③ $tg\ 240^\circ = \sqrt{3}$

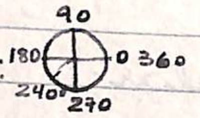
Parte da fórmula derivada:

sec α : $\sec^2\alpha = 1 + tg^2\alpha$

$$\sec^2\alpha = \pm\sqrt{1+tg^2\alpha}$$

$$\sec\alpha = \pm\sqrt{1+3}$$

$$\sec\alpha = \pm 2$$

Como está no 3º quadrante  o sinal é negativo ∴ $\boxed{\sec\alpha = -2}$

cos α : $\frac{1}{\sec\alpha}$ ∴ $\boxed{\cos\alpha = -\frac{1}{2}}$

sen α : $sen^2\alpha + \cos^2\alpha = 1$

$$sen^2\alpha = 1-\cos^2\alpha$$

$$sen\,\alpha = \pm\sqrt{1-\cos^2\alpha}$$

$$sen\,\alpha = \pm\sqrt{1-(\tfrac{1}{2})^2}$$

$$sen\,\alpha = \sqrt{1-\frac{1}{4}} \quad ∴ \quad \boxed{sen\,\alpha = \frac{\sqrt{3}}{2}}$$

$$sen\,\alpha = \sqrt{\frac{3}{4}}$$

cosec α : $\frac{1}{sen\,\alpha}$

$$\cos\alpha = \frac{1}{-\frac{\sqrt{3}}{2}} \quad ∴ \quad \cos\alpha = \frac{1.\sqrt{3/2}}{\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}} = \frac{-2\sqrt{3}}{3}$$

$$\boxed{cosec\,x = \frac{-2\sqrt{3}}{3}}$$

$$\text{cotg}\,\alpha = \frac{1}{tg\alpha} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}.\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} \quad ∴ \quad \boxed{\text{cotg}\,\alpha = \frac{\sqrt{3}}{3}}$$

Segunda Aplicação

na 2ª aplicação temos a espressão das linhas trigonométricas em função de uma delas.

① Exprimir todas as linhas trigonométricas em função do $\boxed{sen\,x}$

Obs. → no 2º membro da equação; variável independente, só pode aparecer o **seno**.

$$\cos x = \pm\sqrt{1-sen^2x} \qquad \sec x = \frac{1}{\pm\sqrt{1-sen^2x}} = \frac{\sqrt{1-sen^2x}}{1-sen^2x}$$

$$tg\,x = \frac{sen\,x}{\pm\sqrt{1-sen^2x}} \qquad \text{cotg}\,x = \frac{\pm\sqrt{1-sen^2x}}{sen\,x}$$

$$cosec\,x = \frac{1}{sen\,x}$$

② Esprimir todas as linhas trigonométricas em função do coseno.

Partindo de: $\boxed{sen^2x + \cos^2x = 1}$ temos:

$$sen\,x = \pm\sqrt{1-\cos^2x} \qquad \sec x = \frac{1}{\cos x}$$

$$tg\,x = \frac{\pm\sqrt{1-\cos^2x}}{\cos x} \ \frac{(\text{seno})}{(\text{coseno})} \qquad cosec\,x = \frac{1}{+\sqrt{1-\cos^2x}} =$$

$$\text{cotg}\,x = \frac{\cos x}{\pm\sqrt{1-\cos^2x}} = \frac{\pm\cos x\sqrt{1-\cos^2x}}{1-\cos^2x} \qquad \frac{\pm\sqrt{1-\cos^2x}}{1-\cos^2x}$$

③ Exprimir todas as linhas trigonométricas em função da $\boxed{\text{tangente de } x}$.

$$\sec x = \pm\sqrt{1+tg^2x}$$

$$\cos x = \pm\frac{1}{\pm\sqrt{1+tg^2x}} = \frac{\pm\sqrt{1+tg^2x}}{1+tg^2x}$$

$$\operatorname{cosec} x = \sqrt{1+cotg^2x} = \pm\sqrt{1+\frac{1}{\cancel{co}tg^2x}} = \frac{\pm\sqrt{tg^2x+1}}{tg\,x}$$

inverso da csc.

$$\operatorname{sen} x = \frac{tg\,x}{\pm\sqrt{tg^2x+1}} = \frac{\pm tg\sqrt{tg^2x+1}}{tg^2x+1}$$

18/9/72

Exercícios

① Dado: $\cos x = -2 \quad x \in 3^\circ Q$

$$\operatorname{sen} x = -\frac{1}{2}$$

$$\operatorname{sen}^2 x + \cos^2 x = 1$$

$$\cos^2 x = \sqrt{1-\operatorname{sen}^2 x}$$

$$\cos^2 x = \sqrt{1-\frac{1}{4}}$$

$$\cos^2 x = \frac{\sqrt{3}}{2}$$

$$tg\,x = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$tg\,x = -\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}$$

$$tg\,x = \frac{1}{\sqrt{3}}$$

$$tg\,x = \frac{\sqrt{3}}{3}$$

$$cotg\,x = \frac{3}{\sqrt{3}} \quad \therefore \quad cotg\,x = \frac{3\sqrt{3}}{\sqrt{3}.\sqrt{3}} \quad \therefore \quad cotg\,x = \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = \sqrt{3}$$

$$\sec x = \frac{-2}{\sqrt{3}} \quad \therefore \quad \sec x = \frac{-2\sqrt{3}}{3}$$

---

② $\dfrac{2\operatorname{sen} x + tg\,x}{2\cos x - cotg\,x} \qquad x \in IQ \qquad \csc x = \sqrt{2}$

$$\operatorname{Sen} x = \frac{1}{\sqrt{2}} \quad \therefore \quad \operatorname{sen} x = \frac{\sqrt{2}}{2}$$

$$\frac{2}{4} + \cos^2 x = 1$$

$$\cos x = \sqrt{1-\operatorname{sen} x}$$

$$\cos x = \sqrt{1-\frac{1}{2}} \quad \therefore \quad \cos x = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$$

$$\frac{(\sqrt{2}+1)\,(\sqrt{2}+1)}{(\sqrt{2}-1)\,(\sqrt{2}+1)} = \frac{2+2\sqrt{2}+1}{2-1} =$$

$$\boxed{3+2\sqrt{2}}$$

---

③ Simplificar a expressão: $y = tg\,x . \operatorname{sen} x + \cos x$

$$y = \frac{\operatorname{sen} x}{\cos x} . \operatorname{sen} x + \cos x$$

$$y = \frac{\operatorname{sen}^2 x}{\cos x} + \cos x$$

$$y = \frac{\operatorname{sen}^2 x + \cos^2 x}{\cos x} \quad \therefore \quad \boxed{y = \frac{1}{\cos x}}$$

---

④ Verificar a seguinte identidade: $\cos x - \operatorname{sen} x = cotg\,x . \cos x$

$$\frac{1}{\operatorname{sen} x} - \operatorname{sen} x = \frac{\cos x}{\operatorname{sen} x} . \cos x$$

$$\frac{1-\operatorname{sen}^2 x}{\cancel{\operatorname{sen} x}} = \frac{\cos^2 x}{\cancel{\operatorname{sen} x}}$$

$$\boxed{\cos^2 x = \cos^2 x}$$

⑤ Sendo $\cos x = \frac{1}{4}$ $x \in IQ$, calcular as demais funções do arco x.

$\sec x = 4$

$\text{sen}^2 x + \cos^2 x = 1$

$\text{sen}^2 x = 1 - \cos^2 x$

$\text{sen}\, x = \sqrt{1 - \cos^2 x}$

$\text{sen}\, x = \sqrt{1 - \frac{1}{16}}$

$\text{sen}\, x = \sqrt{\frac{15}{4}}$

$\text{tg}\, x = \frac{\frac{\sqrt{15}}{4}}{\frac{1}{4}} = \sqrt{15}$

$\text{cosec}\, x = \frac{4}{\sqrt{15}}$

$\text{cosec}\, x = \frac{4 . \sqrt{15}}{\sqrt{15} . \sqrt{15}}$

$\text{cosc}\, x = \frac{4\sqrt{15}}{15}$

$\text{cotg}\, x = \frac{1}{\sqrt{15}}$

$\text{cotg}\, x = \frac{1\sqrt{15}}{15}$

---

⑥ Escrever a expressão $\frac{\text{tg}\,x - \sec x}{\text{cosc}\, x}$ em função de $\text{sen}\, x$.

$$\text{Sen}\, x = \frac{\frac{\sqrt{1-\text{sen}^2 x}}{1-\text{sen}^2 x} - \frac{\sqrt{1-\text{sen}^2 x}}{1-\text{sen}^2 x}}{\frac{1}{\text{sen}^2 x}} =$$

$$\frac{(\text{sen}\, x - 1)\sqrt{1-\text{sen}^2 x}}{1-\text{sen}^2 x} \cdot \text{sen}\, x =$$

$$\frac{\text{sen}\, x \left[(\text{sen}\, x - 1)\sqrt{1-\text{sen}^2 x} \right]}{1-\text{sen}^2 x}$$

---

⑦ Calcular: $\text{sen}^2 a$ e $\cos^2 a$ sabendo-se que $\text{tg}\, a = 1 + \sqrt{2}$

$1 + \sqrt{2} = \frac{\text{sen}\, a}{\cos a}$

$\text{sen}\, a = (1 + \sqrt{2}) \cos a$

$\text{sen}^2 a + \cos^2 a = 1$

$[(1 + \sqrt{2}) \cos a]^2 + \cos^2 a = 1$

$(1 + 2\sqrt{2} + 2) \cos^2 a + \cos^2 a = 1$

$(1 + 2\sqrt{2} + 2 + 1) \cos^2 a = 1$

$(4 + 2\sqrt{2}) \cos^2 a = 1$

$\cos^2 a = \frac{1}{4 + 2\sqrt{2}} = \frac{4 - 2\sqrt{2}}{16 - 8}$

$\frac{4 - 2\sqrt{2}}{8} = \frac{2 - \sqrt{2}}{4} \therefore \boxed{\cos^2 a = \frac{2 - \sqrt{2}}{4}}$

$\text{sen}^2 a + \cos^2 a = 1$

$\text{sen}^2 a = \sqrt{1 - \cos^2 a}$

$\text{sen}^2 a = 1 - \left(\frac{2 - \sqrt{2}}{4}\right)$

$\text{sen}^2 a = \frac{4 - 2 + \sqrt{2}}{4}$

$\boxed{\text{sen}^2 a = \frac{2 + \sqrt{2}}{4}}$

---

⑧ Calcular o valor da expressão: $y = \frac{2\,\text{sen}^2 x - \text{tg}\, x}{\text{cosc}\, x}$ $x \in IQ$ e $\text{cotg}\, x = 1$

$y = \frac{2\left(\frac{\sqrt{2}}{2}\right)^2 - 1}{\sqrt{2}} \therefore y = \frac{2\,\frac{2}{4} - 1}{\sqrt{2}} = \frac{1 - 1}{\sqrt{2}} = \frac{0}{\sqrt{2}} = \boxed{\text{zero}}$

⑨ Sabe-se que a sec. de um arco do 2ºQ é -2
Calcular o seno e a tangente desse arco

$\sec x = -2$

$\cos x = \frac{1}{\sec x} \therefore \cos x = -\frac{1}{2}$

$\text{sen}\, x = \sqrt{1-\cos^2 x}$

$\text{sen}\, x = \sqrt{1-(-\frac{1}{2})^2}$

$\text{sen}\, x = \sqrt{1-\frac{1}{4}}$

$\text{sen}\, x = \frac{\sqrt{3}}{2}$

$\text{tg}\, x = \frac{\text{sen}\, x}{\cos x} \therefore \text{tg}\, x = \frac{\frac{\sqrt{3}}{\cancel{2}}}{-\frac{1}{\cancel{2}}} \therefore \text{tg}\, x = -\sqrt{3}$

23/9/72

Terceira Aplicação

Demonstração ou Dedução de Identidades

Consiste em fazer transformações sucessivas em um dos membros até se igualar ao outro.

Partir sempre da expressão maior até chegar à menor e partindo tb. das linhas mais simples.

$$\boxed{\text{tg}\, x + \text{cotg}\, x \equiv \sec x \cdot \text{cosc}\, x}$$

1ª solução → $\text{tg}\, x + \text{cotg}\, x = \frac{\text{sen}\, x}{\cos x} + \frac{\cos x}{\text{sen}\, x} = \frac{\text{sen}^2 x + \cos^2 x}{\text{sen}\, x \cos x} =$

$$\frac{1}{\text{sen}\, x \cos x} = \frac{1}{\text{sen}\, x} \cdot \frac{1}{\cos x} = \boxed{\text{cosc}\, x \cdot \sec x.}$$

2ª solução → $\sec x . \cos x = \frac{1}{\cos x} \cdot \frac{1}{\text{sen}\, x} = \frac{1}{\cos x \cdot \text{sen}\, x} =$

$$\frac{\text{sen}^2 x + \cos^2 x}{\text{sen}\, x \cdot \cos x} = \frac{\text{sen}^{\cancel{2}} x}{\cancel{\text{sen}\, x} \cos x} * \frac{\cos^{\cancel{2}} x}{\text{sen}\, x \cancel{\cos x}}$$

$$\frac{\text{sen}\, x}{\cos x} + \frac{\cos x}{\text{sen}\, x} \Rightarrow \boxed{\text{tg}\, x + \text{cotg}\, x}$$

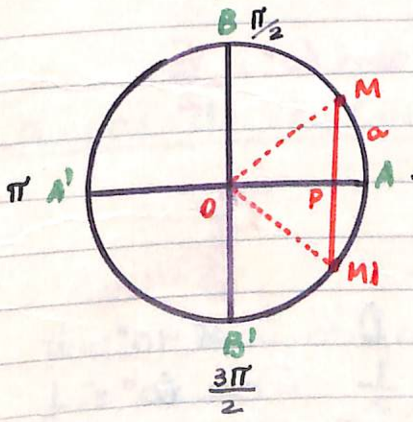
$$\boxed{\frac{1+\text{sen}^2 x}{1-\text{sen}^2 x} \equiv 1 + 2\,\text{tg}^2 x}$$

$$\frac{1+\text{sen}^2 x}{1-\text{sen}^2 x} = \frac{1+\text{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} + \frac{\text{sen}\, x}{\cos^2 x} =$$

$$\sec^2 x + \text{tg}^2 x = 1 + \text{tg}^2 x + \text{tg}^2 x = \boxed{1+2\,\text{tg}^2 x}$$

Cálculo das linhas dos arcos da forma π/2

① **Teorema**: "O seno de um arco positivo é menor que 90° é igual à metade da corda do arco duplo."



MM' → corda que subtende 2 arcos (2a)

$\overline{AM}$ → a

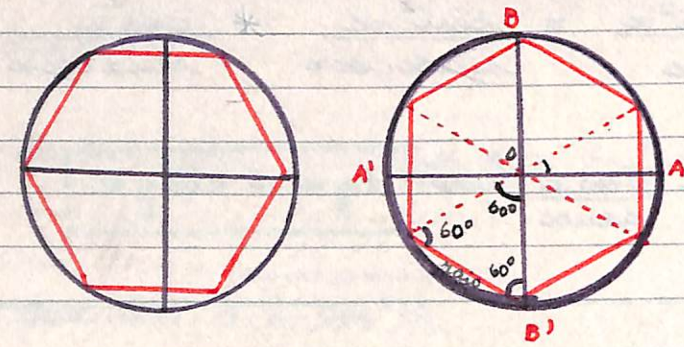
$\overline{MM'}$ → $2\,\overline{MP} = 2\,\text{sen}\, a$

sen a → $\frac{1}{2}\,\overline{MM'}$

Ary Quintella → 2º ano col.

pag. 111.

seno 30° → hexágono

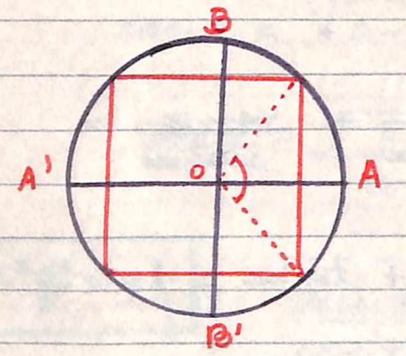


lado do hexágono = raio

$\text{sen}\,30° = \frac{l_{⬡}}{2} = \frac{R}{2} = \frac{1}{2}$

baseado no teorema anterior

seno 45° → quadrado



Pitágoras

$(2R)^2 = l^2 + l^2$

$4R^2 = 2l^2$

$l^2 = 2R^2$

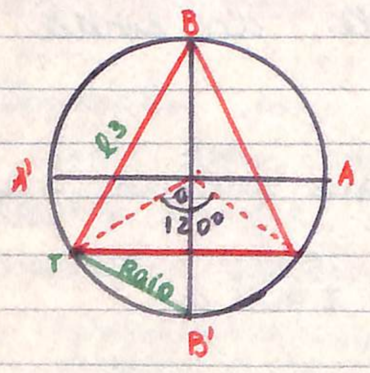
$l = R\sqrt{2}$

$R = \frac{l\sqrt{2}}{2}$

$\text{Sen}\,45° = \frac{l_4}{2} = \frac{R\sqrt{2}}{2}$

$\text{Sen}\,45° = \frac{\sqrt{2}}{2}$

seno 60° → triângulo



$l_3 = R\sqrt{3}$

$l^2 = 3R^2$

$\text{Sen}\,60° = \frac{l_3}{2} = \frac{R\sqrt{3}}{2}.$

$l_3 = R\sqrt{3}$

$\text{sen}\,60° = \frac{\sqrt{3}}{2}$

Tabela Prática

$\text{Sen}\,0° = 0$

$\text{sen}\,30° = \sqrt{1}/2$

$\text{sen}\,45° = \sqrt{2}/2$

$\text{Sen}\,60° = \sqrt{3}/2$

$\text{Sen}\,90° = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$

$\text{tg}\,30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\text{tg}\,45° = 1$

$\text{tg}\,60° = \sqrt{3}$

$\text{tg}\,90° = \nexists$

$\text{Sen}\,0° = 0$

$\text{sen}\,30° = \frac{1}{2}$

$\text{sen}\,45° = \frac{\sqrt{2}}{2}$

$\text{sen}\,60° = \frac{\sqrt{3}}{2}$

$\text{sen}\,90° = 1$

$\cos 90° = 0$

$\cos 60° = \frac{1}{2}$

$\cos 45° = \frac{\sqrt{2}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\cos 0° = 1$

$\text{tg}\,0° = 0$

$\text{tg}\,30° = \frac{\frac{1}{2}\,\text{s.}}{\sqrt{3}/2\,\text{c.}} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

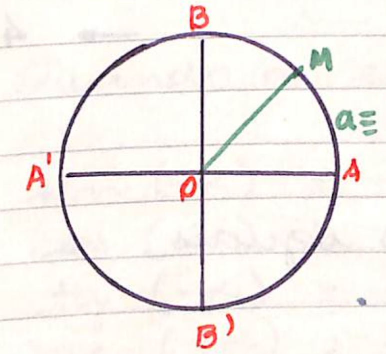
$\text{tg}\,45° = 1$

$\text{tg}\,60° = \frac{\sqrt{3}/2}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$

$\text{tg}\,90° = \frac{1\ \text{seno}}{0\ \text{cos.}}$ = descontinua $\nexists$

Arcos de extremidades associadas

① arcos côngruos



$a: \overset{\frown}{AM}$

$b = 2K\pi + \overset{\frown}{AM}$

$a \equiv b$

$a \cong 360k + \gamma$

$a \cong 2k\pi + \alpha$

Arcos côngruos são arcos da mesma origem e mesma extremidade, mas não são arcos iguais.

Ex: $60 \cong 420° \cong 760°$

1 volta   2 voltas

② arcos de extremidades associadas

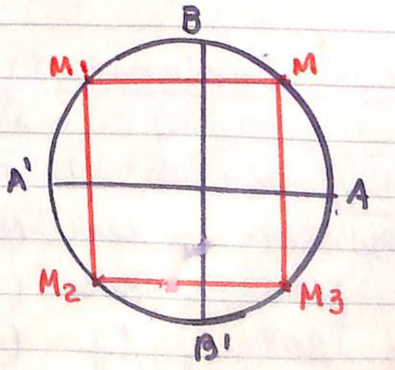
Todos tem origem em A

São eles: $\overset{\frown}{AM}$

$\overset{\frown}{AM_1}$

$\overset{\frown}{AM_2}$

$\overset{\frown}{AM_3}$



$\overset{\frown}{AM}$ - extremidade associada a $\overset{\frown}{AM_1}$
$\overset{\frown}{AM}$ - extremidade associada a $\overset{\frown}{AM_2}$
$\overset{\frown}{AM}$ - extremidade associada a $\overset{\frown}{AM_3}$
$\overset{\frown}{AM} = \overset{\frown}{M_1A} = \overset{\frown}{A'M_2} = \overset{\frown}{M_3A}$

II Q arcos suplementares → $\overset{\frown}{AM_1} = 180 - \alpha$ ou $\pi - \alpha$
III Q " explementares → $\overset{\frown}{AM_2} = 180 + \alpha$ ou $\pi + \alpha$
IV Q " replementares → $\overset{\frown}{AM_3} = 360 - \alpha$ ou $2\pi - \alpha$
I Q → $\overset{\frown}{AM} = \alpha$ ~~ou~~

Quando $K = 0$, temos a menor determinação do arco.

Generalidades de um arco

$\overset{\frown}{AM} = 360K + \alpha = 2K\pi + \alpha$ → 1º Q.
$\overset{\frown}{AM_1} = 360K + (180 - \alpha) = 2K\pi + (\pi - \alpha)$ → 2º Q.
$\overset{\frown}{AM_2} = 360K + (180 + \alpha) = 2K\pi + (\pi + \alpha)$ → 3º Q.
$AM_3 = 360K - \alpha = 2K\pi - \alpha$ → 4º Q.

a) Arcos Côngruos

Sem as linhas replementares iguais.
Sejam os arcos: $\alpha = 765°$
$\beta = 45°$

sen 765° = sen(2 × 360° + 45°) = sen 45°
cos 765° = cos(2 × 360° + 45°) = cos 45°
tg 765° = tg (2 × 360° + 45°) = tg 45°

b) Arcos Suplementares (redução do II Q → I Q)

$\boxed{AM_1}$ $\alpha = 120°$
sen 120° = sen (180° - 120°) = sen 60°
cos 120° = cos (180° - 120°) = -cos 60°
tg 120° = tg (180° - 120°) = -tg 60°
cosc 120° = csc (180° - 120) = csc 60°
sec 120° = sec (180° - 120) = -sec 60°
cotg 120° = ctg (180° - 120) = -ctg 60°

c) Arcos Explementares (do 3º Q para o 1º Q)

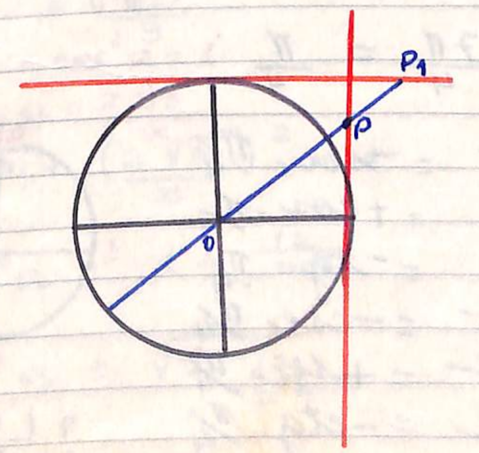
$\boxed{AM_2}$ sen 225° = sen (225° - 180°) = -sen 45°
cos 225° = cos (225° - 180°) = -cos 45°
tg 225° = tg (225° - 180°) = tg 45°
cse 225° = cse (225° - 180°) = -cse 45°
sec 225° = sec (225° - 180°) = -sec 45°
ctg 225° = ctg (225° - 180°) = ctg 45°

d) Arcos Replementares (do 4º Q para o 1º Q)

$\boxed{AM_3}$ sen 330° = sen (360° - 330°) = -sen 30°
cos 330° = cos (360° - 330°) = cos 30°
tg 330° = tg (360° - 330°) = -tg 30°
csc 330° = csc (360° - 330°) = -csc 30°
sec 330° = sec (360° - 330°) = sec 30°
ctg 330° = ctg (360° - 330°) = -ctg 30°

Obs. - Quando α é negativo $(-\alpha)$

sen $(-\alpha)$ = -sen α
cos $(-\alpha)$ = cos α
tg $(-\alpha)$ = -tg α
csc $(-\alpha)$ = -cosc α
sec $(-\alpha)$ = sec α
ctg $(-\alpha)$ = -ctg α



secante → OP (da origem até a linha da tangente)
cossecante → OP1 (da origem até a linha da cotangente)

Exercícios

$2^{\circ}Q: \pi - \alpha$
$3^{\circ}Q: \alpha - \pi$
$4^{\circ}Q: 2\pi - \alpha$

Reduzir ao 1º quadrante, para todas as funções circulares:

① 345° (4º Q → 1º Q)

$360^{\circ} - 345^{\circ} = 15^{\circ}$

$\text{sen}\, 345^{\circ} = -\text{sen}\, 15^{\circ}$
$\cos 345^{\circ} = +\cos 15^{\circ}$
$\text{tg}\, 345^{\circ} = -\text{tg}\, 15^{\circ}$
$\csc 345^{\circ} = -\csc 15^{\circ}$
$\sec 345^{\circ} = +\sec 15^{\circ}$
$\text{cotg}\, 345^{\circ} = -\text{ctg}\, 15^{\circ}$

② $\frac{4\pi}{3}$ (2º Q → 1º Q)

$\frac{4\pi}{3} - \pi = \frac{\pi}{3}$

$\text{sen}\, \frac{4\pi}{3} = +\text{sen}\, \frac{\pi}{3}$
$\cos \frac{4\pi}{3} = -\cos \frac{\pi}{3}$
$\text{tg}\, \frac{4\pi}{3} = -\text{tg}\, \frac{\pi}{3}$
$\csc \frac{4\pi}{3} = +\csc \frac{\pi}{3}$
$\sec \frac{4\pi}{3} = -\sec \frac{\pi}{3}$
$\text{ctg}\, \frac{4\pi}{3} = -\text{ctg}\, \frac{\pi}{3}$

③ 135° (2º Q → 1º Q)

$180^{\circ} - 135^{\circ} = 45^{\circ}$

$\text{sen}\, 135^{\circ} = \text{sen}\, 45^{\circ}$
$\cos 135^{\circ} = -\cos 45^{\circ}$
$\text{tg}\, 135^{\circ} = -\text{tg}\, 45^{\circ}$
$\csc 135^{\circ} = \text{ese}\, 45^{\circ}$
$\sec 135^{\circ} = -\sec 45^{\circ}$
$\text{etg}\, 135^{\circ} = -\text{ctg}\, 45^{\circ}$

④ $\frac{2\pi}{3}$ (2º Q → 1º Q)

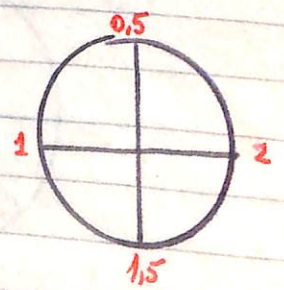
$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\text{sen}\, \frac{2\pi}{3} = +\text{sen}\, \frac{\pi}{3}$
$\cos \frac{2\pi}{3} = -\cos \frac{\pi}{3}$
$\text{tg}\, \frac{2\pi}{3} = -\text{tg}\, \frac{\pi}{3}$
$\csc \frac{2\pi}{3} = +\csc \frac{\pi}{3}$
$\sec \frac{2\pi}{3} = -\sec \frac{\pi}{3}$
$\text{cotg}\, \frac{2\pi}{3} = -\text{cotg}\, \frac{\pi}{3}$

⑤ $\frac{7\pi}{4}$ (4º Q → 1º Q)

$2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

$\text{sen}\, 7\pi/4 = -\text{sen}\, \pi/4$
$\cos 7\pi/4 = +\cos \pi/4$
$\text{tg}\, 7\pi/4 = -\text{tg}\, \pi/4$
$\csc \frac{7\pi}{4} = -\text{cse}\, \pi/4$
$\sec \frac{7\pi}{4} = +\sec \pi/4$
$\text{ctg}\, \frac{7\pi}{4} = -\text{ctg}\, \pi/4$

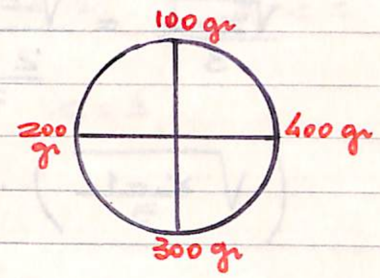


$7 \,\underline{|\, 4}$
30 1,75
20
0

⑥ 320 gr (4º Q → 1º Q)

$400\text{ gr} - 320\text{ gr} = 80\text{ gr}$

$\text{sen}\, 320\text{ gr} = -\text{sen}\, 80\text{ gr}$
$\cos 320\text{ gr} = +\cos 80\text{ gr}$
$\text{tg}\, 320\text{ gr} = +\text{tg}\, 80\text{ gr}$
$\csc 320\text{ gr} = -\csc 80\text{ gr}$
$\sec 320\text{ gr} = +\sec 80\text{ gr}$
$\text{ctg}\, 320\text{ gr} = +\text{ctg}\, 80\text{ gr}$



⑦ $\frac{21\pi}{5}$ (arco no 1º Q. $2k\pi + \alpha$)

$\text{sen}\, \frac{21\pi}{5} = +\text{sen}\, \frac{\pi}{5}$
$\cos \frac{21\pi}{5} = +\cos \frac{\pi}{5}$
$\text{tg}\, \frac{21\pi}{5} = +\text{tg}\, \frac{\pi}{5}$
$\csc \frac{21\pi}{5} = +\csc \frac{\pi}{5}$
$\sec \frac{21\pi}{5} = +\sec \frac{\pi}{5}$
$\text{ctg}\, \frac{21\pi}{5} = +\text{ctg}\, \frac{\pi}{5}$

$21 \,\underline{|\, 5}$
1 $4\frac{\pi}{5}$ (4 voltas + $\frac{\pi}{5}$)

Exercícios

Calcular $\underline{x}$ no sistema:

$$\begin{cases} \cos a = \frac{2\sqrt{3}}{x} \\ \text{tg}\, a = \sqrt{\frac{x-1}{3}} \end{cases}$$

$\text{sen}^2 a + \cos^2 a = 1$

$\cos a = \frac{2\sqrt{3}}{x}$

1) $\text{sen}^2 a + \left(\frac{2\sqrt{3}}{x}\right)^2 = 1$

$\text{sen}^2 a = 1 - \frac{12}{x^2}$

$\text{sen}\, a = \sqrt{1 - \frac{12}{x^2}}$

② $tg\,a = \frac{sen}{cos}$

$$tg\,a = \frac{\sqrt{1-\frac{12}{x^2}}}{\frac{2\sqrt{3}}{x}}$$

$$\frac{\sqrt{x-1}}{3} = \frac{\sqrt{\frac{x^2-12}{x^2}}}{\frac{2\sqrt{3}}{x}}$$

$$\left(\sqrt{\frac{x-1}{3}}\right)\cdot\left(\frac{\frac{1}{x}\cdot\sqrt{x^2-12}}{\frac{1}{x}\cdot 2\sqrt{3}}\right)^2$$

$\frac{x-1}{9} = \frac{x^2-12}{12}$ m.m.c = 36

$4x-4 = 3x^2-36$

$3x^2-4x-32=0$

$$x^{2} = \frac{4 \pm \sqrt{16+384}}{6}$$

$x = \frac{4 \pm 20}{60}$ $\begin{cases} x' \rightarrow 4 \\ x'' \rightarrow -\frac{8}{3} \end{cases}$

---

① $\cos a = \frac{2\sqrt{3}}{4}$

$\cos a = \frac{\sqrt{3}}{2}$

$\cos a = \frac{1{,}73}{2}$ $\therefore \cos a = 0{,}86$

$1 \leq \cos \leq 1$

② $tg\,a = \frac{2}{3-x}$ $sen\,a = \frac{-1}{\sqrt{x^2+1}} + \cos^2 a = 1$

$cosc. = \sqrt{x^2+1}$

$$\frac{1}{x^2+1} + \cos^2 a = 1$$

$\cos^2 a = 1 - \frac{1}{x^2+1}$ $tg\,a = \frac{\frac{1}{\sqrt{x^2+1}}}{\frac{x}{\sqrt{x^2+1}}} = \frac{1}{x}$

$\cos a = \sqrt{\frac{x^2+1-1}{x^2+1}}$ $\frac{1}{x} = \frac{2}{3-x}$

$\cos a = \sqrt{\frac{x^2}{x^2+1}}$ $3-x = 2x$ $3x = 3$

$x = 1.$

$\cos a = \frac{x}{\sqrt{x^2+1}}$

Simplificar :

1) $\cos(-a) + sen(90° - a) - tg(90° + a)$

$\cos a + \cos a + cotg\,a$

$2\cos a + cotg\,a.$

2) $5\,sen\,x - 2/3\,sen(\pi - x) - sen(\pi/2 + x)$

$5\,sen\,x - 2/3\,sen\,x - \cos x$ m.m.c.

$\frac{13}{3}\,sen\,x - \cos x.$

3) $\cos\frac{21\pi}{2} + \cos\left(\frac{\pi}{2} - a\right) \times tg\left(\frac{3\pi}{2} + a\right)$

$\cos\frac{\pi}{2} + sen\,a \times (-cotg\,a)$

$\cos\frac{\pi}{2} = 0$

$\cancel{sen\,a} \times \left(\frac{-\cos a}{\cancel{sen\,a}}\right)$

4) $\frac{5}{4}\,cose\,(\pi - x) + \frac{2}{3}\cos\left(\frac{\pi}{2} - x\right) - sen(2\pi - x)$

R: $\frac{5}{12}(3\,csc.x + 4\,sen\,x)$

$\frac{5}{4}\cdot\frac{1}{sen\,x} + \frac{2}{3}\,sen\,x + sen\,x$

$\frac{5}{4\,sen\,x} + \frac{2}{3}\,sen\,x + sen\,x =$

$$\frac{15 + 8\,\text{sen}^2 x + 12\,\text{sen}^2 x + 12\,\text{sen}^2 x}{12}$$

$$\underline{20 + 8\,\text{sen}^2 x + 12\,\text{sen}^2 x + 12\,\text{sen}^2 x}$$ (nulo)

$$\frac{20\,\text{sen}^2 x + 15}{12\,\text{sen}\,x} = \frac{20\,\text{sen}^x}{12\,\text{sen}\,x} + \frac{\overset{3}{\cancel{15}}}{\underset{4}{\cancel{12}}\,\text{sen}\,x} = \frac{5}{3}\,\text{sen}\,x + \frac{5}{4.\,\text{sen}\,x}.$$

② $$\frac{\text{tg}\,x}{\text{cotg}\left(\frac{\pi}{2}+x\right)\sec(\pi-x)\,\text{sen}\left(a\frac{\pi}{2}-x\right)}$$

$$\frac{-\text{tg}\,x}{-\text{tg}\,x - \sec x.\cos x.} \Rightarrow \frac{\text{tg}\,x}{\overset{-}{(+)}\text{tg}\,x\,x - \left(\pm\frac{1}{\cancel{\cos} x}.\cancel{\cos} x\right)} = ①$$

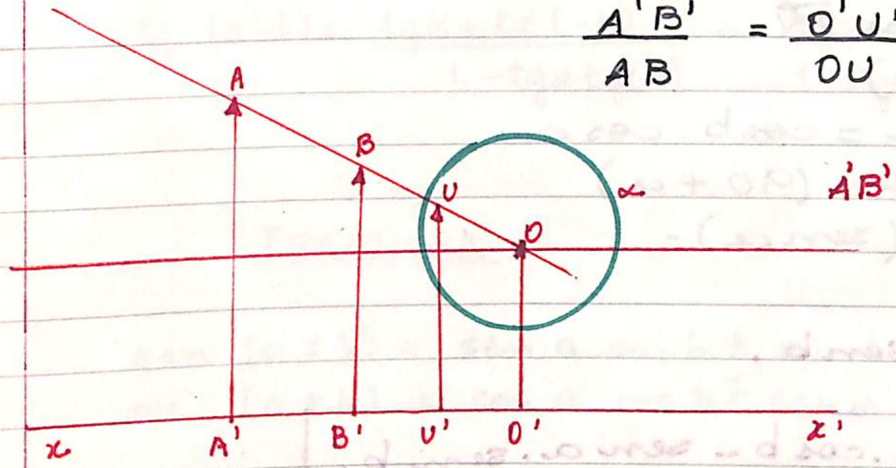
Questões da Prova:

28/10

Operação com Arcos

① medida algébrica da projeção de um vetor.:

A medida algébrica da projeção ortogonal de um vetor sobre um eixo, é igual ao produto da medida algébrica do vetor p/ cosseno do ângulo que seu suporte forma com o eixo.



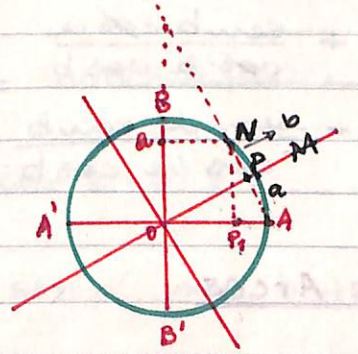
$$\frac{\overline{A'B'}}{AB} = \frac{\overline{O'U'}}{OU} \qquad \frac{A'B'}{AB} = \frac{\cos\alpha}{1}$$

$$A'B' = AB \times \cos\alpha$$

② adição de arcos

$\text{sen}\,(a+b) \neq \text{sen}\,a + \text{sen}\,b$.

③ fórmulas de Adição de Arcos



$\overset{\frown}{AM} = a$

$\overset{\frown}{MN} = b$

$\text{sen}(a+b) = \overline{P_1 N}$

$\cos(a+b) = \overline{OP_1}$

$\text{sen}\,b = \overline{PN}$

$\cos b = \overline{OP}$

$\overrightarrow{OP} + \overrightarrow{PN} = \overrightarrow{ON}$

proj. N = proj. $\overrightarrow{OP}$ + pr. $\overrightarrow{PN}$ (chasles) sobre o eixo BB'

$pr.\ \overrightarrow{ON} = OQ = \text{sen}\,(a+b)$

$pr\ \overrightarrow{OP} = \overline{OP}\cos(90-a) = \cos b \times \text{sen}\,a$

$pr\ \overline{PN} = \overline{P}N \cos(\hat{O}B;\hat{P}N) = \text{sen}\,b \cos a$

$$\boxed{\text{sen}\,(a+b) = \text{sen}\,a \cdot \cos b + \text{sen}\,b \cdot \cos a}$$

$\cos(a+b)$

$\overline{O}N = \overrightarrow{OP} + PN$

projetando sobre $A\overline{A}'$

$pr\ \overrightarrow{ON} = pr\ \overrightarrow{OP} + pr\ \overrightarrow{PN}$

$pr\ \overrightarrow{ON} = \cos(a+b)$

$pr\ \overline{OP} = \overline{O}P \cos a = \cos b \cdot \cos a.$

$pr\ \overline{PN} = pr\ \overline{PN} \cdot \cos(90+a)$

$pr\ \overrightarrow{PN} = \text{sen}\,b \cdot (\text{sen}\,a) =$

$-\text{sen}\,a\ \text{sen}\,b.$

$$\boxed{\cos(a+b) = \cos a \cdot \cos b - \text{sen}\,a \cdot \text{sen}\,b.}$$

$$tg\,(a+b) = \frac{\text{sen}\,(a+b)}{\cos(a+b)}$$

$\frac{\text{sen}\,a \cos b + \text{sen}\,b \cos a}{\cos a \cos b - \text{sen}\,a\,\text{sen}\,b}$ dividindo-se: por $\cos a \cos b$.

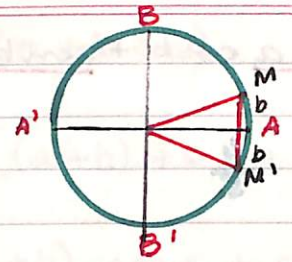
$$tg\,(a+b)\ \frac{\frac{\text{sen}\,a\,\cancel{\cos b}}{\cos a\,\cancel{\cos b}} + \frac{\text{sen}\,b\,\cancel{\cos a}}{\cancel{\cos a}\cos b}}{\frac{\cos a \cos b}{\cos a \cos b} - \frac{\text{sen}\,a\,\text{sen}\,b}{\cos a \cos b}} = \boxed{\frac{tg\,a + tb}{1 - tga\,tgb}}.$$

Subtração de Arcos

$a+b = a + (-b)$ $\qquad \cos(-b) = \cos b.$

$\text{sen}\,[a+(-b)] = \text{sen}\,a \quad \cos(-b)\ \cos a$

$$\boxed{\text{sen}\,(a-b) = \text{sen}\,a \cos b - \text{sen}\,b \cos a}$$



$\cos[a+(-b)] = \cos a \ \cos(-b) - \text{sen}\,a\ \text{sen}\,(-b) =$

$$\boxed{\cos a \cos b + \text{sen}\,a\ \text{sen}\,b}$$

$$tg\,(a-b) = \frac{tga + tg\,(-b)}{1 - tga\,tg(-b)} = \frac{tg\,a - tb}{1 + tga\,tgb}$$

Fórmulas

$$\boxed{\begin{aligned} \text{sen}\,(a \pm b) &= \text{sen}\,a \cos b \pm \text{sen}\,b \cos a \\ \cos(a \pm b) &= \cos a \cos b \mp \text{sen}\,a\ \text{sen}\,b \\ tg\,(a \pm b) &= \frac{tga \pm tgb}{1 \mp tga\,tgb} \end{aligned}}$$

Exercícios

① Desenvolver e simplificar

$\cos(a+b) + \cos(a-b) =$

$\cos a \cos b - \text{sen}\,\cancel{a}\ \cancel{\text{sen}\,b} + \cos a \cos b + \cancel{\text{sen}\,a}\ \cancel{\text{sen}\,b} =$

$2 \cos a \cdot \cos b$

② Sendo a e b do 1º Q e

$\text{sen}\,a = \frac{3}{5}$ $\qquad \text{sen}\,b = \frac{5}{13}$

Calcular : $\text{sen}\,(a+b)$

e $\cos(a-b)$

Segue a fórmula: $\text{sen}(a+b) = \underline{\text{sen}\,a}\cos b + \underline{\text{sen}\,b}\cos a$

$$\cos a = \pm\sqrt{1-\text{sen}^2 a} = +\sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

$$\cos b = \sqrt{1-\frac{25}{169}} = +\frac{12}{13}$$

$$\text{sen}(a+b) = \frac{3}{5} \times \frac{12}{13} + \frac{5}{13} \times \frac{4}{5} = \frac{36}{65} + \frac{4}{13} = \frac{36+20}{65} = \frac{56}{65}$$

$\cos(a-b) = \cos a \cos b + \text{sen}\,a\,\text{sen}\,b$

$$\cos(a-b) = \frac{4}{5} \times \frac{12}{13} + \frac{3}{5} \times \frac{5}{13} = \frac{48+15}{65} = \frac{63}{65}$$

---

① → dados
$\text{sen}\,a = \frac{3}{5}$
$\text{sen}\,b = \frac{5}{13}$

② fórmulas
$\text{sen}(a+b)$
$\text{sen}(a-b)$

③ incógnitas

④ resolução

---

③ Demonstrar a identidade sendo o arco do 1:Q

$$\text{arc}\,\text{tg}\,\frac{1}{2} + \text{arc}\,\text{tg}\,\frac{1}{3} \equiv \frac{\pi}{4}$$

$\text{arc}\,\text{tg}\,\frac{1}{2} = a$
$\text{arc}\,\text{tg}\,\frac{1}{3} = b$
$\text{tg}\,a = \frac{1}{2}$
$\text{tg}\,b = \frac{1}{3}$

$a+b = \frac{\pi}{4} \quad \text{tg}(a+b) = \text{tg}\,\frac{\pi}{4}$
$\text{tg}(a+b) = 1$
$\text{tg}(a+b) = \frac{\text{tg}\,a + \text{tg}\,b}{1 - \text{tg}\,a\,\text{tg}\,b}$

$$\text{tg}(a+b) = \frac{1/2 + 1/3}{1 - 1/2 \times 1/3} = ①$$

---

④ Baseando-se nas fórmulas:
$\text{sen}(a+b) = \text{sen}\,a \cdot \cos b + \cos a \cdot \text{sen}\,b$
$\cos(a+b) = \cos a \cdot \cos b + \text{sen}\,a \cdot \text{sen}\,b$
deduzir a formula para:

$\text{sen}(a+b+c)$

$$\text{sen}\left[(a+b)+c\right] =$$

$$\text{sen}(a+b) \cdot \cos c + \cos(a+b) \cdot \text{sen}\,c =$$

$$(\text{sen}\,a \cdot \cos b + \cos a\,\text{sen}\,b) \cdot \cos c + (\cos a \cos b - \text{sen}\,a \cdot \text{sen}\,b) \cdot \text{sen}\,c =$$

$\text{sen}\,a \cdot \cos b \cdot \cos c + \cos a \cdot \text{sen}\,b \cdot \cos c + \cos a \cdot \cos b \cdot \text{sen}\,c - \text{sen}\,a \cdot \text{sen}\,b \cdot \text{sen}\,c$.

$\text{tg}(a+b+c)$

1- $\text{tg}(a+b) = \dfrac{\text{tg}\,a + \text{tg}\,b}{1 - \text{tg}\,a - \text{tg}\,b}$

2- $\text{tg}\left[(a+b)+c\right] = \left[\dfrac{\text{tg}(a+b) + \text{tg}\,c}{1 - \text{tg}(a+b) \cdot \text{tg}\,c}\right]$

3- $\text{tg}\left[\dfrac{\dfrac{\text{tg}\,a + \text{tg}\,b}{1 - \text{tg}\,a \cdot \text{tg}\,b} + \text{tg}\,c}{1 - \left(\dfrac{\text{tg}\,a + \text{tg}\,b}{1 - \text{tg}\,a \cdot \text{tg}\,b}\right) \cdot \text{tg}\,c}\right]$

4- $\dfrac{\dfrac{\text{tg}\,a + \text{tg}\,b + \text{tg}\,c\,(1 - \text{tg}\,a\,\text{tg}\,b)}{1 - \text{tg}\,a\,\text{tg}\,b}}{\dfrac{1 - \text{tg}\,a\,\text{tg}\,c + \text{tg}\,b \cdot \text{tg}\,c}{1 - \text{tg}\,a \cdot \text{tg}\,b}}$

5- $\dfrac{\dfrac{\text{tg}\,a + \text{tg}\,b + \text{tg}\,c - \text{tg}\,a\,\text{tg}\,b\,\text{tg}\,c}{\cancel{1 - \text{tg} \cdot \text{tg}\,b}}}{\dfrac{1 - \text{tg}\,a\,\text{tg}\,b - (\text{tg}\,a\,\text{tg}\,c + \text{tg}\,b \cdot \text{tg}\,c)}{\cancel{1 - \text{tg}\,a \cdot \text{tg}\,b}}} \div$

6- $\dfrac{\text{tg}\,a + \text{tg}\,b + \text{tg}\,c - \text{tg}\,a\,\text{tg}\,b\,\text{tg}\,c}{1 - \text{tg}\,a \cdot \text{tg}\,b - \text{tg}\,a\,\text{tg}\,c - \text{tg}\,b\,\text{tg}\,c}$

⑤ Sendo : sen $45° = \frac{\sqrt{2}}{2}$ e sen $30° = \frac{1}{2}$

Calcular as funções trigonométricas de 15° e de 75°.

$$\boxed{\text{sen}(a+b) = \text{sen}\,a \cdot \cos b + \text{sen}\,b \cos a}$$

$$\text{sen}(45°+30°) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\text{sen}\,75° = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4} \qquad \text{sen}\,15° = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos(a-b) = \cos a \cos b + \text{sen}\,a\ \text{sen}\,b$$

$$\cos(45°-30°) = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4} \qquad \text{tg}\,15° = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

---

⑥ Calcular sen 105° e cos 105°

$$\text{sen}\,105° = \text{sen}(60+45)$$

$$\text{sen}\,105° = \text{sen}\,60 \cdot \cos 45 + \text{sen}\,45 \cdot \cos 60$$

$$\text{sen}\,105° = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\text{sen}\,105° = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4} \qquad \text{sen}\,15° = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos 105° = \frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{4}$$

$$\cos 105° = (a+b) = \cos a \cdot \cos b - \text{sen}\,a \cdot \text{sen}\,b$$

$$\cos(60+45) = 1/2 \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} =$$

$$\cos 105 = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4} \qquad \cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\text{tg}\,105° = \frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{2}+\sqrt{6}}{4}} = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}+\sqrt{6}}$$

Calcular csc (x − y) dados :

sec $x = \frac{5}{4}$ e sen $y = \frac{5}{13}$

sendo x e y arcos do 1º Quadrante

$$\csc(x-y) = \frac{1}{\text{sen}(x-y)} = \frac{1}{\text{sen}\,x \cos y - \text{sen}\,y \cdot \cos x}$$

$$\frac{1}{\text{sen}\,x \cdot \cos y - \text{sen}\,y \cdot \cos x}$$

$$\frac{1}{\frac{3}{5} \cdot \frac{12}{13} - \frac{15}{13} \cdot \frac{4}{5}} =$$

$$\frac{1}{\frac{36}{65} - \frac{20}{65}} = \frac{1}{\frac{16}{65}} =$$

$$1 \times \frac{65}{16} = \frac{65}{16} \rightarrow \csc(x-y)$$

$$\text{sen}^2 y + \cos^2 y = 1$$

$$\frac{25}{169} + \cos^2 y = 1$$

$$\cos^2 y = 1 - \frac{25}{169}$$

$$\cos^2 y = \frac{144}{169}$$

$$\cos^2 y = \frac{144}{169}$$

$$\cos y = \pm\sqrt{\frac{144}{169}}$$

$$\boxed{\cos y = \pm\frac{12}{13}.}$$

$$\text{sen}^2 x + \cos^2 x = 1$$

$$\text{sen}^2 x + \left(\frac{4}{5}\right)^2 = 1$$

$$\text{sen}^2 x + \frac{16}{25} = 1$$

$$\text{sen}^2 x = 1 - \frac{16}{25}$$

m.d.c

$$\text{sen}^2 x = \frac{9}{25}$$

$$\text{sen}\,x = \pm\sqrt{\frac{4}{25}} = \pm\frac{3}{5}$$

Sendo : tg $a = 2$ e
tg $b = 6$

Calcular: cotg (a+b)

$$\text{cotg}(a+b) = \frac{1}{\text{tg}(a+b)} =$$

$$\frac{1}{\frac{\text{tg}a - \text{tg}\,b}{1 - \text{tg}a \cdot \text{tg}\,b}}$$

$$\frac{1 - \text{tg}a\ \text{tg}\,b}{\text{tg}a + \text{tg}\,b} = \frac{1 - 2 \cdot 6}{2+6} =$$

$$\frac{1-12}{8} = -\frac{11}{8}$$

Dados:

$\cos a = \frac{5}{6}$

$\text{tg } b = 3$, sendo a do 1º Q. e $b \in$ 3º Q.

Calcular: $\text{sen}(a+b)$ e $\cos(b-a)$

$\text{sen}(a+b) = \text{sen } a \cdot \cos b + \cos a \cdot \text{sen } b.$

$\cos(b-a) =$

$\cos b = -\frac{\sqrt{10}}{10}$

$\text{sen}^2 b + \cos^2 b = 1$

$\text{sen}^2 b + \left(-\frac{\sqrt{10}}{10}\right)^2 = 1$

$\text{sen}^2 b + \frac{10}{100} = 1$

$\text{sen}^2 b = 1 - \frac{1}{10}$

$\text{sen}^2 b = \frac{10-1}{10} = \sqrt{\frac{9}{10}}$

$\sec^2 x = 1 + \text{tg}^2 x$

$\frac{1}{\cos^2 x} = 1 + 3^2$

$\frac{1}{\cos^2 x} = 10$

$1 = 10 \cos^2 x$

$\cos^2 x = \frac{1}{10}$

$\cos x = \pm\sqrt{\frac{1}{10}}$

$\cos x = -\frac{1}{\sqrt{10}} = \frac{-\sqrt{10}}{10}$

$\text{sen } b = \frac{\sqrt{9}}{10}$

$\text{sen } b = \pm\frac{3}{\sqrt{10}}$

$\text{sen } b = -\frac{3\sqrt{10}}{10}$

$\text{sen}^2 a + \cos^2 a = 1$

$\text{sen}^2 a + \left(\frac{5}{6}\right)^2 = 1$

$\text{sen}^2 a + \frac{25}{36} = 1$

$\text{sen}^2 a =$ ~~36~~ $1 - \frac{25}{36}$

$\text{sen}^2 a = \frac{36-25}{36} = \frac{11}{36}$

$\text{sen } a = \frac{\sqrt{11}}{36} = \frac{\sqrt{11}}{6}$

4/11

Multiplicação de Arcos

Dado o arco a acha-se na

$\text{sen}(2a) = \text{sen}(a+a)$

$\text{sen } 2a = \text{sen}(a+a) = \text{sen } a \cos a + \text{sen } a \cos a = 2 \text{sen } a \cos a$

$$\boxed{\text{sen } 2a = 2 \text{sen } a \cos a}$$

$\cos 2a = \cos(a+a) = \cos a \cos a - \text{sen } a \text{ sen } a = \cos^2 a - \text{sen}^2 a$

$$\boxed{\cos 2a = \cos^2 a - \text{sen}^2 a}$$

$\text{tg } 2a = \text{tg}(a+a) = \frac{\text{tg} a + \text{tg} a}{1 - \text{tg} a \, \text{tg} a} = \text{tg } 2a = \frac{2 \text{tg} a}{1 - \text{tg}^2 a}$

$$\boxed{\text{tg } 2a = \frac{2 \text{tg} a}{1 - \text{tg}^2 a}}$$

$na = [\underset{a}{\underset{\downarrow}{a}} + \underbrace{(n-1)a}_{b}]$ $a[\cancel{1} + n - \cancel{1}] = na$

$\text{sen } na = \text{sen}[a + (n-1)a] = \text{sen } a \cos[(n-1)a] + \text{sen}(n-1)a \cdot \cos a$

$\text{sen } 5a = \text{sen } a \cos 4a + \text{sen } 4a \cos a$ $\text{sen } 4a = \text{sen}(\overset{a}{\overset{\uparrow}{2a}} + \overset{b}{\overset{\uparrow}{2a}})$

$\cos na = \cos[a + (n-1)a] = \cos a \cos[(n-1)a] - \text{sen } a \text{ sen}[(n-1)a]$

$\text{tg } na = \text{tg}[a + (n-1)a] = \frac{\text{tg} + \text{tg}[(n-1)a]}{1 - \text{tg} a \, \text{tg}[(n-1)a]}$

Consequências

$\cos 2a = \cos^2 a - \text{sen}^2 a$

$\cos 2a = \cos^2 a - (1 - \cos^2 a) = 2\cos^2 a - 1$

$\cos 2a = \cos^2 a - \text{sen}^2 a = 1 - \text{sen}^2 a - \text{sen}^2 a$

$\cos 2a = 1 - 2\,\text{sen}^2 a$

$2\,\text{sen}^2 a = 1 - \cos 2a$

$\text{sen}^2 a = 1 - \cos 2a$

$\text{sen}\, a = \pm\sqrt{\dfrac{1 - \cos 2a}{2}}$

$\text{sen}\, \cancel{2}a = 2\,\text{sen}\,\dfrac{a}{2}\,\cos\dfrac{a}{2}$

$\cos a = \cos^2\dfrac{a}{2} - \text{sen}^2\dfrac{a}{2}$

$\text{tg}\, a = \dfrac{2\,\text{tg}\, a/2}{1 - \text{tg}^2 a/2}$

Exercícios

① Sendo $\text{sen}\, a = 3/5$, calcular as linhas do arco duplo.

$\text{sen}\, 2a = 3/5$
$\cos 2a = 4/5$
$\text{tg}\, 2a = 3/4$

⑴ **Dados** e **incógnitas**

$\text{sen}\, a = 3/5$

$\text{sen}\, 2a \quad \cos 2a \quad \text{tg}\, 2a$

② **fórmulas**

$\text{sen}\, 2a = 2\,\text{sen}\, a \cos a$
$\cos 2a = \cos^2 a - \text{sen}^2 a$
$\text{tg}\, 2a = \dfrac{2\,\text{tg}\, a}{1 - \text{tg}^2 a}$

$\text{sen}\, 2a = 2\,\text{sen}\, a \cos a$
$\cos 2a = \cos^2 a - \text{sen}^2 a$
$\text{tg}\, 2a = \dfrac{2\,\text{tg}\, a}{1 - \text{tg}^2 a}$

$\text{sen}^2 a + \cos^2 a = 1$

③ **Cálculos**

3.1. auxiliares

$\cos a = \sqrt{1 - \text{sen}^2 a}$

$\cos a = \sqrt{1 - 9/25} = \boxed{\dfrac{4}{5}}$

$\text{tg}\, a = \dfrac{\text{sen}\, a}{\cos a}$

$\text{tg}\, a = \dfrac{3/5}{4/5} = \boxed{3/4}$

3.2. definitivos:

$\text{sen} 2a = 2\,\text{sen}\, a \cos a$
$\text{sen}\, a = 2\,(3/5 \cdot 4/5)$

$\text{sen} 2a = 2 \cdot \dfrac{12}{25}$

$\text{sen} 2a = \boxed{\dfrac{24}{25}}$ → não pode dar > que 1.

$\cos 2a = \cos^2 a - \text{sen}^2 a$
$\cos 2a = \dfrac{16}{25} - \dfrac{9}{25}$

$\cos 2a = \boxed{7/25}$

$\text{tg}\, 2a = \dfrac{2 \times 3/4}{1 - 9/16}$

$\text{tg}\, 2a = \dfrac{6/4}{7/16} = \dfrac{6}{\cancel{4}_1} \times \dfrac{\overset{4}{\cancel{16}}}{7}$

$\text{tg}\, 2a = \boxed{\dfrac{24}{7}}$

② **Sendo** $\boxed{\text{sen } a - \cos a = 0{,}50}$

Calcular **sen 2a**

① Dados:

$$\begin{cases} \text{sen } a - \cos a = 0{,}50 \\ \text{sen}^2 a + \cos^2 a = 1 \end{cases}$$

② fórmulas

$\text{sen } 2a = 2\text{sen } a \cos a$

③ incógnitas

$\text{sen } 2a = 2\text{sen}a\cos a$.

④ resolução

$(\text{sen}a - \cos a)^2 - \text{sen}^2 a + \cos^2 a - 2\text{sen}a.\cos a$

$(0{,}50)^2 = 1 - 2\text{ sen } a \cos a$

$0{,}25 = 1 - \text{sen } 2a$

$\text{sen } 2a = 1 - 0{,}25$

$\text{sen } 2a = 0{,}75 = 3/4$

resolver o sistema $\begin{cases} \text{sen } a - \cos a = 0{,}50 \\ \text{sen}^2 a + \cos^2 a = 1 \end{cases}$

---

③ Achar a fórmula do : sen 3a em funçãs do sen a (seguir desenvolvi/ das fórmulas)

$$\text{sen } 3a = \text{sen}(a + 2a) = \text{sen } a \cos 2a + \text{sen } 2a \cos a$$
$$= \text{sen } a(\cos^2 a - \text{sen}^2 a) + 2 \text{ sen } a \cos a . \cos a =$$

$$\text{sen } a \cos^2 a - \text{sen}^3 a + 2 \text{sen } a \cos^2 a = 3 \text{sen } a \cos^2 a - \text{sen}^3 a$$

$$\text{sen } 3a = 3 \text{sen} a \cos^2 a - \text{sen}^3 a$$

$$\text{sen } 3a = 3 \text{sen } a(1 - \text{sen}^2 a) - \text{sen}^3 a = 3 \text{sen}a - 3\text{sen}^3 a - \text{sen}^3 a$$

$$\boxed{\text{sen } 3a = 3 \text{ sen}a - 4\text{sen}^3 a.}$$

Aplicações

Teorema. As linhas de um arco podem ser expressas em funções racionais da tangente de sua metade.

$\text{sen } a = f(\text{tg } a/2)$
$\cos a = f(\text{tg } a/2)$
$\text{tg } a = f(\text{tg } a/2)$

$$\text{sen } a = \frac{2\text{ sen } a/2 \cos /a2}{1}$$

$$\text{sen } a = \frac{2\text{ sen } a/2 \cos a/2}{\text{sen}^2 a/2 + \cos^2 a/2} : \cos^2 \frac{a}{2}$$

$$\text{sen } a = \frac{\dfrac{2 \text{ sen } a/2 \cos a/2}{\cos^2 a/2}}{\dfrac{\text{sen}^2 a/2 \cos 2 a/2}{\cos^2 a/2}} =$$

$$\text{sen } a = \frac{\dfrac{2 \text{ sen } a/2 \cos a/2}{\cos^2 a/2}}{\dfrac{\text{sen}^2 a/2}{\text{sen}^2 a/2} + \dfrac{\cos^2 a/2}{\cos^2 a/2}} = \frac{2 \text{ tg } a/2}{1 + \text{tg}^2 a/2}$$

↗ simplificando :

$$\boxed{\text{sen } a = \frac{2 \text{ tg } a/2}{1 + \text{tg}^2 a/2}}$$

cos a: $\dfrac{\cos^2 a/2 - \text{sen}^2 a/2}{\text{sen}^2 a/2 + \cos^2 a/2} \div \cos^2 a/2$

$$\cos a = \dfrac{\dfrac{\cos^2 a/2}{\cos^2 a/2} - \dfrac{\text{sen}^2 a/2}{\cos^2 a/2}}{\dfrac{\text{sen}^2 a/2}{\cos^2 a/2} + \dfrac{\cos^2 a/2}{\cos^2 a/2}}$$

$$\cos a = \dfrac{1 - tg^2 a/2}{tg^2 a/2 + 1} \therefore \boxed{\cos a = \dfrac{1 - tg^2 a/2}{tg^2 a/2 + 1}}$$

Fórmulas da bissecção em função do cos a.

Partindo de:

$\cos 2a = 2\cos^2 a - 1$

$\cos 2a = 1 - \text{sen}^2 a - \text{sen}^2 a = 1 - 2\,\text{sen}^2 a$

$$\begin{cases} \cos 2a = 2\cos^2 a - 1 \\ \cos 2a = 1 - 2\,\text{sen}^2 a \end{cases}$$

Substituindo:

$\cos a = 2\cos^2 a/2 - 1$

$\cos a = 1 - 2\,\text{sen}^2 a/2$

$2\cos^2 a/2 = 1 + \cos a$

$\cos^2 a/2 = \dfrac{1 + \cos a}{2}$

$$\boxed{\cos a/2 = \pm\sqrt{\dfrac{1 + \cos a}{2}}}$$

P/a se achar o cos. de um arco de 15° dado o arco de 30°, aplica-se esta fórmula.

$2\,\text{sen}^2 a/2 = 1 - \cos a$

$\text{Sen}\, a/2 = \pm\sqrt{\dfrac{1 - \cos a}{2}}$

$$\boxed{tg\, a/2 = \pm\sqrt{\dfrac{1 - \cos a}{1 + \cos a}}}$$

6/11

Exercícios

Sendo: $\begin{cases} tg\, a = 3 & e \quad a \in \text{III Q} \\ \sec b = 2 & e \quad b \in \text{IV Q} \end{cases}$

Calcular: 1→ cos (a + b)

2→ sen (a − b)

3→ sen 2a + sen 3b

① $1 + tg^2 a = \sec^2 a$

$1 + 3^2 = \sec^2 a$

$\sec a = \sqrt{1+9} \therefore \boxed{\sec a = \pm\sqrt{10}}$

$\cos a = \dfrac{1}{\sqrt{10}} \cdot \dfrac{\sqrt{10}}{\sqrt{10}} = \boxed{\cos a = -\dfrac{\sqrt{10}}{10}}$

$\text{sen}^2 a + \cos^2 a = 1$

$\text{sen}\, a = \sqrt{1 - \dfrac{10}{100}} = \sqrt{\dfrac{90}{100}} = \dfrac{3}{\sqrt{10}} \therefore \boxed{\text{sen}\, a = \dfrac{-3\sqrt{10}}{10}}$

Uma vez, calculados os resultados, aplica-se a fórmula:

$$\boxed{\cos(a+b) = \cos a \cdot \cos b - \text{sen}\, a \cdot \text{sen}\, b.}$$

$$\sec b = 2 \quad \therefore \quad \boxed{\cos b = \frac{1}{2}}$$

$$\operatorname{sen}^2 b + \cos^2 b = 1$$

$$\operatorname{sen} b = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} \quad \therefore \quad \boxed{\operatorname{sen} b = -\frac{\sqrt{3}}{2}}$$

① Logo: $\cos(a+b) = \frac{-\sqrt{10}}{10} \cdot \frac{1}{2} - \left(-\frac{3\sqrt{10}}{10}\right) \cdot \left(-\frac{\sqrt{3}}{2}\right)$

$$\cos(a+b) = \frac{-\sqrt{10}}{20} - \frac{3\sqrt{30}}{20}$$

$$\boxed{\cos(a+b) = \frac{-\sqrt{10} - \sqrt{10}\,.\,3\sqrt{3}}{20}}$$

Verificar a identidade:

$$\frac{1 + 2\operatorname{sen} x - \operatorname{sen} 2x}{2\operatorname{sen} x + \operatorname{sen} 2x} = \sec^2 \frac{x}{2}$$

$$\frac{1 + 2\operatorname{sen} x - 2\operatorname{sen} x \cos x}{2\operatorname{sen} x + 2\operatorname{sen} x \cos x} = \frac{1}{\cos^2 \frac{x}{2}}$$

$$\frac{1 + \cancel{2\operatorname{sen}}\,(1 - \cos x)}{\cancel{2\operatorname{sen}}\,x\,(1 + \cos x)} = \frac{1}{\frac{1+\cos x}{2}}$$

$$1 + \frac{(1 - \cos x)}{(1 + \cos x)} = \frac{2}{1 + \cos x}$$

$$1 + \cancel{\cos x} + 1 - \cancel{\cos x} = 2$$

$$2 = 2$$

---

① Calcular $\operatorname{sen} 2x$, sendo:

$$\begin{cases} \operatorname{sen} x - \cos x = 1/5 \\ \operatorname{sen}^2 x + \cos^2 x = 1 \end{cases}$$

$$\operatorname{sen}^2 x = 1/5 + \cos^2 x$$

$$\operatorname{sen}^2 x = (1/5 + \cos x)^2$$

$$\operatorname{sen}^2 x = \frac{1}{25} + \frac{2}{5}\cos x + \cos^2 x$$

$$\frac{1}{25} + \frac{2}{5}\cos x + \cos^2 x + \cos^2 x = 1$$

$$1 + 10\cos x + 25\cos^2 x + 25\cos^2 x = 25$$

$$1 + 10\cos x + 50\cos^2 x - 25 = 0$$

$$50\cos^2 x + 10\cos x - 24 = 0$$

$$25\cos^2 x + 5\cos x - 12 = 0$$

$$\cos x = \frac{-5 \pm \sqrt{25 + 1.200}}{50}$$

$$\cos x = \frac{-5 \pm \sqrt{1.225}}{50}$$

$$\cos x = \frac{-5 \pm \sqrt{1225}}{50} \quad \therefore \quad \cos x = \frac{-5 \pm 35}{50}$$

$$\cos x = \frac{-5 + 35}{50} \quad \therefore \quad \frac{30}{50} \Rightarrow \boxed{\cos x = 3/5}$$

$$\cos x = \frac{-5 - 35}{50} \quad \therefore \quad \frac{-40}{50} \Rightarrow \boxed{\cos x = -4/5}$$

$$\operatorname{sen} x - \cos x = 1/5$$

$$(\operatorname{sen} x - \cos x)^2 = \left(\frac{1}{5}\right)^2$$

$$\operatorname{sen}^2 x - 2\operatorname{sen} x \cos x + \cos^2 x = 1/25$$

$$-2\operatorname{sen} x \cos x = 1/25 - 1$$

$$-2\operatorname{sen} x \cos x = \frac{1 - 25}{25}$$

$$-2\operatorname{sen} x \cos x = \frac{-24}{25}$$

$$2\operatorname{sen} x \cos x = \frac{24}{25} \quad \therefore \quad \boxed{\operatorname{sen} 2x = \frac{24}{25}}$$

② A cotangente de um ângulo sendo $1+\sqrt{2}$ calcular a secante do dobro deste ângulo.

Calcular: $\sec 2x$.

$$\csc^2 x = 1 + \cot g^2 x \qquad \cot g = 1+\sqrt{2}$$

$$\csc^2 x = 1 + (1+\sqrt{2})^2 \qquad \cot g^2 x = \csc^2 x - 1$$

$$\csc^2 x = 1 + 1 + 2\sqrt{2} + 2$$

$$\csc^2 x = 4 + 2\sqrt{2}$$

$$\csc^2 x = \sqrt{4+2\sqrt{2}}$$

$$\boxed{\csc^2 x = \sqrt{4+2\sqrt{2}}}$$

$$\operatorname{sen} x = \frac{1}{\sqrt{4+2\sqrt{2}}} = \frac{\sqrt{4+2\sqrt{2}}}{4+2\sqrt{2}} = \frac{(4-2\sqrt{2})\sqrt{4+2\sqrt{2}}}{16-8} =$$

$$\boxed{\operatorname{sen} x = \frac{(4-2\sqrt{2})\sqrt{4+2\sqrt{2}}}{8}}$$

$$\cos^2 x + \operatorname{sen}^2 x = 1$$

$$\cos^2 x + \frac{(16-16\sqrt{2}+8)(4+2\sqrt{2})}{64} = 1$$

$$\cos^2 x + \frac{(24-16\sqrt{2})(4+2\sqrt{2})}{64} = 1$$

$$\cos^2 x + \frac{96-16\sqrt{2}-64}{64} - 1$$

$$\cos^2 x + \frac{32-16\sqrt{2}}{64} = 1$$

$$\cos^2 x + \frac{2-\sqrt{2}}{4} = 1$$

$$\cos x + \frac{\sqrt{2-\sqrt{2}}}{2} = 1$$

$$4\cos^2 x + 2 - \sqrt{2} = 4$$

$$4\cos^2 x = 4 - 2 + \sqrt{2}$$

$$\cos^2 x = \frac{2+\sqrt{2}}{4}$$

$$\boxed{\cos x = \frac{\sqrt{2+2}}{2}}$$

$$\sec 2x\,? \qquad \cos 2x = \cos^2 x - \operatorname{sen}^2 x$$

$$\frac{1}{\cos^2 x} = \frac{1}{\cos^2 x - \operatorname{sen}^2 x} \qquad \operatorname{sen} x =$$

$$\operatorname{sen} x = \frac{(4-2\sqrt{2})\sqrt{4+2\sqrt{2}}}{8}$$

$$\sec^2 x = \frac{1}{\frac{2+\sqrt{2}}{4} - \frac{1}{4+2\sqrt{2}}}$$

$$\sec 2x = \frac{1}{\frac{(2+\sqrt{2})(4+2\sqrt{2})-4}{4(4+2\sqrt{2})}}$$

$$\sec 2x = \frac{16+8\sqrt{2}}{8+8\sqrt{2}+4-4}$$

$$\sec 2x = \frac{16+8\sqrt{2}}{8+8\sqrt{2}} = \frac{8(2+\sqrt{2})}{8(1+\sqrt{2})} = \frac{2+\sqrt{2}}{1+\sqrt{2}}$$

$$\sec 2x = \frac{2+\sqrt{2}}{1+\sqrt{2}} \qquad \frac{2+2\sqrt{2}+\sqrt{2}+2}{-1} = \boxed{\frac{4+3\sqrt{2}}{-1}}$$

③- Partindo das linhas conhecidas de 30° e 45°, calcular por adição e subtração as linhas de 75° e 15°:

seno de 75° → $\text{sen}(30° + 45°) = \text{sen } 30 . \cos 45 + \text{sen } 45 . \cos 30 .$

$$\frac{1}{2} . \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} . \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \rightarrow \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

cos 75° → $\cos(30 + 45) = \cos 30 . \cos 45 - \text{sen } 30 . \cos 45$

$$\frac{\sqrt{3}}{2} . \frac{\sqrt{2}}{2} - \frac{1}{2} . \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \rightarrow \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

tg 75° → $\frac{\sqrt{2}+\sqrt{6}}{4} \times \frac{4}{\sqrt{6}-\sqrt{2}} = \frac{(\sqrt{2}+\sqrt{6})(\sqrt{6}+\sqrt{2})}{6-2} = \frac{2+2\sqrt{12}+6}{4}$

$$\frac{8 + 2\sqrt{2}}{4} = \frac{4 + 2\sqrt{3}}{2} = \boxed{2+\sqrt{3}}$$

Os ângulos de 75° e 15° são complementares
logo: $\text{sen } 15° = \cos 75° = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos 15° = \text{sen } 75° = \frac{\sqrt{2}+\sqrt{6}}{4}$

④ Dados seno $\boxed{a = 3/5}$ e seno $\boxed{b = 4/5}$ e sendo a e b do 1º Q. calcular:

seno (a - b) e cos (a + b)

$\cos a = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25}{25} - \frac{9}{25}} \rightarrow \cos a = \frac{4}{5}$

$\cos b = \sqrt{1 - (\frac{4}{5})^2} \rightarrow \sqrt{\frac{25}{25} - \frac{16}{25}} \rightarrow \cos b = \frac{3}{5}$

continua na pag... já está resolvido neste caderno

Quintella - 2º Vol.

11/11

Logaritmos e Equações Exponenciais

① def: operações inversas: adição ⟺ subtração
multiplicação ⟺ divisão
potenciação ⟺ { radiciação
{ logaritmação

Equivalências

$$\text{base} \rightarrow 3^{4 \rightarrow \text{expoente}} = 81 \rightarrow \text{potência} \iff \sqrt[4]{\underset{\text{radicando}}{81}} = 3 \rightarrow \text{raiz}$$

(índice)

$$3^x = 81 \iff \sqrt[x]{81} = 3 \rightarrow \text{1ª apl. de log.}$$

$$\text{base } 3^{\overset{\text{Expoente}}{x}} = 81 \text{ potência} \iff \log_{3 \rightarrow \text{base}} 81 = x \rightarrow \text{logaritmo}$$

(81 → nº ou anti-logaritmo)

$$\log_\alpha N = a \quad \alpha > 0 \quad \alpha \neq 1 \iff \alpha^a = N$$

def. log de N numa base α, é o expoente que se deve elevar a base α para obter o número.

Exemplos

1- Escreva sob a forma logarítmica:

a) $3^4 = 81 \iff \log_3 81 = 4$

b) $4^{\frac{1}{2}} = 2 \iff \log_4 2 = \frac{1}{2}$

c) $5^{-2} = 1/25 \iff \log_5 \frac{1}{25} = -2$

d) $(1/2)^{\frac{1}{3}} = x \iff \log_{\frac{1}{2}} x = \frac{1}{3}$

e) $a^{-b} = y \iff \log_a y = -b$

② sistemas de logarítmos

2.1 → base 10 - sistemas de log. decimais ordinários ou de BRIGGS.

2.2 → sistema de log. naturais, hiperbólicos ou neperianos. e = 2,71828...

$\boxed{\log N \text{ (base 10)} = x}$

$\boxed{lN \text{ (base e)} = y}$ ou $\boxed{\ln N = y}$ (log. nat.)

③ variação dos logarítmos

3.1 → $\log_\alpha 1 = 0 \iff \alpha^0 = 1$

3.2 → $\log_\alpha^\alpha = 1 \iff \alpha^1 = \alpha$ → o log. da base é sempre 1.

3.3 → $\log \alpha^2 = 2$

3.4 → $\log \alpha^3 = 3$

3.5 → $\log \alpha^n = n$

só as potências da base tem log. inteiros.

$\log 10 = 1$

$\log 100 = 2$

$\log 1000 = 3$

$\log 10000 = 4$

③.④ → os logaritmos variam no mesmo sentido dos números quando $\underset{\text{base}}{b} > 1$

$H \{ A > B$ $\qquad$ $T \{ \log A > \log B$

$\left.\begin{array}{l} \log_a A = x \iff a^x = A \\ \log_a B = y \iff a^y = B \end{array}\right\} a^x > a^y \therefore x > y$ ou

$\log A > \log B$ (c.q.d).

consequências :

① Os n^os^ > 1, têm log > 0

$\log_a N > \log_a 1 = \log_a N > 0$

② Os n^os^ < 1, têm log < 0 → N < 1

$\log_a N < \log_a 1 \rightarrow \log_a N < 0$

④ Propriedades Operatórias dos logaritmos

$$\boxed{\begin{array}{ll} \log_\alpha a = x & \therefore \alpha^x = a \\ \log_\alpha b = y & \therefore \alpha^y = b \end{array}}$$

logaritmando:

a) $a \times b = \alpha^{x+y} \rightarrow \log_\alpha a.b = x + y$

b) $a/b = \alpha^{x-y} \rightarrow \log_\alpha a/b = x - y$

c) $a^m = \alpha^{m.x} \rightarrow \log_\alpha a^m = m.x$

d) $\sqrt[m]{a} = a^{x/m} \rightarrow \log_\alpha \sqrt[m]{a} = \dfrac{x}{m}$

Temos portanto as igualdades:

$$\begin{array}{lll} \log_\alpha a.b = x + y & \iff & \log_\alpha a + \log_\alpha b \\ \log_\alpha a/b = x - y & \iff & \log_\alpha a - \log_\alpha b \\ \log_\alpha a^m = m.x & \iff & m.\log_\alpha a \\ \log_\alpha \sqrt[m]{a} = \dfrac{x}{m} & \iff & \dfrac{\log_\alpha a}{m} \end{array}$$

Exemplos

① $\log 3.2 = \log 3 + \log 2$

$\log 3/2 = \log 3 - \log 2$

$\log 3^2 = 2 . \log 3$

$\log \sqrt[2]{3} = \frac{\log 3}{2}$

② Sendo dados:

$\boxed{\begin{array}{l}\log 2 = 0,3010 \\ \log 3 = 0,4771\end{array}}$

Calcular:

a) $\log 6 = \log 2.3 = \log 2 + \log 3 = 0,3010 + 0,4771 = \boxed{0,7781}$

b) $\log 3/2 = \log 3 - \log 2 = 0,471 - 0,3010 = \boxed{0,1761}$

$\begin{array}{r|l} 108 & 2 \\ 54 & 2 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$

c) $\log 108 = \log (2^2 \times 3^3) = 2 \log 2 + 3 \log 3 =$

$2 . 0,3010 + 3 . 0,4771 =$

$0,6020 + 1,4313 = \boxed{2,0333}$

---

⑤ Cologarítmo

é o inverso do logarítmo.

$\boxed{\text{colog } a = \frac{1}{\log a}} = \boxed{\log 1 - \log a}$ $\log 1 = 0$ log [illegible] = 1

$\boxed{\text{colog } a = -\log a}$

$\log \frac{a}{b} = \log a - \log b = \log a + \text{colog } b$

Ex: Exprimir o log. da expressão:

$\frac{b^2c^3}{\sqrt{a}}$ sendo dados log de a, b, c

$\log \frac{b^2c^3}{\sqrt{a}} = \log b^2c^3 - \log \sqrt{a} =$

$= \log b^2 + \log c^3 - \frac{\log a}{2} = 2 \log b + 3 \log c + \frac{\text{colog } a}{2}$

$= \log \frac{b^2c^3}{\sqrt{a}} = 2 \log b + 3 \log c + \frac{\text{colog } a}{2}$

13/ a) Calcular o log de 3 na base 243

b) calcular o log de $\sqrt[5]{2^3}$ na base $\sqrt{2}$

c) O log de um nº de uma certa base é 3 e o log. deste mesmo nº numa base igual ao dobro da anterior é 2. Calcular o nº.

b→ $\log_{\sqrt{2}} \sqrt[5]{2^3} = (\sqrt{2})^x = \sqrt[5]{2^3}$

$(2^{\frac{1}{2}})^x = 2^{\frac{3}{5}}$ $2^{\frac{x}{2}} = 2^{\frac{3}{5}}$ $\frac{x}{2} = \frac{3}{5}$ $5x = 6$

$x = \frac{6}{5}$

a→ $\log_{243}^{3} = x$

$243^x = 3$

$(3^5)^x = 3$ $3^{5x} = 3$

$5x = 1 \Leftrightarrow x = \frac{1}{5}$

c→ $\log N = 3 \Rightarrow b^3 = N$

$\log_{2b} N = 2 \Rightarrow (2b)^2 = N$

$b^3 = (2b)^2$

$b^3 = 4b^2 \Rightarrow b^2$

$b = 4$

$b^3 = N$

$4^3 = N \rightarrow N = 64$

Exercícios:

① Escrever na forma logarítmica:

$2^3 = 8$ R. $\log_2 8 = 3$ $10^3 = 0,001$

$10^2 = A$ R. $\log_{10} A = a$ $3^{-2} = \frac{1}{9}$

② Escrever na forma esponencial:

$\log_4 16 = 2$ $\log_4^{2} = 16$

$\log_5 25 = 2$ $\log_5^{2} = 25$

$\log_2 2 = 1$ $\log_2^{1} = 2$

$\log 3 = 0,4771$ $\log^{0,4771} = 3$

③- Achar os log. dos seguintes n.^{os} na base 2.

$8 \; , \; 16 \; , \; \frac{1}{4} \; , \; \frac{1}{32}$

$b^x = 8 \quad b^x = 2^3 \therefore x = 3 \qquad b^x = 16 \quad b^x = 2^4 \therefore \boxed{x = 4}$

④- Conhecidos $\log 2 = 0,3010$ e $\log 3 = 0,4771$, calcular: $b^x = \frac{1}{4} \therefore b^x = \frac{1}{2^2} \quad x = \frac{1}{2}$

$\log \frac{4}{9} \qquad \log 4 - \log 9 = \log 2 \times \log 2 - \log 3 + \log$

$\log 0,12$

$\log \frac{\sqrt{243}}{\sqrt[3]{81}}$

$\log \frac{\sqrt[3]{3}}{\sqrt{2}}$

Esprimir os log. das formulas e indentificá-los:

$S = \pi R^2$

$S = \frac{l^2 \sqrt{3}}{}$

$S = \frac{\pi R^2 h}{3}$

Calcular

① O log da base aumentada de 6 é 2. Calcule a base.

$\log_6 b + 6 = 2$

$b^2 = b + 6$

$b^2 - b - 6 = 0 \qquad$ base = 3

② Desenvolver:

$\log b \left(\frac{A \times B}{C}\right) \qquad \log_b (A \times B) - \log_b C$

$\log_b A + \log_b B - \log_6 C$

Propriedades

$\log_x (a \times b) = \log_x a + \log_x b$

$\log x \left(\frac{a}{b}\right) = \log x^a - \log x^b$

$\log x \, a^b = b . \log a$

$\log x \sqrt[a]{b} = \frac{1 . \log x^b}{a}$

Exemplos:

$\log x \frac{ab^2 \sqrt[3]{a^2}}{\sqrt[4]{ab^3}}$

$\log_x a + 2 \log_x b + \frac{2 \log a}{3} - \frac{\log a . b}{4} =$

$\log_x a + 2 \log b + \frac{2 \log a}{3} - \frac{1 \log a}{4} - \frac{3 \log b}{4}$

$\boxed{\frac{17}{12} \log_x a + \frac{5}{4} \log_x b}$

③ Provar que:

$\log y \left(\frac{\sqrt[4]{5} \times \sqrt[10]{2}}{\sqrt[3]{18} \times \sqrt{2}}\right) = \frac{1}{4} \log y^5 - \frac{2}{5} \log y^2 - \frac{2}{3} \log y^3$

$\log y \left(\sqrt[4]{5} \times \sqrt[10]{2}\right) - \log y \left(\sqrt[3]{18} \times \sqrt{2}\right) =$

$\log y \sqrt[4]{5} + \log \sqrt[10]{2} - \frac{1}{3} \log y \left(\sqrt{18} \times \sqrt{2}\right)$

$\frac{1}{4}\log y^5 + \frac{1}{10}\log^2_y - \frac{1}{3}\left[\log y^{18} + \frac{1}{2}\log y^2\right)$

$\frac{1}{4}\log y5 + \frac{1}{10}\log^2 y - \frac{1}{3}\log y18 - \frac{1}{6}\log y^2 =$

$\frac{1}{4}\log y^5 + \frac{1}{10}\log y^2 - \frac{1}{3}\log 2 - \frac{2}{3}\log 3 - \frac{1}{6}\log y^2$

$\frac{1}{4}\log 5 - \frac{2}{3}\log_y^3 + \frac{3\log y^2 - 10\log y^2 - 5\log y^2}{30}$

$\frac{1}{4}\log 5 - \frac{2}{3}\log_y^3 - \frac{12\log y^2}{30}$

$\frac{1}{4}\log y^5 - \frac{2}{3}\log y^3 - \frac{2}{5}\log y^2$

---

④ Um nº, o seu log. que é 2 e a base do log, formam nessa ordem uma progressão geométrica. Determinar o nº.

$\log_b N = 2$ $\quad \begin{cases} b^2 = N \\ N.2 = 2 - b \end{cases}$ $\quad N + r = 2$

$2 + r = b$

$N . 2 . b . . . .$ $\quad b^2 - 2 = 2 - b$ $\quad N - 2 = 2 - b$

$N - 2 = 2 - b$

$N = 2 - b + 2$ $\quad$ ou $\begin{cases} b^2 - 2 = 2 - b \\ b^2 + b + 4 = 0 \end{cases}$

$b = \frac{1 \pm \sqrt{17}}{2}$

$x + r = 2$

$2 + r = b$ $\quad b = 4 - x$

$x - 2 = 2 - b$

$\log_{4-x} x = 2$

$(4-x)^2 = x$

$16 - 8x + x^2 = x$ $\quad x = \frac{9 + \sqrt{17}}{2}$ $\quad N = \frac{1 \pm 2\sqrt{17} + 17}{4}$

$x^2 - 8x - x + 16 = 0$

$x^2 - 9x + 16 = 0$ $\quad = \frac{18 \pm 2\sqrt{17}}{4} = x = \boxed{\frac{9 \pm \sqrt{17}}{2}}$

$x = \frac{9 \pm \sqrt{81 - 64}}{2}$ $\quad \cancel{x} =$

⑤ Qual o nº cujo log. na base $\sqrt{2}$ é -6

$\log_{\sqrt{2}} x = -6$

$(\sqrt{2})^{-6} = x$

$\frac{1}{(\sqrt{x})^6} = x$

$\frac{1}{2^3} = x$ $\quad \frac{1}{8} = x$

⑥ Desenvolver:

$\log b\left(\sqrt[3]{\frac{A^2\sqrt{B}}{C^2}} \times \sqrt[15]{\frac{A8}{\sqrt[8]{B6}}}\right)$

$\log_b\left(\frac{A \times \sqrt{b}}{3}\right) - \frac{2\log bc}{3} + \frac{8\log bA}{15} - \frac{\log b\sqrt[8]{B^6}}{15}$

$\frac{2\log b A}{3} + \frac{1\log b B}{\substack{\cancel{2.3} \\ 6}} - \frac{2\log bc}{3} + \frac{8\log bA}{15} - \frac{6\log_b B}{120}$

$\frac{10\log bA + 8\log bA}{15} + \frac{20\log_b B - 6\log B}{120} - \frac{2\log C}{3}$

$\frac{\overset{6}{\cancel{18}}\log A}{\underset{5}{\cancel{15}}} + \frac{\overset{7}{\cancel{14}}\log B}{\underset{60}{\cancel{120}}} - \frac{2\log C}{3}$

$\frac{6}{5}\log A + \frac{7}{60}\log B - \frac{2}{3}\log C$

18/11/72 **Logarítmos Decimais**

característica → parte inteira

mantissa → parte decimal

Cálculo da característica:

A característica do logarítmo de um nº maior do que 1 é igual ao nº de algarismos constituintes da parte inteira do nº considerado, diminuido de uma unidade.

Excemplo: log 2 = $\overset{c}{0},3010 \rightarrow m$
log 305,2 = 2, . . . (3 alg. da mant. - 1 = 2)
log 40,72 = 1, ... (2 alg da mant - 1 = 1)

A característica do logaritmo de um nº positivo menor que 1 é um nº negativo, constituido de tantas unidades, quantos forem os zeros que antecederem seu 1º algarismo significativo.

Excemplo: log 0,1 = -1, + fração (0 → 1 alg = -1)
log 0,003 = -3, + fração
log 0,005 = - 3, ± fração } acha-se na tábua.

Cálculo da mantissa

log 3120 = 3,491546
log 4370 = 3,6404814
log 216 = 2,33445375
log 471 = 2,67302091
log 4320/7581 = 7,635 4837

20/11/72

Achar na táboa os logaritmos dos seguintes nºs:

log 1071 = 3,02979
log 0,0032 = $\bar{3}$,50515
log 4,37 = 0,64048
log 23,50 = 1,37107
log 350,1 = 2,54419
log 43,512 = 1,63861
log 0,0874712 = $\bar{2}$,94187
log 0,01 = $\bar{2}$,0 = -2 (log de 1 é zero)

log 3,47214 = 0,54060
log 3,672421 = 0,56495
log 472,126 = 2,67406
log 32,681 = 1,51429
log 0,0132 = $\bar{2}$, 12057

log 43,51② = 1,63859
2
1,63861 10

1 → 10
0,2 → x

1,63859
2
1,63861

x = 2

log 0,0874712 = $\bar{2}$,94187
0,1
0,02

8747 → 94186 dif. tabular = 5
0,1 → 1
0,02 → 0,1
$94187,\cancel{1}$

log 3,472114 = 0,54060

3472 → 54058 dif. tabular = 12.
0,1 → 1
0,04 → 0,5
$54059,\cancel{5}$ (arredonda de 5 em diante
60

log 3,672421 = 0,56495 dif. tabular 12
4 → 5
2 → 2
1 → 1

3672/421 = 56490
521
$56495,\cancel{81}$

$\log 472,126 = 2,67406$

4721 | 26 → 67403

$\underline{26}$

67405,~~6~~ → 2,67406

$\log 32,681 = 1,51429$

3268 | 1 → 51428

$\underline{1}$

51429

dif. tab. → 13

1 → 1

$\log N = 2,67471 \Rightarrow N = 472,83333\ldots$ (2+1) → caracterís

67468 → 4728

$\underline{+\ 0,3333}$

4728,3333

67471

$\underline{67468}$

3 → dif.

1 9

x 3

$9x = 3$

$x = \frac{3}{9} = 0,3333\ldots$

$\log N = \bar{2},426812 \Rightarrow N = 0,0267183$

42667 → 2671 (nº que corresponde)

$\underline{0,83}$

0,0267183

1 → 17

x → 14,2

$x = \frac{14,2}{17} = 0,83$

2,426812

$\underline{42667}$

14,2 (dif.).

$\log N = 0,83216 \Rightarrow N = 6,7945$

83213 → 6794

$\underline{0,5}$

6,7945

6 → 1

3 → x

$x = 0,5$

$\log N = 10,81232 \Rightarrow N = 64911428571,42.$

81232 → não tem na táboa

81231 → 6491

$\underline{0,14}$

649114|285714,2

a característica é 10, então o nº terá 11 casas decimais

7 → 1

1 → x

$7x = 1$

$x = \frac{1}{7} = 0,14$

$\log N = 5,47712 \Rightarrow N = 300.000$

$\log N = \bar{5},47712 \Rightarrow N = 0,00003$

$\log N = 0,32621 \Rightarrow N = 2,119$[illegible]

32613 → 2119

21 → 1

8 → x $\therefore x = \frac{8 \times 1}{21}$ $\therefore x = 0,38$

2119

$\underline{0,38}$

2119,~~38~~

A característica é zero, então 0+1 = 1 casa inteira → 2,119

$\log N = 0,32141 \Rightarrow N = 2,096$

32139 → 2096

21 → 1

2 → x $\therefore x = \frac{2}{21} = 0,095$

32141

$\underline{32139}$

00002

2096

$\underline{0,095}$

2,096,~~095~~

$\log N = \bar{2},87421 \Rightarrow N = 0,07485$

$87421 \longrightarrow$
$87419 \longrightarrow 7485$

$6 \longrightarrow 1$
$2 \longrightarrow x \therefore x = \frac{2}{6} \therefore x = 0,333\ldots$

7485
$0,333$
$7485333 \Rightarrow$

a característica é $\bar{2}$, então dará 2 zeros e ficará:
$0,07485\cancel{333}$

$\log N = \bar{1},21287 \Rightarrow N = 0,1633.$

$21287 \longrightarrow$
$21272 \longrightarrow 1632$

$27 \longrightarrow 1$
$15 \longrightarrow x \therefore x = \frac{15 \times 1}{27} \therefore x = 0,555\ldots$

1632
$0,555$
$1632,555 \Rightarrow 1633$ ´(arredonda)

Característica = 1 → um zero

N = 0,1633.

Logarítmo preparado e negativo

log. preparado: $\log 0,02 = \bar{2},30103$
$-2 + 0,30103$

log. negativo : $-1,69897$
$-1 - 0,69897$

Transformações

n → p $\quad -1 - 0,69897 \ (+1 - 1)$

o um (+) vai para a mantissa
o um (−) vai para a característica

$(-1-1) + (+1 - 0,69897)$
$-2 + 0,30103$
$\bar{2},30103 \rightarrow$ log. preparado.

Outro método:

$\bar{2},30103$	$-1,69897$
$-1,69897$	$\bar{2},30103$

25| $\log 0,03 = \bar{2},47712 = -2 + 0,47712$

$\log 2 = 0,30103$

$\log 0,002 = \bar{3},30103 = -3 + 0,30103$

$\log 0,002 = \bar{3},30103 = -2,69897 = -2,00000 - 0,69897$

$-3,00000$
$+0,30103$
$-2,69897$

~~preparado p/a negativo~~

negativo p/a preparado

$\log 0,002 = -2,69897 = -2 - 0,69897 = (-2-1) + 1 - 0,69897$

$\log 0,002 = -3 + 0,30103 = \boxed{\bar{3},30103}$

$\log 0,03 = -1,52288 = -1 - 0,52288 =$

$(-1-1) + 1 - 0,52288 = -2 + 0,47712 =$

log 0,003 = $-2 + 0,47712 = \boxed{\bar{2},47712}$

Regra prática p/a achar o cologaritmo:

É só transformar log. negativo em preparado.

log 2 = 0,30103   colog 2 = −0,30103

transformando em preparado:

$\boxed{\bar{1},69897}$ subtrai-se de: 9 e 10.

log 2 = 0,30103

colog. 2 = $\bar{1},69897$

"Soma-se 1 positivo e troca-se o sinal e subtrai-se a mantissa de 9, com exceção do 1º à direita que subtrai-se de 10"

log 3 = 0,47712

colog 3 = $\bar{1},52288$

log 0,003 = $\bar{3},47712 = -3 + 0,47712$

(adiciona-se 1: −3+1 = −2 e troca-se o sinal = +2)

colog 0,003 = 2,52288

Operações com logaritmos

$\bar{3},5847 + 2,8010 + \bar{4},9897 =$

$$\begin{array}{r} {\scriptstyle +2} \\ \bar{3},5847 \\ +\ 2,8010 \\ \bar{4},9897 \\ \hline \bar{3},3754 \end{array}$$

Multiplicação

ⓐ logaritmo positivo

2,74483 × 3 = 8,23449

ⓑ logaritmo preparado: (separa-se)

$\bar{1},36821 \times 2 \Rightarrow (-1+0,36821) \times 2 = \boxed{-2+0,73642}$

$\bar{2},9321 \times 3 = (-2+0,9321) \times 3 \Rightarrow -6+2,79633 =$

$\boxed{\bar{4},79633}$

Divisão de logaritmos

a) logaritmo positivo

$3,74836 \div 2 = \boxed{1,87718}$

b) logaritmo preparado.

$\bar{6},32673 \div 3 = (-6+0,32673) \div 3 =$

$-2 + 10891 \Rightarrow \boxed{\bar{2},10891}$

c) quando a característica não é divisível pelo número (3÷5)

$\bar{3},42673 \div 5 \Rightarrow$

$(-3+0,42673) \div 5 \Rightarrow (-3-2+2+0,42673) \div 5 \Rightarrow$

$(-5+2,42673) \div 5 = \boxed{-\bar{1},48555}$

d) divisão de logaritmo por logaritmo

$\bar{3},4850 \quad \div \quad \bar{1},3010$

$-2,5150 \quad \div -0,6990 \cong \boxed{2,5}$

Uso das tábuas

de 1 a 10.000

log 235 = 2,37107

log 2476 = 3,39375

log 0,0726 = $\bar{2},86094$

log 12356 = 4,09188

$$1\left\langle\begin{array}{lcl} 1235 & \longrightarrow & 0,9167 \\ & & \underline{\quad 21} \\ 12356 & \longrightarrow & 0,9188 \\ 1236 & \longrightarrow & 0,9292 \end{array}\right\rangle 35$$

$\frac{1}{0.6} \quad \frac{35}{x} \quad \therefore x = 35 \times 0,6 = 2.$

log x = 2,59994

$$\begin{array}{rlcl} 6 & 59988 & \longrightarrow & 3980 \\ & & & \underline{\quad 06} \\ 11 & 59994 & & 3.986 \\ & 59999 & \longrightarrow & 3981 \end{array}\Big\rangle 1$$

$11 \longrightarrow 1 \qquad x = \frac{6}{11} = 0,6$

$6 \longrightarrow x$

$\boxed{x = 398,06}$

Cálculo de expressões

Calcular por log. a 5 casas decimais de:

$$x = \sqrt[4]{\frac{427 \times 37000}{341^3}}$$

① → logaritmar a expressão:

$$\log x = \frac{\log \frac{427 \times 37000}{341^3}}{4}$$

$$\log x = \frac{\log 427 + \log 37000 + 3\,\text{colog}\,341}{4}$$

② cálculos auxiliares:

log 427 = 2,63043

log 37000 = 4,58820

log 341 = 2,53275

colog 341 = $\bar{3},46725$

$-9,+1,40175 = \bar{8},40175$

3·colog 341 = $\bar{8},40175$

③ operações:

$$\log x = \frac{2,63043 + 4,56820 + \bar{8},40175}{4}$$

$$\log x = \frac{\overset{(-3)}{\bar{1}},\overset{+(3)}{600}38}{4} = \frac{(-4+3,60038)}{4} =$$

$\boxed{\bar{1},90010}$

④ $x =$ (consultar a táboa)

$\log x = \bar{1},90010$

1 $90010 \longrightarrow$ —

$90009 \longrightarrow 7945$

$90015 \longrightarrow 7946$ 1

6 — 1

1 — x

$x = \frac{1}{6} = 0,16$

$$\begin{array}{r} 7945 \\ 0,16 \\ \hline 794516 \end{array}$$

$\therefore\ x = 0,7945\overset{2}{\underline{1}}\cancel{6}$