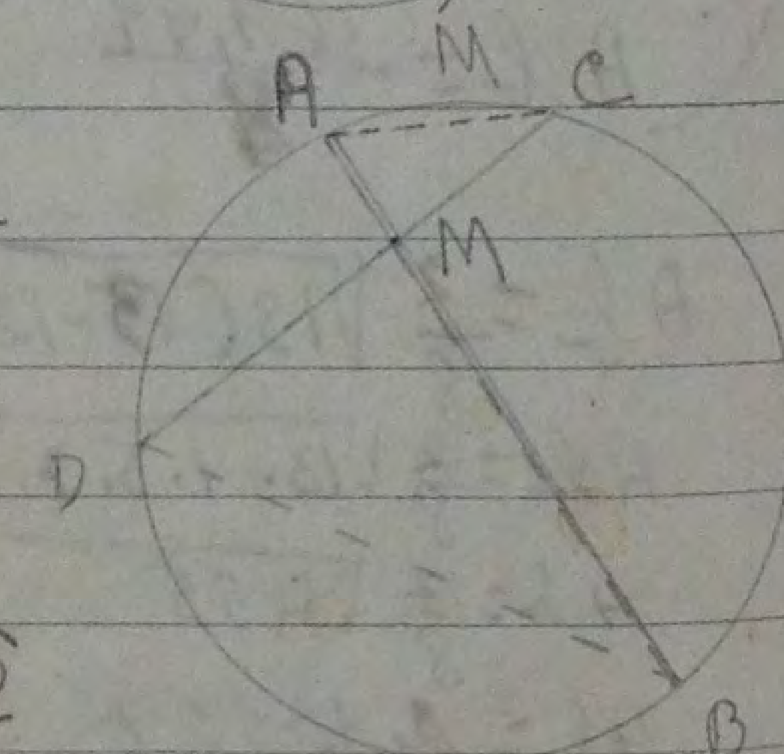
1 { T{ $\overline{AM}^2 = AB \cdot Am$
H{ AM = corda
D{ $A\hat{M}B$ = retângulo
$\overline{AM}^2 = AB \cdot Am$



2 { T{ $\overline{MC}^2 = m \cdot n$
D{ $\overline{MC}^2 = m \cdot n$



3 { H{ AB e CD = cordas
T{ $\overline{AM} \times \overline{MB} = DM \times MC$
△ACM e △DBM
1º caso: $\hat{A} = \frac{\overline{CB}}{2}$
$\hat{D} = \frac{\overline{CB}}{2}$
$\hat{C} = \frac{\overline{AD}}{2}$ $\hat{B} = \frac{\overline{AD}}{2}$



$\frac{AM}{MD} = \frac{CM}{BM}$

$AM \cdot BM = MD \cdot CM$.

7.10.61.

As cordas que unem o ponto C da circunferência, as extremidades A e B medem 9 e 12 m.
Calcular a distância C-H, C ao diâmetro AB as projeções AH e HB das cordas sôbre o diametro

$H^2 = b^2 + a^2$

$H^2 = 81 + 144$

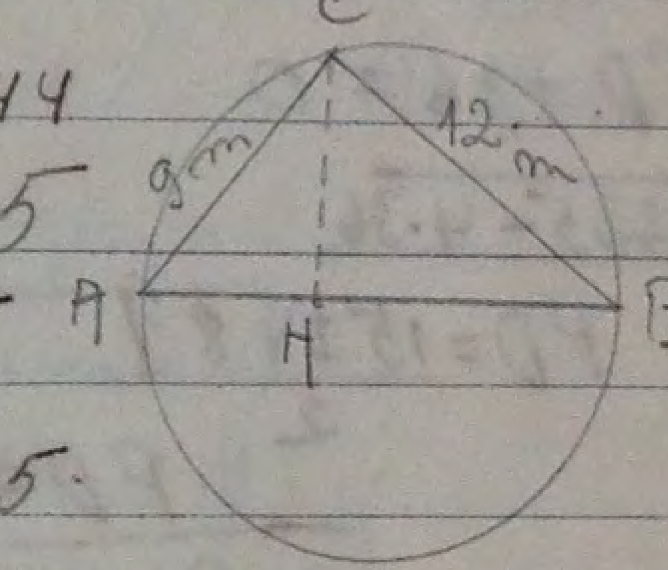
$h^2 = 225$

$h = 15$

raio = 7,5

AB = 15

AH = 5,4 } CH = 7,2
HB = 9,6 }



$\overline{CH}^2 = AH \cdot BH$

AH = 5,4

$\overline{AC}^2 = AB \cdot AH$

$\overline{AC}^2 = 81$, AB = 15

$AH = \frac{81}{15} = 5,4$

$\overline{CB}^2 = D \cdot HB$

$144 = 15 \cdot HB$

$HB = \frac{144}{15} = 9,6$

2 cordas AB e CD interceptam em M. Sabendo-se que AM = 5 dm MB = 6 dm MC = 7,5 dm

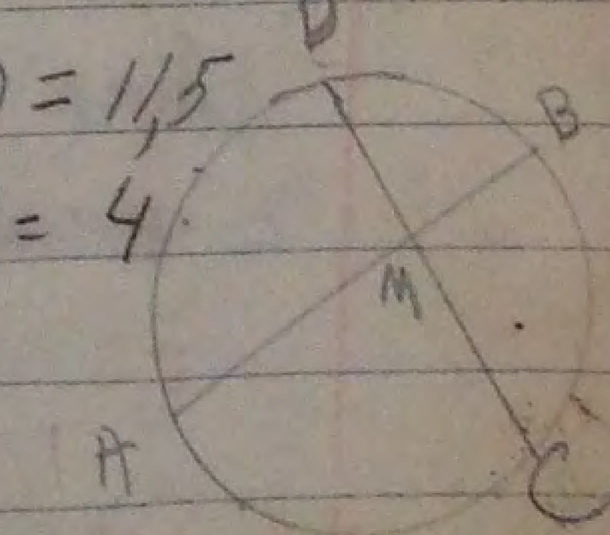
$AM \cdot MB = DM \cdot MC$

$5 \times 6 = DM \cdot 7,5$

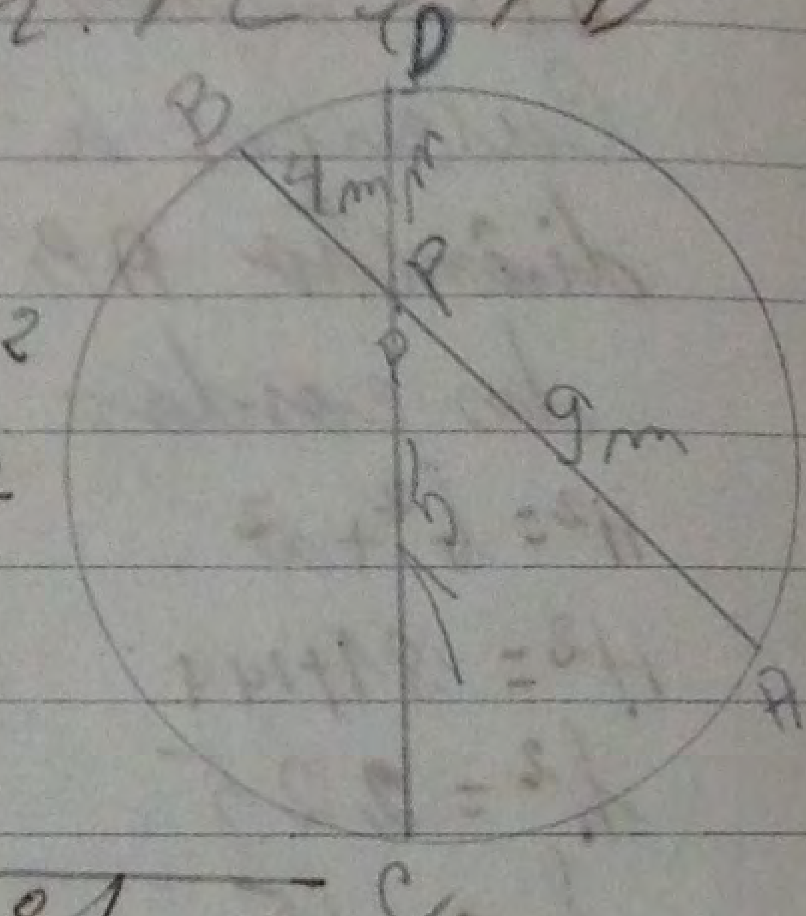
$DM = \frac{30}{7,5} = 4$ DM = 4

CD = 11,5

DM = 4



2 cordas AB e CD interceptam-se em P. Sabendo que AP = 9
PB = 4
CD = 15 Calcular PC e PD



$AP \cdot PB = CP \cdot PD$

$9 \times 4 = (15 - PD) PD$

$9 \times 4 = 15PD - \overline{DP}^2$

$36 = 15PD - \overline{PD}^2$

$PD^2 - 15PD + 36 = 0$

$PD = \frac{15 \pm \sqrt{225 - 4 \cdot 36}}{2}$

$PD = \frac{15 \pm \sqrt{81}}{2}$

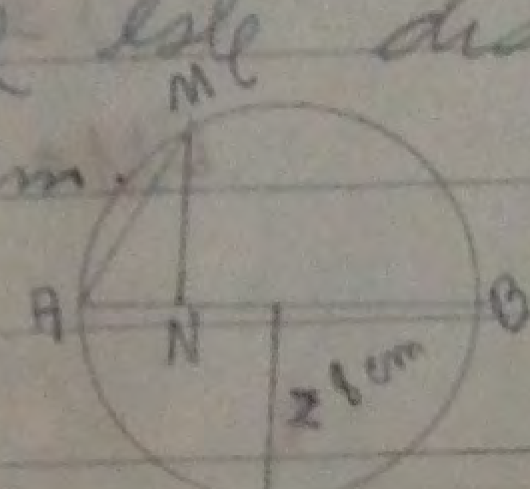
$PD = \frac{15 \pm 9}{2}$ $\quad PD = 12$

$15 - 12 = 3$ $\quad$ **CP = 3**

$PD = 12$

9-10-61

Raio = 8cm, traça-se pela extremidade de um diametro uma corda cuja a projeção sôbre este diametro é igual a 4 cm.



$\overline{AM}^2 = AB \cdot AN$

$16 \times 4 = \overline{AM}^2$

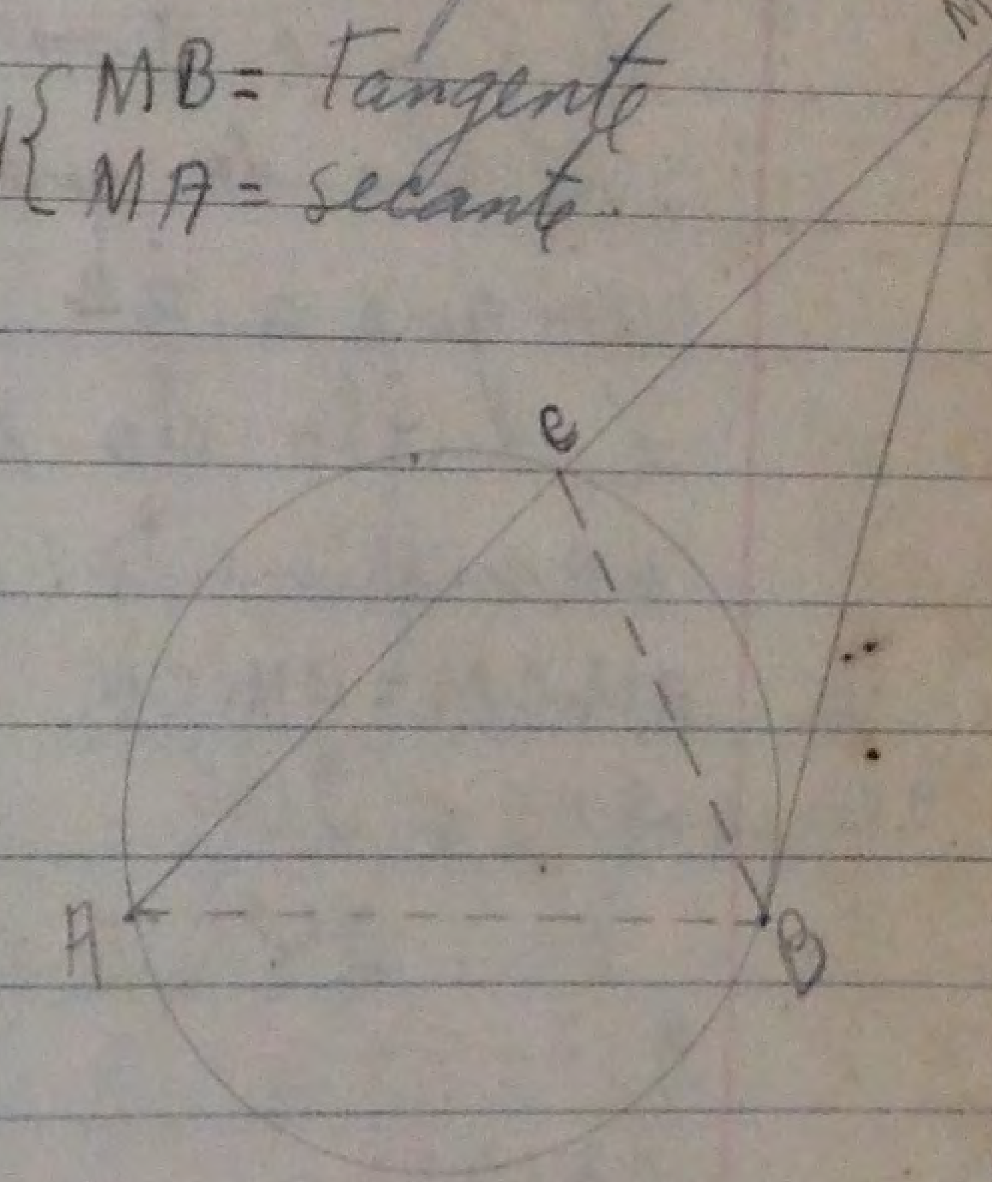
$64 = \overline{AM}^2$

$AM = \sqrt{64}$ $\quad AM = 8$

10-10-61

De um ponto b são traçadas a uma circunferência, de raio igual a 5cm, uma tangente e uma secante. Sabendo que a secante mede 16 m. tg = 12 m. Calcular o valor da parte externa da secante.

H{ MB = tangente
MA = secante



4ª relação

H{ MB = tangente
MA = secante

T{ $\overline{MB}^2 = MA \cdot CM$

$\triangle MCB \quad \triangle MAB$

$\hat{A} = \frac{\text{arco } CB}{2}$

$\hat{B} = \frac{\text{arco } CA}{2}$

$\hat{M}$ comum nos 2 $\triangle$s

$\hat{A} = \hat{B}$

$\frac{MA}{MB} = \frac{MB}{MC}$ $\quad MA \cdot MC = \overline{MB}^2$

$\overline{MB}^2 = MA \cdot MC$

tg = 12 $\quad$ Secante = 16 m.

AM=16 MB=12 $\ell = \frac{\overline{MB}^2}{AM}$

$\frac{AM}{MB} = \frac{MB}{\ell}$ $\ell = CM$

$\ell = \frac{144}{16} = 9$ $\ell = 9$ $\ell = CM = 9$

Uma corda é dividida por uma outra em 2 segm. de 8 m. e 9 m.

Sabendo que um dos segm. em que a 2ª divide a 1ª é duplo do outro. Determinar o comprimento da corda.

AM.CM = BM.CM

$9.8 = \ell.\ell \rightarrow 8 \times 9 = 2\ell^2$

$72 = 2\ell^2$

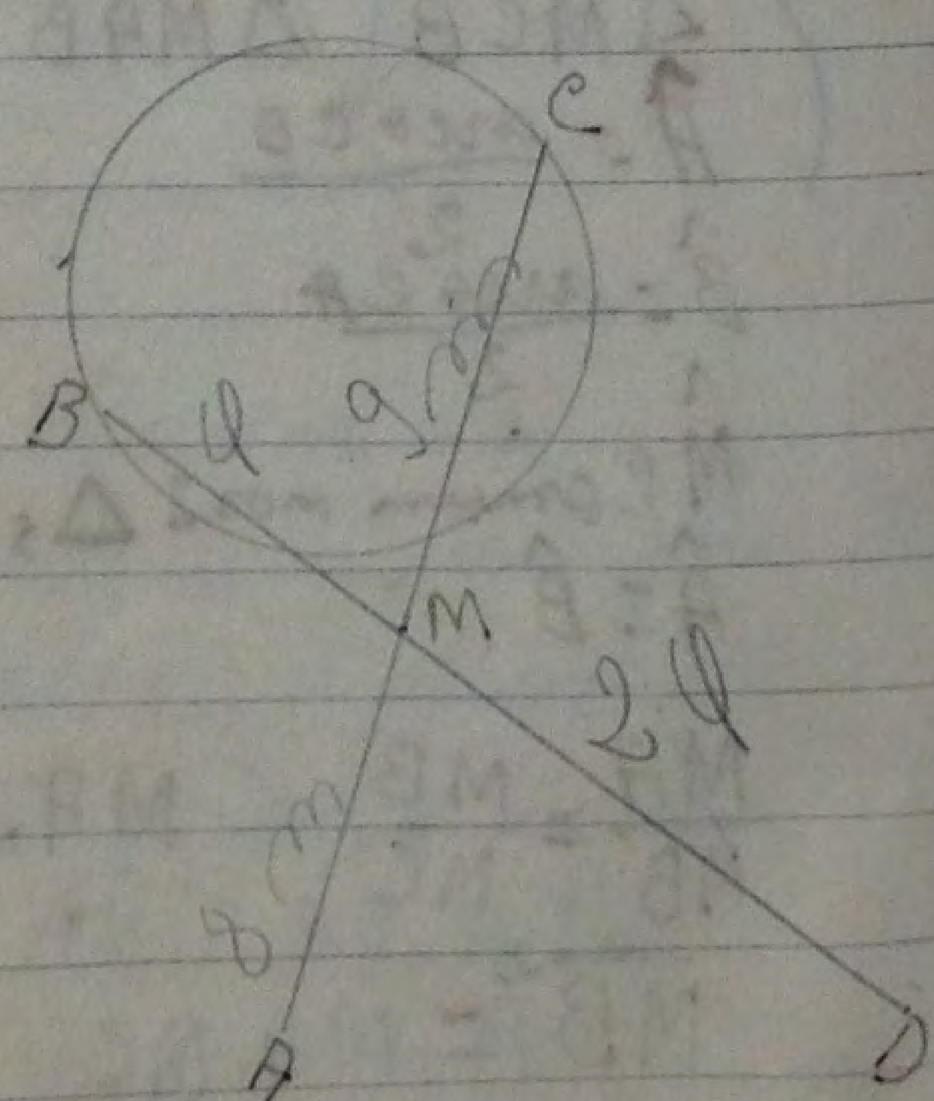
$2\ell^2 = 72$

$\ell^2 = \frac{72}{2} = 36$

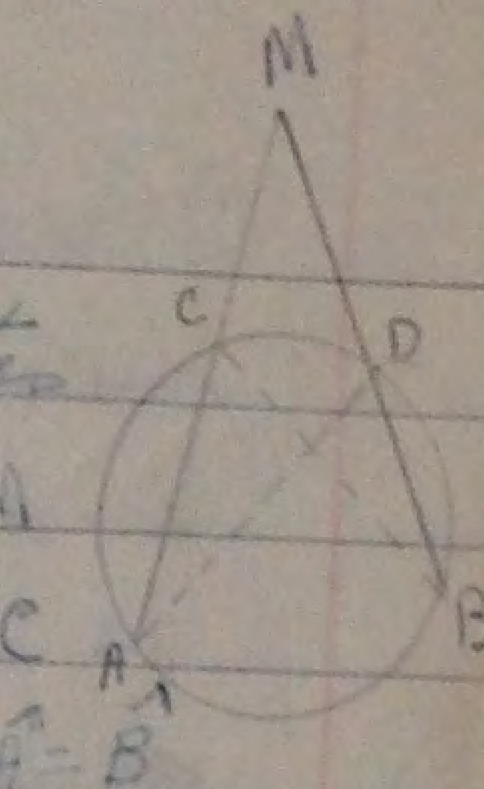
$\ell = 6$

$2\ell = 12$

BD = 18



13-10-61



4ª relação: H{ AM e BM = secantes
T{ AM.CM = BM.DM
D{ Δ AMD Δ BMC
$\hat{M}$: é comum $\hat{A} = \hat{B}$

$\hat{A} = \frac{\overset{\frown}{CD}}{2}$ $\hat{B} = \frac{\overset{\frown}{CD}}{2}$ $\hat{A} = \hat{B}$

$\frac{AM}{MB} = \frac{DM}{CM}$ **AM.CM = BM.DM**

Uma corda e um diâmetro se interceptam num círculo de 8 dm de raio.

As duas menores partes que resultem dessa intercessão medem respectivamente 3 dm na corda e 2 dm no diâmetro. Quanto mede a Corda?

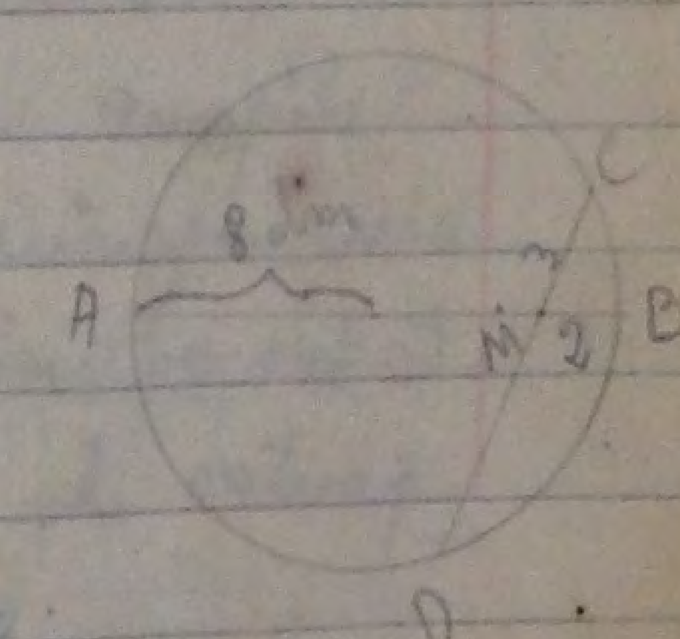
AM.MB = CM.MD

14.2 = 3 MD

28 = 3 MD

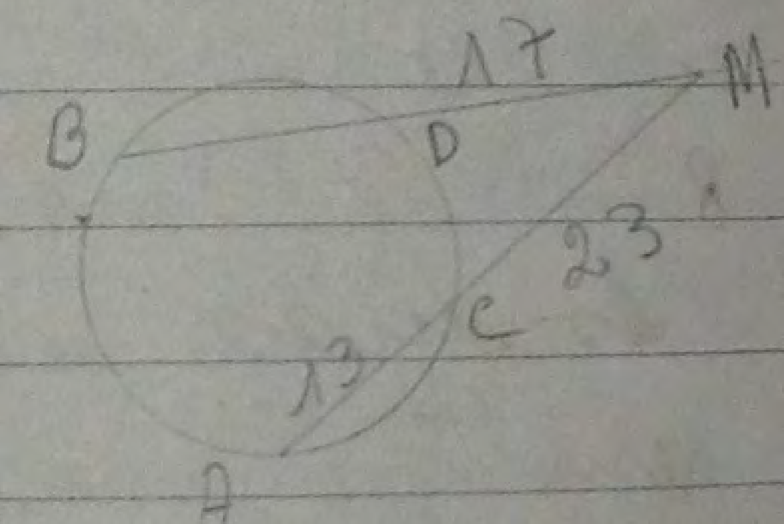
$MD = \frac{28}{3} = 9,333$

$CD = 9,333 + 3 = 12\frac{1}{3}$ dm



$CD = 12\frac{1}{3}$ dm

2 secantes são traçadas dum mesmo ponto exterior à uma circunferência. As partes interna e externa de uma delas medem 13 dm e 23 dm, e a parte externa da outra mede 17 dm. Calcular a parte interna da última.



$AM.CM = BM.DM$

$36.23 = BM.17$

$17BM = 828$

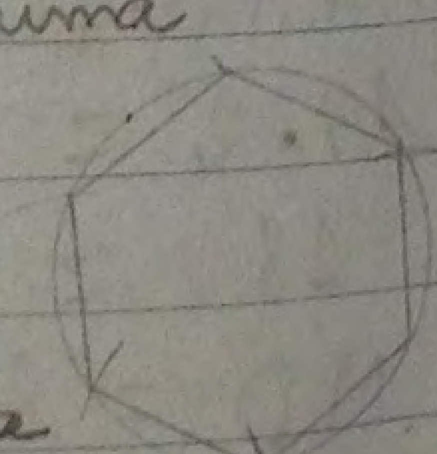
$BM = \frac{828}{17} = 48,7$

$BM = 48,7$ $\quad 48,7 - 17 = 31,7$

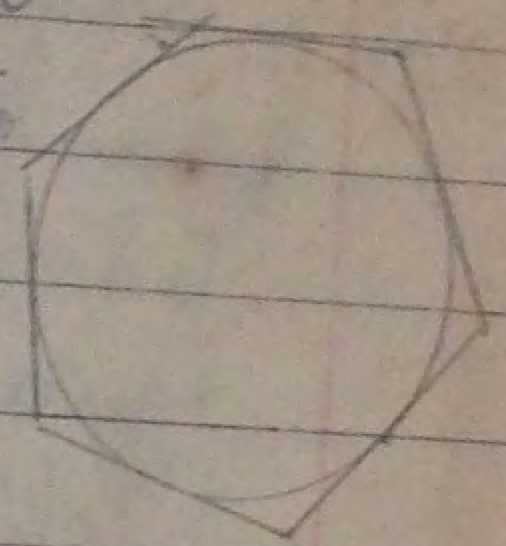
17-10-61

Quadriláteros convexos inscritos

Polígono inscrito em uma circunferência quando todos os vértices são pontos da circunferência



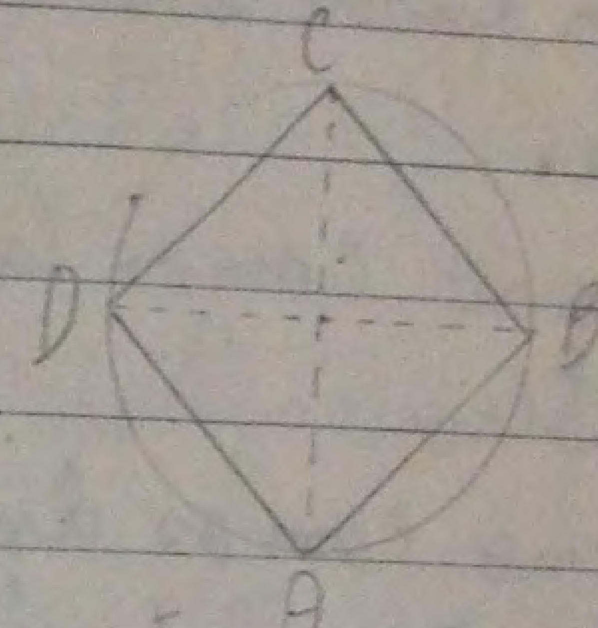
Polígono circunscrito os lados são tangentes à circunferência



H{ ABCD quadrilátero inscrito

T{ $\hat{D} = \hat{B} = 180°$

$\hat{C} = \hat{A} = 180°$



Os ângulos opostos são suplementares

D{ $\hat{D} = \frac{\overset{\frown}{CB} + \overset{\frown}{BA}}{2}$

$\hat{B} = \frac{\overset{\frown}{AD} + \overset{\frown}{DC}}{2}$

$\hat{D} + \hat{B} = \frac{\overset{\frown}{CB} + \overset{\frown}{BA} + \overset{\frown}{AD} + \overset{\frown}{DC}}{2} = \frac{360}{2} = 180°$

Quadrilátero circunscritível

H{ABCD: quadrado circunscrito

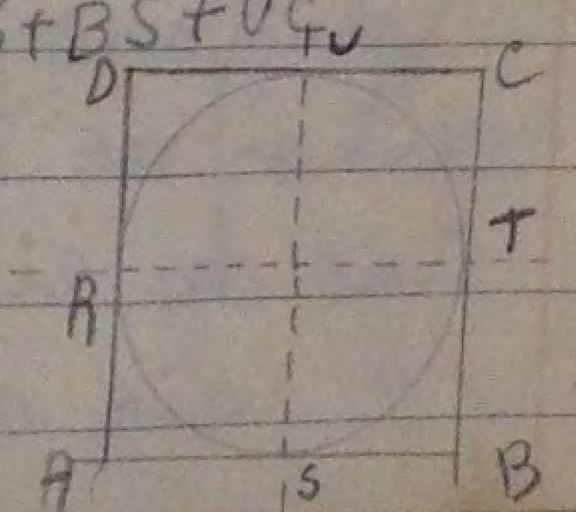
Enun: a soma de 2 lados opostos é igual a soma dos outros dois

T{ $AD + BC = AB + DC$

$DR = DU \quad AR = AS \quad BP = BS \quad CT = UC$

$DR + AR + BT + CT = DU + AS + BS + UC$

D{ $AD + BC = DC + AB$

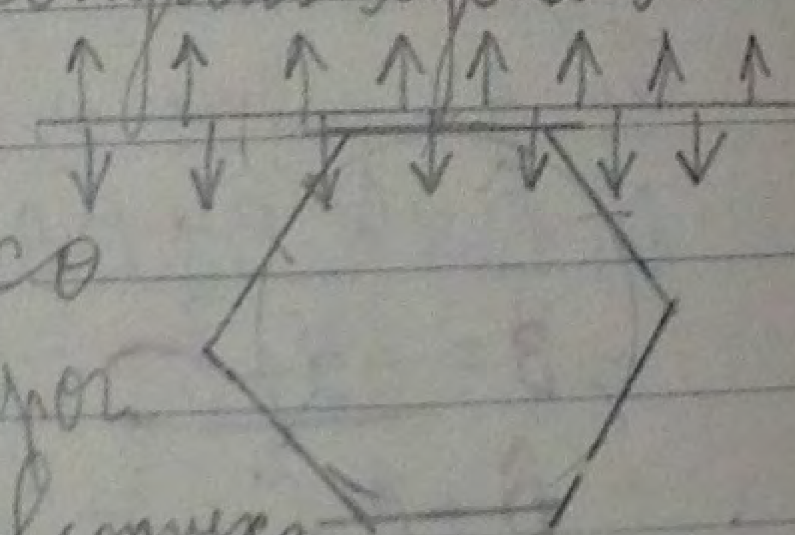


18-10-61.

Polígono convexo e regular: quando for limitado por todos os lados e todos os ângulos iguais.

Polígono com os lados iguais = = equilátero

Polígono com os ângulos iguais = equiângulo.



Polígono convexo quando limitado por uma linha poligonal convexa

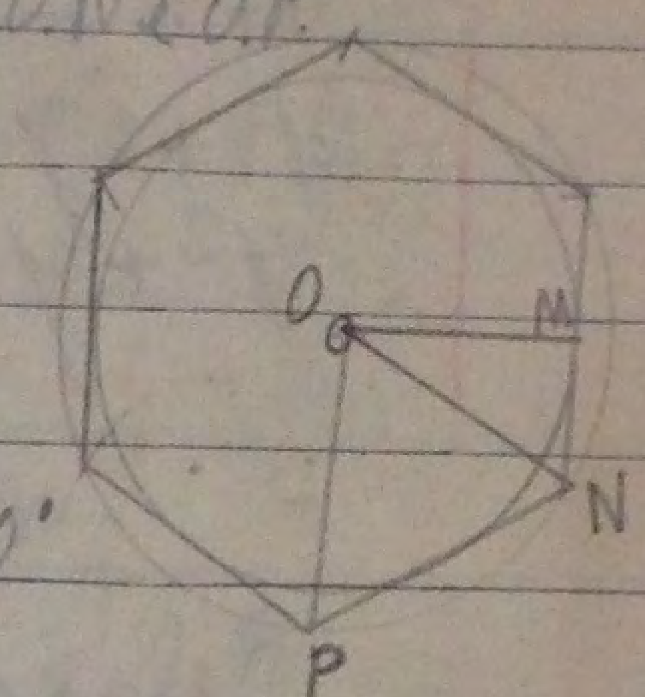
Polígono concavo:

Apótema do polígono é o raio O.M. da circunferência inscrita no polígono, é a distância do centro ao lado do polígono

Raio da circunferência circunscrita é o raio do polígono O.N.

Raios consecutivos O.N. e O.P.

Ângulo central NÔP



Quando os lados são raios consecutivos

o ângulo central $= \frac{360°}{6} = 60°$

$a = 60°$

Qual é o polígono cujo ângulo central é 72°

$a_n = a_{72°} = \frac{360°}{n} \therefore a_n = \frac{360°}{72} = 5$ lados

$2x^2 - 9x - 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \therefore x = \frac{9 \pm \sqrt{81 + 40}}{4}$

$x = \frac{9 \pm \sqrt{121}}{4}$ $\quad x = \frac{9 \pm 11}{4}$

$x' = \frac{9 + 11}{4} = \frac{20}{4}$ $\quad x' = 5$

$x'' = \frac{9 - 11}{4} = -\frac{2}{4} = -\frac{1}{2}$ $\quad x'' = -\frac{1}{2}$

$(x+5)(x+2) = 40$

$x^2 + 7x + 10 - 40 = 0 \therefore x^2 + 7x - 30 = 0$

$x' = 3$ $\quad x'' = -10$

$(y+1)^2 = 3+y$

$y^2+2y+1=3+y$

$y^2+y-2=0$

$y'=1 \qquad y''=-2$

20-10-61.

Relações métricas entre o lado, raio e apótema de um polígono regular inscrito

lado = l   raio = R   apótema = a

$R^2 = \frac{l^2}{4}+a^2$

$R^2 = \frac{4a^2+l^2}{4}$ ΔABC → retângulo

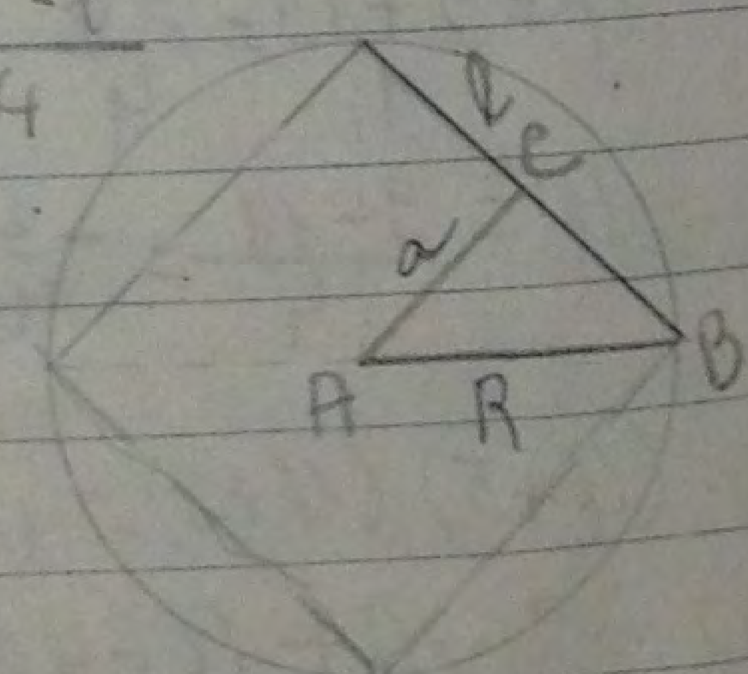
$R = \frac{\sqrt{4a^2-l^2}}{2}$

$R=\frac{1}{2}\sqrt{4a^2+l^2}$

$a^2 = R^2 - \frac{l^2}{4}$ $a^2=\frac{4R^2-l^2}{4}$

$a=\frac{\sqrt{4R^2-l^2}}{2}$

$a=\frac{1}{2}\sqrt{4R^2-l^2}$.



$\frac{l^2}{4}=R^2-a^2 \therefore l^2=4(R^2-a^2)$

$l=2\sqrt{R^2-a^2}$

$\frac{u}{7}+\frac{21}{u+5}=\frac{47}{7}$

$u^2+5u+147=47u+235$

$u^2+5u-47u+147-235=0$

$u^2-42u-88=0$

$u=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \therefore u=\frac{42\pm\sqrt{1764+352}}{2}$

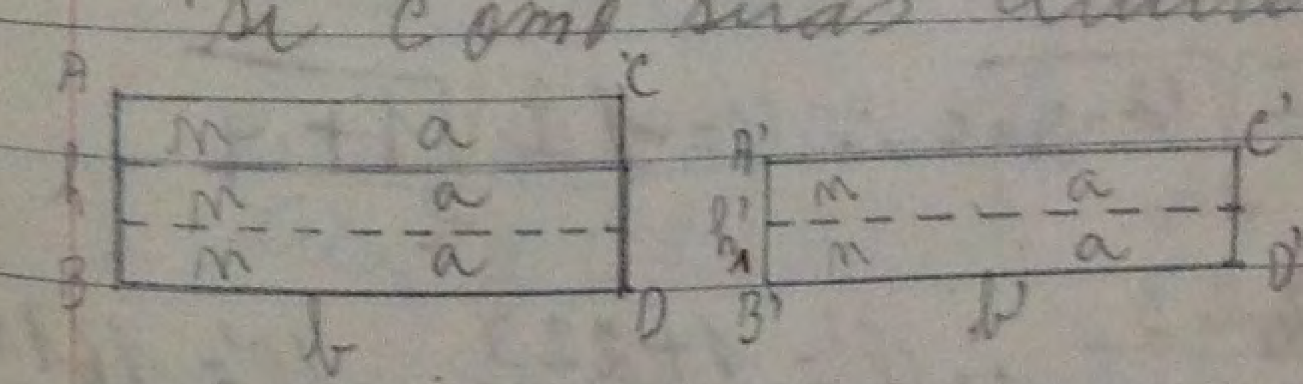
$u=\frac{42\pm\sqrt{2116}}{2} \therefore u=\frac{42\pm 46}{2}$

$u'=\frac{42+46}{2} \therefore u'=\frac{88}{2}$ $u'=44$

$u''=\frac{42-46}{2} \therefore u''=-\frac{4}{2}$ $u''=-2$

24-10-61.

Áreas: as áreas de 2 retângulos que tem a mesma base estão entre si como suas alturas



Hip{ h = altura do retângulo ABCD

T{ $\frac{\text{Área } ABCD}{\text{área } A'B'C'D'} = \frac{h}{h'}$

H{ h = altura do retângulo ABCD
h' = altura do retângulo A'B'C'D'
b = base dos retângulos

$h = 3\,m$

$h' = 2\,m$

$\frac{h}{3m} = \frac{h'}{2m}$ $\left(\frac{h}{h'} = \frac{3}{2}\right)$ (1ª)

área ABCD = 3a

área A'B'C'D' = 2a

$\frac{\text{área } ABCD}{\text{área } A'B'C'D'} = \frac{h}{h'}$

$\frac{\varphi+1}{\varphi} + 1 = \frac{\varphi}{\varphi-1} \therefore \frac{\varphi^2 - 1 + \varphi(\varphi-1) = \varphi^2}{\varphi\cdot(\varphi-1)}$

$\varphi^2 + \varphi^2 - \varphi - \varphi^2 - 1 = 0$

$\varphi^2 - \varphi - 1 = 0$

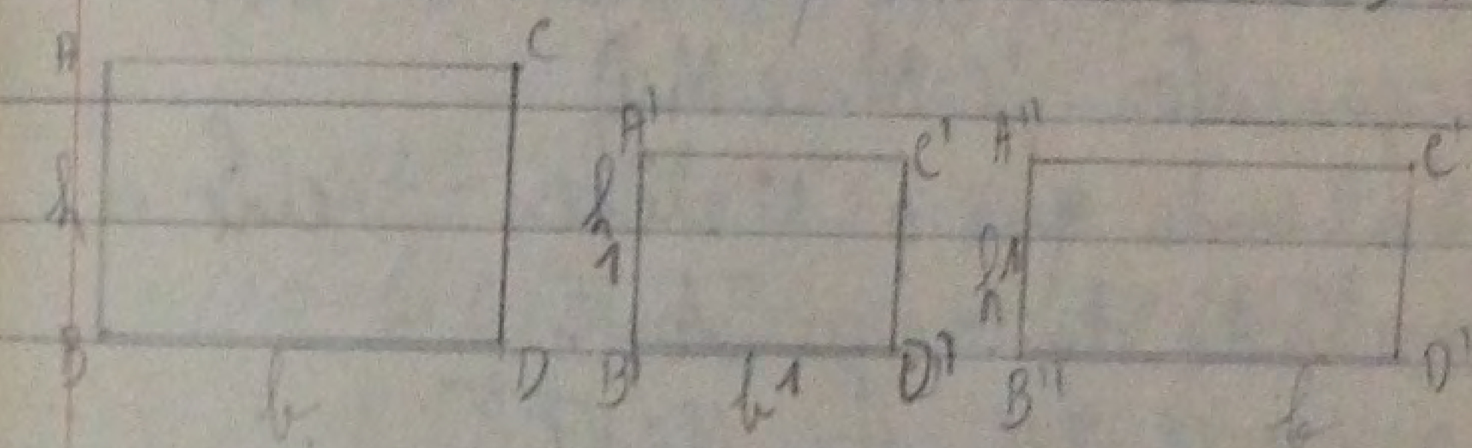
$\varphi = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \therefore \varphi = \frac{1 \pm \sqrt{1+4}}{2}$

$\varphi = \frac{1 \pm 2{,}23}{2} \therefore \varphi' = \frac{1+2{,}23}{2}$ $\varphi' = 1{,}611$

$\varphi'' = \frac{1-2{,}23}{2}$ $\varphi'' = -0{,}61$

25-10-61

As áreas de 2 retângulos quaisquer estão entre si como os produtos das bases pelas alturas.



T{ $\frac{\text{área } ABCD}{\text{área } A'B'C'D'} = \frac{b \times h}{b' \times h'}$

H{ h = altura de ABCD
b = base de ABCD
h' = altura de A'B'C'D'
b' = base de A'B'C'D'

$\frac{\text{área } ABCD}{\text{área } A''B''C''D''} = \frac{h}{h'}$

$\frac{\text{área } A''B''C''D''}{\text{área } A'B'C'D'} = \frac{b}{b'}$

$\frac{\text{área } ABCD}{\text{área } A''B''C''D''} \times \frac{\text{área } A''B''C''D''}{\text{área } A'B'C'D'} = \frac{h}{h'} \times \frac{b}{b'}$

$\frac{\text{área } ABCD}{\text{área } A'B'C'D'} = \frac{h}{h'} \times \frac{b}{b'}$

Uma corda CD intercepta
um diâmetro AB = 8 cm
Ponto médio M do raio
Determinar os valores dos
segmentos CM e MD
Sabendo que o 1º dêles vale a
metade do segundo.

$2 \cdot 6 = x \cdot 2x$ $\qquad x = \sqrt{6}$

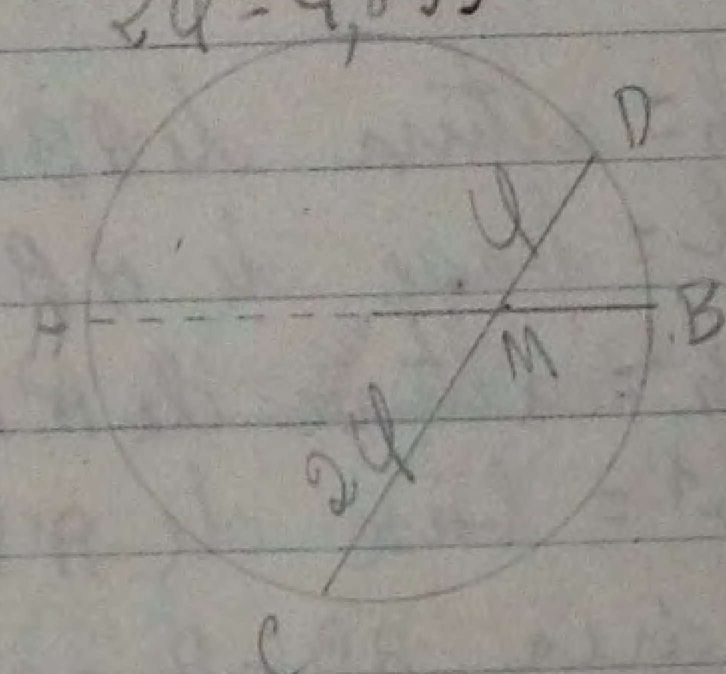
$12 = 2x^2$ $\qquad x = 2{,}4495$

$2x^2 = 12$ $\qquad 2x = 4{,}899$

$2x^2 - 12 = 0$

$x^2 = \frac{12}{2}$

$x^2 = 6$



27-10-61.

$\frac{x-1}{2} - \frac{3x - x^2}{3} = x + \frac{1}{3}$

$3x - 3 - 6x + 2x^2 = 6x + 2$

$2x^2 - 9x - 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \therefore x = \frac{9 \pm \sqrt{81 + 40}}{4} \therefore x = \frac{9 \pm \sqrt{121}}{4}$

$x = \frac{9 \pm 11}{4}$ $\qquad x' = \frac{9 + 11}{4} \therefore x' = 5$

$x'' = \frac{9 - 11}{4} \therefore x'' = -\frac{1}{2}$

$\frac{3x-1}{7} - \frac{9x-3}{16} = \frac{x^2-4}{7} - \frac{(2x-1)^2}{4}$

$\frac{48x - 16 - 63x - 21 = 16x^2 - 64 - 28x^2 + 56x - 28}{112}$

$48x - 16 - 63x - 21 - 16x^2 + 64 + 28x^2 - 56x + 28 = 0$

$12x^2 - 71x + 55 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \therefore x = \frac{71 \pm \sqrt{5041 - 2640}}{24}$

$x = \frac{71 \pm 49}{24}$ $\qquad x' = \frac{71 + 49}{24} = 5$ $\quad x' = 5$

$x'' = \frac{71 - 49}{24} \therefore x'' = \frac{11}{12}.$

7-11-61

$\frac{2x-1}{x+1} = \frac{x+1}{x-1} \therefore \frac{(x-1)(2x-1) = (x+1)(x+1)}{(x+1)(x-1)}$

$(x-1)(2x-1) = (x+1)(x+1)$

$2x^2 - x^2 - 2x - x + 1 = x^2 + 2x + 1.$

$2x^2 - x^2 - 2x - x - 2x - 1 - 1 = 0$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0 \therefore x' = 0$

$x - 5 = 0 \therefore x'' = 5$

Área do retângulo:
$S = b \times h$.
Área do quadrado:
$S = b^2$
Área do triângulo.
$S = \frac{b \times h}{2}$
Área do trapézio.
$S = \frac{B+b}{2} \times h$
Área do lozango:
$S = \frac{D \times d}{2}$
Área da circunferência
$S = \pi R^2$

8-11-61.

$P = 24$ m — P = perímetro
$B = 2h$. — B = base maior
$2h + h = \frac{24}{2}$ ∴ $3h = 12$
$h = \frac{12}{3}$ ∴ $h = 4$
$2h = B$ — $2h = 2 \times 4$ ∴ $B = 8$

$S = B \cdot h$ — $P = 24$
$P = 2(B+h)$ — $24 = 2 \cdot 3h$
$24 = 2(B+h)$ — $24 = 6h$
$h = \frac{24}{6} = 4$ m — $B = 2h$ ∴ $B = 8$ m.

A diagonal de um retângulo é igual a 10 metros, um de seus lados = 8 m.
$B^2 + h^2 = d^2$ — $B = 8$
$64 + h^2 = 100$
$h^2 = 100 - 64$
$h = \sqrt{100-64}$ ∴ $h = \sqrt{36}$ ∴ $h = 6$
$S = 8 \times 6$ ∴ superf $= 48 m^2$.

Calcular a área de um retângulo. O perímetro é igual a 28 dm, a diag = 10 dm.
$P = 2(B+h)$
$28 = 2(B+h)$. — $\frac{28}{2} = B + h$
$B + h = 14$
$B = 14 - h$
$100 = h^2 + (14-h)^2$
$100 = h^2 + 196 - 28h + h^2$
$2h^2 - 28h + 196 - 100 = 0$
$2h^2 - 28h + 96 = 0$
$h = \frac{28 \pm \sqrt{784 - 768}}{4}$ — $h = \frac{28 \pm \sqrt{16}}{4}$
$h = \frac{28 \pm 4}{4}$ — $h' = 8$ — $h'' = 6$
$h = 6$ — $b = 8$

Superf. = 48 dm^2

Superficie de um triângulo é igual 400 m^2, altura = 20 m

$\frac{Bh}{2}$ = Suf do triângulo

400x2 = 20B

800 = 20B

$B = \frac{800}{20} = 40\,m$ B = 40 m.

Num lozango, de lado igual a 10 m. a diagonal maior = 16 m.
Calcular a área.

diag. menor^2 + 8^2 = 100

diag menor = $\sqrt{100-64}$

diag menor = 6 ∴ 2x6 = 12

16 x 12 = 192

192 ÷ 2 = 96 m^2.

As bases de um trapézio medem 12 m e 8 m.
altura = 5 m.

$S = \frac{B+b}{2} \times h$ $S = \frac{12+8}{2} \times 5 = 50\,m^2$

10-11-61.

Calcular a área de um circulo que tem 14 m de diâmetro.

$S = \pi R^2$

$S = 3{,}14 \times \frac{14}{2}^2$ ∴ $S = 3{,}14 \times 49 = 153{,}94\,m^2$

A soma das diagonais de um lozango é 8,4 m.
Sabe-se que as diagonais são proporcionais aos nºs 3 e 4

$S = \frac{D \times d}{2}$ d + D = 8,4

$\frac{d}{D} = \frac{3}{4}$ $\frac{8{,}4}{d} = \frac{7}{3}$ $d = \frac{8{,}4 \times 3}{7} = \frac{25{,}2}{7}$

d = 3,6

$\frac{8{,}4}{D} = \frac{7}{4}$ $D = \frac{8{,}4 \times 4}{7} = \frac{33{,}6}{7} = 4{,}8$

D = 4,8

$\frac{3{,}6 \times 4{,}8}{2} = \frac{17{,}28}{2} = 8{,}64\,m^2$

Determinar a área de um lozango que tem 3,4 m de lado e a diagonal menor igual a 2 m.

$l^2 = 11,56$

$d = 1$ $\quad D^2 = l^2 - d^2$

$D^2 = 11,56 - 1 \therefore D^2 = 10,56$

$D = \sqrt{10,56} \therefore D = 3,24$

$S = \frac{D \times d}{2} \therefore S = \frac{~~6,48 \times 1~~}{} \quad S = \frac{6,48 \times 2}{2}$

$S = 6,48\ m^2$

Calcular o valor da outra diagonal de um lozango, sabendo-se que a área do lozango é equivalente a área de um triângulo que possue 3,4 m de base e 2,6 m de altura.

A diagonal maior do lozango mede 4 m.

$S_{triângulo} = \frac{B \times h}{2} \quad S^{tr.} = \frac{3,4 \times 2,6}{2}$

$S^t = 4,42\ m^2$

$S_{lozango} = 4,42\ m^2$

$d = \frac{S \times 2}{D} \quad d = \frac{4,42 \times 2}{4\ 2} \quad d = 2,21\ m$

$d = 2,21\ m$

Calcular a base menor de um trapézio, tendo a base maior 12 m e 5 m de altura, a superfície igual a 50 m^2

$S_{trap.} = \frac{B+b}{2} \times h \therefore 2S = (B+b) \times h$

$100 = (B+b) \times h$

$b = \frac{100}{5} - 12 \quad b = 20 - 12$

$b = 8\ m.$

14-11-61.

Determinar a área de um triângulo equilátero cujo o lado mede 6 cm.

$S = \frac{B \cdot h}{2} \quad S = \frac{l \times \frac{l\sqrt{3}}{2}}{2} \quad S = \frac{l^2\sqrt{3}}{2} \times \frac{1}{2}$

$h^2 = l^2 - \frac{l^2}{4} \therefore h^2 = \frac{4l^2 - l^2}{4} = \frac{3l^2}{4}$

$h = \frac{l\sqrt{3}}{2} \quad S = \frac{l^2\sqrt{3}}{4}$

$h = \frac{6\sqrt{3}}{2}$ $h = \frac{3\not{6} \times 1,732}{\not{2}\,1} = 5,196$

$S = \frac{5,196 \times \not{6}^{3}}{\not{2}} = 15,588\ cm^2$

Calcular a área de um triângulo equilátero que tem 8 cm de lado.

$S = \frac{B \times h}{2}$ $h^2 = l^2 - \frac{l^2}{4}$

$h^2 = \frac{4l^2 - l^2}{4}$ $h^2 = \frac{3l^2}{4}$

$h = \frac{l\sqrt{3}}{2}$ $h = \frac{\not{8}^{4} \times 1,732}{\not{2}} = 6,928\ cm$

$S = \frac{6,928 \times \not{8}\,4}{\not{2}\,1} = 27,712\ cm^2$

Calcular o comprimento de uma circunferência que tem 8 cm de diâmetro

perímetro da circ $= 2r.\pi$

perímetro da circ $= 3,14 \times 8 = 25,12\ cm$

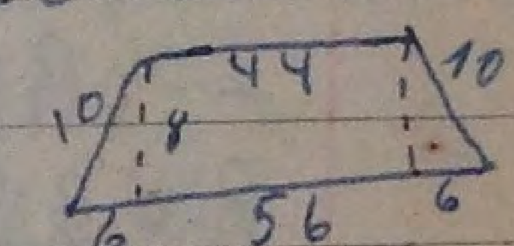
Calcular o diâmetro de uma circunferência que tem 25,12 cm de comprimento

$D = P \div \pi$

$D = 25,12 \div 3,14 = 8$. $D = 8\ cm$

Determinar a área de um trapézio isósceles, sabendo que as bases medem 44 e 56 dm, os lados não paralelos somam 20 dm

$S = \frac{B + b}{2} \times h$



$h^2 = 100 - 36 = 64$

$56 - 44 = 12$ $\frac{12}{2} = 6$

$h^2 = 64 \therefore h = 8$

$S = \frac{(56 + 44)}{2} \times 8$ $\frac{100}{2} \times 8 = 400$

$S = 400\ dm^2$

17-11-61.

$l^2 = R^2 + R^2$

$l^2 = 2R^2$

$l = R\sqrt{2}$

$l = 2a$ a = apótema

$a = \frac{l}{2}$

$a = \frac{R\sqrt{2}}{2}$

O lado de um quadrado inscrito num círculo mede $3\sqrt{2}$ cm.

Calcular o raio do círculo

$L = R\sqrt{2}$ $3\sqrt{2} = R\sqrt{2}$

$R = \frac{3\sqrt{2}}{\sqrt{2}} = 3$ $R = 3$

Um quadrado tem o apótema igual a 5 dm.

Calcular o perímetro do quadrado e o diametro do circulo.

$l = 2a$ $l = 2 \times 5$ $l = 10$

Perímetro $= 4 \times 10 = 40$ dm

$R = \frac{l}{\sqrt{2}}$ $R = \frac{10}{1{,}414} = 7{,}05$ $D = 14{,}1$ dm

Um quadrado está inscrito num círculo de raio igual a 4 dm. Calcular o lado, o apótema e a diagonal.

$l^2 = 2R^2 \therefore l = R\sqrt{2} \therefore l = 4 \cdot 1{,}414$

$l = 5{,}656$ dm

$a = \frac{l}{2} \therefore a = \frac{5{,}656}{2}$ $a = 2{,}828$ dm

$D = 2R \therefore D = 8$ dm

Um quadrado está inscrito num círculo de raio igual a 6 dm. Calcular o apótema e perímetro.

$l = R\sqrt{2} \therefore l = 6\sqrt{2} \therefore l = 6 \cdot 1{,}414 \therefore l = 8{,}484$ dm

$a = \frac{l}{2} \therefore a = \frac{8{,}484}{2} \therefore a = 4{,}242$

$P = l \times 4 \therefore P = 8{,}484 \times 4 = 33{,}936$ dm.

21-11-61.

$\ell - 2\sqrt{\ell} = 15$

$-2\sqrt{\ell} = 15 - \ell$

$4\ell = 225 - 30\ell + \ell^2$

$\ell^2 - 30\ell + 225 - 4\ell = 0$

$\ell^2 - 34\ell + 225 = 0$

$\ell = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \therefore \ell = \frac{34 \pm \sqrt{1156 - 900}}{2}$

$\ell = \frac{34 \pm 16}{2}$ $\ell' = \frac{34 + 16}{2} = 25$ serve

$\ell'' = \frac{34 - 16}{2} = 9$ não serve

$\sqrt{x-13}=2$

$x-13=4$

$x=4+13$

$x=17$

$\sqrt{17-13}=2$

$\sqrt{4}=2$

$2=2$

$\sqrt{18-2x}=\sqrt{x+6}$

$18-2x=x+6$

$2x+x=-6+18$

$3x=12$

$x=\frac{12}{3}$

$x=4$

$\sqrt{18-8}=\sqrt{4+6}$

$\sqrt{10}=\sqrt{10}$

$2\sqrt{5x-1}=3\sqrt{3x-2}$

$4(5x-1)=9(3x-2)$

$20x-4=27x-18$

$-7x=-14$

$x=\frac{-14}{-7}$ $x=2$

$\sqrt{5x+6}=2+\sqrt{5x-6}$

$5x+6=4+4\sqrt{5x-6}+5x-6$

$5x+6-4-5x+6=4\sqrt{5x-6}$

$8=4\sqrt{5x-6}$

$64=16.(5x-6$

$64=80x-96$

$80x=64+96$

$80x=160$ $x=\frac{160}{80}=2.$ $x=2$

24-11-61.

$x^2-mx+36=0$ $x'=9$

$m=9+4=13$ $x'+x''=m$

$x'\cdot x''=36$

$9.x''=36$

$x''=4$

$x^2-15+(8m+2)=0$

$x'+x''=5$ $x'+x''=15$

$x'\cdot x''=8m+2$ $x'=x''+5$

$5+x''+x''=15$ $2x''=10$

$x''=5$ $x'=10$

$50=8m+2$ $50-2=8m$

$8m=48$ $m=\frac{48}{8}=6$ $m=6$

1) raizes reais desiguais $m<9$ $\Delta>0$

2) raizes reais iguais $\Delta=0$

3) não tem raizes reais $\Delta<0$

$x^2-6x+m=0$ $\Delta=b^2-4ac$ $\therefore \Delta>0$

$36-4m>0$ $-4m>-36$ (-1) $\therefore 4m<36$

$m<\frac{36}{4}$ $m<9$

$36-4m=0$ $-4m=-36$ (-1)

$4m=36$ $\therefore$ $m=\frac{36}{4}$ $m=9$

$36-4m<0$ $-4m<-36$ (-1)

$4m>36$ $\therefore$ $m>9$