$$\int_1^5 \sqrt{126-x^3}\,dx \qquad n=4$$

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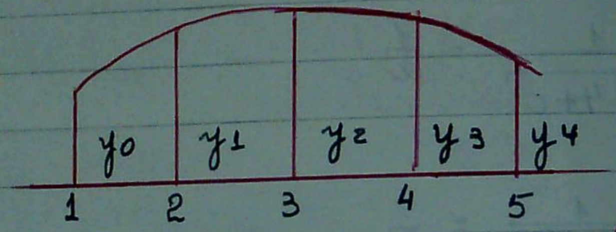
$$\Delta x = \frac{b-a}{N} = \frac{5-1}{4} = \frac{4}{4} = 1$$

$$y = \sqrt{126-x^3}$$



$\Delta x = 1$

$$y_0 = \sqrt{126-1} = \sqrt{125} = 11{,}1$$

$$y_1 = \sqrt{126-8} = \sqrt{118} = 10{,}67$$

$$y_2 = \sqrt{126-27} = \sqrt{99} = 9{,}96$$

$$y_3 = \sqrt{126-64} = \sqrt{62} = 7{,}80$$

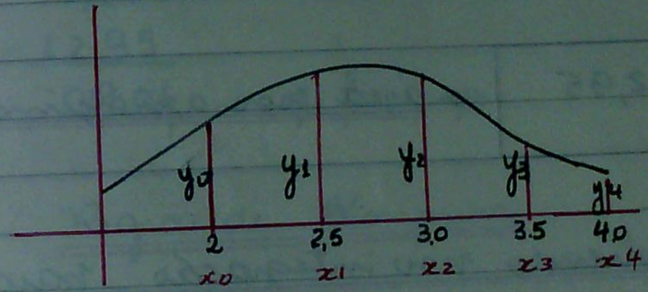
$$y_4 = \sqrt{126-125} = \sqrt{1} = 1$$

$$A = 1\,(5{,}55 + 10{,}67 + 9{,}96 + 7{,}80 + 0{,}5)$$

$$\boxed{A = 34{,}48}$$

$$A = \Delta x\left(\tfrac{1}{2}y_0 + y_1 + y_2 + y_3 + \tfrac{1}{2}y_4\right)$$

$$\int_2^4 x\sqrt{16-x^2}\,dx \qquad N=4$$



$$\Delta x = \frac{4-2}{4} = 0{,}5$$

$$y = x\sqrt{16-x^2}$$

$$y_0 = 2\sqrt{16-4} = 2\sqrt{12} = 4.\sqrt{3} = 4.\,1{,}732 = 6{,}928 = 7$$

$$y_1 = 2{,}5\sqrt{16-6{,}25} = 2{,}5\sqrt{9{,}75} = 2{,}5 \cdot 3{,}12 = 7.8$$

$y_2 = 3\sqrt{16-9} = 3\sqrt{7} = 3 . 2,6 = 7,8$

$y_3 = 3,5\sqrt{16-12,25} = 3,5 . \sqrt{3,75} = 3,5 . 1,94 = \boxed{6,79}$

6,80

$y_4 = 4\sqrt{16-16} = \boxed{0}$

½ de 7

$A = 0,5\left(3,5 + 7,8 + 7,8 + 6,8\right) =$

$A = 25,9 \times 0,5$

$\boxed{A = 12,95}$ → área por aproximação

resolvendo por integração para verificar o erro.

$$-\frac{1}{2}\int_2^4 (16-x^2)^{1/2}\, 2x\,dx$$

$$A = -\frac{1}{\cancel{2}}\left[\frac{(16-x^2)^{3/2}}{3/\cancel{2}}\right]_2^4 =$$

$$A = -\left[\frac{\sqrt[2]{(16-x^2)^3}}{3}\right]_2^4 =$$

$$A = -\left[\frac{\overbrace{\sqrt{(16-16)^3}}^{=0}}{3}\right] + \left[\frac{\sqrt{(16-4)^3}}{3}\right]$$

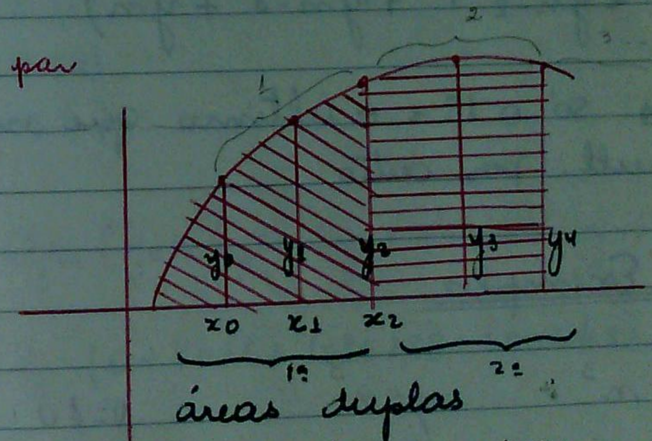
$$A = \frac{\sqrt{12^3}}{3} = \frac{\sqrt{2^6 3^3}}{3} = \frac{2^3 . \cancel{3}\sqrt{3}}{\cancel{3}} =$$

$A = 8 . 1,732 = \boxed{13,856}$ → área exata.

$$\begin{array}{r} 13,856 \\ -\ 12,95 \\ \hline 0,906 \end{array}$$ → diferença

Regra de Simpson

n → par



áreas duplas

n → deve ser par

fórmula:

1º) $\frac{\Delta x}{3}(y_0 + 4y_1 + y_2)$

2º) $\frac{\Delta x}{3}(y_2 + 4y^3 + y^4)$

fórmula geral:

$$\frac{\Delta x}{3}(y_{n-2} + 4y_{n-1} + y_n)$$

$$A = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y^2 + 4y^3 + 2y_4 \ldots$$

$$+ 2y_{n-2} + 4y_{n-1} + y_n)$$

obs: só o 1º e o último que não é mult. por outro.

Exemplo

$\int_0^{10} x^3$ $\qquad y = x^3$ $\qquad N = 10$

$\Delta x = \frac{10-0}{10} = 1$ $\qquad$ O $\underline{x}$ varia de 0 a 10.

$y_0, y_1, \ldots y_{10}$

$y_0 = 0$ $(x)^3$

$y_1 = 1$ $(1)^3$

$x = (0, 1, 2, \ldots 10) \rightarrow$ variação do x

$y_2 = 8 \cdot (2)$ $(2)^3$ $\qquad$ os pares $\times$ 2

$y_3 = 27 \cdot (4)$ $(3)^3$ $\qquad$ os ímpares $\times$ 4

$y_4 = 64 \cdot (2)$ $(4)^3$

$y_5 = 125 \cdot (4)$ $(5)^3$

$y_6 = 216 \cdot (2)$ $(6)^3$

$y_7 = 343 \cdot (4)$ $(7)^3$

$y_8 = 512 \cdot (2)$ $(8)^3$

$y_9 = 729 \cdot (4)$ $(9)^3$

$y_{10} = 1000 \cdot (\;)$ $(10)^3$

$$A = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 \ldots + y_n)$$

$$A = \frac{1}{3}(0 + 4 + 16 + 108 + 128 + 500 + 432 + 1372 +$$

$$1024 + 2916 + 1000) =$$

$A = \frac{1}{3}(7.500) =$

$\boxed{A = 2500}$ → (exata)

por integração:

$\int_0^{10} x^3 dx =$

$\left[\frac{x^4}{4}\right]_0^{10} = \frac{10^4}{4} - \frac{0}{4} = \frac{\cancel{10000}^{5000}}{\cancel{4}_2} = \boxed{2.500}$

$\int_0^8 3x^2$ $n = 8$

$y = 3x^2$

$\Delta x = \frac{8-0}{8} = 1$

$\begin{cases} x = (0, 1, 2 \dots 8) \\ y_0, y_1 \dots y_8 \end{cases}$ → variação do y.

$y_0 = 0$
$y_1 = 3.1 = 3 \cdot (4)$
$y_2 = 3.2^2 = 12 \cdot (2)$
$y_3 = 3.3^2 = 27 \cdot (4)$
$y_4 = 3.4^2 = 48\ (2)$
$y_5 = 3.5^2 = 75\ (4)$
$y_6 = 3.6^2 = 108\ (2)$
$y_7 = 3.7^2 = 147\ (4)$
$y_8 = 3.8^2 = 192$

$A = \frac{1}{3}(0 + 12 + 24 + 108 + 96 + 300$
$+ 216 + 588 + 192) =$

$A = \frac{1}{3}(1536)$

$\boxed{A = 512}$

Obs: y_0 e y_8 → não mult. por nada.

Calculando por integração

$\int_0^8 3x^2 dx =$

$3\left[\frac{x^3}{3}\right]_0^8 = \left[x^3\right]_0^8 \to 512 - 0 = \boxed{512}$

$\int_2^4 x\sqrt{16 - x^2}\, dx$

$y = x\sqrt{16 - x^2}$
$n = 4$
var. do x: 0 a 4

$\Delta x = \frac{4-2}{4} = \boxed{0,5}$

do exercício anterior (Trapézio)

$2 \cdot \sqrt{16-(2)^2} = 2\sqrt{12} = 4\sqrt{3} = 4 \times 1{,}732 = 6{,}928$ (?)

$y_0 = 7$

$y_1 = 7.8$ (4)

$y_2 = 7.8$ (2)

$y_3 = 6.8$ (4)

$y_4 = 0$

$$A = \frac{0{,}5}{3}(7 + 31{,}2 + 15{,}6 + 27{,}2 + 0)$$

$$A = \frac{0.5}{\cancel{3}}(\cancel{81}^{27}) =$$

$$A = 0{,}5 \times 27$$

$$\boxed{A = 13{,}5}$$

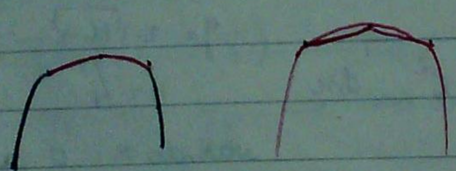
$A_{simpson} = 13{,}5$ (+ aprox)

$A_{p/ integral} = 13{,}85$ (exata)

$A_{\text{trapézio}} = 12{,}95$ (menos apr.)

Conclusão:

A regra de Simpson aproxima mais do que a do trapézio, isto porque ligando-se 3 pontos de uma parábola por um arco é mais exato do que ligar por meio de uma linha reta.



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Integrais Impróprias

Limites Infinitos

$$\int_a^\infty f(x)\,dx =$$

$$\lim_{b\to\infty}\int_a^b f(x)\,dx.$$

$$\int_{-\infty}^b \varphi(x)\,dx = \lim_{a\to-\infty}\int_a^b \varphi(x)\,dx$$

① $\displaystyle\int_1^\infty \frac{dx}{x^2}$ → transforma-se em limite:

$$\lim_{b\to\infty}\int_1^b \frac{dx}{x} = \lim_{b\to\infty}\int_1^b x^{-2}\,dx =$$

$$\lim_{b\to\infty}\left[\frac{x^{-1}}{-1}\right]_1^b = \lim_{b\to\infty}\left[-\frac{1}{x}\right]_1^b =$$

$$\lim_{b\to\infty}\left[-\frac{1}{b} + \frac{1}{1}\right] = \boxed{1}$$

② $\int_0^{+\infty} \frac{8a^3\,dx}{x^2+4a^2}$ →

1º) transforma-se em limite

2º) aplica-se a integral.

$$\lim_{b\to\infty} \int_0^b \frac{8a^3\,dx}{x^2+4a^2} =$$

$$\lim_{b\to\infty} \left[8a^3 \int_0^b \frac{dx}{x^2+4a^2}\right] =$$

pela fórmula: $\int \frac{dv}{v^2+a^2}$

$$\lim_{b\to\infty} \left[\overset{4}{\cancel{8}}\, \overset{2}{\cancel{a^3}} \left(\frac{1}{\cancel{2a}} \cdot \operatorname{arctg} \frac{x}{2a}\right)\right]_0^b =$$

$$\lim_{b\to\infty} \left[4a^2 \operatorname{arctg} \frac{x}{2a}\right]_0^b =$$

$$\lim_{b\to\infty} \left[4a^2 \operatorname{arctg} \frac{b}{2a} - 4a^2 \operatorname{arctg} \overset{=0}{\frac{0}{2a}}\right]$$

$$\overset{2}{\cancel{4}}a^2 \cdot \frac{\pi}{\cancel{2}} = \boxed{2a^2\pi}$$

③ $\int_0^{\infty} \frac{dx}{x^2+1} =$

cálculos e fórmulas auxiliares:

$$\int \frac{dv}{v^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{v}{a} + C$$

$$tg \to \infty = \frac{\pi}{2}$$

$$tg\, 0 = 0$$

resolução

$$\lim_{b\to\infty} \int_0^b \frac{dx}{x^2+1} = \lim_{b\to\infty} \left[\frac{1}{1} \operatorname{arctg} \frac{x}{1}\right]_0^b =$$

$$\lim_{b\to\infty} \left[\cancel{1} \operatorname{arctg} \frac{b}{\cancel{1}} - \cancel{1} \operatorname{arctg} \frac{0}{\cancel{1}}\right] = \boxed{\frac{\pi}{2}}$$

④ $\int_1^{+\infty} \frac{dx}{x\sqrt{2x^2-1}}$ integração por parte

$$\begin{cases} u = \sqrt{2}\,.\,x \\ x = \dfrac{u}{\sqrt{2}} \end{cases} \qquad dx = \frac{du}{\sqrt{2}}$$

$$\int_1^{\infty} \frac{\frac{du}{\sqrt{2}}}{\frac{u}{\sqrt{2}}\sqrt{u^2-a^2}} =$$

$$\int_1^{\infty} \frac{du}{u\sqrt{u^2-a^2}} =$$

$u = a \sec z$

$du = a \sec z \, tg\, z$

$\sec z = \dfrac{u}{a}$

$z = arc.\sec \dfrac{u}{a}$

$$\int_1^{\infty} \frac{a \cancel{\sec z}\, \cancel{tg}\, z\, dz}{a \cancel{\sec z}\; a\cancel{tg}\, z} =$$

$$\int_1^{\infty} \frac{dz}{a} =$$

$$\lim_{b\to\infty} \int_1^{b} \frac{dz}{a} = \lim_{b\to\infty} \left[\frac{z}{a}\right]_1^b =$$

$$\lim_{b\to\infty} \left[\frac{arc\,\sec\,\frac{u}{a}}{a}\right]_1^b =$$

$$\lim_{b\to\infty} \left[arc\,\sec\,\sqrt{2}\,.\,x\right]_1^b =$$

$$\lim_{b\to\infty} \left[arc\,\sec\,\sqrt{2}\,b - arc\,\sec\,\sqrt{2}\,.\right] =$$

$$\frac{\pi}{2} - \frac{\pi}{4} = \frac{2\pi}{4} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$$

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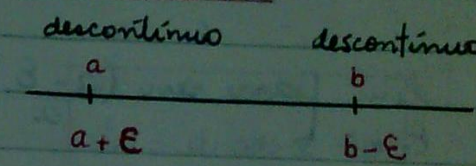
Integrais Impróprias

$$\int_a^b f(x)\,dx$$



$$\lim_{\varepsilon\to 0} \int_{a+\varepsilon}^{b} f(x)\,dx$$

$$\int_a^b f(x)\,dx \qquad \int f(x) \text{ é descontínua}$$

para $x = b$

$$\lim_{\varepsilon\to 0} \int_a^{b-\varepsilon} f(x)\,dx$$

* $\int_0^a \frac{dx}{\sqrt{a^2-x^2}}$

descontínua no ponto $x = a$ $\rightarrow$ substituindo o x por a dá $\frac{dx}{\sqrt{a^2-a^2}} = \frac{dx}{0} = \frac{1}{0}$ ind.

$$\lim_{\varepsilon \to 0} \int_0^{a-\varepsilon} \frac{dx}{\sqrt{a^2-x^2}} =$$

$$\lim_{\varepsilon \to 0} \left[\text{arc sen } \frac{x}{a}\right]_0^{a-\varepsilon}$$

$$\lim_{\varepsilon \to 0} \left[\text{arc sen } \left(\frac{a-\varepsilon}{a}\right) - \text{arc sen } \frac{0}{a}\right] =$$

$$\lim_{\varepsilon \to 0} \left[\text{arc sen } \left(1 - \frac{\varepsilon}{a}\right)\right] = \boxed{\frac{\pi}{2}}$$

$\int_0^1 \frac{dx}{x^2}$ $\qquad x = 0 \rightarrow$ função descontínua

$\frac{dx}{x^2} = \frac{dx}{1^2} = \frac{1}{1}$ def lim.

$$\lim_{\varepsilon \to 0} \int_{0+\varepsilon}^{1} \frac{dx}{x^2} =$$

$\frac{dx}{x^2} = \frac{1}{0^2} = \frac{1}{0}$ ind $(0+\varepsilon$

$$\lim_{\varepsilon \to 0} \left[\frac{x^{-1}}{-1}\right]_{0+\varepsilon}^{1} =$$

$$\lim_{\varepsilon \to 0} \left[-\frac{1}{x}\right]_{0+\varepsilon}^{1} =$$

$$\lim_{\varepsilon \to 0} \left[-\frac{1}{1} + \frac{1}{0+\varepsilon}\right] = \boxed{\infty}$$

$\int_1^2 \frac{dx}{x^2\sqrt{4-x^2}}$

$u = x$

$dx = du$

$a = 2$

$\boxed{u = a \text{ sen } z \quad du = a \cos z \, dz}$

$x = 2$

$y = \infty$

$$\int_1^2 \frac{du}{u^2\sqrt{a^2-u^2}} = \int_1^2 \frac{a \cos z \, dz}{a^2 \text{ sen}^2 z}$$

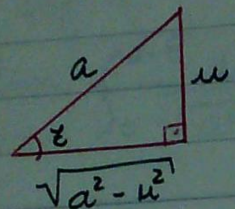
$$\int_1^2 \frac{a \cos z \, dz}{a^2 \text{ sen}^2 z \sqrt{a^2 - a^2 \text{ sen}^2 z}} =$$

$\sqrt{a^2(1-\text{sen}^2 z)} = a \cos z$

$$\int_1^2 \frac{\cancel{a} \cancel{\cos z} \, dz}{a^2 \text{ sen}^2 z \, \cancel{a} \cancel{\cos z}} =$$

$$\int_1^2 \frac{dz}{a^2 \operatorname{sen}^2 z} = \lim_{\varepsilon \to 0} \frac{1}{a^2} \int_1^{2-\varepsilon} \operatorname{cossec}^2 z \, dz =$$

$$\lim_{\varepsilon \to 0} \left[-\frac{1}{a^2} \operatorname{ctg} z \right]_1^{2-\varepsilon} =$$



$$\lim_{\varepsilon \to 0} \left[-\frac{1}{a^2} \frac{\sqrt{a^2 - u^2}}{u} \right]_1^{2-\varepsilon} =$$

$$\lim_{\varepsilon \to 0} \left[-\frac{1}{4} \frac{\sqrt{4 - x^2}}{x} \right]_1^{2-\varepsilon} =$$

$$\lim_{\varepsilon \to 0} \left[-\frac{1}{4} \frac{\overset{0}{\sqrt{4-(2-\varepsilon)^2}}}{2-\varepsilon} + \frac{1}{4} \frac{\sqrt{4-1}}{1} \right] =$$

$$\boxed{\frac{\sqrt{3}}{4}}$$

$$\int_0^4 \frac{dx}{\sqrt{4-x}}$$

$x = 4 \rightarrow$ indet. no pt 4

$$\lim_{4 \to 0} \int_0^{4-\varepsilon} \frac{dx}{\sqrt{4-x}} =$$

$\frac{dx}{\sqrt{4-x}} = \frac{1}{\sqrt{4-4}} = \frac{1}{0} \rightarrow$ ind.

$$\lim_{\varepsilon \to 0} - \int_0^{4-\varepsilon} (4-x)^{-1/2} \, -dx$$

$$\lim_{\varepsilon \to 0} \left[\frac{-(4-x)^{1/2}}{1/2} \right]_0^{4-\varepsilon} =$$

$$\lim_{\varepsilon \to 0} \left[-2 \sqrt{4-x} \right]_0^{4-\varepsilon} =$$

$$\lim_{\varepsilon \to 0} \left[\underbrace{-2\sqrt{4-(4-\varepsilon)}}_{0} + \underbrace{2\sqrt{4-0}}_{+2 \cdot 2 = 4} \right] = \boxed{4}$$

$$\int_0^1 \frac{dx}{\sqrt{x}}$$

$y = \frac{1}{\sqrt{x}} = \frac{1}{0}$

é ind. no pt. $0 \rightarrow 0+\varepsilon$

$$\lim_{\varepsilon \to 0} \int_{0+\varepsilon}^1 \frac{dx}{\sqrt{x}} =$$

$$\lim_{\varepsilon \to 0} \int_{0+\varepsilon}^1 (x)^{-1/2} \, dx =$$

$$\lim_{\varepsilon \to 0} \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_{0+\varepsilon}^1 = \lim_{\varepsilon \to 0} \left[2\sqrt{x} \right]_{0+\varepsilon}^1 =$$

$$\lim_{\varepsilon \to 0} \left[2\sqrt{1} - 2\underbrace{\sqrt{0+\varepsilon}}_{0} \right] = \boxed{2}$$

$$\int_0^{2a} \frac{dx}{(x-a)^2} =$$

$$\frac{dx}{(x-a)^2} = \frac{1}{(2a-a)^2}$$

$$\frac{dx}{(x-a)^2} = \frac{1}{(0-a)^2} =$$

$$\lim_{\varepsilon \to 0} \int_0^{a-\varepsilon} (x-a)^{-2}\,dx + \lim_{\varepsilon \to 0} \int_{a+\varepsilon}^{2a} (x-a)^{-2}\,dx$$

$$\lim_{\varepsilon \to 0} \int_0^{a-\varepsilon} (x-a)^{-2}\,dx + \lim_{\varepsilon \to 0} \left[\frac{(x-a)^{-1}}{-1} \right]^{-2} dx$$

$$\lim_{\varepsilon \to 0} \left[\frac{(x-a)^{-1}}{-1} \right]_0^{a} + \lim_{\varepsilon \to 0} \left[\frac{(x-a)^{-1}}{-1} \right]_{a+\varepsilon}^{2a}$$

$$\lim_{\varepsilon \to 0} \left[\frac{-1}{a-\varepsilon-a} + \frac{1}{0-a} \right] + \lim \left[-\frac{1}{2a-a} + \frac{1}{a+\varepsilon-a} \right] =$$

$$\lim_{\varepsilon \to 0} \left[\frac{1}{\varepsilon} - \frac{1}{a} \right] + \lim_{\varepsilon \to 0} \left[-\frac{1}{a} + \frac{1}{\varepsilon} \right] =$$

$$\lim_{\varepsilon \to 0} \left[0 - \frac{1}{a} \right] + \lim_{\varepsilon \to 0} \left[-\frac{1}{a} + 0 \right] =$$

$$-\frac{1}{a} - \frac{1}{a} = -\frac{2}{a} \quad \text{(absurdo)}$$

Obs: neste caso os limites não são finitos e a integral não tem sentido.

$\int_a^{2a} \frac{x^2\,dx}{\sqrt{x^2-a^2}}$ $\quad y = \frac{x^2}{\sqrt{x^2-a^2}}$ substitue o x por a pra ver se é descontínua

$a^2 - a^2 = 0$

27/4/73

$du = dx$

$u = x$

$a+\varepsilon \quad 2a$ em a função desc.

$\int_a^{2a} \frac{u^2\,du}{\sqrt{u^2-a^2}}$ $\quad \begin{cases} u = a \sec z \\ du = a \sec z . tg z . dz \end{cases}$

$\int_a^{2a} \frac{a^2 \sec^2 z . a \sec z \; tg z \; dz}{\sqrt{a^2 \sec^2 z - a^2}} \rightarrow \sqrt{a^2(\sec^2 - 1)} = \sqrt{a^2 tg^2 z} = a\,tg z$

$\int_a^{2a} \frac{a^2 \sec^2 z . \cancel{a} \sec z . \cancel{tg z} . dz}{\cancel{a}\,\cancel{tg z}}$

$a^2 \int_a^{2a} \sec^3 z \; dz$ assim não é possível resolver.

Portanto

$a^2 \int_a^{2a} \sec^2 z . \sec z \; dz$

$\int u\,dv = vu - \int v\,du$ (por partes)

$\begin{cases} v = tg z & \text{integra} \\ dv = \sec^2 z \; dz \end{cases}$ $\quad \begin{cases} u = \sec z & \text{deriva} \\ du = \sec z \; tg z \; dz \end{cases}$

$\lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z \, dz =$ $\quad y = \frac{\sec^2}{\sqrt{x^2-a^2}}$ $\quad a^2 - a^2 = 0 \quad \frac{a^2}{0}$ = desc.

$\lim_{\varepsilon \to 0} a^2 \left[tg z . \sec z \right]_{a+\varepsilon}^{2a} - \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} tg z .$

$tg z . \sec z . dz =$

$- \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} (\sec^2 z - 1) . \sec z =$

$- \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z + \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec z \, dz$

$\lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z \, dz =$

$\lim_{\varepsilon \to 0} a^2 \left[tg z . \sec z + \ln(\sec z + tg z) \right]_{a+\varepsilon}^{2a}$

$- \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z \, dz$

$$\int_a^{2a} \frac{x^2\,dx}{\sqrt{x^2-a^2}}$$

$$y = \frac{x^2}{\sqrt{x^2-a^2}}$$ substitue o x por a pla ver se é descontínua

$a^2 - a^2 = 0$

27/4/73

$du = dx$

$u = x$

em a função desc.

$$\int_a^{2a} \frac{u^2 \cdot du}{\sqrt{u^2-a^2}} \qquad \begin{cases} u = a \sec z \\ du = a \sec z . tg z . dz \end{cases}$$

$$\int_a^{2a} \frac{a^2 \sec^2 z . a \sec z \; tg z \; dz}{\sqrt{a^2 \sec^2 z - a^2}} \rightarrow \sqrt{a^2(\sec^2-1)} = \sqrt{a^2 tg^2 z} = a \, tg z$$

$$\int_a^{2a} \frac{a^2 \sec^2 z . \cancel{a} \sec z . \cancel{tg z} . dz}{\cancel{a} \cancel{tg z}}$$

$$a^2 \int_a^{2a} \sec^3 z \; dz$$ assim não é possível resolver.

Portanto

$$a^2 \int_a^{2a} \sec^2 z . \sec z \; dz$$

$\int u\,dv = vu - \int v\,du$ (por partes)

$\begin{cases} v = tg z \text{ integra} \\ dv = \sec^2 z \; dz \end{cases} \qquad \begin{cases} u = \sec z \text{ deriva} \\ du = \sec z \; tg z \; dz \end{cases}$

no sec.

$$\lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z \, dz =$$

$$\lim_{\varepsilon \to 0} a^2 \left[tg z . \sec z \right]_{a+\varepsilon}^{2a} - \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} tg z .$$

$tg z . \sec z . dz =$

$$- \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} (\sec^2 z - 1) . \sec z =$$

$$- \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z + \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec z \, dz$$

$$\lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z \, dz =$$

$$\lim_{\varepsilon \to 0} a^2 \left[tg z . \sec z + \ln(\sec z + tg z) \right]_{a+\varepsilon}^{2a} -$$

$$- \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z \, dz$$

$$\int_a^{2a} \frac{x^2\,dx}{\sqrt{x^2-a^2}}$$

$$y = \frac{x^2}{\sqrt{x^2-a^2}}$$ substitue o x por a pla vê-se é descontínua

$a^2 - a^2 = 0$

em a função desc.

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$du = dx$

$u = x$

$$\int_a^{2a} \frac{u^2\,du}{\sqrt{u^2-a^2}} \qquad \begin{cases} u = a \sec z \\ du = a \sec z . tg z . dz \end{cases}$$

$$\int_a^{2a} \frac{a^2 \sec^2 z . a \sec z \; tg z \; dz}{\sqrt{a^2 \sec^2 z - a^2}} \rightarrow \sqrt{a^2(\sec^2 - 1)} = \sqrt{a^2 tg^2 z} = a\,tg z$$

$$\int_a^{2a} \frac{a^2 \sec^2 z . \cancel{a} \sec z . \cancel{tg z} . dz}{\cancel{a}\,\cancel{tg z}}$$

$a^2 \int_a^{2a} \sec^3 z \; dz$ assim não é possível resolver.

Portanto:

$a^2 \int_a^{2a} \sec^2 z . \sec z \; dz$

$\int u\,dv = vu - \int v\,du$ (por partes)

$\begin{cases} v = tg z \\ dv = \sec^2 z\,dz \end{cases}$ integra $\qquad \begin{cases} u = \sec z \\ du = \sec z \; tg z \; dz \end{cases}$ deriva

$$\lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z\,dz =$$

$$\lim_{\varepsilon \to 0} a^2 \Big[tg z . \sec z \Big]_{a+\varepsilon}^{2a} - \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon} tg z .$$

$tg z . \sec z . dz =$

$$- \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} (\sec^2 z - 1) . \sec z =$$

$$- \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z + \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec z\,dz$$

$$\lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z\,dz =$$

$$\lim_{\varepsilon \to 0} a^2 \Big[tg z . \sec z + \ln(\sec z + tg z) \Big]_{a+\varepsilon}^{2a}$$

$$- \lim_{\varepsilon \to 0} a^2 \int_{a+\varepsilon}^{2a} \sec^3 z\,dz$$

$$-\lim_{\varepsilon\to 0} a^2\int_{a+\varepsilon}^{2a} (\sec^2 z-1).\sec z =$$

$$-\lim_{\varepsilon\to 0} a^2\int_{a+\varepsilon}^{2a} \sec^3 z + \lim_{\varepsilon\to 0} a^2\int_{a+\varepsilon}^{2a} \sec z\, dz =$$

$$\lim_{\varepsilon\to 0} a^2\int_{a+\varepsilon}^{2a} \sec^3 z\, dz =$$

$$\lim_{\varepsilon\to 0} a^2\Big[\operatorname{tg} z.\sec z + \ln(\sec z+\operatorname{tg} z)\Big]_{a+\varepsilon}^{2a} -$$

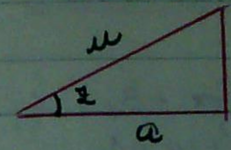
$$-\lim_{\varepsilon\to 0} a^2\int_{a+\varepsilon}^{2a} \sec^3 z\, dz =$$

$$2.\lim_{\varepsilon\to 0} a^2\int_{a+\varepsilon}^{2a} \sec^3 z\, dz =$$

$$\lim_{\varepsilon\to 0} a^2\left[\frac{\operatorname{tg} z.\sec z + \ln(\sec z+\operatorname{tg} z)}{2}\right]_{a+\varepsilon}^{2a} =$$

$$\lim_{\varepsilon\to 0} a^2\left[\frac{\frac{\sqrt{u^2-a^2}}{a}\cdot\frac{u}{a} + \ln\left(\frac{u}{a}+\frac{\sqrt{u^2-a^2}}{a}\right)}{2}\right]_{a+\varepsilon}^{2a}$$

$$\boxed{\operatorname{sen} z = \frac{u}{a}} \rightarrow \cos z = \frac{a}{u}$$



$$\sqrt{u^2-a^2}$$

ε→0 anula

anula

$$\lim_{\varepsilon\to 0} a^2\left[\frac{\frac{x\sqrt{x^2-a^2}}{a^2} + \ln\left(\frac{x}{a}+\frac{\sqrt{x^2-a^2}}{a}\right)}{2}\right]_{a+\varepsilon}^{2a} =$$

$$\frac{a^2}{2}\left[-\ln 1^{0} + \frac{2a\sqrt{4a^2-a^2}}{a^2} + \ln\left(\frac{2a}{a}+\frac{\sqrt{4a^2-a^2}}{a}\right)\right] =$$

$$\frac{a^2}{2}\left[\frac{2a^2}{a^2}\sqrt{3} + \ln(2+\sqrt{3})\right] =$$

2 + 1,732 = 3,732

$$\boxed{a^2\sqrt{3} + \frac{a^2}{2}\ln(3,732)}$$

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Integração por Artifícios

① Por partes

② Decomposição das funções racionais

③ Substituição conveniente da variável.

Exemplos:

① $\dfrac{x^4+3x^3}{x^2+2x+1} = x^2+x-3+\dfrac{5x+3}{x^2+2x+1}$ (resultado da divisão)

Este caso é o da divisão pois o numerador é maior que o denominador.

② Numerador menor que o denom.

Não dá pra aplicar integração imediata. Exemplo:

$$\int \frac{(2x+3)\,dx}{x^3+x^2-2x}$$

A função será: $\dfrac{2x+3}{x^3+x^2-2x}$ que é uma função racional (não pode aparecer radical no denominador)

O artifício é: decompor o denominador em fatores do 1º grau.

Fatorando:

$x^3+x^2-2x = x\,(x^2+x-2)$

raízes: $\begin{cases} x' = -2 \\ x'' = 1 \end{cases}$

fatoração: $\boxed{x\,(x-1)\,(x+2)}$

$$\frac{2x+3}{x(x-1)(x+2)} =$$

decompondo em somas: $\dfrac{A}{x} + \dfrac{B}{x-1} + \dfrac{C}{x+2}$

desenvolvendo: (acha-se o m.m.c)

$2x+3 = (x-1)(x+2)A + x(x+2)B + x(x-1)C$

$2x+3 = (x^2+x-2)A + (x^2+2x)B + (x^2-x)C =$

$2x+3 = Ax^2+Ax-2A+Bx^2+2Bx+Cx^2-Cx =$

$0x^2+2x+3 = (A+B+C)x^2+(A+2B-C).x-2A =$

Isto é uma identidade.

$$\begin{cases} A+B+C = 0 \\ A+2B-C = 2 \\ -2A = 3 \end{cases} \quad \therefore \quad \boxed{A = -\frac{3}{2}}$$

$$\begin{array}{l} A+B+C=0 \\ A+2B-C=2 \\ \hline 2A+3B=2 \end{array} \quad \therefore \quad 2\cdot\left(-\frac{3}{2}\right)+3B=2$$

$$-3+3B=2$$

$$3B=5 \quad \therefore \quad \boxed{B=\frac{5}{3}}$$

$$A+B+C=0$$

$$C=-A-B$$

$$C=\frac{3}{2}-\frac{5}{3} \quad \therefore \quad C=\frac{9-10}{6}=-\frac{1}{6} \quad \therefore$$

$$\boxed{C=-\frac{1}{6}}$$

Volta no ponto em que separou a soma:

$$\frac{2x+3}{x(x-1)(x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$$

$$\frac{2x+3}{x(x-1)(x+2)} = \frac{-\frac{3}{2}}{x} + \frac{\frac{5}{3}}{x-1} + \frac{(-)\frac{1}{6}}{x+2} \Rightarrow$$

$$\frac{2x+3}{x(x-1)(x+2)} = -\frac{3}{2x} + \frac{5}{3(x-1)} - \frac{1}{6(x+2)}$$

portanto:

$$\int \frac{(2x+3)\,dx}{x(x-1)(x+2)} = -\frac{3}{2}\int\frac{dx}{x} + \frac{5}{3}\int\frac{dx}{x-1} - \frac{1}{6}\int\frac{dx}{x+2} =$$

$$= -\frac{3}{2}\ln x + \frac{5}{3}\ln(x-1) - \frac{1}{6}\ln(x+2) + C$$

ou

colocando em evidência:

$$= \boxed{\ln \frac{(x-1)^{5/3}}{x^{3/2}\cdot(x+2)^{1/6}}}$$

---

① $$\int \frac{(4x-2)\,dx}{x^3-x^2-2x}$$

função: $\dfrac{4x-2}{x^3-x^2-2x}$

fatorando: $x(x-x')(x-x'')$ ou $x(x'-a)\cdot(x''-a)$

$x^3 - x^2 - 2x = x(x^2 - x - 2)$

raízes: $\begin{cases} x' = 2 \\ x'' = -1 \end{cases}$

fatoração: $\boxed{x\,(x-2)\,(x+1)}$

$\dfrac{4x-2}{x(x-2)(x+1)} = \dfrac{A}{x} + \dfrac{B}{x-2} + \dfrac{C}{x+1}$ m.m.c

$4x-2 = A(x-2)\cdot(x+1) + B\,(x)\cdot(x+1) + C\cdot(x)\cdot(x-2)$

$4x-2 = A\,(x^2-x-2) + B\,(x^2+x) + C(x^2 \cancel{-2x})$

$4x-2 = Ax^2 - Ax - 2A + Bx^2 + Bx + Cx^2 - 2Cx$

$0x^2 + 4x - 2 = (A+B+C)x^2 + (-A+B-2C)\,x - 2A =$

$A + B + C = 0$

$-A + B - 2C = 4$

$-2A = -2 \quad \therefore \quad \boxed{A = 1}$

$B = -A - C$

$B = -1 + 2$

$\boxed{B = 1}$

$\cancel{A} + B + C = 0$

$A - B + 2C = -4$

$2A \quad + 3C = -4$

$3C = -4 - 2$

$C = -\dfrac{6}{3} \quad \therefore \quad \boxed{C = -2}$

$\dfrac{4x-2}{x^3-x^2-2x} = \dfrac{1}{x} + \dfrac{1}{x-2} - \dfrac{2}{x+1}$

$\displaystyle\int \frac{(4x-2)\,dx}{x^3-x^2-2x} = \int\frac{dx}{x} + \int\frac{dx}{x-2} - 2\int\frac{dx}{x+1}$

divisão

$= \ln x + \ln(x-2) - 2\ln(x+1) + c$

vai p/a expoente

$\boxed{= \ln \dfrac{x\cdot(x-2)}{\cancel{2}\,(x+1)^2} + c}$

---

② $\displaystyle\int \frac{(5x^2-3)\,dx}{x^3-x}$

função: $\dfrac{5x^2-3}{x^3-x}$

fatoração: $x\,(x^2-1)$

$x^2 - 1 = 0$

$x' = +1$

$x'' = -1$

$x\,(x'-a)\,(x''-a)$

$x\,(x-1)\,(x+1)$

$$\frac{5x^2-3}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$5x^2-3 = (x-1)(x+1)A + x(x+1)B + x(x-1)C$$
$$5x^2-3 = (x^2-1)A + (x^2+x)B + (x^2-x)C$$
$$5x^2-3 = Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx =$$
$$5x^2-3 = (A+B+C)x^2 + (B-C)x + (-A)$$

$$\begin{aligned} A+B+C &= 5 \\ B-C &= 0 \\ -A &= -3 \end{aligned} \quad \therefore \boxed{A=3}$$

$$\begin{aligned} A+B+C&=5 \\ B-C&=0 \\ \hline A+2B\ /&=5 \end{aligned}$$

$$3+2B=5$$
$$2B=5-3$$
$$2B=2$$
$$B=\frac{2}{2} \quad \therefore \boxed{B=1}$$

$$B-C=0$$
$$1-C=0$$
$$-C=-1 \quad \therefore \boxed{C=1}$$

$$\frac{5x^2-3}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$\frac{5x^2-3}{x(x-1)(x+1)} = \frac{3}{x} + \frac{1}{x-1} + \frac{1}{x+1}$$

$$\int \frac{(5x^2-3)\,dx}{x(x-1)(x+1)} = 3\int \frac{dx}{x} + \int \frac{dx}{x-1} + \int \frac{dx}{x+1} =$$

$$3\ln x + \ln(x-1) + \ln(x+1)$$

$$\ln\left[x^3 \cdot (x-1) \cdot (x+1)\right] + C$$

$$\boxed{\ln x^3 (x^2-1) + C}$$

$$\int \frac{(3x^2+5x)\,dx}{(x-1)(x+1)^2} =$$

$$\frac{3x^2+5x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)^2} + \frac{C}{x+1}$$

vai dar os expoentes

$$\begin{cases} x' = 1 \\ x'' = -1 \\ x''' = -1 \end{cases}$$

Caso especial em que 2 raízes são iguais.

$3x^2+5x = A(x+1)^2 + B(x-1) + C(x-1)(x+1)$

$3x^2+5x = Ax^2 + 2Ax + A + Bx - B + Cx^2 - C$

$3x^2+5x = (A+C)x^2 + (2A+B)x - B - C + A$

$$\begin{cases} A+C=3 \\ 2A+B=5 \\ A-B-C=0 \end{cases} \qquad \begin{aligned} -A-C &= -3 \\ A-C-B &= 0 \\ \hline -2C-B &= -3 \end{aligned}$$

$$\begin{aligned} -2A-2C &= -6 \\ 2A+B &= 5 \\ \hline B-2C &= -1 \end{aligned} \qquad \begin{aligned} -B-2C &= -3 \\ B-2C &= -1 \\ \hline -4C &= -4 \end{aligned}$$

$\therefore \boxed{C=1}$

$\boxed{A=2}$ $\boxed{B=1}$

$$\frac{3x^2+5x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)^2} + \frac{C}{x+1}$$

$$\frac{3x^2+5x}{(x-1)(x+1)^2} = \frac{2}{x-1} + \frac{1}{(x+1)^2} + \frac{1}{x+1}$$

$$\int \frac{(3x^2+5x)\,dx}{(x-1)(x+1)^2} = 2\int\frac{dx}{x-1} + \int\frac{dx}{(x+1)^2} + \int\frac{dx}{x+1}$$

$$= 2\ln(x-1) - \frac{1}{x+1} + \ln(x+1) + C$$

$$= \boxed{\ln(x-1)^2(x+1) - \frac{1}{x+1} + C}$$

III Caso:

30/4/73 O denominador contem fatores do 2º grau mas nenhum é repetido.

Ex: x^2+4 (não pode mais fatorar porque ficaria $x = \pm\sqrt{-4}$ → entraria no campo dos nºs complexos. As raízes seriam irreais ou complexas.

→ p/ cada termo do 2º grau

Exemplo: $\boxed{\dfrac{Ax+B}{x^2+px+q}}$ usa-se esta fórmula sempre que tiver termo do 2º grau

$$\int \frac{4\,dx}{x^3+4x}$$

fatorando o denominador:

$$\int \frac{4\,dx}{x(x^2+4)} = \frac{4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

não pode mais fatorar

quadrado da soma

$$3x^2+5x = A(x+1)^2 + B(x-1) + C(x-1)(x+1)$$
$$3x^2+5x = Ax^2+2Ax+A+Bx-B+Cx^2-C$$
$$3x^2+5x = (A+C)x^2+(2A+B)x - B - C + A$$

$$\begin{cases} A+C=3 \\ 2A+B=5 \\ A-B-C=0 \end{cases}$$

$$\begin{array}{r} -A-C = -3 \\ A-C-B = 0 \\ \hline / \ -2C-B=-3 \end{array}$$

$$\begin{array}{r} -2A-2C=-6 \\ 2A+B=5 \\ \hline / \ B-2C=-1 \end{array} \qquad \begin{array}{r} -B-2C=-3 \\ B-2C=-1 \\ \hline -4C=-4 \end{array}$$

$$\therefore \boxed{C=1}$$

$$\boxed{A=2} \qquad \boxed{B=1}$$

$$\frac{3x^2+5x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)^2} + \frac{C}{x+1}$$

$$\frac{3x^2+5x}{(x-1)(x+1)^2} = \frac{2}{x-1} + \frac{1}{(x+1)^2} + \frac{1}{x+1}$$

$$\int \frac{(3x^2+5x)\,dx}{(x-1)(x+1)^2} = 2\int \frac{dx}{x-1} + \int \frac{dx}{(x+1)^2} + \int \frac{dx}{x+1}$$

$$= 2\ln(x-1) - \frac{1}{x+1} + \ln(x+1) + C$$

$$= \boxed{\ln (x-1)^2 (x+1) - \frac{1}{x+1} + C}$$

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III Caso:

O denominador contem fatores do 2º grau mas nenhum é repetido.

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p/ o termo do 2º grau

Exemplo: $\boxed{\dfrac{Ax+B}{x^2+px+q}}$ usa-se esta fórmula sempre que tiver o termo do 2º grau

$$\int \frac{4\,dx}{x^3+4x}$$

fatorando o denominador:

$$\int \frac{4\,dx}{x(x^2+4)} = \frac{4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

não pode mais fatorar

fatorando: $\dfrac{1}{(x+2)(x^2-2x+4)}$

O $\underline{x^3+8}$ admite raízes complexas porque o polinômio $\boxed{x^2-2x+4}$ não pode mais ser fatorado, do contrário dá raízes complexas. Portanto, aplica-se o $\underline{3^o \text{ caso}}$.

$$\frac{1}{(x+2)(x^2-2x+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4}$$

$$\frac{1}{\cancel{(x+2)(x^2-2x+4)}} = \frac{Ax^2-2Ax+4A+Bx^2+2Bx+Cx+2C}{\cancel{(x+2)(x^2-2x+4)}}$$

$$1 = (A+B)x^2 + (-2A+2B+C)x + 4A+2C$$

$$\begin{cases} A+B=0 \\ -2A+2B+C=0 \\ 4A+2C=1 \end{cases}$$

determinando os valores:

1º) $A+B=0$

$A=-B$

2º) subst. A na 2ª e 3ª:

$$-2A+2B+C=0$$
$$4A+2C=1 \Rightarrow \begin{array}{r} \cancel{+2B}+\cancel{2B}+C=0 \\ \cancel{-4B}\quad +2C=1 \\ \hline +3C=1 \end{array}$$

$\therefore \boxed{C=\frac{1}{3}}$

3º) $4A+2C=1$

$4A=1-2C$

$A=\dfrac{1-2C}{4} \Rightarrow A=\dfrac{1-\frac{2}{3}}{4} \therefore A=\dfrac{\frac{1}{3}}{4}$

$\boxed{A=\frac{1}{12}}$

$\boxed{B=\frac{-1}{12}}$

$$\int\frac{dx}{x^3+8} = \int\frac{\frac{1}{12}dx}{x+2} + \int\frac{\left(-\frac{1}{12}x+\frac{1}{3}\right)dx}{x^2-2x+4} =$$

$$\int\frac{dx}{x^3+8} = \frac{1}{12}\ln(x+2) + \underbrace{\int\frac{\left(\frac{-x+4}{12}\right)dx}{x^2-2x+4}}_{\Delta}$$

$$\Delta = \frac{1}{12}\int\frac{(-x+4)\,dx}{x^2-2x+4}$$

→ transforma-se em expressão quadr.

$A+B=9 \rightarrow B=-8$

$C=0 \qquad C=0$

$9A=9 \rightarrow A=1$

$$\int \Delta\, dx = \int x^2 dx + \int \frac{A}{x} + \int \frac{Bx+C}{x^2+9}$$

$$\frac{x^3}{3} + \int \frac{1}{x} dx + \int \frac{-8x}{x^2+9}$$

$$\frac{x^3}{3} - \ln x - 4 \ln (x^2+9)$$

$$\frac{x^3}{3} - (\ln x + 4 \ln (x^2+9))$$

$$\boxed{\frac{x^3}{3} - \ln x (x^2+9)^4 + C}$$

(18) $$\int_1^4 \frac{(5x^2+4)\,dx}{x^3+4x} =$$

$$\frac{5x^2+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx}{x^2} + \frac{C}{4} =$$

$$\int 5x^2+4 = Ax^2+4A+Bx^2+Cx$$

$$\int 5x^2+4 = (A+B)x^2 + 4A + Cx$$

$A+B=5$

$4A=4$

$C=0$

$$\begin{cases} A=1 \\ B=4 \\ C=0 \end{cases}$$

$$\int_1^4 \Delta = \int_1^4 \frac{1}{x} dx + \int_1^4 \frac{Bx+C}{x^2+4}$$

$$\int_1^4 \Delta = \ln x + 2\int_1^4 \frac{\cancel{4}^2 x}{x^2+4} \rightarrow$$ derivada de $x^2 = 2x$. divide o 4 por 2 e põe o outro 2 antes de $\int$.

$$\int_1^4 \Delta = \ln x \Big|_1^4 2\ln(x^2+4)\Big|_1^4$$

$$\int_1^4 \Delta = \ln 4 - \ln 1 + 2\ln(16+4)$$

$$-2\ln(1+4)$$

$$\int_1^4 \Delta = \ln 4 + 2\ln 20 - 2\ln 5$$

$$\int_1^4 \frac{\ln 4 \times 400}{25} \rightarrow \frac{20^2}{5^2}$$

$$\int_1^4 \frac{5x^2+4}{x^3+4x} = \boxed{\ln 64.}$$

fatorando: $\dfrac{1}{(x+2)(x^2-2x+4)}$

O $\underline{x^3+8}$ admite raízes complexas porque o polinômio $\boxed{x^2-2x+4}$ não pode mais ser fatorado, do contrário dá raízes complexas. Portanto, aplica-se o $\underline{3º\ caso}$.

$$\frac{1}{(x+2)(x^2-2x+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4}$$

$$\frac{1}{\cancel{(x+2)(x^2-2x+4)}} = \frac{Ax^2-2Ax+4A+Bx^2+2Bx+Cx+2C}{\cancel{(x+2)(x^2-2x+4)}}$$

$$1 = (A+B)x^2 + (-2A+2B+C)x + 4A+2C$$

$$\begin{cases} A+B=0 \\ -2A+2B+C=0 \\ 4A+2C=1 \end{cases}$$

determinando os valores:

1º) $A+B=0$

$A=-B$

2º) subst. A na 2ª e 3ª:

$$-2A+2B+C=0 \qquad\qquad +\cancel{2B}+\cancel{2B}+C=0$$
$$4A+2C=1 \Rightarrow \qquad -\cancel{4B}\quad +2C=1$$
$$+3C=1$$

$\therefore \boxed{C=\frac{1}{3}}$

3º) $4A+2C=1$

$4A=1-2C$

$A=\dfrac{1-2C}{4} \Rightarrow A=\dfrac{1-\frac{2}{3}}{4} \therefore A=\dfrac{\frac{1}{3}}{4} \therefore$

$\boxed{A=\frac{1}{12}}$

$\boxed{B=\frac{-1}{12}}$

$$\int \frac{dx}{x^3+8} = \int \frac{\frac{1}{12}dx}{x+2} + \int \frac{\left(-\frac{1}{12}x+\frac{1}{3}\right)dx}{x^2-2x+4} =$$

$$\int \frac{dx}{x^3+8} = \frac{1}{12}\ln(x+2) + \underbrace{\int \frac{\left(\frac{-x+4}{12}\right)dx}{x^2-2x+4}}_{\Delta}$$

$$\Delta = \frac{1}{12}\int \frac{(-x+4)\,dx}{x^2-2x+4} \rightarrow$$ transforma-se em expressão quadr.

transformação:

pode ser escrito: u^2+3

$$x^2-2x+4 = (x^2-2x+1)+3 = (x-1)^2+3$$

$(x-1)^2+3$ (troca-se por u) → artifício

$x-1 = u$

$\boxed{x = u+1}$

$\boxed{dx = du}$

$$\Delta = \frac{1}{12}\int \frac{(-u+3)\,du}{u^2+3}$$

aplica-se a fórmula: arc tg.

$$\Delta = -\frac{1}{12}\int \frac{u\,du}{u^2+3} + \frac{1}{12}\int \frac{3\,du}{u^2+3}$$

subst

$u=x-1$

pela transformação

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$$\int \frac{(4x^2+6)\,dx}{x^3+3x}$$

$$\frac{4x^2+6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$$\frac{4x^2+6}{\cancel{x(x^2+3)}} = \frac{A(x^2+3)+(Bx+C).x}{\cancel{x(x^2+3)}}$$

$$4x^2+6 = Ax^2+3A+Bx^2+Cx$$

$$4x^2+6 = (A+B)x^2+Cx+3A$$

$A+B = 4 \quad \therefore B=2$

$C = 0 \quad \therefore C=0$

$3A = 6 \quad \therefore A=2$

$$\int \frac{(4x^2+6)\,dx}{x^3+3x} = 2\int \frac{dx}{x} + \int \frac{2x\,dx}{x^2+3}$$

$$\int \frac{(4x^2+6)\,dx}{x^3+3x} = 2\ln x + \ln(x^2+3) + C$$

$$= \ln\left[x^2(x^2+3)\right] + C$$

$$\boxed{= \ln\left[x^4+3x^2\right] + C}$$

$$\int \frac{(x^2+x)\,dx}{(x-1)(x^2+1)} \Rightarrow \Delta$$

$$\frac{(x^2+x)}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$\frac{x^2+x}{\cancel{(x-1)(x^2+1)}} = \frac{A.(x^2+1)+(Bx+C)(x-1)}{\cancel{(x-1)(x^2+1)}}$$

$$x^2+x = Ax^2 + A + Bx^2 - Bx + Cx - C$$

$$x^2+x = (A+B)x^2 + (-B+C)x + (A-C)$$

$$A+B=1$$
$$-B+C=1$$
$$A-C=0$$

$$A=C \Rightarrow \begin{array}{r} A+B=1 \\ A-B=1 \\ \hline 2A\ / = 2 \end{array}$$

$$\begin{cases} A=1 \\ C=1 \\ B=0 \end{cases}$$

$$A+B=1$$
$$B=1-A$$
$$B=1-1$$
$$B=0$$

$$\int \Delta = \int \frac{dx}{x-1} + \int \frac{dx}{x^2+1} \rightarrow \text{fórmula } \int \frac{dv}{a^2+v^2}$$

$$\Delta = \ln(x-1) + \frac{1}{1}\,\text{arc tg}\,\frac{x}{1} + C$$

$$\boxed{\Delta = \ln(x-1) + \text{artg}\,x + C}$$

$$\int \frac{(2t^2-8t-8)\,dt}{(t-2)(t^2+4)} =$$

$$\frac{2t^2-8t-8}{(t-2)(t^2+4)} = \frac{A}{t-2} + \frac{Bt+C}{t^2+4} =$$

$$\frac{2t^2-8t-8}{\cancel{(t-2)(t^2+4)}} = \frac{A(t^2+4)+(Bt+C).(t-2)}{\cancel{(t-2).(t^2+4)}}$$

$$2t^2-8t-8 = At^2 + 4A + Bt^2 + Ct - 2Bt - 2C$$

$$2t^2-8t-8 = (A+B)t^2 + (C-2B)t + (4A-2C)$$

$A=2-B$

$$A+B=2$$
$$C-2B=-8$$
$$4A-2C=-8$$

$$\begin{array}{r} 2C-4B=-16 \\ -2C+4A=-8 \\ \hline /\ 4A-4B=-24 \end{array}$$

$4.(2-B)-4B=-24$
$8-4B-4B=-24$
$-8B=-24-8$
$-8B=-32$
$\boxed{B=4}$

$A+B=2$
$A+4=2$
$A=2-4$ ∴
$\boxed{A=-2}$

$4A-2C=-8$
$4.-2-2C=-8$
$-8-2C=-8$
$-2C=-8+8$
$-2C=0$
$C=\frac{0}{2}$ ∴ $\boxed{C=0}$

$A=-2$
$B=4$
$C=0$

$$\int \Delta = -2\int \frac{dt}{t-2} + 2\int \frac{2t\,dt}{t^2+4}$$

$$\int \Delta = -2\ln(t-2) + 2\ln(t^2+4) + C$$

$$\int \Delta = -\ln(t-2)^2 + \ln(t^2+4)^2 + C$$

$$\boxed{\int \Delta = \ln\frac{(t^2+4)^2}{(t-2)^2} + C}$$

$$\int \frac{(x^2+x-10)\,dx}{(2x-3)(x^2+4)} = \Delta$$

$$\frac{(x^2+x-10)}{(2x-3)(x^2+4)} = \frac{A}{2x-3} + \frac{Bx+C}{x^2+4}$$

$$\frac{x^2+x-10}{\cancel{(2x-3)(x^2+4)}} = \frac{A.(x^2+4)+(Bx+C).(2x-3)}{\cancel{(2x-3)(x^2+4)}}$$

$$x^2+x-10 = Ax^2+4A+2Bx^2-3Bx+2Cx-3C$$
$$x^2+x-10 = (A+2B)x^2+(-3B+2C)x+(4A-3C)$$

$A+2B=1$
$-3B+2C=1$
$4A-3C=-10$

$A=1-2B$
subst. na 3ª:
$4.(1-2B)-3C=-10$
$4-8B-3C=-10$
$-8B-3C=-14$

$2C-3B=1$ (3)
$-3C-8B=-14$ (2)

$6C-9B=3$
$-6C-16B=-28$
$/\ -25B=-25$

$B=\frac{-25}{-25}$ ∴ $\boxed{B=1}$

$A = 1 - 2B$

$A = 1 - 2 \therefore \boxed{A = -1}$

$A = -1$

$B = 1$

$C = 2$

$2C - 3B = 1$

$2C = 3B + 1$

$C = \frac{3.1 + 1}{2}$

$C = \frac{4}{2} \therefore \boxed{C = 2}$

$$\int \Delta = -\frac{1}{2}\int \frac{2dx}{2x-3} + \int \frac{(x+2)\,dx}{x^2+4} =$$

$$\int \Delta = -\frac{1}{2} \ln (2x-3) + \frac{1}{2}\int \frac{2x}{x^2+4} dx + 2\int \frac{dx}{x^2+4} \cancel{dx} \rightarrow \text{fórmula}$$

$$\int \Delta = -\frac{1}{2} \ln (2x-3) + \frac{1}{2} \ln (x^2+4) + 2 . \frac{1}{2} \text{arc.} \operatorname{tg} \frac{x}{2} + C$$

$$\boxed{\int \Delta = \frac{1}{2} \ln \frac{x^2+4}{2x-3} + \operatorname{arc\,tg} \frac{x}{2} + C}$$

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$$\int_2^4 \frac{(x^3-2)\,dx}{x^3 - x^2}$$ dividindo o numerador pelo denominador.

$$\int_2^4 \left(\frac{1 + x^2 - 2}{x^3 - x^2} \right) dx$$

integral da soma:

$$1\int_2^4 dx + \int \frac{(x^2-2)}{x^3 - x^2} dx$$

$$\frac{x^2-2}{x^2(x-1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-1}$$

$$\frac{x^2-2}{x^2(x+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-1}$$

$x^2 - 2 = A(x-1) + Bx(x-1) + Cx^2$

$x^2 - 2 = Ax - A + Bx^2 - Bx + Cx^2$

$x^2 - 2 = (B+C)x^2 + (A-B)x - A$

$B + C = 1 \rightarrow C = -1$

$A - B = 0 \rightarrow A = B \therefore B = 2$

$-A = 2 \rightarrow A = 2$

$\int_2^4 dx + \int_2^4 \Delta = x \int_2^4 + 2\int_2^4 \frac{dx}{x^2} + 2\int_2^4 \frac{dx}{x} -$

$\int_2^4 \frac{dx}{x-1} =$

$x \int_2^4 - 2\frac{1}{x}\int_2^4 + 2\ln x \int_2^4 - \ln(x-1) \Big]_2^4$

$= 2 + \frac{1}{2} + 2\ln 2 - \ln 3 =$

$\int_1^4 dx + \int_2^4 \Delta = \boxed{\frac{5}{2} + \ln\frac{4}{3}} \to$ resposta

calculando:

$-2\,\frac{1}{x}\int_2^4 = -2\left[\frac{1}{4} - \frac{1}{2}\right] - 2\left(-\frac{1}{4}\right) = \frac{1}{2}$

$2\ln x \int_2^4 = 2\ln(4) - 2\ln 2$

$2\ln\frac{4}{2} = 2\ln 2$

resposta: $\boxed{-\ln(x-1)\int_2^4 - \ln 3 - \ln 1}$

$\int_1^2 \frac{(x-3)\,dx}{x^3+x^2} =$

$\frac{x-3}{x^2(x+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+1}$

$x-3 = A(x+1) + Bx(x+1) + Cx^2$

$x-3 = Ax + A + Bx^2 + Bx + Cx^2$

$x-3 = x^2 (A+B)x + A$

$B+C=0$

$A+B=1 \to 1-A \quad \therefore \quad A=-3$

$B = 3+1 \;(\therefore\;) \quad B=4$

$C=-B \quad \therefore \quad C=-4$

$\int_1^2 \frac{x-3\,dx}{x^3-x^2} \to \Delta$:

$-3\int_1^2 \frac{dx}{x^2} + 4\int_1^2 \frac{dx}{x} - 4\int_1^2 \frac{dx}{x+1}$

$\int_1^2 \Delta = +3.\frac{1}{x}\int_1^2 + 4\ln x\int_1^2 - 4\ln(x+1)\Big]_1^2$

$A+B=9 \rightarrow B=-8$

$C=0 \quad C=0$

$9A=9 \rightarrow A=1$

$\int \Delta\, dx = \int x^2 dx + \int \frac{A}{x} + \int \frac{Bx+C}{x^2+9}$

$\frac{x^3}{3} + \int \frac{1}{x} dx + \int \frac{-8x}{x^2+9}$

$\frac{x^3}{3} - \ln x - 4 \ln (x^2+9)$

$\frac{x^3}{3} - (\ln x + 4 \ln (x^2+9)$

$$\boxed{\frac{x^3}{3} - \ln x \,(x^2+9)^4 + C}$$

18 $\int_1^4 \frac{(5x^2+4)dx}{x^3+4x} =$

$\frac{5x^2+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx}{x^2} + \frac{C}{4} =$

$\int 5x^2+4 = Ax^2+4A+Bx^2+Cx$

$\int 5x^2+4 = (A+B)x^2+4A+Cx$

$A+B=5$

$4A=4$

$C=0$

$\begin{cases} A=1 \\ B=4 \\ C=0 \end{cases}$

$\int_1^4 \Delta = \int_1^4 \frac{1}{x} dx + \int_1^4 \frac{Bx+C}{x^2+4}$

$\int_1^4 \Delta = \ln x + 2 \int_1^4 \frac{\cancel{4}^{2} x}{x^2+4} \rightarrow$ derivada de $x^2 = 2x$. Divide o 4 por 2 e põe o outro 2 antes de $\int$.

$\int_1^4 \Delta = \ln x \Big|_1^4 2 \ln (x^2+4) \Big|_1^4$

$\int_1^4 \Delta = \ln 4 - \ln 1 + 2 \ln (16+4)$

$-2\ln(1+4)$

$\int_1^4 \Delta = \ln 4 + 2\ln 20 - 2\ln 5$

$\int_1^4 \frac{\ln 4 \times 400}{25} \rightarrow \frac{20^2}{5^2}$

$\int_1^4 \frac{5x^2+4}{x^3+4x} = \boxed{\ln 64.}$

$A+B=9 \rightarrow B=-8$

$C=0 \qquad C=0$

$9A=9 \rightarrow A=1$

$\int \Delta\, dx = \int x^2 dx + \int \frac{A}{x} + \int \frac{Bx+C}{x^2+9}$

$\frac{x^3}{3} + \int \frac{1}{x} dx + \int \frac{-8x}{x^2+9}$

$\frac{x^3}{3} - \ln x - 4\ln(x^2+9)$

$\frac{x^3}{3} - (\ln x + 4\ln(x^2+9))$

$$\boxed{\frac{x^3}{3} - \ln x\,(x^2+9)^4 + C}$$

18 $\int_1^4 \frac{(5x^2+4)\,dx}{x^3+4x} =$

$\frac{5x^2+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx}{x^2} + \frac{C}{4} =$

$\int 5x^2+4 = Ax^2 + 4A + Bx^2 + Cx$

$\int 5x^2+4 = (A+B)x^2 + 4A + Cx$

$A+B=5 \qquad A=1$

$4A=4 \qquad B=4$

$C=0 \qquad C=0$

$\int_1^4 \Delta = \int_1^4 \frac{1}{x} dx + \int_1^4 \frac{Bx+C}{x^2+4}$

$\int_1^4 \Delta = \ln x + 2\int_1^4 \frac{\cancel{4}^2 x}{x^2+4} \rightarrow$ derivada de $x^2 = 2x$. Divide o 4 por 2 e põe o outro 2 antes de $\int$.

$\int_1^4 \Delta = \ln x \Big|_1^4 2\ln(x^2+4)\Big|_1^4$

$\int_1^4 \Delta = \ln 4 - \ln 1 + 2\ln(16+4)$

$-2\ln(1+4)$

$\int_1^4 \Delta = \ln 4 + 2\ln 20 - 2\ln 5$

$\int_1^4 \ln \frac{4 \times 400}{25} \rightarrow \frac{20^2}{5^2}$

$\int_1^4 \frac{5x^2+4}{x^3+4x} = \boxed{\ln 64.}$

Integração por Substituição

08/05/73

Substitue-se o x por: $x = z^N$, sendo N o m.m.c dos denominadores dos escpoentes.

$N = 4$

$x = z^4 \rightarrow z = x^{\frac{1}{4}}$

$x^{\frac{1}{2}} = z^2 \qquad z^3 = x^{3/4}$ (mult. o numerador de $\frac{1}{4}$ e o escpoente 1 por 3)

$x^{3/4} = z^3$

$dx = 4z^3 dz$

$$\int \frac{z^2 4z^3 dz}{1+z^3} = \int \frac{4z^5 dz}{1+z^3} = 4\int \frac{z^5 dz}{1+z^3}$$

$$\begin{array}{r|l} z^5 & z^3+1 \\ \hline -z^5 - z^2 & z^2 - \frac{z^2}{z^3+1} \\ / \quad z^2 & \\ / & \end{array}$$

$$4\int \left(z^2 - \frac{z^2}{z^3+1}\right) dz = 4\int z^2 dz - \frac{4}{3}\int \frac{z^2\, dz}{z^3+1}$$

$$\frac{4z^3}{3} - \frac{4}{3} \ln (z^3+1) + C$$

$z^3 = x^{3/4}$

$$\boxed{\frac{4x^{3/4}}{3} - \frac{4}{3} \ln (x^{3/4}+1) + C}$$

$$\int \frac{(5x+9)\,dx}{(x-9)\,x^{3/2}}$$

$N = 2$

$x = z^N$

$x = z^2$

$z = x^{1/2}$

$z^3 = x^{3/2}$

$dx = 2z\,dz$

$$\int \frac{(5z^2+9)\; 2\cancel{z}\, dz}{(z^2-9)\,z^{\cancel{3}2}}$$

$$\int \frac{(5z^2+9)\, 2dz}{(z^2-9)\,z^2}$$

$$2\int \frac{(5z^2+9)\,dz}{(z^2-9)\,z^2}$$

$$\frac{10z^2+18}{(z+3)(z-3)\,z^2} = \frac{A}{z+3} + \frac{B}{z-3} + \frac{C}{z^2} + \frac{D}{z}$$

$$10z^2 + 18 = Az^2(z-3) + Bz^2(z+3) + C(z+3)$$

$$(z-3)+D(z+3)\cdot(z-3) =$$

$$10z^2+18 = Az^3-3Az^2+Bz^3+3Bz^2+Cz^2-9C+Dz^3-9Dz$$

$$10z^2+18 = (A+B+D)z^3+(-3A+3B+C)z^2-9Dz-9C$$

$A+B+D=0$

$-3A+3B+C=10$

$-9D=0$

$-9C=18$

$C=-2$

$D=0$

$$\begin{array}{r} 3A+3B=0 \\ -3A+3B=12 \\ \hline 6B=12 \end{array}$$

$B=\frac{12}{6}$

$B=2$

$A=-B \quad \therefore A=-2$

$A+B=0\ (3)$

$-3A+3B=12$

$$\int\Delta = -2\int\frac{dz}{z+3}+2\int\frac{dz}{z-3}-2\int\frac{dz}{z^2} =$$

$$\int\Delta = -2\ln(z+3)+2\ln(z-3)+2\cdot\frac{1}{z}$$

$$\int\Delta = 2\ln\frac{z-3}{z+3}+\frac{2}{z}$$

$$-2\int z^{-2}dz = -2\frac{z^{-1}}{-1} = 2\cdot\frac{1}{z}$$

$$\int\frac{(5x+9)}{(x-9)x^{3/2}}dx = \boxed{2\ln\frac{\sqrt{x}-3}{\sqrt{x}+3}+\frac{2}{\sqrt{x}}}$$

$$\int\frac{\sqrt{x}\,dx}{x^3+2x^2-3x} =$$

$$\int\frac{x^{\frac{1}{2}}dx}{x^3+2x^2-3x}$$

$x=z^N$

$N=2$

$x=z^2$

$x^{\frac{1}{2}}=z$

$x^2=z^4 \quad x^3=z^6$

$dx=2z\,dz$

$$\int\frac{z^2\,2z\,dz}{z^6+2z^4-3z^2} = \int\frac{2z^{\cancel{3}}dz}{\cancel{z^2}(z^4+2z^2-3)} =$$

$$2\int\frac{dz}{z^4+2z^2-3} =$$

$x'=\pm1$

$z=-1$

$z=+1$

$$\begin{array}{l|l} z^4+2z^2-3 & z+1 \\ \hline -z^4-z^3 & z^3-z^2+3z-3 \\ \quad -z^3+2z^2 & \\ \quad +z^3+z^2 & \\ \quad\quad +3z^2-\cancel{3z} & \\ \quad\quad -3z^2-3z & \\ \quad\quad\quad -3z-3 & \\ \quad\quad\quad +3z+3 & \\ \quad\quad\quad\ \ 0\quad 0 & \end{array}$$

$$\begin{array}{l|l} z^3 - z^2 + 3z - 3 & z - 1 \\ \hline -z^3 + z^2 & z^2 + 3 \\ \quad / \quad / \quad 3z - 3 & \\ \qquad\quad -3z + 3 & \\ \qquad\quad\; / \quad / & \end{array}$$

$$\frac{2}{(z+1)(z-1)(z^2+3)} =$$

$$\frac{A}{z+1} + \frac{B}{z-1} + \frac{B}{z^2} + \frac{D}{3}$$

$$2 = A(z-1)(z^2+3) + B(z+1)(z^2+3)(Dz+C)(z+1)(z-1)$$

$$2 = (A+B+D)z^3 + (-A+B+C)z^2 + (3A+3B-D)z$$

$$-3A + 3B - C$$

$$\begin{array}{l} A+B+D=0 \\ 3A+3B-D=0 \\ \hline 4A+4B \;/ = 0 \end{array}$$

$8B = 2 \;\therefore\; B = \frac{1}{4}$

$D = 0$

$A = -B$ $\qquad C = \frac{-1}{2}$

$A = -\frac{1}{4}$ $\qquad A = -\frac{1}{4}$

$$\int \Delta = -\frac{1}{4}\int \frac{dz}{z+1} + \frac{1}{4}\int \frac{dz}{z-1} - \frac{1}{2}\int \frac{dz}{z^2+3}$$

$$\int \Delta = -\frac{1}{4}\ln(z+1) + \frac{1}{4}\ln(z-1) - \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{z}{\sqrt{3}} + C$$

$$\int \Delta = \frac{1}{4}\ln\frac{\sqrt{x}-1}{\sqrt{x}+1} - \frac{1}{2\sqrt{3}}\,\mathrm{arctg}\,\frac{\sqrt{x}}{\sqrt{3}} + C$$

$$\boxed{\int \Delta = \frac{1}{4}\ln\frac{\sqrt{x}-1}{\sqrt{x}+1} - \frac{\sqrt{3}}{6}\,\mathrm{arctg}\sqrt{\frac{x}{3}} + C}$$

$$\int \frac{(x^{3/2} - x^{1/3})\,dx}{6x^{1/4}}$$

$x = z^N \rightarrow N = 12$

$x = z^{12} \rightarrow z = x^{1/12}$

$x^{3/2} = z^{18}$

$x^{1/3} = z^4$

$x^{1/4} = z^3$

$dx = 12z^{11}dz$

$$\int \frac{(z^{18} - z^4)\,\cancel{12}^{2}\, z^{\cancel{11}^{8}}\,dz}{\cancel{6}\,\cancel{z^3}} =$$

$$2\int (z^{18} - z^4)\, z^8\, dz =$$

$$2\int (z^{26} - z^{12})\, dz =$$

$$2\int z^{26}\,dz - 2\int z^{12}\,dz$$

$$2.\frac{z^{27}}{27} - 2\frac{z^{13}}{13} + C$$

$$\frac{2}{27}z^{27} - \frac{2z^{13}}{13}$$

$$\frac{2}{27}z^{27} - \frac{2z^{13}}{13}$$

$$\frac{2}{27}x^{\frac{27}{12}\,\frac{9}{4}} - \frac{2}{13}x^{\frac{13}{12}} + C$$

$$\boxed{\int\frac{\left(x^{3/2} - x^{1/3}\right)}{6x^{1/4}}\,dx = \frac{2}{27}x^{9/4} - \frac{2}{13}x^{13/12} + C}$$

Integração qdo $a+bx$ contem expoente fracionário:

11/05/73 ① $\displaystyle\int\frac{dx}{(1+x)^{3/2}+(1+x)^{1/2}} =$

$\boxed{a+bx = z^N}$ com $N =$ M.M.C dos denominadores dos expoentes.

$1+x = z^2 \to$ m.m.c dos expoentes

$$(1+x)^{3/2} = z^3 \Rightarrow (1+x)^{3/2} = (z^2)^{3/2} = (1+x)^{3/2} = z^3$$

$$(1+x)^{1/2} = z \Rightarrow (1+x)^{1/2} = (z^2)^{1/2} = (1+x)^{1/2} = z$$

faz-se esta transformação para passar expoente fracionário em inteiro.

$x = z^2 - 1$

$dx = 2z\,dz$

substituindo os valores x e dx temos:

$$\int\frac{dx}{(1+x)^{3/2}+(1+x)^{1/2}} = \int\frac{2\cancel{z}\,dz}{z^{\cancel{3}\,2}+\cancel{z}\,1} =$$

$$\int\frac{2\,dz}{z^2+1} = 2\int\frac{dz}{z^2+1}$$ fórmula arc tg

$$2.\frac{1}{1}\,\text{arc.tg}\,\frac{z}{1} = 2\,\text{arc.tg}\,z$$

voltando à substituição:

$$z = (1+x)^{1/2} = \boxed{2\,\text{arc tg}\sqrt{1+x} + C}$$

$$z = \sqrt{1+x}$$

② $\int \frac{x^2\,dx}{(4x+1)^{5/2}} =$

$4x+1 = z^2 \Rightarrow 4x+1 = z^2 \Rightarrow$

$\boxed{a+bx = zN}$ $\Rightarrow z = \sqrt{4x+1}$

$x = \frac{z^2-1}{4} \therefore x = \frac{1}{4}z^2 - \frac{1}{4}$

$dx = \frac{1}{\cancel{4}_2}\cdot \cancel{2} z\,dz = \frac{z}{2}dz$

$(4x+1)^{5/2} = (z^2)^{5/2}$

$(4x+1)^{5/2} = z^5$

Partindo de $\int \frac{x^2\,dx}{(4x+1)^{5/2}}$ temos:

11/05

$\int \frac{\left(\frac{z^2-1}{4}\right)^2 \cdot \frac{z^1}{2}\,dz}{z^{\cancel{5}\,4}} =$

$\int \frac{(z^4 - 2z^2 + 1)\,dz}{32 z^4}$ ($4^2 \cdot 2$ que passa a mult. z^4) =

$\int \frac{\cancel{z^4}\,dz}{32 \cancel{z^4}\,dz} - \int \frac{\cancel{2}^{1} z^2\,dz}{\cancel{32}_{16}\, z^{\cancel{4}\,2}} + \int \frac{dz}{z^4} =$

$\frac{1}{32}\int dz - \frac{1}{16}\int \frac{dz}{z^2} + \frac{1}{32}\int \frac{dz}{z^4} =$

$-\frac{1}{16}\int z^{-2}\,dz = -\frac{1}{16}\cdot\frac{z^{-1}}{-1} = \frac{1}{16z}$

$\frac{1}{32}\int z^{-4}\,dz = \frac{1}{32}\cdot\frac{z^{-3}}{-3} = -\frac{1}{96z^3}$

$= \frac{z}{32} + \frac{1}{16z} - \frac{1}{96z^3}$

m.m.c $= 96z^3$

$\frac{3z^4 + 6z^2 - 1}{96z^3}$, substituindo:

$\frac{3\cdot\left(\sqrt{4x+1}\right)^4 + 6\left(\sqrt{4x+1}\right)^2 - 1}{96\cdot\left(\sqrt{4x+1}\right)^3} =$

$\frac{3(4x+1)^2 + 6(4x+1) - 1}{96\sqrt{(4x+1)^3}} =$

$\frac{3(16x^2+8x+1) + 24x + 6 - 1}{96\sqrt{(4x+1)}^3} =$

$$\frac{48x^2+24x+3+24x+6-1}{96\left(\sqrt{(4x+1)^3}\right)} =$$

$$\frac{48x^2+48x+8}{96\left(\sqrt{4x+1}\right)^3} =$$ (simpl. por 8)

$$\boxed{\frac{6x^2+6x+1}{12(4x+1)^{3/2}} + C}$$

(3) $$\int \frac{(\sqrt{x+1}+1)\,dx}{\sqrt{x+1}-1} =$$

$$\int \frac{\left[(x+1)^{1/2}+1\right]dx}{(x+1)^{1/2}-1} =$$

$x+1 = z^N \quad \therefore \quad x+1 = z^2$

$(x+1)^{1/2} = (z^2)^{1/2} \quad \therefore (x+1)^{1/2} = z$

$(x+1) = z^2 \quad \therefore \boxed{x = z^2-1}$

$dx = 2z\,dz$

$(x+1)^{1/2} = z$

$$\int \frac{(z+1)\,2z\,dz}{z-1} = 2\int \frac{(\cancel{2}z^2+\cancel{2}z)\,dz}{z-1}$$

z^2+z	$z-1$
$-z^2+z$	$z+2+\frac{2}{z-1}$
$/ \quad 2z$	
$-2z+2$	
$/ \quad +2$	
-2	
$/$	

$$2\int \left(z+2+\frac{2}{z-1}\right)dz =$$

$$2\int z + 4\int dz + 4\int \frac{dz}{z-1}$$

$$= \frac{\cancel{2}z^2}{\cancel{2}} + 4z + 4\ln(z-1)$$

completar

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$$\int_0^3 \frac{dt}{(t+2)\sqrt{t+1}} =$$

$(t+2)^{1/2} = z$

$t+1 = z^2$

$(t+1)^{1/2} = (z^{\cancel{2}})^{1/\cancel{2}}$

$(t+1)^{1/2} = z \Rightarrow t = z^2 - 1$

$dt = 2z\,dz$

$$\int_0^3 \frac{dt}{(t+2)(t+1)^{1/2}} = \int_0^3 \frac{2\cancel{z}\,dz}{(\underbrace{z^2-1}_{t}+2)\cancel{z}} =$$

$$\int_0^3 \frac{2\,dz}{z^2+1} = 2\int_0^3 \frac{dz}{t^2+1} =$$

$$2 \cdot \frac{1}{1} \operatorname{arc\,tg} \frac{z}{1} + C = 2 \operatorname{arc\,tg} z + C$$

$$\int_0^3 \frac{dt}{(t+2)\sqrt{t+1}} = \left[2 \operatorname{arc\,tg} \sqrt{t+1}\right]_0^3 =$$

$$2 \operatorname{arc\,tg} 2 - 2 \operatorname{arc\,tg} 1 = \boxed{2 \operatorname{arc\,tg} 2 - \frac{\pi}{2}}$$

(13) $$\int_0^4 \frac{dx}{1+\sqrt{x}} =$$

$x = z^2$

$x^{1/2} = (z^2)^{1/2}$

$x^{1/2} = z$

$dx = 2z\,dz$

$$\int_0^4 \frac{dx}{1+x^{1/2}} =$$

$$\int_0^4 \frac{2z\,dz}{1+z} = 2\int_0^4 \frac{z\,dz}{1+z} =$$

$$2\int_0^4 dz - 2\int_0^4 \frac{dz}{z+1} =$$

$z \;|\; 1+z$

$-z-1 \quad 1 - \frac{1}{1+z}$

$/ \; -1$

$+1$

$/$

$$\int_0^4 \frac{z\,dz}{1+z} =$$

$$\left[2z - 2\ln(z+1)\right]_0^4 =$$

$$\int_0^4 \frac{dx}{1+\sqrt{x}} = \left[2\sqrt{x} - 2\ln(\sqrt{x}+1)\right]_0^4 =$$

$$2.\sqrt{4} - 2\ln(\sqrt{4}+1) =$$

$$4 - 2\ln(3) + 2\ln 1 = \boxed{4 - 2\ln 3}$$

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$$\int_0^3 \frac{dt}{(t+2)\sqrt{t+1}} =$$

$(t+2)^{1/2} = z$

$t+1 = z^2$

$(t+1)^{1/2} = (z^2)^{1/2}$

$(t+1)^{1/2} = z \Rightarrow t = z^2 - 1$

$dt = 2z\,dz$

$$\int_0^3 \frac{dt}{(t+2)(t+1)^{1/2}} = \int_0^3 \frac{2\cancel{z}\,dz}{(\underbrace{z^2-1+2}_{t+2})\cancel{z}} =$$

$$\int_0^3 \frac{2\,dz}{z^2+1} = 2\int_0^3 \frac{dz}{t^2+1} =$$

$$2 \cdot \frac{1}{1} \operatorname{arc\,tg} \frac{z}{1} + C = 2 \operatorname{arc\,tg} z + C$$

$$\int_0^3 \frac{dt}{(t+2)\sqrt{t+1}} = \left[2 \operatorname{arc\,tg} \sqrt{t+1}\right]_0^3 =$$

$$2 \operatorname{arc\,tg} 2 - 2 \operatorname{arc\,tg} 1 = \boxed{2 \operatorname{arc\,tg} 2 - \frac{\pi}{2}}$$

(13) $$\int_0^4 \frac{dx}{1+\sqrt{x}} =$$

$x = z^2$

$x^{1/2} = (z^2)^{1/2}$

$x^{1/2} = z$

$dx = 2z\,dz$

$$\int_0^4 \frac{dx}{1+x^{1/2}} =$$

$$\int_0^4 \frac{2z\,dz}{1+z} = 2\int_0^4 \frac{z\,dz}{1+z} =$$

$$2\int_0^4 dz - 2\int_0^4 \frac{dz}{z+1} =$$

$$\begin{array}{r|l} z & 1+z \\ \hline -z-1 & 1 - \frac{1}{1+z} \\ \hline -1 & \\ +1 & \\ \hline 1 & \end{array}$$

$$\int_0^4 \frac{z\,dz}{1+z} =$$

$$\left[2z - 2\ln(z+1)\right]_0^4 =$$

$$\int_0^4 \frac{dx}{1+\sqrt{x}} = \left[2\sqrt{x} - 2\ln(\sqrt{x}+1)\right]_0^4 =$$

$$2 \cdot \sqrt{4} - 2\ln(\sqrt{4}+1) =$$

$$4 - 2\ln(3) + 2\ln 1 = \boxed{4 - 2\ln 3}$$

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(15) $\displaystyle\int_0^{1/2} \frac{dt}{\sqrt{2t}\cdot\left(9+\sqrt[3]{2t}\right)} =$

$\displaystyle\int_0^{1/2} \frac{dt}{(2t)^{1/2}\left(9+(2t)^{1/3}\right)} =$

$2t = z^6$

$t = \frac{z^6}{2}$

$dt = \frac{\cancel{6}^{3}}{\cancel{2}} z^5 \cdot dz$

$\boxed{t = \frac{z^6}{2}}$ $\boxed{dt = 3z^5 dz}$

$(2t)^{1/2} = \left(z^{\cancel{6}^{3}}\right)^{1/\cancel{2}}$ $(2t)^{1/3} = \left(z^{\cancel{6}^{2}}\right)^{1/\cancel{3}}$

$(2t)^{1/2} = z^3$ $(2t)^{1/3} = z^2$

$\downarrow \sqrt[6]{2t}$ $\downarrow \sqrt[6]{2t}$

$\displaystyle\int_0^{1/2} \frac{dt}{(2t)^{1/2}\cdot\left(9+(2t)^{1/3}\right)} = \int_0^{1/2} \frac{3z^{\cancel{5}^{2}} dz}{z^{\cancel{3}}\cdot\left(9+z^2\right)} =$

$$\begin{array}{r|l} z^2 & z^2+9 \\ \hline -z^2-9 & 1-\frac{9}{z^2+9} \\ \hline /\ -9 & \\ +9 & \\ \hline / & \end{array}$$

$\displaystyle 3\int_0^{\frac{1}{2}} \frac{z^2\, dz}{(9+z^2)} = 3\int_0^{\frac{1}{2}} \left(1-\frac{9}{z^2+9}\right) dz$

$\displaystyle 3\int_0^{\frac{1}{2}} dz - 3.9\int_0^{1/2} \frac{dz}{z^2+9} =$

$\left[3z - \cancel{27}^{9} \cdot \frac{1}{3} \operatorname{arc\,tg} \frac{z}{3}\right]_0^{1/2} =$

$\left[3z - 9 \operatorname{arc\,tg} \frac{z}{3}\right]_0^{1/2} =$

$\left[3\sqrt[6]{2t} - 9 \operatorname{arc\,tg} \frac{\sqrt[6]{2t}}{3}\right]_0^{1/2} =$

$3.\sqrt[6]{2.\frac{1}{2}} - 9 \operatorname{arc\,tg} \frac{\sqrt[6]{2.1/2}}{3} =$

$\boxed{3 - 9 \operatorname{arc.tg} \frac{1}{3} + C}$

---

(18) $\displaystyle\int_3^{29} \frac{(x-2)^{2/3}\, dx}{(x-2)^{2/3}+3} =$

$x-2 = z^3 \implies x = z^3+2$

$(x-2)^{2/3} = \left(z^{\cancel{3}}\right)^{2/\cancel{3}} = z^2$ $dx = 3z^2 dz$

fatorando: $\dfrac{1}{(x+2)(x^2-2x+4)}$

O $\underline{x^3+8}$ admite raízes complexas porque o polinômio $\boxed{x^2-2x+4}$ não pode mais ser fatorado, do contrário dá raízes complexas. Portanto, aplica-se o $\underline{3^o \text{ caso}}$.

$$\frac{1}{(x+2)(x^2-2x+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4}$$

$$\frac{1}{\cancel{(x+2)(x^2-2x+4)}} = \frac{Ax^2-2Ax+4A+Bx^2+2Bx+Cx+2C}{\cancel{(x+2)(x^2-2x+4)}}$$

$$1 = (A+B)x^2 + (-2A+2B+C)x + 4A+2C$$

$$\begin{cases} A+B=0 \\ -2A+2B+C=0 \\ 4A+2C=1 \end{cases}$$

determinando os valores:

1º) $A+B=0$

$A=-B$

2º) subst. A na 2ª e 3ª:

$$\begin{aligned} -2A+2B+C&=0 \\ 4A+2C&=1 \end{aligned} \Rightarrow \begin{aligned} +\cancel{2B+2B}+C&=0 \\ \underline{-4B\quad +2C}&\underline{=1} \\ +3C&=1 \end{aligned}$$

$\therefore \boxed{C=\frac{1}{3}}$

3º) $4A+2C=1$

$4A=1-2C$

$A=\dfrac{1-2C}{4} \Rightarrow A=\dfrac{1-\frac{2}{3}}{4} \therefore A=\dfrac{\frac{1}{3}}{4} \therefore$

$\boxed{B=\frac{-1}{12}}$ $\boxed{A=\frac{1}{12}}$

$$\int \frac{dx}{x^3+8} = \int \frac{\frac{1}{12}dx}{x+2} + \int \frac{\left(-\frac{1}{12}x+\frac{1}{3}\right)dx}{x^2-2x+4} =$$

$$\int \frac{dx}{x^3+8} = \frac{1}{12}\ln(x+2) + \underbrace{\int \frac{\left(\frac{-x+4}{12}\right)dx}{x^2-2x+4}}_{\Delta}$$

$$\Delta = \frac{1}{12}\int \frac{(-x+4)\,dx}{x^2-2x+4} \rightarrow \text{transforma-se em expressão quadr.}$$

$$\int_3^{29} \frac{(x-2)^{2/3}\,dx}{(x-2)^{2/3}+3} = \int_3^{29} \frac{z^2 \cdot 3z^2\,dz}{z^2+3}$$

$$3\int_3^{29} \frac{z^4\,dz}{z^2+3}$$

$$\begin{array}{l|l} z^4 & z^2+3 \\ \hline -z^4-3z^2 & z^2-3+\frac{9}{z^2+3} \\ \quad -3z^2 & \\ \quad +3z^2+9 & \\ \hline \quad\quad +9 & \end{array}$$

$$3\int_0^{29} \left(z^2-3+\frac{9}{z^2+3}\right)dz =$$

$$3\int_3^{29} z^2\,dz - 9\int_3^{29} dz + 27\int_3^{29} \frac{dz}{z^2+3} =$$

$$= \left[\frac{3z^3}{3} - 9z + 27 \cdot \frac{1}{\sqrt{3}} \cdot \operatorname{arctg} \frac{z}{\sqrt{3}}\right]_3^{29} =$$

$$\left[x-2-9\sqrt[3]{x-2} + \frac{27}{\sqrt{3}} \operatorname{arctg} \frac{\sqrt{x-2}}{\sqrt{3}}\right]_3^{29} =$$

$$= \left[\underbrace{29-2}_{27} - 9\sqrt[3]{27} + \frac{27}{\sqrt{3}} \operatorname{arctg} \frac{\sqrt[3]{27}}{\sqrt{3}}\right] -$$

$27 - 9.3$

$27 - 27$

$\frac{27\sqrt{3}}{\sqrt{3}.\sqrt{3}} = \frac{27\sqrt{3}}{\sqrt{9}} = \frac{27\sqrt{3}}{3}$ $9\sqrt{3}$

$\frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{9}} = \frac{3\sqrt{3}}{3}$

$\frac{27\sqrt{3}}{\sqrt{3}.\sqrt{3}} = \frac{27\sqrt{3}}{3}$ $9\sqrt{3}$

$$-\left[3-2-9\sqrt[3]{1} + \frac{27}{\sqrt{3}} \operatorname{arctg} \frac{\sqrt{1}}{\sqrt{3}}\right] =$$

$$\left[\frac{27}{\sqrt{3}} \operatorname{arctg} \sqrt{3} + 8 - \frac{27}{\sqrt{3}} \operatorname{arctg} \frac{1}{\sqrt{3}}\right] =$$

$$9\sqrt{3} \operatorname{arctg} \sqrt{3} + 8 - 9\sqrt{3} \operatorname{arctg} \frac{\sqrt{3}}{3} =$$

$$9\sqrt{3}\left(\operatorname{arctg}\sqrt{3} - \operatorname{arctg}\frac{\sqrt{3}}{3}\right) + 8 =$$

$$\cancel{9}^{3}\sqrt{3} \cdot \frac{\pi}{\cancel{6}_2} + 8 = \boxed{\frac{3\sqrt{3}\,\pi}{2} + 8}$$

Observação: tg = sen/cos

$$\operatorname{tg} 60° = \frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}} = \sqrt{3}$$

	30°	45°	60°
tg	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
sen	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$

$\operatorname{tg} 1 = \frac{\pi}{2}$

$\operatorname{tg} 2 = \frac{\pi}{4}$

$\operatorname{tg} 3 = \frac{\pi}{6}$

não falta nada.

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Transformações de Diferenciais Trigonométricas

sen u cos u

$tg \frac{u}{2} = z$

partindo destas relações temos:

pla deduzir as relações

$\text{sen } u = \frac{2z}{1+z^2}$
$\cos u = \frac{1-z^2}{1+z^2}$
$du = \frac{2dz}{1+z^2}$
$tg \frac{u}{2} = z$

$$\begin{cases} \text{sen} \frac{u}{2} = \pm \sqrt{\frac{1-\cos u}{2}} \\ \cos \frac{u}{2} = \pm \sqrt{\frac{1+\cos u}{2}} \\ tg \frac{u}{2} = \frac{\text{sen} \frac{u}{2}}{\cos \frac{u}{2}} \end{cases}$$

Aplicações

1) $\int \frac{d\theta}{1+\text{sen}\,\theta + \cos\theta}$

Subst:

$\theta = u$

$d\theta = du$

$\int \frac{du}{1+\text{sen}\,u + \cos u}$

Subst. pelas relações dadas:

$$\int \frac{\frac{2\,dz}{1+z^2}}{1+\underbrace{\frac{2z}{1+z^2}}_{\text{sen }\theta}+\underbrace{\frac{1-z^2}{1+z^2}}_{\cos\theta}}$$

tirando o m.m.c:

$$\int \frac{\frac{2\,dz}{\cancel{1+z^2}}}{\frac{1+\cancel{z^2}+2z+1-\cancel{z^2}}{\cancel{1+z^2}}} = \int \frac{\cancel{2}\,dz}{\cancel{2}+\cancel{2}z} =$$

$$\int \frac{dz}{1+z} = \ln(1+z)+C$$

$z = tg\,\frac{u}{2}$, então:

$$\ln(1+z) = \ln\left(1+tg\,\frac{u}{2}\right)+C$$

$$= \boxed{\ln\left(1+tg\,\frac{\theta}{2}\right)+C}$$

② $\displaystyle\int \frac{dx}{\text{sen}\,x + tg\,x}$ $x = u$, $dx = du$, $tg\,x = \frac{\text{sen}\,x}{\cos x}$

$$\int \frac{du}{\text{sen}\,u + \frac{\text{sen}\,u}{\cos u}} =$$

$$\int \frac{\frac{2\,dz}{1+z^2}}{\frac{2z}{1+z^2}+\frac{\frac{2z}{\cancel{1+z^2}}}{\frac{1-z^2}{\cancel{1+z^2}}}}$$ tira-se o m.m.c

$$\int \frac{\frac{2\,dz}{\cancel{1+z^2}}}{\frac{2z-\cancel{2z^3}+2z+\cancel{2z^3}}{\cancel{(1+z^2)}\cdot(1-z^2)}} = \int \frac{2\,dz}{\frac{4z}{1-z^2}} =$$

$$\cancel{2}\int \frac{(1-z^2)\,dz}{\underset{2}{\cancel{4}}z} = \frac{1}{2}\int \frac{(1-z^2)\,dz}{z}$$

decompondo:

$$\frac{1}{2}\int \frac{dz}{z} - \frac{1}{2}\int \frac{z^{\cancel{2}}\,dz}{\cancel{z}}$$

$$\frac{1}{2}\ln z - \frac{1}{2}\int z\,dz$$

$$\frac{1}{2}\ln z - \frac{1}{2}\cdot\frac{z^2}{2} + C = \boxed{\frac{1}{2}\ln z - \frac{1}{4}z^2 + C}$$

$$\boxed{\frac{1}{2}\ln\left(tg\frac{u}{2}\right)-\frac{1}{4}tg^2\frac{u}{2}+C}$$

$$\boxed{\frac{1}{2}\ln\left(tg\frac{x}{2}\right)-\frac{1}{4}tg^2\frac{x}{2}+C}$$ resposta

③ $\int \frac{dx}{5+4\,\text{sen}\,2x}$

$2x = u$

$x=\frac{u}{2} \therefore x=\frac{1}{2}u$

$dx=\frac{1}{2}du$

$$\int \frac{\frac{1}{2}du}{5+4\,\text{sen}\,u} =$$

$$= \int \frac{\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}dz}{1+z^2}}{5+4.\frac{2z}{1+z^2}} = \int \frac{\frac{dz}{\cancel{1+z^2}}}{\frac{5+5z^2+8z}{\cancel{1+z^2}}}=$$

fatoração:

$$\int \frac{dz}{5z^2+8z+5} =$$

$5z^2+8z+5=$

$5\left(z^2+\frac{8}{5}z\right)+5$ ou $\cancel{5}\left[\left(z^2+\frac{8}{5}z\right)+1\right]=$

$\frac{z}{2}\cdot 2\cdot\frac{4}{5}$ ao quadrado

$$\left(z^2+\frac{8}{5z}+\frac{16}{25}\right)+1-\frac{16}{25} =$$

$$\left(z+\frac{4}{5}\right)^2+\frac{9}{25}$$

fórmula arctg

$$\frac{1}{5}\int\frac{dz}{\left(z+\frac{4}{5}\right)^2+\frac{9}{25}} = \frac{1}{5}\cdot\frac{1}{\frac{3}{5}}\cdot arctg\frac{z+\frac{4}{5}}{\frac{3}{5}}+C$$

$$=\frac{1}{3}\,arc\,\cancel{tg}\left(tg\frac{\frac{u}{2}+\frac{4}{5}}{\frac{3}{5}}\right)+C$$

$$=\frac{1}{3}\,arc\,tg\left(\frac{\frac{5\,tg\,x+4}{\cancel{5}}}{\frac{3}{\cancel{5}}}\right.$$

$$\boxed{=\frac{1}{3}\,arc\,tg\left(\frac{5\,tg\,x+4}{3}\right)+C}$$

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$$\int\frac{dx}{4+5\cos x}$$

$x = u$

$dx = du$

$$\int\frac{\frac{2dz}{1+z^2}}{4+5\left(\frac{1-z^2}{1+z^2}\right)} =$$

$$\int\frac{\frac{2dz}{1+z^2}}{4+\left(\frac{5-5z^2}{1+z^2}\right)} = \int\frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{4+4z^2+5-5z^2}{\cancel{1+z^2}}} =$$

$$\int \frac{2\,dz}{4+4z^2+5-5z^2} =$$

$$\int \frac{2\,dz}{-z^2+9} = -2\int \frac{dz}{9-z^2} = -2 \cdot \frac{1}{6} \ln \frac{3+z}{3-z}$$

$$-\frac{1}{3} \ln \frac{3+z}{3-z} = -\frac{1}{3} \ln\left(\frac{tg\,\frac{x}{2} \cdot 3}{tg\,\frac{x}{2}+3}\right) + C$$

$$-\left[\frac{1}{3} \ln\left(tg\,\frac{x}{2}-3\right) - \frac{1}{3} \ln\left(tg\,\frac{x}{2} \pm 3\right)\right]$$

$$-\frac{1}{3} \ln\left(tg\,\frac{x}{2}-3\right) + \frac{1}{3} \ln\left(tg\,\frac{x}{2}+3\right)$$

$$\frac{1}{3} \ln\left(\frac{tg\,\frac{x}{2}+3}{tg\,\frac{x}{2}-3}\right)$$

$$\int_{-1}^{0} \frac{dx}{4x^2-9} = \qquad dv = 2\,dx \quad v = 2x$$

$$v^2 \le 4 \therefore v^2 \le a \Rightarrow 9-4x^2$$

$$\frac{1}{2}\int \frac{dx}{(2x)^2-(3)^2} = -\frac{1}{2}\int_{-1}^{3} \frac{2\,dx}{9-4x^2} =$$

$$+\left[\frac{1}{2} \cdot \frac{1}{6} \ln \frac{3+2x}{3-2x}\right]_{-1}^{0} + C$$

$$-\left[\frac{1}{12} \ln \frac{3-2}{3+2}\right] = \left[-\frac{1}{12} \ln \frac{1}{5}\right] =$$

$$-\frac{1}{12} \ln 1 - \ln 5 = +\frac{1}{12} \ln 5$$

$$\int_0^{3a} \frac{2x\,dx}{(x^2-a^2)^{2/3}}$$

$a+\varepsilon$ $\quad$ $3a$

$$3\sqrt[3]{(a+\varepsilon)^2-a^2}$$

$$3\sqrt[3]{a^2-a^2} = 0$$

$$\int_0^{3a} \frac{2x\,dx}{(x^2-a^2)^{2/3}} = \lim_{\varepsilon \to 0} \int_0^{a-\varepsilon} \frac{2x\,dx}{(x^2-a^2)^{2/3}} +$$

$$\lim_{\varepsilon \to 0} \int_{a+\varepsilon}^{3a} \frac{2x\,dx}{(x^2-a^2)^{2/3}}$$

$$\lim_{\varepsilon \to 0} \int_0^{a-\varepsilon} (x^2-a^2)^{-2/3}\, x\,dx + \lim_{\varepsilon \to 0} \int_{a+\varepsilon}^{3a} (x^2-a^2)^{-2/3}$$

$$\lim_{\varepsilon \to 0} \left[\frac{(x^2-a^2)^{1/3}}{1/3}\right]_0^{a-\varepsilon} + \lim_{\varepsilon \to 0} \left[3\sqrt[3]{x^2-a^2}\right]_{a+\varepsilon}^{3a}$$

$$-3\sqrt[3]{-a^2}$$

$$\lim_{\varepsilon \to 0} \left[3\sqrt[3]{9a^2-a^2}\right]$$

$$3\sqrt{a^2(a-1)} = 3\sqrt[3]{8a^2}$$

$$6\sqrt[3]{a^2} + 3a^{2/3} =$$

$$6a^{2/3} + 3a^{2/3} = 9a^{2/3}$$

$$\int \frac{(x^3+1)\,dx}{x(x-1)^3} =$$

*

$$\frac{x^3+1}{x(x-1)^3} = \frac{A}{x} + \frac{B}{(x-1)^3} + \frac{C}{(x-1)^2} + \frac{D}{x-1}$$

$$x^3+1 = A(x-1)^3 + Bx + Cx(x-1) + Dx(x-1)^2$$

$$Ax^3 - 3Ax^2 + 3Ax - A + Bx + Cx^2 - Cx + Dx^3 -$$

$$2Dx^2 + Dx$$

$$(A+D)x^3 + (-3A+C-2D)x^2 + (3a+B-C+D)x - A$$

$A+D=1$ $\quad$ $D = 1-A$

$-3A+C-2D=0$ $\quad$ $D = 1+1 \therefore D=2$

$3A+B-C+D=0$

$-A=1 \quad A=-1$ $\quad$ $C = 3A+2D$

$C = -3+4$

$C=1$

$B = C-3A-D$

$B = 1+3-2$

$B=2$

$$\int \Delta = -\int\frac{dx}{x} + 2\int\frac{dx}{(x-1)^3} + \int\frac{dx}{(x-1)^2} + 2\int\frac{dx}{x-1}$$

$$\int \Delta = -\ln x + \cancel{2}\frac{(x-1)^{-2}}{-\cancel{2}} + \frac{(x-1)^{-1}}{-1} + 2(x-1)^{-1}$$

agrupando $\to 2\ln(x-1)$

$$\int \Delta = -\ln\frac{(x-1)^2}{x} - \frac{1}{(x-1)^2} - \frac{1}{x-1}$$

$$\int \Delta = \ln\frac{(x-1)^2}{x} - \frac{1+x-1}{(x-1)^2}$$

$$\int \Delta = \ln\frac{(x-1)^2}{x} - \frac{x}{(x-1)^2}$$

$$\int \frac{(4x^2+2x+8)\,dx}{x(x^2+2)^2} = x(x^2+2)^2(x+2)$$

$$\frac{(4x^2+2x+8)}{x(x^2+2)^2} = \frac{A}{x} + \frac{Bx+C}{(x^2+2)^2} + \frac{Dx+E}{x^2+2}$$

$= A(x^2+2)^2 + (Bx+C).x + (Dx+E)x.(x^2+2)$

$= Ax^4 + 4Ax^2 + 4A + Bx^2 + Cx + Dx^4 + 2Dx^2$

$+ Ex^3 + 2Ex$

$(A+D)x^4 + Ex^3 \; (4A+B+2D)x^2 + (C+2E)x + 4A$

$A+D=0 \qquad A=2$

$4A+B+2D=4 \qquad B=0$

$C+2E=2 \qquad C=2$

$4A=8 \qquad E=0$

$$\int \Delta = 2\int \frac{dx}{x} + 2\int \frac{dx}{(x^2+2)^2} - \int \frac{x\,dx}{x^2+2}$$

$$\int \Delta = 2\ln x - \ln(x^2+2) + \frac{1}{4}\left[\frac{x}{x^2+2} + \int \frac{dx}{x^2+2}\right]$$

$$\int \Delta = 2\ln x - \ln(x^2+2) + \frac{x}{4x^2+8} + \frac{1}{4}\cdot\frac{1}{\sqrt{2}}\,\mathrm{arctg}\,\frac{x}{\sqrt{2}}$$

$$\int \Delta = 2\ln x - \ln(x^2+2) + \frac{x}{4x^2+8} + \frac{1}{4\sqrt{2}}\,\mathrm{arctg}\,\frac{x\sqrt{2}}{2}$$

$$\int \Delta = \ln\frac{x^2}{x^2+2} + \frac{x}{4x^2+8} + \frac{\sqrt{2}}{8}\,\mathrm{arctg}\,\frac{x\sqrt{2}}{2}$$

$$\int_3^{29} \frac{(x-2)^{2/3}\,dx}{(x-2)^{2/3}+3}$$ (no caderno já foi resolvido)

28/05/73

$$\int_0^1 \frac{(4x^2+2x)\,dx}{(x^2+1)(x+1)^2} =$$

$$\frac{(4x^2+2x)\,dx}{(x^2+1)(x+1)^2} =$$

$$\frac{4x^2+2x}{(x^2+1)(x+1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{(x+1)^2} + \frac{D}{x+1}$$

$$\frac{4x^2+2x}{(x^2+1)(x+1)^2} = \frac{(Ax+B)(x+1)^2 + C(x^2+1) + D(x^2+1)(x+1)}{(x^2+1)(x+1)^2}$$

$\boxed{A=2 \quad D=-2 \quad B=1 \quad C=1}$

$$\int_0^1 \Delta = \int_0^1 \frac{(2x+1)\,dx}{x^2+1} + \int_0^1 \frac{dx}{(x+1)^2} - \int_0^1 \frac{2dx}{x+1}$$

$$\int_0^1 \Delta = \int_0^1 \frac{2x\,dx}{x^2+1} + \int_0^1 \frac{dx}{x^2+1} + \int_0^1 \frac{dx}{(x+1)^2}.$$

TD

$$-2\int_0^1 \frac{dx}{x+1} =$$

$$\int_0^1 A = \left[\ln(x^2+1) + \text{arc tg } x - \frac{1}{x+1} - 2\ln(x+1)\right]_0^1$$

substituindo:

$$\left[\ln(1+1) + \text{arc tg } 1 - \frac{1}{1+1} - 2\ln(1+1)\right] -$$

$$\left[\ln(0+1) + \text{arc tg } 0 - \frac{1}{0+1} - 2\ln(0+1)\right]$$

$$\left[\ln 2 + \text{arctg } 1 - \frac{1}{2} - 2\ln 2\right] - \left[\cancel{\ln 1} +\right.$$

$$\left.\cancel{\text{arc tg } 0} - \frac{1}{1} - \cancel{2\ln 1}\right] =$$

$$\boxed{-\ln 2 + \frac{\pi}{4} + \frac{1}{2}}$$

Oração

São Judas

eu possa louvar a Deus convosco e com todos os eleitos por toda a eternidade.

Eu vos prometo, ó bendito Judas lembrar-me sempre deste grande favor e nunca deixar de vos honrar, como meu especial e poderoso patrono, e fazer tudo o que estiver à meu alcance para incentivar a devoção para convosco. Amem. São Judás, rogai por nós e por todos os que vos honram e invocam o vosso auxílio! (3 Padre Nossos, 3 Ave Marias, 3 Glória Patri).