fórmulas do arco duplo

$$\boxed{\begin{aligned} \operatorname{sen}^2 u &= \frac{1}{2} - \frac{1}{2}\cos 2u \\ \cos^2 u &= \frac{1}{2} + \frac{1}{2}\cdot\cos 2u \\ \operatorname{sen} u \cdot \cos u &= \frac{1}{2}\cdot \operatorname{sen} 2u \end{aligned}}$$

para qdo o m e n são pares

Ex: $\int \cos^2 x \operatorname{sen}^4 x$ (resolver)

continuando:

sen do arco duplo

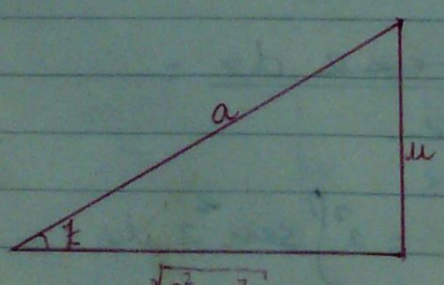
$$a^2\int\left(\frac{1}{2} - \frac{1}{2}\cos 2z\right) =$$

$$\frac{a^2}{2}\int dz - \frac{a^2}{2\cdot 2}\int \cos 2z \, 2\, dz =$$

$$\boxed{\frac{a^2}{2}\cdot z - \frac{a^2}{4}\operatorname{sen} 2z + c}$$

partindo da 2ª subst.

$$\boxed{\begin{aligned} \operatorname{sen} z &= \frac{u}{a} \\ z &= \operatorname{arc\,sen} \frac{u}{a} \end{aligned}}$$



$$\frac{a^2}{2}\cdot z - \frac{a^2}{4}\operatorname{sen} 2z + c = \frac{4}{2}\cdot \operatorname{arc\,sen}\frac{t}{2} - \frac{4}{4}\cdot 2\frac{u}{a}\cdot\frac{\sqrt{a^2-u^2}}{a} + c$$

$$\boxed{\operatorname{sen} 2z = 2\operatorname{sen} z \cdot \cos z}$$

$$\boxed{2\operatorname{arc\,sen}\frac{t}{2} - \frac{t}{2}\sqrt{4-t^2} + c}$$

Revisão

① $$\int \frac{t^3\,dt}{\sqrt{a^4+t^4}} = \int (a^4+t^4)^{-1/2}\, t^3\,dt =$$

$$\frac{1}{4}\int (a^4+t^4)^{-1/2}\, 4t^3\,dt =$$

$$\frac{1}{4}\left[\frac{a^4+t^4}{\frac{1}{2}}\right]^{\frac{1}{2}} + c = \frac{2\sqrt{a^4+t^4}}{4_2} + c$$

$$\boxed{\frac{\sqrt{a^4+t^4}}{2} + c}$$

fórmulas do arco duplo

$$\boxed{\begin{aligned} sen^2 u &= \frac{1}{2} - \frac{1}{2}\cos 2u \\ \cos^2 u &= \frac{1}{2} + \frac{1}{2}\cdot\cos 2u \\ sen\, u \cdot \cos u &= \frac{1}{2}\cdot sen\, 2u \end{aligned}}$$

para qdo m e n são pares

Ex: $\int \cos^{2} x \; sen^{4} x$ (resolver)

continuando:

sen do arco duplo

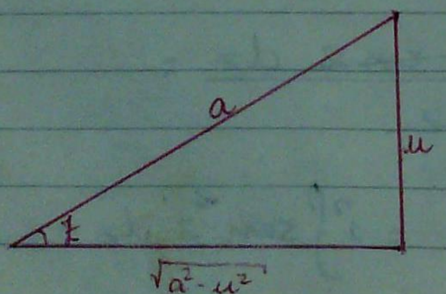
$$a^2\int\left(\frac{1}{2} - \frac{1}{2}\cos 2z\right) =$$

$$\frac{a^2}{2}\int dz - \frac{a^2}{2\cdot 2}\int \cos 2z \; 2\,dz =$$

$$\boxed{\frac{a^2}{2}\cdot z - \frac{a^2}{4}\, sen\, 2z + c}$$

partindo da 2ª subst.

$$\boxed{\begin{aligned} sen\, z &= \frac{u}{a} \\ z &= arc\; sen\, \frac{u}{a} \end{aligned}}$$



$$\frac{a^2}{2}\cdot z - \frac{a^2}{4}\, sen\, 2z + c = \frac{4}{2}\cdot arc\; sen\,\frac{t}{2} - \frac{4}{4}\cdot 8\,\frac{u}{a}\cdot\frac{\sqrt{a^2-u^2}}{a} + c$$

$$\boxed{sen\, 2z = 2\, sen\, z \cdot \cos z}$$

$$\boxed{2\; arc\; sen\, \frac{t}{2} - \frac{t}{2}\sqrt{4-t^2} + c}$$

Revisão

① $$\int \frac{t^3\,dt}{\sqrt{a^4+t^4}} = \int (a^4+t^4)^{-1/2}\, t^3\,dt =$$

$$\frac{1}{4}\int (a^4+t^4)^{-1/2}\, 4t^3\,dt =$$

$$\frac{1}{4}\,\frac{[a^4+t^4]^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{\cancel{2}\sqrt{a^4+t^4}}{\cancel{4}_{2}} + c =$$

$$\boxed{\frac{\sqrt{a^4+t^4}}{2} + c}$$

② $\int \frac{(2x+3)\,dx}{\sqrt{x^2+3x}} =$

$\int (x^2+3x)^{-1/2} \cdot (2x+3)\,dx =$

$\int v^n dv = \frac{v^{n+1}}{n+1} \Rightarrow \frac{(x^2+3x)^{1/2}}{\frac{1}{2}} + c =$

$\boxed{2\sqrt{x^2+3x} + c}$

---

③ $\int \frac{(2x+3)}{x+2}\,dx =$

$\int (2x+3) : (x+2) = \int 2\,dx - \frac{1}{x+2}\,dx \Rightarrow$

$$\begin{array}{r|l} 2x+3 & x+2 \\ \hline -2x-4 & 2 - \frac{1}{x+2} \\ \hline / \quad -1 & \\ +1 & \\ \hline 0 & \end{array}$$

$\int 2\,dx - \int \frac{dx}{x+2} =$

$2\int dx - \int \frac{dx}{x+2} =$

$\boxed{2x - \ln(x+2) + c}$

---

④ $\int \frac{x^2\,dx}{(2+x^3)} = \frac{1}{3}\int \frac{3x^2\,dx}{(2+x^3)} \Rightarrow \frac{1}{3}\ln(2+x^3)$

$\frac{1}{3}\int (2+x^3)^{-1} \cdot 3x^2\,dx =$

$\boxed{\frac{1}{3}\ln(2+x^3) + c}$

---

⑤ $\int 6e^{3x}dx$

fórmula: $\int e^v dv = e^v + c$

$\frac{6}{3}\int e^{3x}\, 3\,dx = \boxed{2e^{3x} + c}$

② $\int \frac{(2x+3)\,dx}{\sqrt{x^2+3x}} =$

$\int (x^2+3x)^{-1/2} \cdot (2x+3)\,dx =$

$\int v^n dv = \frac{v^{n+1}}{n+1} \Rightarrow \frac{(x^2+3x)^{1/2}}{\frac{1}{2}} + c =$

$\boxed{2\sqrt{x^2+3x} + c}$

---

③ $\int \frac{(2x+3)}{x+2}\,dx =$

$\int (2x+3) : (x+2) = \int 2\,dx - \frac{1}{x+2}\,dx \Rightarrow$

$$\begin{array}{r|l} 2x+3 & x+2 \\ \hline -2x-4 & 2 - \frac{1}{x+2} \\ \hline / \quad -1 & \\ +1 & \\ \hline 0 & \end{array}$$

$\int 2\,dx - \int \frac{dx}{x+2} =$

$2\int dx - \int \frac{dx}{x+2} =$

$\boxed{2x - \ln(x+2) + c}$

---

④ $\int \frac{x^2\,dx}{(2+x^3)} = \frac{1}{3}\int \frac{3x^2\,dx}{(2+x^3)} \Rightarrow \frac{1}{3}\ln(2+x^3)+c$

$\frac{1}{3}\int (2+x^3)^{-1} \cdot 3x^2\,dx =$

$\boxed{\frac{1}{3}\ln(2+x^3) + c}$

---

⑤ $\int 6e^{3x}dx$

fórmula: $\int e^v dv = e^v + c$

$\frac{6}{3}\int e^{3x}\,3\,dx = \boxed{2e^{3x}+c}$

outra solução: $2\int e^{3x} \cdot 3\,dx =$

$$\boxed{2e^{3x} + C}$$

⑥ $\int (ctg^2 2\theta + ctg^4 2\theta)\,d\theta =$

12/3/73

sen z = $\dfrac{\text{cateto oposto}}{\text{hipotenusa}} \to \dfrac{b}{a}$

cosseno z = $\dfrac{\text{cateto adjacente}}{\text{hipotenusa}} \to \dfrac{c}{a}$

tangente z = $\dfrac{\text{cateto oposto}}{\text{cateto adjacente}} \to \dfrac{b}{c}$

cotangente z = $\dfrac{\text{cateto adjacente}}{\text{cateto oposto}} \to \dfrac{c}{b}$

$\int (tg\,bt - ctg\,bt)^3\,dt$

$\int tg^3\,bt - 3\int tg^2\,bt \cdot ctg\,bt\,dt + 3\int tg\,bt.$

$ctg^2\,bt - \int ctg^3\,bt.$

desenv. de $\int tg^3 bt\,dt \to \int tg^2\,bt \cdot tg\,bt\,dt =$

$\int (sec^2 - 1)\,tg\,bt\,dt.$

$\frac{1}{b}\int tg\,bt\,sec^2\,bt \cdot b\,dt - \frac{1}{b}\int tg\,bt\,dt_{(b)}$

$$\boxed{\frac{1}{b}\,\frac{tg^2 bt}{2} + \frac{1}{b}\ln \cos bt.}$$

desenv. de: $-3\int tg^2 bt . ctg\, bt =$

$-3\int tg^2 bt . \frac{1}{tg\, bt}\, dt =$

$-\frac{3}{b}\int tg\, bt . b\, dt =$

$\frac{3}{b}\ln \cos bt.$

desenv. de: $+3\int tg\, bt . ctg^2 bt =$

$3\int \frac{1}{ctg\, bt} . ctg^2 bt\, dt =$

$= \frac{3}{b}\int ctg\, bt\; b\, dt =$

$\frac{3}{b}\ln sen\, bt$

$-\int ctg^3 bt = \int ctg^2 bt . ctg\, bt\, dt$

$\int (cossec^2 bt - 1)\, ctg\, bt\, dt =$

$\frac{1}{b}\int ctg\, bt . (-b)\, cossec^2 bt\, dt + \frac{1}{b}\int ctg\, bt\, b\, dt$

$+\frac{1}{b}\,\frac{ctg^2 bt}{2} + \frac{1}{b}\ln sen\, dt.$

Resultado do exercício:

$\frac{1}{2b} . tg^2 bt + \frac{1}{b}\ln \cos bt + \frac{3}{b}\ln \cos bt$

$+ \frac{3}{b}\ln sen\, bt + \frac{1}{2b} ctg^2 bt + \frac{1}{b}\ln sen\, bt + c$

somando os termos semelhantes:

$\frac{1}{2b}(tg^2 bt + ctg^2 bt) + \frac{4}{b}(sen\, bt + \frac{4}{b}$

$\ln \cos bt + \frac{4}{b}\ln\ sen\, bt + c$

desenvolvimento

$\ln a + \ln b = \ln(ab)$

$\frac{4}{b}(\ln \cos bt + \ln sen\, bt)$

$\frac{4}{b}\ln \cos bt . sen\, bt = \frac{4}{b}\ln \frac{sen\, 2bt}{2}$

resposta:

$$\boxed{\frac{1}{2b}(tg^2 bt + ctg^2 bt) + \frac{4}{b} . \ln sen\, bt - \ln 2 + c}$$

$=$

$\frac{1}{2b}(tg^2 bt + ctg^2 bt) + \frac{4}{b}\ln sen\, 2bt + c$

$$\int \frac{dx}{x\sqrt{x^2+4}}$$

1

$u = a\,tg\,z$

$a \sec z$

02

13/3/1973

$\text{sen}\, u \cdot \cos u = \frac{1}{2}\,\text{sen}\, u$

$\text{sen}^2 u = \frac{1}{2} - \frac{1}{2}\cos 2u$

$\cos^2 u = \frac{1}{2} + \frac{1}{2}\cos 2u$

qdo for escpoente par usa-se esta substituição.

$$\int \cos^2 u \cdot du$$

$$\int \left(\frac{1}{2} + \frac{1}{2}\cos 2u\right) du$$

$$\frac{1}{2}\int du + \frac{1}{2 \cdot 2}\int \cos 2u\,(2)\frac{1}{2}\,du$$

$$\boxed{\frac{1}{2}u + \frac{1}{4}\,\text{sen}\, 2u + c}$$

$$\int \text{sen}^2 x \cos^2 a\, dx$$

$\text{sen}^2 u \cdot \cos^2 u = \frac{1}{4}\,\text{sen}^2 2u$ (substituição)

$$\frac{1}{4}\int \text{sen}^2 2u\, du =$$

$$\frac{1}{4}\int \left(\frac{1}{2} - \frac{1}{2}\cos 2(2u)\right) du$$

$$\frac{1}{4}\int\left(\frac{1}{2}-\frac{1}{2}\cos 4u\right)du =$$

$$\frac{1}{8}\int du - \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{1}{4}\int \cos 4u \, du =$$

$$\frac{u}{8} - \frac{1}{32}\,\text{sen}\, 4u + c$$

$$\boxed{\frac{1}{8}\left(u - \frac{1}{4}\,\text{sen}\,4u\right) + c}$$

$$\int \text{sen}^2 x \, dx = \int\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx$$

$$\frac{1}{2}\int dx - \frac{1}{2.2}\int \cos 2x \, dx$$

$$\boxed{\frac{1}{2}x - \frac{1}{4}\,\text{sen}\,2x + c}$$

$$\int \text{sen}^4 x \, dx =$$

$$\int (\text{sen}^2 x)^2 dx$$

$$\int\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)^2 dx$$

(obs: ao quadrado é + difícil, partir p/a outro)

pg 270

$$\int \text{sen}^2 x \,.\, \text{sen}^2 x \, dx =$$

$$\int (1-\cos^2)\,\text{sen}^2 \, dx$$

$$\int \text{sen}^2 x \, dx - \int \cos^2 x \, \text{sen}^2 x \, dx$$

subst. na fórmula

$$\int\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx - \int\left(\frac{1}{2}\,\text{sen}\,2x\right)^2 dx$$

$$\frac{1}{2}\int dx - \frac{1}{2.2}\int \cos 2x\,(2)\,dx - \frac{1}{4}\int \text{sen}^2 2x \, dx =$$

$$\frac{x}{2} - \frac{1}{4}\,\text{sen}\,2x - \frac{1}{4}\int\left(\frac{1}{2}-\frac{1}{2}\cos 2.2x\right)dx =$$

$$\frac{x}{2} - \frac{1}{4}\,\text{sen}\,2x - \frac{1}{4}\cdot\frac{1}{2}\int dx + \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{2}\,\text{sen}\,4x + c$$

$$\frac{x}{2} - \frac{1}{4}\,\text{sen}\,2x - \frac{x}{8} + \frac{1}{32}\,\text{sen}\,4x + c$$

$$\boxed{\frac{3x}{8} - \frac{\text{sen}\,2x}{4} + \frac{\text{sen}\,4x}{32} + c}$$

$$\int \cos^4 x \, dx$$

$$\int (\cos^2 x).(\cos^2 x)\, dx$$

$\frac{1}{4}\int \left(\frac{1}{2} - \frac{1}{2}\cos 4u\right) du =$

$\frac{1}{8}\int du - \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{1}{4}\int \cos 4u \, du =$

$\frac{u}{8} - \frac{1}{32}\operatorname{sen} 4u + c$

$\boxed{\frac{1}{8}\left(u - \frac{1}{4}\operatorname{sen} 4u\right) + c}$

$\int \operatorname{sen}^2 x \, dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$\frac{1}{2}\int dx - \frac{1}{2.2}\int \cos 2x \, dx$

$\boxed{\frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x + c}$

$\int \operatorname{sen}^4 x \, dx =$

$\int (\operatorname{sen}^2 x)^2 dx$

$\int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)^2 dx$ (obs: ao quadrado é + difícil, partir p/a outro)

pg 270 $\int \operatorname{sen}^2 x \,.\, \operatorname{sen}^2 x \, dx =$

$\int (1 - \cos^2) \operatorname{sen}^2 dx$

$\int \operatorname{sen}^2 x \, dx - \int \cos^2 x \operatorname{sen}^2 x \, dx$

subst. na fórmula

$\int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx - \int \left(\frac{1}{2}\operatorname{sen} 2x\right)^2 dx$

$\frac{1}{2}\int dx - \frac{1}{2.2}\int \cos 2x^{(2)} dx - \frac{1}{4}\int \operatorname{sen}^2 2x \, dx =$

$\frac{x}{2} - \frac{1}{4}\operatorname{sen} 2x - \frac{1}{4}\int \left(\frac{1}{2} - \frac{1}{2}\cos 2.2x\right) dx =$

$\frac{x}{2} - \frac{1}{4}\operatorname{sen} 2x - \frac{1}{4}\cdot\frac{1}{2}\int dx + \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{2}\operatorname{sen} 4x + c$

$\frac{x}{2} - \frac{1}{4}\operatorname{sen} 2x - \frac{x}{8} + \frac{1}{32}\operatorname{sen} 4x + c$

$\boxed{\frac{3x}{8} - \frac{\operatorname{sen} 2x}{4} + \frac{\operatorname{sen} 4x}{32} + c}$

$\int \cos 4x \, dx$

$\int (\cos^2 x).(\cos^2 x) \, dx$

$$\int (1-\operatorname{sen}^2 x)\cdot \cos^2 x\, dx =$$

$$\int \cos^2 x\, dx - \int \operatorname{sen}^2 x \cdot \cos^2 x =$$

$$\int \left(\frac{1}{2}+\frac{1}{2}\cos 2x\right) dx - \int \left(\frac{1}{2}\cdot \operatorname{sen} 2x\right)^2 dx$$

$$\frac{1}{2}\int dx + \frac{1}{2.2}\int \cos 2x\,(2)\, dx - \frac{1}{4}\int (\operatorname{sen}^2 2x)\, dx$$

$$\frac{x}{2} + \frac{1}{2.2}\operatorname{sen} 2x - \frac{1}{4}\int\left(\frac{1}{2}-\frac{1}{2}\cdot\cos 4x\right) dx =$$

$$\frac{x}{2} + \frac{\operatorname{sen} 2x}{4} - \frac{1}{4}\cdot\frac{1}{2}\int dx + \frac{1}{8}\cdot\frac{1}{4}\int \cos 4x\, dx$$

$$\frac{x}{2} + \frac{\operatorname{sen} 2x}{4} - \frac{x}{8} + \frac{1}{32}\cdot \operatorname{sen} 4x + c$$

$$\boxed{-\frac{3x}{8} + \frac{\operatorname{sen} 2x}{4} + \frac{\operatorname{sen} 4x}{32} + c}$$

$$\int \operatorname{sen}^6 x\, dx$$

$$\int \operatorname{sen}^2 x \cdot \operatorname{sen}^4 x\, dx =$$

$$\int (1-\cos^2 x)\, \operatorname{sen}^4 x\, dx =$$

$$\int \operatorname{sen}^4 x\, dx - \int \cos^2 x \cdot \operatorname{sen}^2 x\, dx$$

$$-\int \cos^2 x \cdot \operatorname{sen}^2 x\, dx =$$

$$-\int \left(\frac{1}{4}\operatorname{sen}^2 2x \left(\frac{1}{2}-\frac{1}{2}\cos 2x\right) dx\right.$$

$$-\frac{1}{8}\int \operatorname{sen}^2 2x\, dx + \frac{1}{8}\int \operatorname{sen}^2 2x \cos 2x\, dx.$$

$$\Downarrow$$

$$\frac{1}{2}(\operatorname{sen} 2x)^2 \cdot \cos 2x \cdot 2dx$$

$$-\frac{1}{8}\int\left(\frac{1}{2}-\frac{1}{2}\cdot\cos 2x\right) dx + \frac{1}{16}\int \operatorname{sen}^2 2x \cdot \cos 2x\, dx$$

$$+\int \operatorname{sen}^4 x\, dx - \frac{1}{8}\int (\operatorname{sen}^2 2x)^2\, dx + \frac{1}{16}\int (\operatorname{sen} 2x)^2 \cdot$$

$$\cos x \cdot 2\, dx =$$

$$+\frac{3x}{8} + \frac{\operatorname{sen} 2x}{4} + \frac{\operatorname{sen} 4x}{32} - \int\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)$$

$$dx + \frac{1}{16}\,\frac{\operatorname{sen}^3 2x}{3} =$$

$$\frac{3x}{8} + \frac{\operatorname{sen} 2x}{4} + \frac{\operatorname{sen} 4x}{32} - \frac{x}{2} - \frac{1}{2.2}\operatorname{sen} 2x \rightarrow$$

$+ \frac{1}{16} \frac{sen^3 2x}{3} + c$

$-\frac{x}{8} + \frac{\cancel{sen} 2x}{4} + \frac{sen 4x}{32} - \frac{sen 2x}{\cancel{4}} +$

$\frac{sen^3 2x}{48} - \frac{x}{8} + \frac{sen 4x}{32} + \frac{sen^3 2x}{48} + c$

16/3/73 Resolução do exercício anterior:

$\int sen^6 x \, dx$

$\int sen^2 x . sen^4 x \, dx$

$\int (1 - cos^2 x) \, sen^4 x \, dx$

$\int sen^4 x \, dx - \int cos^2 x \, sen^4 x \, dx$

resolve-se separadamente cada integral.

$\underbrace{\int sen^2 x \, sen^2 x \, dx}_{I} - \underbrace{\int (cos^2 x . sen^2 x) sen^2 x \, dx}_{II}$

(I) $\int sen^2 x \, sen^2 x \, dx =$

$\int (1 - cos^2 x) \, sen^2 x \, dx$

$\int sen^2 x \, dx - \int sen^2 x . cos^2 x \, dx$

arco duplo

$sen^2 x$ $\frac{1}{2} sen 2x$

$\int \left(\frac{1}{2} - \frac{1}{2} cos 2x\right) dx - \frac{1}{4} sen^2 2x \, dx$

$cos 2x = cos 2x + 4x$

$\frac{1}{2}\int dx - \frac{1}{2} \cdot \frac{1}{2} \int cos 2x \cdot 2 \, dx - \frac{1}{4} \int \left(\frac{1}{2} - \frac{1}{2} cos 4x\right) dx$

integral de

$\frac{x}{2} - \frac{1}{4} sen 2x - \frac{x}{8} + \frac{sen 4x}{32}$

auxiliar $\left[\frac{1}{2} \cdot \frac{1}{4} \int cos 4x \, dx\right] \Rightarrow \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} \int cos 4x \, 4 \, dx = \frac{1}{32} sen 4x$

$\boxed{\frac{3x}{8} - \frac{sen 2x}{4} + sen \frac{4x}{32}}$ (m.m.c) resultado de I

Resolvendo a II:

$\int (cos^2 x . sen^2 x) \, sen^2 x \, dx$

$-\int \left(\frac{1}{2} . sen 2x\right)^2 . (sen^2 x \, dx$

$-\frac{1}{4}\ sen^2 2x \cdot sen^2 x\ dx$

$-\frac{1}{4}\int\left(\frac{1}{2}-\frac{1}{2}\cos 4x\right)\cdot sen^2 x\ dx =$

$-\frac{1}{8}\int sen^2 x\ dx + \frac{1}{8}\int \cos 4x\ sen^2 x\ dx$

$\int sen^2 x\ dx \rightarrow \frac{x}{2} - \frac{1}{4}\cdot 2x$ (já resolvido anterior/)

continuando: $\boxed{-\frac{1}{8}\left(\frac{x}{2} - \frac{sen 2x}{4}\right)} + \boxed{\frac{1}{8}\int \cos 4x\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx} =$

A B

$\rightarrow B$

desenv. de A

$\frac{1}{16}\int \cos 4x - \frac{1}{16}\int \cos 4x \cdot \cos 2x$

processo mto complicado

16

mais simples:

$-\frac{1}{4}\int sen^2 2x\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$-\frac{1}{8}\int sen^2 2x\ dx + \frac{1}{8\cdot 2}\int sen^2 2x \cos 2x\ dx$

$-\frac{1}{8}\int\left(\frac{1}{2} - \frac{1}{2}\cos 4x\right)dx + \frac{1}{16}\cdot\frac{sen^3 2x}{3}$

$\int \cos 4x$

$-\frac{x}{16} + \frac{sen 4x}{64} + \frac{sen^3 2x}{48}$ (II)

$+\frac{3x}{8} + \frac{sen 4x}{32} - \frac{sen 2x}{4}$ (I)

$$\boxed{\frac{5x}{16} + \frac{3\ sen 4x}{64} - \frac{sen 2x}{4} + \frac{sen^3 2x}{48} + C}$$

tira-se o m.m.c e soma-se

resposta:

Integração por partes

$$\boxed{d(uv) = u\,dv + v\,du}$$

$$\boxed{u\cdot dv = -v\cdot du + d(u\cdot v)}$$

Aplicações:

$\int dx = x$

$\int d(uv) = uv$

$\int u\,dv = uv - \int v\,du$

$-\frac{1}{4}\ \text{sen}^2 2x \cdot \text{sen}^2 x\, dx$

$$\cancel{-\frac{1}{4}\int\left(\frac{1}{2}-\frac{1}{2}\cos 4x\right)\cdot \text{sen}^2 x\, dx =}$$

$$\cancel{-\frac{1}{8}\int \text{sen}^2 x\, dx + \frac{1}{8}\int \cos 4x\ \text{sen}^2 x\, dx}$$

$\cancel{\int \text{sen}^2 x\, dx \Rightarrow \frac{x}{2} - \frac{1}{4}\cdot 2x}$ ~~(já resolvido anteriormente)~~

~~continuando:~~ $\cancel{\boxed{-\frac{1}{8}\left(\frac{x}{2}-\frac{\text{sen}\,2x}{4}\right)}_A + \boxed{\frac{1}{8}\int \cos 4x \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx}_B =}$

~~desenv. de A~~

$$\cancel{\frac{1}{16}\int \cos 4x - \frac{1}{16}\int \cos 4x \cdot \cos 2x}$$

16 processo mto complicado

mais simples:

$$-\frac{1}{4}\int \text{sen}^2 2x\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx$$

$$-\frac{1}{8}\int \text{sen}^2 2x\, dx + \frac{1}{8\cdot 2}\int \text{sen}^2 2x \cos 2x^2\, dx$$

$$-\frac{1}{8}\int\left(\frac{1}{2}-\frac{1}{2}\cos 4x\right)dx + \frac{1}{16}\cdot\frac{\text{sen}^3 x}{3}$$

$\int \cos 4x$

$$-\frac{x}{16} + \frac{\text{sen}\,4x}{64} + \frac{\text{sen}^3 2x}{48} \quad (\text{II})$$

$$+\frac{3x}{8} + \frac{\text{sen}\,4x}{32} - \frac{\text{sen}\,2x}{4} \quad (\text{I})$$

$$\boxed{\frac{5x}{16} + \frac{3\,\text{sen}\,4x}{64} - \frac{\text{sen}\,2x}{4} + \frac{\text{sen}^3 2x}{48} + C}$$

tira-se o m.m.c e soma-se

resposta:

Integração por partes

$$\boxed{d(uv) = u\,dv + v\,du}$$

$$\boxed{u\cdot dv = -v\cdot du + d(u\cdot v)}$$

Aplicações:

$\int dx = x$

$\int d(uv) = uv$

$$\int u\,dv = uv - \int v\,du$$

$\int x \cos x \, dx$

$dv = \cos x$

$u = x$

$v = \operatorname{sen} x$ (derivada do cos.)

$\begin{cases} u = x \\ du = dx \end{cases}$

aplicando a fórmula:

$\boxed{\int u\,dv = u\,v - \int v\,du}$

$\int x . \cos x \, dx = x \operatorname{sen} x - \int \operatorname{sen} x . dx$

$\boxed{= x \operatorname{sen} x + \cos x + c}$

$\int x \operatorname{sen} x \, dx$

17/03/73

$u = x$

$v = -\cos x$

$du = dx$

$dv = \operatorname{sen} x$.

fórmula: $\boxed{\int u\,dv = uv - \int v\,du}$

$\int x \operatorname{sen} x \, dx = x . (-\cos x) - \int -\cos x \, dx$

$\int x \operatorname{sen} x \, dx = -x . \cos x + \int \cos x \, dx$

$\int x \operatorname{sen} x \, dx = \boxed{-x . \cos x + \operatorname{sen} x + c}$

$\int \underbrace{\operatorname{arc\,tg} x}_{u} \underbrace{dx}_{dv}$

$u = \operatorname{arc\,tg} x$ $\quad du = \dfrac{dx}{1+x^2}$ → derivada de arc tg

$dv = dx$ $\quad v = x$

fórmula: $\boxed{\int u\,dv = u.v - \int v\,du}$

$\int \operatorname{arc\,tg} x = x . \operatorname{arctg} x - \int x . \dfrac{dx}{1+x^2} =$

$\int \operatorname{arctg} x = x . \operatorname{arc\,tg} x - \frac{1}{2} \int \dfrac{2x\,dx}{1+x^2} =$

$\int \operatorname{arc\,tg} x = \boxed{x \operatorname{arc\,tg} x - \frac{1}{2} . \ln(1+x^2) +}$

$\int x^2 e^x \, dx =$

$u = x^2$ $\quad du = 2x\,dx$ (diferencial e id

$dv = e^x dx$ $\quad v = e^x$

$$\int x^2 e^x \, dx = x^2 \cdot e^x - \boxed{\int e^x \cdot 2x \, dx}$$

$u = 2x$ $\quad dv = e^x \, dx$

$du = 2dx$ $\quad v = e^x$

$x^2 e^x - 2x e^x + 2\int e^x \cdot dx$

$x^2 e^x - 2x e^x + 2 \cdot e^x + C$

$$\boxed{e^x (x^2 - 2x + 2) + C}$$

$$\int \ln x \, dx$$

dif $\begin{cases} u = \ln x \\ du = \dfrac{dx}{x} \end{cases}$ $\quad \begin{cases} v = x \\ dv = dx \end{cases}$ derivada

$$\int \ln x \, dx = \ln x \cdot x - \int \cancel{x} \frac{dx}{\cancel{x}}$$

$$\int \ln x \, dx = x \cdot \ln x - \int dx$$

$$\int \ln x \, dx = \boxed{x (\ln x - 1) + C}$$

$$\int \underset{u}{x} \, \underbrace{\operatorname{sen} \frac{x}{2} \, dx}_{dv}$$

derivar $\begin{cases} u = x \\ du = dx \end{cases}$ integrar $\begin{cases} v = -2 \cos \dfrac{x}{2} \\ dv = \operatorname{sen} \dfrac{x}{2} \end{cases}$ $\quad -2 \int \operatorname{sen} \dfrac{x}{2} \cdot \dfrac{1}{2} dx$

$\boxed{-2 \cos \dfrac{x}{2}} \, dv$

<u>resolvendo:</u>

$$\int x \operatorname{sen} \frac{x}{2} \, dx = -2x \cdot \cos \frac{x}{2} + 2 \int \cos \frac{x}{2} \frac{1}{2} dx =$$

constante

$$\int x \operatorname{sen} \frac{x}{2} \, dx = -2x \cos \frac{x}{2} + 2 \cdot 2 \operatorname{sen} \frac{x}{2} + C$$

$$\int x \operatorname{sen} \frac{x}{2} \, dx = \boxed{-2x \cos \frac{x}{2} + 4 \operatorname{sen} \frac{x}{2} + C}$$

$$\int \overset{u}{u} \, \overset{dv}{\sec^2 u \, du}$$

$\begin{cases} u = u \\ du = du \end{cases}$ $\quad \begin{cases} dv = \sec^2 u \, dx \\ v = \operatorname{tg} u \end{cases}$

resolvendo:

$$\int u \sec^2 u . du = u . tg\, u - \int tg\, u\, du$$

$$\int u \sec^2 u\, du = \boxed{u\, tg\, u + \ln \cos u + c}$$

$$\int v \operatorname{sen}^2 3v\, dv =$$

$\begin{cases} u = v \\ du = dv \end{cases}$

$dv = \operatorname{sen}^2 3v$ (pra integrar precisa desmembrar)

$v = \frac{v}{2} - \frac{1}{12} \operatorname{sen} 6v$

$\int \operatorname{sen}^2 = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6v\right) dv$

$= \frac{v}{2} - \frac{1}{12} \operatorname{sen} 6v$

OBS: Se fosse: sen^3 a integração seria:

$$\int (1 - \cos^2 v) \operatorname{sen} v =$$

$$\int \operatorname{sen} v - \int \underset{v^N}{\cos^2 v} . \underset{dv}{\operatorname{sen} v}$$

Substituindo:

$+\frac{1}{12}\int \operatorname{sen} 6v . 6dv$

$$\int u\, dv = v . \left(\frac{v}{2} - \frac{\operatorname{sen} 6v}{12}\right) - \int \left(\frac{v}{2} - \frac{\operatorname{sen} 6v}{12}\right)^{6} dv$$

falta

$$\frac{v^2}{2} - v \frac{\operatorname{sen} 6v}{12} - \frac{v^2}{4} + \frac{\cos 6v}{72} + c$$

$$\boxed{\frac{v^2}{4} - v\frac{\operatorname{sen} 6v}{12} - \frac{\cos 6v}{72} + c}$$

$$\int y^2 \operatorname{sen} ny\, dy$$

$\begin{cases} u = y^2 \\ du = 2y\, dy \end{cases}$ $\begin{cases} dv = \operatorname{sen} ny\, dy \\ v = -\frac{\cos ny}{N} \end{cases}$

$\int \operatorname{sen} ny\, dy =$

$\frac{1}{n} \int \operatorname{sen} ny . n\, dy$

$-\frac{1}{n} \cos ny$

fórmula: $\boxed{\int u\, dv = uv - \int v\, du}$

$$\int y^2 \operatorname{sen} ny\, dy = -\frac{y^2 . \cos ny}{N} + \frac{1}{N} \int \underset{dv}{\cos ny}\ \underset{u}{2y}\, dy$$

integrando novamente!

$u = 2y$ $\quad v = \frac{\operatorname{sen} Ny}{N}$

$du\ 2dy$ $\quad dv = \cos ny\, dy$

substituindo:

$$-\frac{y^2 \cdot \cos ny}{N} + \frac{1}{N}\left[\frac{2y \cdot \operatorname{sen} Ny}{N} - \int \frac{\operatorname{sen} Ny}{N} \cdot 2dy\right]$$

resp.

$$\boxed{-\frac{y^2 \cos ny}{n} + \frac{2y \operatorname{sen} ny}{n^2} + \frac{2}{n^3}\cos ny + c}$$

$$\boxed{-\frac{1}{N}\int \operatorname{sen} ny\, Ndy} \Rightarrow \frac{2}{N^2}\left(\frac{1}{N}\cos ny\right) = \frac{2}{N^3} \cdot \cos Ny + C$$

$$\int x \operatorname{arc\,tg} x\, dx \Rightarrow \int \underbrace{\operatorname{arc\,tg} x}_{u}\ \underbrace{x\, dx}_{dv}$$

$u = \operatorname{arc\,tg} x$ $\quad v = \frac{x^2}{2}$

$du\ \frac{dx}{1+x^2}$ $\quad dv = x\, dx$

Substituindo

$$\int \operatorname{arc\,tg} x\ x\,dx = \frac{x^2}{2}\operatorname{arc\,tg} x - \int \frac{x^2}{2}\frac{dx}{1+x^2}$$

derivada de arc tg $= \frac{1}{x^2+1}$ diferencial de arc tg $\Rightarrow d(\operatorname{arc\,tg}) = \int \frac{dx}{1+x^2}$

Obs:

$\frac{1}{2}\int \frac{x^2}{1+x^2}\, dx$ $\quad \frac{d}{dx}(1+x^2) = 2x$

não está na forma $\frac{du}{u}$, nem $v^n dv$ pois isso divide-se:

$$\begin{array}{r|l} x^2 & x^2+1 \\ -x^2-1 & 1 - \frac{1}{x^2+1} \\ \hline -1 & \\ \hline 0 & \end{array}$$

$$= \frac{x^2}{2}\operatorname{arc\,tg} x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{x^2}{2}\operatorname{arc\,tg} x - \frac{x}{2} + \frac{1}{2}\operatorname{arc\,tg} x + c$$

colocando em evidência:

$$\boxed{\frac{x^2+1}{2} \cdot \operatorname{arc\,tg} x - \frac{x}{2} + c}$$

2ª resolução

$u = x$ $\quad dv = \operatorname{arc\,tg} x\, dx$

$du = dx$ $\quad v = \frac{1}{1+x^2}$

Fórmula da Integração por partes

19/03/78

$\int \text{arc tg}\, x\, dx = x \text{ arc tg}\, x - \frac{1}{2} \ln(1+x^2) + c$

$\int \text{arc tg}\, v\, dv = v.\text{arc tg}\, v - \ln \sqrt{1+v^2} + c$

$\frac{1}{2} \ln v \qquad \ln v^{\frac{1}{2}} = \ln \sqrt{v}$

Deduções de fórmulas

$\boxed{\int u.dv = u.v - \int v\, du}$

$d(u.v) = u.d(v) + v\, du$

$u.dv = d(u.v) - v.du$ e colococando-~~se~~

as integrais:

$\int u\, dv = \int d(uv) - \int v du$

$\int u\, dv = uv - \int v\, du.$

$\int u\, dv = uv - \int v\, du$

$\int \text{arc sen}\, x\, dx =$

$\begin{cases} u = \text{arc sen}\, x \\ du = \frac{dx}{\sqrt{1-x^2}} \end{cases}$

$\begin{cases} v = x \\ dv = dx \end{cases}$

$\int \text{arc sen}\, x\, dx = \text{arc sen}\, x . x - \int x . \frac{dx}{\sqrt{1-x^2}}$

$\int \text{arc sen}\, x\, dx = x \text{ arc sen}\, x - \int x \frac{dx}{\sqrt{1-x^2}}$

$\int \text{arc sen}\, x\, dx = x \text{ arc sen}\, x - \int \frac{x\, dx}{\sqrt{1-x^2}} -$

$\frac{1}{2} \int (1-x^2)^{-1/2} - 2x\, dx$

$\int \text{arc sen}\, dx = x.\text{arc sen}\, x + \frac{1}{2} \int (1-x^2)^{-1/2} (-2x)\, dx$

$= x \text{ arc sen}\, x + \frac{1}{2} . \frac{(1-x^2)^{1/2}}{1/2}$

$\int \text{arc sen}\, x\, dx = x . \text{arc sen}\, x + \sqrt{1-x^2}$

pg 106 $\int \text{arc sen}\, v\, dv = v \text{ arc sen}\, v + \sqrt{1-v^2}$

$\int \text{arc sen}\, \boxed{4x}\, \boxed{dx} = \frac{1}{4} \int \text{arc sen}\, 4x\; 4\, dx$

$v = 4x$

$\boxed{= \frac{1}{4} \left[4x . \text{arc . sen}\, 4x + \sqrt{1-16x^2} \right]}$

19/03/7

$$\int arc\cos v\, dv$$

$$\begin{cases} u = arc\cos v \\ du = -\frac{dv}{\sqrt{1-v^2}} \end{cases}$$

$$\begin{cases} v = v \\ dv = dv \end{cases}$$

$$\int arc\cos v\, dv = arc\cos v . v - \int v . \left(-\frac{dv}{\sqrt{1-v^2}}\right)$$

$$\int arc\cos v\, dv = v . arc\cos v + \int \frac{v . dv}{\sqrt{1-v^2}}$$

$$\int arc\cos v\, dv = v . arc\cos v - \frac{1}{2}\int (1-v^2)^{-\frac{1}{2}} (-2v)\, dx$$

$$= v\, arc\cos v - \frac{1}{\cancel{2}} . \frac{(1-v^2)^{1/2}}{1/\cancel{2}} + C$$

$$\boxed{\int arc\cos v\, dv = v . arc\cos v - \sqrt{1-u^2} + C}$$

$$\int arc\sec x\, dx = \qquad \frac{dv}{dx} = 1$$

$$\begin{cases} u = arc\sec x \\ du = \frac{dx}{x\sqrt{x^2-1}} \end{cases} \qquad \begin{cases} v = x \\ dv = dx \end{cases}$$

$$\int arc\sec x\, dx = x\, arc\sec x - \int \frac{\cancel{x}\, dx}{\cancel{x}\sqrt{x^2-1}}$$

$$\int arc\sec x\, dx = x\, arc\sec x - \int \frac{dx}{\sqrt{x^2-1}}$$

$$\int arc\sec x\, dx = x . arc\sec x - \ln\left(x + \sqrt{x^2-1}\right) + C$$

$$\boxed{\int arc\sec v\, dv = v\, arc\sec v - \ln\left(v + \sqrt{v^2-1}\right) + C}$$

$$\int x^2 . arc\,sen\, x\, dx$$

$$\begin{cases} u = arc\,sen\, x \\ du = \frac{dx}{\sqrt{1-x^2}} \end{cases}$$

$$v = \frac{x^3}{3}$$

$$dv = x^2\, dx$$

20/03/73

Revisão:

$$\int tg^3 \alpha . \sec^{5/2} \alpha \, d\alpha$$

$$\int tg^2 \alpha . tg\,\alpha . \sec^{5/2} \alpha . d\alpha$$

$$\int (\sec^2 \alpha - 1)\, tg\,\alpha . \sec^{5/2} \alpha \, d\alpha$$

$$\int \sec^2 \alpha \, \sec^{5/2} \alpha . tg\,\alpha . d\alpha - \int tg\,\alpha . \sec^{5/2} \alpha \, d\alpha$$

$\frac{5}{2} + 1 = \frac{7}{2}$ $\quad \frac{5}{2} - 1 = \frac{3}{2}$

$$\int \underbrace{\sec^1 . \sec^{5/2} \alpha}_{u^N} . \underbrace{\sec\alpha \, tg\,\alpha \, d\alpha}_{du} - \int \sec^{3/2} \alpha \underbrace{\sec\alpha \, tg\,\alpha \, d\alpha}_{du}$$

$$\int (\sec\alpha)^{7/2} . \sec\alpha \, tg\,\alpha \, d\alpha - \int (\sec\alpha)^{3/2} . \sec\alpha \, tg\,\alpha \, d\alpha$$

$$\frac{\sec^{9/2} \alpha}{9/2} - \frac{\sec^{5/2} \alpha}{5/2} + c$$

$$\boxed{\frac{2 \sec^{9/2} \alpha}{9} - \frac{2 \sec^{5/2} \alpha}{5} + c}$$

Ex. 26 → pag. 241:

$$\int \frac{(\sqrt{a} - \sqrt{x})^2}{\sqrt{x}} =$$

10/0/11

$\sqrt{x} = (x^{\frac{1}{2}}) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$-2\int (\sqrt{a} - \sqrt{x})^2 \cdot \left(-\frac{1}{2\sqrt{x}}\right) dx$

$-\frac{2(\sqrt{a} - \sqrt{x})^3}{3} + c$

$\int (\sqrt{a} - \sqrt{x}) \cdot x^{-\frac{1}{2}} =$

$-2\int (\sqrt{a} - \sqrt{x}) \cdot \frac{-1}{2} x^{-\frac{1}{2}}$

$\boxed{-\frac{\cancel{2}(\sqrt{a} - \sqrt{x})^{2}}{\cancel{2}} + c}$

$\int \frac{dx}{\sqrt{9 - 16x^2}} = \frac{1}{4} \text{arc sen} \frac{4x}{3} + c$

$+\frac{1}{4}\int + \frac{4\,dx}{\sqrt{3^2 - (4x)^2}} \Rightarrow \boxed{\frac{1}{4} \text{arc sen} \frac{4x}{3} + c}$

$\int \text{sen}\, 2x \cdot \cos 4x \, dx$

$\int \text{sen}\, mx \cdot \cos nx \, dx =$

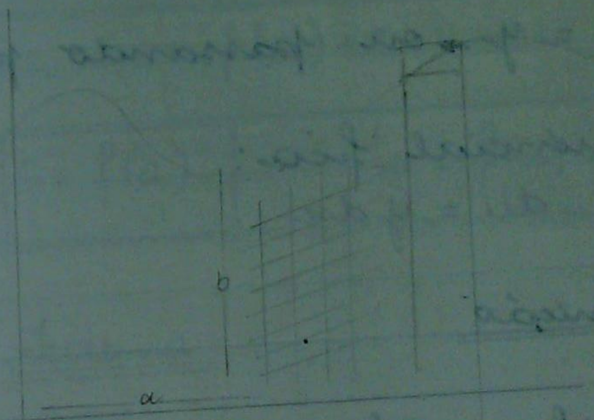
$-\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + c$

9/4/73

Integral Definida

Diferencial da área sob uma curva.

desenho sobre dif.



área de: CDMP = U

$\boxed{y = \phi(x)}$ → Equação da Curva

Para um acréscimo Δx corresponde a um acréscimo Δu.

$PMRN < PQMN < MNSQ$.

$PM \; \Delta x < \Delta u < NQ \, \Delta x$

Dividindo-se por Δx

$PM < \frac{\Delta u}{\Delta x} < NQ$

Fazendo Δx tender a zero, teremos o limite:

$PM = NQ = \frac{du}{dx}$

$\frac{du}{dx} = y$ ou passando para a diferencial fica:

$du = y\,dx$

Definição

A diferencial de uma área limitada por uma curva, o eixo dos x uma coordenada variável é igual ao produto da ordenada variável pela diferencial da abscissa correspondente.

$y = \phi(x)$

$du = y\,dx$

$du = \phi(x)\,dx$

$u = \int \phi(x)\,dx$

Fazendo: $\int \phi(x)\,dx = f(x) + C$

$u = f(x) + C$

$x = a$

$0 = f(a) + C$

$\boxed{C = f(a)}$ $\qquad u = f(x) = f(a)$

Teorema:

As diferenças entre os valores $\int y\,dx$ para: $x = b$ e $x = a$, dá a área limitada pela curva cuja a ordenada é y o eixo dos x e as ordenadas correspondentes a

$x = a$ e $x = b$.

$x = b$ — $b \rightarrow$ limite superior

$u = f(b) - f(a)$ — $a \rightarrow$ limite inferior

$$\int_a^b \phi(x)\,dx$$

$$\int_a^b y\,dx = \left[f(x) + ¢ \right]_a^b =$$

$$\left[f(b) + ¢ \right] - \left[f(a) + ¢ \right] =$$

$$\boxed{u = f(b) - f(a)}$$

① Cálculo de uma integral definida

$$\int_a^b y\,dx$$

① Integrar a diferencial dada.

② Soma-se a integral definida e substitue a variável primeiro pelo limite superior e depois pelo inferior substituindo este último pelo primeiro.

1) $\int_1^4 x^2\,dx =$

$$\left[\frac{x^3}{3} + C \right]_1^4 = \frac{4^3}{3} - \frac{1^3}{3} = \frac{63}{3} = \boxed{21}$$

2) $\int_0^{\pi} \operatorname{sen} x\,dx =$

$$\left[-\cos x \right]_0^{\pi} =$$

$\begin{cases} \cos \pi = -1 \\ \cos 0 = 1 \end{cases}$

$$\left[-(-1) \right] - \left[-1 \right] = 1 + 1 = \boxed{2}$$

3) $\int_0^a \frac{dx}{a^2 + x^2}$

$$\left[\frac{1}{a} \operatorname{arc\,tg} \frac{x}{a} \right]_0^a =$$

$$\left[\frac{1}{a} \operatorname{arc\,tg} \frac{a}{a} \right] - \left[\frac{1}{a} \operatorname{arc\,tg} \frac{0}{a} \right] =$$

$$\frac{1}{a} \underbrace{\operatorname{arc\,tg} 1} - \frac{1}{a} \operatorname{arc\,tg} 0 =$$

$\begin{cases} \operatorname{tg} 1 = \frac{\pi}{4} \\ \operatorname{tg} 0 = 0 \end{cases}$

$$\frac{1}{a} \cdot \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4a}}$$

$x = a$ e $x = b$.

$x = b$ $b \rightarrow$ limite superior

$u = f(b) - f(a)$ $a \rightarrow$ limite inferior

$$\int_a^b \phi(x)\,dx$$

$$\int_a^b y\,dx = \Big[f(x) + \cancel{c}\Big]_a^b =$$

$$\big[f(b) + \cancel{c}\big] - \big[f(a) + \cancel{c}\big] =$$

$$\boxed{u = f(b) - f(a)}$$

① Cálculo de uma integral definida

$$\int_a^b y\,dx$$

① Integrar a diferencial dada.
② Soma-se a integral definida e substitue a variável primeiro pelo limite superior e depois pelo inferior substituindo este último pelo primeiro.

1) $\int_1^4 x^2\,dx =$

$$\left[\frac{x^3}{3} + c\right]_1^4 = \frac{4^3}{3} - \frac{1^3}{3} = \frac{63}{3} = \boxed{21}$$

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$$\big[-\cos x\big]_0^{\pi} =$$

$$\big[-(-1)\big] - \big[-1\big] = 1 + 1 = \boxed{2}$$

$\begin{cases} \cos \pi = -1 \\ \cos 0 = 1 \end{cases}$

3) $\int_0^a \frac{dx}{a^2 + x^2}$

$$\left[\frac{1}{a} \operatorname{arc\,tg} \frac{x}{a}\right]_0^a =$$

$$\left[\frac{1}{a} \operatorname{arc\,tg} \frac{a}{a}\right] - \left[\frac{1}{a} \operatorname{arc\,tg} \frac{0}{a}\right] =$$

$$\frac{1}{a} \operatorname{arc\,tg} 1 - \frac{1}{a} \operatorname{arc\,tg} 0 =$$

$$\frac{1}{a} \cdot \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4a}}$$

$\begin{cases} \operatorname{tg} 1 = \frac{\pi}{4} \\ \operatorname{tg} 0 = 0 \end{cases}$

$$\int_{-1}^{0} \frac{dx}{4x^2-9}$$

$$-\frac{1}{2}\int_{-1}^{0}\frac{2\,dx}{9-4x^2} =$$

$$\left[-\frac{1}{2}\left(\frac{1}{6}\ln\frac{3+2x}{3-2x}\right)\right]_{-1}^{0} = -\frac{1}{12}\left[\ln\frac{3+2x}{3-2x}\right]_{-1}^{0} =$$

$[\ln 1 - \ln 1 - \ln 5]$

$$-\frac{1}{12}\left[\ln 1 - \ln\frac{1}{5}\right] = -\frac{1}{12}\left[\cancel{\ln 1} - \cancel{\ln 1} + \ln 5\right] =$$

$$\boxed{-\frac{1}{12}\ln 5}$$

5) $\int_{0}^{2} (a^2x - x^3)\,dx =$

$$a^2\int_0^2 x\,dx - \int_0^2 x^3\,dx =$$

$$\left[\frac{a^2x^2}{2} - \frac{x^4}{4}\right]_0^2 =$$

$$\left[\frac{2^2a^2}{2} - \frac{2^4}{4}\right] \text{ ou } \left[\frac{a^2\,4}{2} - \frac{16}{4}\right] - \left[\frac{a^2 0}{2} - \frac{0}{4}\right]$$

$$\boxed{2a^2 - 4}$$

3) $\int_{1}^{e} \frac{dx}{x} =$

$$[\ln x]_1^e = \ln e - \ln 1 = 1 - 0 = \boxed{1}$$

4) $\int_{0}^{1} \frac{dx}{\sqrt{3-2x}} =$

$$-\frac{1}{2}\left[(3-2x)^{-1/2}\cdot(-2)\,dx\right]_0^1 =$$

$$-\frac{1}{2}\left[\left(\frac{3-2x)^{1/2}}{1/2}\right]_0^1 = -\left[(3-2x)^{1/2}\right]_0^1 =$$

$$\left[3 - 2^{1/2}\right] - \left[3 - 2.0\right] = \left[3 - \frac{2}{2}\right] - \left[3 - 2.0\right]$$

sem valor

5) $-\frac{1}{2}\int_{0}^{1} (3-2x)^{-1/2}(-2)\,dx$

$$-\frac{1}{2}\left[\frac{(3-2x)^{1/2}}{1/2} = -\left[\sqrt{3-2x}\right]_0^1 =\right.$$

$$-\left[\sqrt{3-2}\right] + \left[\sqrt{3-0}\right] = \boxed{\sqrt{3} - 1 =}$$

$$\sqrt{3-2} = \sqrt{1} = 1$$

⑥ $\int_2^3 \frac{2t\,dt}{1+t^2}$

$\left[\ln(1+t^2)\right]_2^3 =$

$\ln(1+9) - \ln(1+4) =$

$\ln 10 - \ln 5 =$

$\ln \frac{10}{5} = \boxed{\ln 2}$

⑦ $\int_0^2 \frac{x^3\,dx}{x+1} =$

$$\begin{array}{l|l} x^3 & x+1 \\ \hline -x^3 - x^2 & x^2 - x + 1 - \frac{1}{x+1} \\ \quad -x^2 & \\ +x^2 + x & \\ \quad x & \\ -x - 1 & \\ \quad -1 & \\ +1 & \\ 0 & \end{array}$$

$\int_0^2 \left(x^2 - x + 1 - \frac{1}{x+1}\right)dx$

$\left[\frac{x^3}{3} - \frac{x^2}{2} + x - \ln(x+1)\right]_0^2 =$

$\left[\frac{8}{3} - \cancel{\frac{4}{2}}^{2} + \cancel{2} - \ln 3\right] - 0$

$\boxed{\frac{8}{3} - \ln 3}$

⑧ $\int_0^r \frac{r\,dx}{\sqrt{r^2 - x^2}}$:

$r\int_0^r \frac{dx}{\sqrt{r^2-x^2}} = r\left[\text{arc sen}\,\frac{x}{r}\right]_0^r =$

$r\,(\text{arc sen}\,1 - \text{arc sen}\,0) =$

$\boxed{r \cdot \frac{\pi}{2} \quad \text{ou} \quad \frac{\pi r}{2}}$

⑨ $\int_0^a (\sqrt{a} - \sqrt{x})^2\,dx =$

$\int_0^a (a - 2\sqrt{ax} + x)\,dx =$

$\int_0^a (a - 2a^{\frac{1}{2}}x^{\frac{1}{2}} + x)\,dx =$

$\left[ax - 2a^{1/2}\,\frac{x^{3/2}}{3/2} + \frac{x^2}{2}\right]_0^a$

$$4\left[\frac{z^3}{3} - z + \text{arc tg } z\right]_0^2$$

$$4\left[\frac{8}{3} - 2 + \text{arc tg } 2\right] - 4\left[0\right] =$$

$$4\left[\frac{2}{3} + \text{arc tg } 2\right]$$

$$\boxed{\frac{8}{3} + 4 \text{ arc tg } 2}$$

$$\int_0^{\frac{\pi}{2}} \cos\theta \, d\theta$$

1º) integra normal/
2º) substitue pelo lim sup
3º) " " " inf
4º) diminue

<u>integração imediata:</u>

limite superior   limite inferior

$$= \left[\text{sen } \theta\right]_0^{\frac{\pi}{2}} = \text{sen}\frac{\pi}{2} - \text{sen } 0 = 1 - 0 = \boxed{1}$$

$$\int_0^{\pi} \sqrt{2 + 2\cos\theta} \, . \, d\theta$$ (incomp)

$$\int_0^{\pi} (2 + 2\cos\theta)^{\frac{1}{2}} . \, d\theta$$

<u>integração por partes:</u>

$$\int u \, dv = vu - \int v \, du$$

$$\begin{cases} dv = d\theta \\ v = \theta \end{cases}$$

$$\begin{cases} u = (2 + 2\cos\theta)^{1/2} \\ du = \frac{1}{2}(2 + 2\cos\theta)^{-1/2} . (-2 \text{ sen}\theta) = \end{cases}$$

$$\searrow \; -\frac{\text{sen}\theta}{\sqrt{2+2\cos\theta}} \, d\theta$$

$$\int_0^{\pi} \sqrt{2 + 2\cos\theta} \, d\theta = \theta . \sqrt{2 + 2\cos\theta} - .$$

$$- \int \frac{-\theta \, . \, \text{sen}\,\theta \, d\theta}{\sqrt{2 + 2\cos\theta}}$$

incompleto

$$\int_0^{\frac{\pi}{2}} sen^3 x \cos^3 x \, dx =$$

$$\int_0^{\frac{\pi}{2}} sen^2 x \cdot sen\, x \cdot \cos^3 x \, dx =$$

$$\int_0^{\frac{\pi}{2}} (1-\cos^2 x) \cdot sen\, x \cdot \cos^3 x \, dx =$$

$$\int_0^{\frac{\pi}{2}} sen\, x \cos^3 x \, dx - \int_0^{\frac{\pi}{2}} sen\, x \cdot \cos^5 x \, dx =$$

$$-\int_0^{\frac{\pi}{2}} \cos^3 x\,(-sen\, x)\,dx + \int_0^{\frac{\pi}{2}} \cos^5 x\,(-sen\, x)\,dx$$

$$= \left[-\frac{\cos^4 x}{4} + \frac{\cos^6 x}{6}\right]_0^{\pi/2}$$ substituindo:

$$= \left[-\frac{(\cos\frac{\pi}{2})^4}{4} + \frac{(\cos\frac{\pi}{2})^6}{6}\right] - \left[-\frac{(\cos 0)^4}{4} + \frac{(\cos 0)^6}{6}\right]$$

$\cos\frac{\pi}{2} = 0$

$$\left[0 + 0\right] - \left[-\frac{1}{4} + \frac{1}{6}\right] =$$

$\cos 0 = 1$

$$0 - \left[-\frac{3+2}{12}\right] =$$

$$-\left[-\frac{1}{12}\right] = \boxed{\frac{1}{12}}$$

$$\int_0^{\frac{\pi}{4}} sec^4 x \, dx = \frac{4}{3}$$

$$\int_0^{\frac{\pi}{4}} sec^2 x \cdot sec^2 x \, dx$$

$$\int_0^{\frac{\pi}{4}} (1 + tg^2 x)\, sec^2 x \, dx$$

$$\int_0^{\frac{\pi}{4}} sec^2 x \, dx + \int_0^{\frac{\pi}{4}} (tg\, x)^2 sec^2 x \, dx$$

$$\int_0^{\frac{\pi}{2}} sen^3 x \cos^3 x \, dx =$$

$$\int_0^{\frac{\pi}{2}} sen^2 x \cdot sen\, x \cdot \cos^3 x \, dx =$$

$$\int_0^{\frac{\pi}{2}} (1-\cos^2 x) \cdot sen\, x \cdot \cos^3 x \, dx =$$

$$\int_0^{\frac{\pi}{2}} sen\, x \cos^3 x \, dx - \int_0^{\frac{\pi}{2}} sen\, x \cdot \cos^5 x \, dx =$$

$$-\int_0^{\frac{\pi}{2}} \cos^3 x\,(-sen\, x)\, dx + \int_0^{\frac{\pi}{2}} \cos^5 x\,(-sen\, x)\, dx$$

$$= \left[-\frac{\cos^4 x}{4} + \frac{\cos^6 x}{6}\right]_0^{\pi/2}$$ substituindo:

$$= \left[-\frac{(\cos\frac{\pi}{2})^4}{4} + \frac{(\cos\frac{\pi}{2})^6}{6}\right] - \left[-\frac{(\cos 0)^4}{4} + \frac{(\cos 0)^6}{6}\right]$$

$\cos\frac{\pi}{2} = 0$

$\cos 0 = 1$

$$[0 + 0] - \left[-\frac{1}{4} + \frac{1}{6}\right] =$$

$$0 - \left[-\frac{3+2}{12}\right] =$$

$$-\left[-\frac{1}{12}\right] = \boxed{\frac{1}{12}}$$

$$\int_0^{\frac{\pi}{4}} sec^4 x \, dx = \frac{4}{3}$$

$$\int_0^{\frac{\pi}{4}} sec^2 x \cdot sec^2 x \, dx$$

$$\int_0^{\frac{\pi}{4}} (1 + tg^2 x)\, sec^2 x \, dx$$

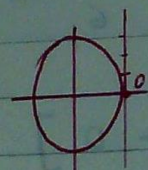
$$\int_0^{\frac{\pi}{4}} sec^2 x \, dx + \int_0^{\frac{\pi}{4}} (tg\, x)^2 sec^2 x \, dx$$

$$= \left[tg\,x + \frac{tg^3 x}{3} \right]_0^{\frac{\pi}{4}}$$

$$= \left[tg\,\frac{\pi}{4} + \frac{\left(tg\,\frac{\pi}{4}\right)^3}{3} \right] - \left[tg\,0 + \frac{(tg\,0)^3}{3} \right]$$

$$= 1 + \frac{1}{3} - 0 =$$

$$= \boxed{\frac{4}{3}}$$



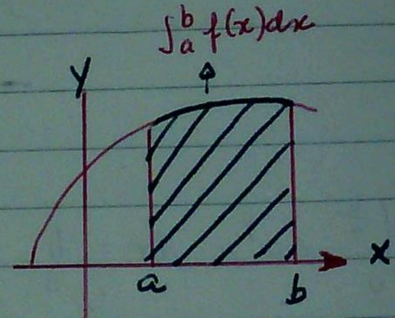
$tg\,0 = 0$

Área de uma curva

7/3/73

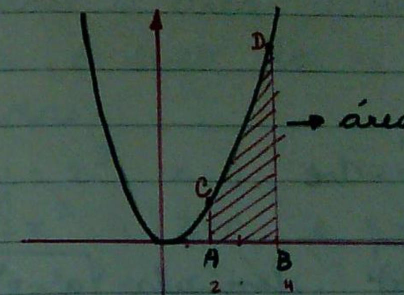
$y = f(x)$

Área $= \int_a^b f(x)\,dx$



① Achar a área limitada pela parábola $y = x^2$, o eixo dos xx e as ordenadas $x = 2$ e $x = 4$

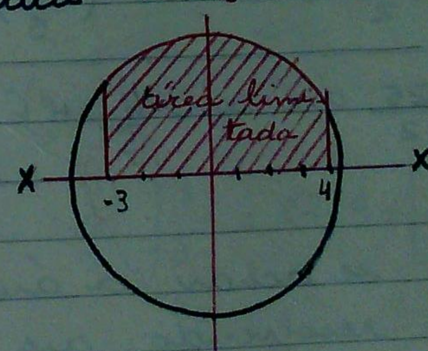
→



$$\text{Área ABCD} = \int_2^4 x^2 = \left[\frac{x^3}{3} \right]_2^4 = \frac{4^3}{3} - \frac{2^3}{3} =$$

$$\frac{64-8}{3} = \frac{56}{3} \therefore \boxed{\text{Área ABCD} = 18,6}$$

② Achar a área limitada pelo círculo $x^2 + y^2 = 25$, o eixo dos xx e as ordenadas $x = -3$ e $x = 4$.



Primeira/ precisamos isolar o y:

$y^2 = 25 - x^2$

$y = \sqrt{25-x^2}$

$A = \int_{-3}^{4} \sqrt{25-x^2} \cdot dx$

$A = \int_{-3}^{4} \sqrt{(5)^2-(x)^2}$ fórmula: $\sqrt{a^2-v^2}\,dv$

$A = \left[\frac{x}{2} \cdot \sqrt{25-x^2} + \frac{25}{2} \text{ arc sen } \frac{x}{5}\right]_{-3}^{4} =$

A = substituindo x pelos limites:

x → limite superior x →

$A = \left[\frac{4}{2} \cdot \sqrt{25-16} + \frac{25}{2} \text{ arc sen} \frac{4}{5}\right] - \left[-\frac{3}{2} \cdot \right.$ →

limite inferior

→ $\left. \cdot \sqrt{25-9} + \frac{25}{2} \text{ arc sen } \left(-\frac{3}{5}\right)\right] =$

$= A: \boxed{6 + \frac{25}{2} \text{ arc sen } \frac{4}{5} + 6 - \frac{25}{2} \text{ arc sen } \left(-\frac{3}{5}\right)}$

=

Obs: Para se achar a área continua-se resolvendo: arc sen $\frac{4}{5}$ =

arc sen x = 0,8

...

③ Achar a área dada pela curva dada, o eixo dos xx e as dadas ordenadas.

a) $y = x^3$, $x = 0$ e $x = 4$

b) $9 - x^2$, $x = 0$ e $x = 3$

c) $y = x^3 + 3x^2 + 2x$, $x = -3$ e $x = 3$

ⓐ $A: \int_0^4 x^3\,dx = \left[\frac{x^4}{4}\right]_0^4 = \frac{4^{\cancel{4}\,3}}{\cancel{4}} - 0 = \boxed{64}$

outra resolução:

$A = \left[\frac{x^4}{4} + C\right] - \left[\frac{0^4}{4} + C\right] =$

$A = \left[\frac{256}{4} + \cancel{C}\right] - \left[0 + \cancel{C}\right] = \boxed{64}$

ⓑ $A: \int_0^3 (9-x^2)\,dx =$ $\int 9dx = 9\int dx$ $9x$.

$\left[9x - \frac{x^3}{3}\right]_0^3 = 27 - 9 = \boxed{18}$

ⓒ $A: \int_{-3}^{3} (x^3 + 3x^2 + 2x)\,dx =$

$\int x^3$, $\int 3x^2$, $\int 2x$ integrais dos termos dados

$$\left[\frac{x^4}{4} + x^3 + x^2\right]_{-3}^{3} =$$

$$\left[\frac{3^4}{4} + 3^3 + 3^2\right] - \left[\frac{(-3)^4}{4} + (-3)^3 + (-3)^2\right] =$$

$$\cancel{\frac{3^4}{4}} + 27 + \cancel{3^2} - \cancel{\frac{3^4}{4}} + 27 - \cancel{9} = \boxed{54}$$

④ $y = x^2 + x + 1$ $x = 2$ $x = 3$

$$A = \int_2^3 x^2 + x + 1 \, dx =$$

$$A = \int x^2 dx + \int x \, dx + \int dx$$

$$A = \left[\frac{x^3}{3} + \frac{x^2}{2} + x\right]_2^3$$

$$A = \left[\frac{3^3}{3} + \frac{3^2}{2} + 3\right] - \left[\frac{2^3}{3} + \frac{2^2}{2} + 2\right] =$$

$$A = \frac{3^3}{3} + \frac{3^2}{2} + 3 - \frac{2^3}{3} - \frac{2^2}{2} - 2$$

$$A = 9 + \frac{9}{2} + 3 - \frac{8}{3} - 2 - 2$$

$$A = 8 + \frac{9}{2} - \frac{8}{3} = \frac{48 + 27 - 16}{6} = \frac{59}{6} = \boxed{9\frac{5}{6}}$$

⑤ $xy = k^2$ $x = a$ $x = b$

$y = \dfrac{k^2}{x}$ → k → constante

$$A = \int_a^b \frac{k^2}{x} dx =$$

$$k^2 \int_a^b \frac{dx}{x} =$$

$$k^2 \left[\ln x\right]_a^b$$

$$k^2 \left[\ln b - \ln a\right] =$$

$$k^2 \left[\ln \frac{b}{a}\right] = \boxed{k^2 \left[\ln \frac{b}{a}\right]}$$

⑥ $y = 2x + \dfrac{1}{x^2}$ $x = 1$ $x = 4$

$$A = \int_1^4 2x + \frac{1}{x^2} \, dx = \left[\frac{2x^2}{2} + \frac{x^{-1}}{-1}\right]_1^4 =$$

$$A = \left[x^2 - \frac{1}{x}\right]_1^4$$

$$A = \left[4^2 - \frac{1}{4}\right] - \left[1^2 - \frac{1}{1}\right] =$$

$$A = \left[16 - \frac{1}{4}\right] - \left[1 - 1\right] =$$

$$A = \left[\frac{63}{4}\right] - \cancel{1} + \cancel{1} = \frac{63}{4} \therefore \boxed{A = 15\frac{3}{4}}$$

⑦ $y = \frac{10}{\sqrt{x+4}}$ $x = 0$ $x = 5$ $= \int_0^5 \frac{10\,dx}{\sqrt{x+4}}$

$$10\int_0^5 (x+4)^{-\frac{1}{2}}\,dy$$

$$10\frac{\sqrt{x+4}}{\frac{1}{2}} = \left[20\sqrt{x+4}\right]_0^5$$

$$20\left[\sqrt{5+4} - \sqrt{0+4}\right]$$

$$20 \cdot 3 - 20 \cdot 2$$

$$\boxed{60 - 40 = \boxed{20}}$$

⑧ $ay = x\sqrt{a^2 - x^2}$ $x = 0$ $x = a$

$$y = \frac{x\sqrt{a^2 - x^2}}{a}$$

$$A = \int_0^a \frac{x}{a}(a^2 - x^2)^{\frac{1}{2}}\,dx$$

$$A = \frac{1}{-2a}\int_0^a (a^2 - x^2)^{\frac{1}{2}} \cdot (-2x)\,dx$$

$$-\frac{1}{2a}\left[\frac{\left[a^2 - x^2\right]^{3/2}}{\frac{3}{2}}\right]_0^a$$

$$-\frac{1}{a}\left[\frac{\sqrt{(a^2 - x^2)^3}}{3}\right]_0^a$$

$$-\frac{1}{a} \cdot \left[\frac{\sqrt{(a^2 - a^2)^3}}{3}\right] + \frac{1}{a}\left[\frac{\sqrt{(a^2 - 0)^3}}{3}\right] =$$

$$\frac{1}{\cancel{a}} \cdot \frac{a^{\cancel{3}\,2}}{3} = \boxed{\frac{a^2}{3}}$$

9) $y^2 + 4x = 0$ $\begin{cases} x = -1 \\ x = 0 \end{cases}$

isolando o y:

$y = f(x)$ $A = \int f(x)\,dx$

$y^2 = -4x$ $\therefore$ $y = \sqrt{-4x}$

$\int \sqrt{-4x}\,dx$ $A = -\frac{1}{4}\int_{-1}^{0} (-4x)^{\frac{1}{2}} 4\,dx$

$$A = -\frac{1}{4}\frac{(-4x)^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{1}{\cancel{4}_2}\frac{(-4x)^{3/2}}{\frac{3}{\cancel{2}}}$$

$$A = -\frac{1}{6}(-4x)^{3/2}$$

$$A = \left[-\frac{1}{6}\sqrt{(-4x)^3}\right]_{-1}^{0}$$

$$\left[-\frac{1}{6}\sqrt{(-4.0)^3}\right] - \left[-\frac{1}{6}.\sqrt{[-4.(-1)]^3}\right] =$$

$$0 + \frac{1}{6}.\sqrt{64} = \frac{1}{\cancel{6}_3}.\cancel{8}^4 = \boxed{\frac{4}{3}}$$

10) $y^2 = 4x + 16$ $\begin{cases} x = -2 \\ x = 0 \end{cases}$

$$y = \sqrt{4x+16}$$

$$\int_{-2}^{0} \sqrt{4x+16}\,dx$$

$$\int_{-2}^{0} (4x+16)^{\frac{1}{2}}$$

$$A = \frac{1}{4}\int_{-2}^{0} (4x+16)^{\frac{1}{2}}.\ 4\,dx$$

$$A = \left[\frac{1}{\cancel{4}_2}.\frac{(4x+16)^{3/2}}{\frac{3}{\cancel{2}}}\right]_{-2}^{0} =$$

subst. pelo lim. sup.

$$A = \frac{1}{6}\left[\sqrt{(0+16)^3} - \sqrt{(8+16)^3}\right]$$

$$A = \frac{1}{16}.4^3 - \frac{1}{6}.\sqrt{2^9}$$

$\sqrt{(4^2)^3} = \sqrt{4^6} = 4^3$

$\sqrt{8^3} = \sqrt{2^9}$

$(2^3)^3 = 2^9$

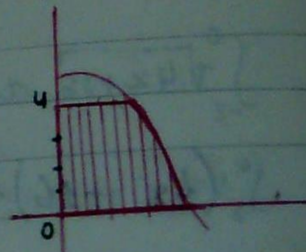
$$A = \frac{64}{6} - \frac{2^4}{6}\sqrt{2}$$

$$A = \frac{\cancel{64}^{32}}{\cancel{6}_3} - \frac{\cancel{16}^{8}}{\cancel{6}_3}\sqrt{2} =$$ simpl.

$$A = \frac{32}{3} - \frac{8}{3}\sqrt{2} = \frac{32-8}{3}\sqrt{2} = \boxed{8\sqrt{2}}$$

Achar a área limitada pela curva dada, o eixo dos yy e as dadas retas.

$y^2 = 4x$ $\quad \begin{cases} y = 0 \\ y = 4 \end{cases}$



$x = \frac{y^2}{4}$

$A = \int_0^4 \frac{y^2}{4}\, dy$

$A = \frac{1}{4} \int_0^4 y^2\, dy$

$A = \frac{1}{4} \cdot \left[\frac{y^3}{3} \right]_0^4$

$A = \frac{1}{4} \cdot \frac{4^3}{3} - \frac{1}{4} \cdot \frac{0^3}{3} =$

$A = \frac{16}{3} \quad \therefore \quad \boxed{A = 5\frac{1}{3}}$

Integral Definida - Cont.

9/4/73

Em função de y

$xy = 8 \quad \rightarrow \quad x = \frac{8}{y}$

$y = 1$

$y = 4$

$A = \int_1^4 \frac{8\, dy}{y} = 8 \int_1^4 \frac{dy}{y}$

$\left[\ln y \right]_1^4 = 8 \left[\ln 4 - \ln 1 \right] = \boxed{8 \ln 4}$

$y^3 = a^2 x$

$y = 0$

$y = a$

$\left. \begin{array}{l} y = \sqrt[3]{a^2 x} \\ \int (a^2 x)^{\frac{1}{3}} \end{array} \right\}$ sem valor

$$a^2x = y^3$$
$$x = \frac{y^3}{a^2}$$
$$A = \int \frac{y^3}{a^2}\,dy$$
$$A = \frac{1}{a^2}\int_0^a y^3\,dy$$
$$A = \frac{1}{a^2}\frac{y^4}{4} = \left[\frac{y^4}{4a^2}\right]_0^a$$
$$A = \left[\frac{a^4}{4a^2}\right] - \left[\frac{0^4}{4a^2}\right]$$
$$\frac{a^{\cancel{4}\,2}}{4\cancel{a^2}} = \boxed{\frac{a^2}{4}}$$

$ay^2 = x^3$

$y = 0$

$y = a$

$$x^3 = ay^2$$
$$x = \sqrt[3]{ay^2}$$
$$A = \int_0^a \sqrt[3]{ay^2}\,dy$$
$$A = \int_0^a (ay^2)^{\frac{1}{3}}\,dy$$
$$\left(\frac{1}{2a}\int_0^a (ay^2)^{\frac{1}{3}}\,2a\,dy\right) \text{ sem valor}$$
$$A = \int_0^a \sqrt[3]{ay^2}\,dy =$$
$$A = \sqrt[3]{a}\int_0^a \sqrt[3]{y^2}\,dy =$$
$$A = \sqrt[3]{a}\int_0^a y^{\frac{2}{3}}\,dy$$
$$A = \sqrt[3]{a}\left[\frac{y^{5/3}}{\frac{5}{3}}\right]_0^a =$$
$$A = \frac{3}{5}\sqrt[3]{a}\left[y^{5/3}\right]_0^a =$$
$$A = \frac{3\sqrt[3]{a}}{5}\left[a^{\frac{5}{3}} - 0\right]$$

$$a^2x = y^3$$

$$x = \frac{y^3}{a^2}$$

$$A = \int \frac{y^3}{a^2}\, dy$$

$$A = \frac{1}{a^2}\int_0^a y^3\, dy$$

$$A = \frac{1}{a^2}\, \frac{y^4}{4} = \left[\frac{y^4}{4a^2}\right]_0^a$$

$$A = \left[\frac{a^4}{4a^2}\right] - \left[\frac{0^4}{4a^2}\right]$$

$$\frac{a^{\cancel{4}\,2}}{4\cancel{a^2}} = \boxed{\frac{a^2}{4}}$$

---

b)

$$ay^2 = x^3$$

$$x^3 = ay^2$$

$$x = \sqrt[3]{ay^2}$$

$$y = 0$$

$$y = a$$

$$A = \int_0^a \sqrt[3]{ay^2}\, dy$$

$$A = \int_0^a (ay^2)^{\frac{1}{3}}\, dy$$

$$\left(\frac{1}{2a}\int_0^a (ay^2)^{\frac{1}{3}}\ 2a\, dy\right) \quad \text{sem valor}$$

$$A = \int_0^a \sqrt[3]{ay^2}\, dy =$$

$$A = \sqrt[3]{a}\int_0^a \sqrt[3]{y^2}\, dy =$$

$$A = \sqrt[3]{a}\int_0^a y^{\frac{2}{3}}\, dy$$

$$A = \sqrt[3]{a}\left[\frac{y^{5/3}}{\frac{5}{3}}\right]_0^a =$$

$$A = \frac{3}{5}\sqrt[3]{a}\left[y^{5/3}\right]_0^a =$$

$$A = \frac{3\sqrt[3]{a}}{5}\left[a^{\frac{5}{3}} - 0\right]$$

$$A = \frac{3\sqrt[3]{a}}{5} \cdot \sqrt[3]{a^5} =$$

$$A = \frac{3\sqrt[3]{a^6}}{5}$$

$$\boxed{A = \frac{3}{5}a^2}$$

$$y = \frac{1a}{2\cancel{a}}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$$ (curva chamada catenária)

$x = a$

$x = -a$

$$A = \frac{1}{2}a\int_{-a}^{a}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)dx$$

$$A = \frac{1}{2a}\int_{-a}^{a} e^{\frac{x}{a}}dx + \frac{1}{2}a\int_{-a}^{a} e^{-\frac{x}{a}}dx$$

$$\int e^{v}dv = e^{v} + C$$

$$A = \frac{1}{2}a^2\int_{-a}^{a} e^{\frac{x}{a}} \cdot \frac{1}{a}dx - \frac{1}{2}a^2\int_{-a}^{a} e^{-\frac{x}{a}} \cdot \left(-\frac{1}{a}\right)dx$$

$$A = \frac{a^2}{2}\left[e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right]_{-a}^{a}$$

$$A = \frac{a^2}{2}\left[\left(e^{\frac{a}{a}} - e^{-\frac{a}{a}}\right) - \left(e^{-\frac{a}{a}} - e^{\frac{a}{a}}\right)\right] =$$

$$A = \frac{a^2}{2}\left[e - \frac{1}{e} - \frac{1}{e} + e\right]$$

$$A = \frac{a^2}{2}\left[2e - \frac{2}{e}\right]$$

$$A = \frac{a^2}{\cancel{2}}\left[\frac{\cancel{2}e^2 - \cancel{2}}{e}\right]$$

$$A = \frac{a^2e^2 - a^2}{e}$$

$$\boxed{A = a^2\left(e - \frac{1}{e}\right)}$$

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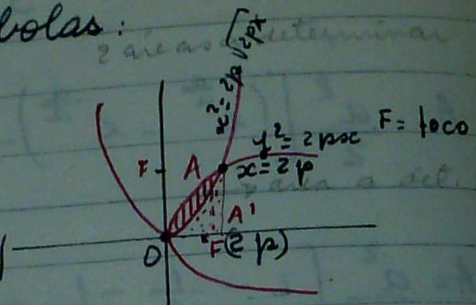
* Achar a área compreendida entre as duas parábolas:

$$\begin{cases} y^2 = 2px \\ x^2 = 2py \end{cases}$$



resolve-se primeira/ o sistema:

Lim. inf = 0

Lim. sup = onde as p. se encontram.

$y = \sqrt{2px}$

$x^2 = 2p\sqrt{2px}$ → eq. irracional

para eliminar o x eleva-se tudo ao quad.

$x^4 = 4p^2(2px)$

$x^3 = 8p^3x$

extraindo a raiz cúbica:

limites $\boxed{\begin{matrix} x = 2p. \\ x = 0 \end{matrix}}$ a área será determinada em função do x.

A: 1) $y = \sqrt{2px}$ tira da 1ª equação: $y^2 = 2px$

A': 2) $y = \dfrac{x^2}{2p}$ " " 2ª " $x^2 = 2py$

1) $A = \int_0^{2p} \sqrt{2px}\, dx$

2) $A' = \int_0^{2p} \dfrac{x^2}{2p}\, dx$

$A - A' =$

resolvendo a 1ª integral:

$$A = \int_0^{2p} (2px)^{1/2}\, dx$$

$$A = \frac{1}{2p}\int_0^{2p} (2px)^{1/2}\, 2p\, dx$$

$$A = \frac{1}{2p}\left[\frac{(2px)^{3/2}}{3/2}\right]_0^{2p} \therefore A = \frac{1}{3p}\left[\sqrt{(2px)^3}\right]_0^{2p}$$

$$A = \frac{1}{3p}\cdot\left(\sqrt{(2p.2p)^3}\right) = \frac{1}{3p}\left(\sqrt{(2^2p^2)^3}\right) =$$

$$A = \frac{1}{3p}\sqrt{2^6p^6} \Rightarrow \frac{1}{3\cancel{p}}\cdot 2^3 p^{\cancel{3}\,2} \Rightarrow \boxed{\frac{8p^2}{3}}$$ 1ª área → A

$$A' = \int_0^{2p} \frac{x^2}{2p}\, dx$$

$$A' = \frac{1}{2p}\int_0^{2p} x^2\, dx = \left[\frac{1}{2p}\ \frac{x^3}{3}\right]_0^{2p}$$

$$A' = \left[\frac{x^3}{6p}\right]_0^{2p} \therefore \frac{(2p)^3}{6p} = \frac{2^{\cancel{3}\,2}\, p^{\cancel{3}\,2}}{\cancel{6}_{3}\,\cancel{p}} = \quad \left(\frac{2^2 \cdot \cancel{2}}{3 \cdot \cancel{2}}\right)$$

$$\boxed{\frac{4p^2}{3}} \rightarrow \text{2ª área } A'$$

$$A - A' = \frac{8p^2}{3} - \frac{4p^2}{3} = \boxed{\frac{4p^2}{3} \text{ ou } \frac{4}{3}p^2}$$

10/4/73 Achar a área compreendida entre as duas parábolas:

parábolas $\begin{cases} y^2 = ax & \text{I} \\ x^2 = by & \text{II} \end{cases}$

$y = \sqrt{ax}$

$x^2 = b\,\overbrace{\sqrt{ax}}^{y}$

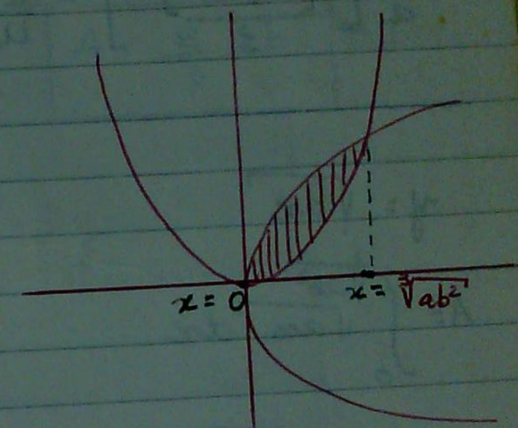
achando o valor de x (eq. irrac.)

$(x^2)^2 = (b\sqrt{ax})^2$

$x^4 = b^2 ax$

$x^4 - ab^2 x = 0$

$x(x^3 - ab^2) = 0$



$\left[\begin{array}{l} x = 0 \rightarrow \text{limite inferior} \\ x^3 = ab^2 \therefore x = \sqrt[3]{ab^2} \rightarrow \text{limite superior} \end{array}\right.$

Integração de I e II

resolução → atic

$y = \sqrt{ax}$

sem valor

$$A = \int_0^{(ab)^{\frac{2}{3}}} (ax)^{\frac{1}{2}}\, dx = \frac{1}{a}\int_0^{(ab)^{\frac{2}{3}}} (ax)^{\frac{1}{2}} \cdot a\, dx$$

$$\frac{1}{a}\left[\frac{ax^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{(ab)^{\frac{3}{2}}}$$

$$\frac{2}{a}\left[\frac{\sqrt{(ax)^3}}{3}\right]_0^{(ab)^{3/2}}$$ sem valor

$$\frac{2}{a}\left[\frac{\sqrt{[a.(ab)^{3/2}]^3}}{3}\right] - \frac{2}{a}\cdot 0$$

$$y = \sqrt{ax}$$

$$A = \int_0^{\sqrt[3]{ab^2}} \sqrt{ax}\, dx$$

$$A = \frac{1}{a}\int_0^{\sqrt[3]{ab^2}} (ax)^{\frac{1}{2}}.a\, dx$$

$$A = \frac{1}{a}\left[\frac{(ax)^{3/2}}{3/2}\right]_0^{\sqrt[3]{ab^2}}$$

$$A = \frac{2}{3a}\left[\sqrt{(ax)^3}\right]_0^{\sqrt[3]{ab^2}}$$

$$A = \frac{2}{3a}\left[\sqrt{a^3(\sqrt[3]{ab^2})^3}\right]$$

$$A = \frac{2}{3a}\left[\sqrt{a^3 a b^2}\right]$$

$$A = \frac{2}{3a}\cdot a^2 b \quad \therefore \quad \boxed{A = \frac{2}{3}ab} \quad \text{I}$$

2ª parábola: $x^2 = by$

$$y = \frac{x^2}{b}$$

$$A' = \int_0^{\sqrt[3]{ab^2}} \frac{x^2}{b}\, dx$$

$$A' = \frac{1}{b}\int^{\sqrt[3]{ab^2}} x^2\, dx$$

$$A' = \frac{1}{b}\left[\frac{x^3}{3}\right]_0^{\sqrt[3]{ab^2}}$$

substituindo pelo

limite superior:

$$A' = \frac{1}{b} \cdot \frac{\left(\sqrt[\cancel{3}]{ab^2}\right)^{\cancel{3}}}{3}$$

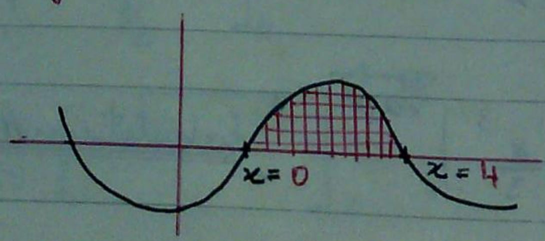
$$A' = \frac{ab^{\cancel{2}}}{3\cancel{b}} \therefore \boxed{A' = \frac{ab}{3}}$$

$$A - A' = \frac{2}{3}ab - \frac{1}{3}ab$$

$$\boxed{A - A' = \frac{1}{3}ab}$$

Achar a área compreendida pelo laço da curva dada pela equação:

$$4y^2 = x^2(4-x)$$



Quando a curva corta o eixo dos xx, o $\boxed{y=0}$

Partindo de $y=0$

$$x^2(4-x)=0$$

$x^2 = 0 \therefore \boxed{x=0}$ → passa na origem

$$4-x=0$$

$$-x=-4 \therefore \boxed{x=4}$$

<u>Integrando</u>

$$4y^2 = x^2(x-4)$$

limites $\begin{cases} x=0 \\ x=4 \end{cases}$

$$y = \sqrt{\frac{x^2(4-x)}{4}}$$

$$y = \frac{x\sqrt{4-x}}{2} \therefore$$

$$A: \int_0^4 x \frac{\sqrt{4-x}}{2}\, dx$$

$$A: \frac{1}{2} \int_0^4 x \sqrt{4-x}\, dx$$ int. por partes

$$\int u\, dv = vu - \int v.du$$

$$\begin{cases} u = x & dv = \sqrt{4-x}\, dx \\ du = dx & v = -\dfrac{2\sqrt{(4-x)^3}}{3} \end{cases}$$

determinando v

$$(-)\int (4-x)^{1/2}.(-)\, dx$$

$$-\frac{(4-x)^{3/2}}{3/2}$$

$$\boxed{-\frac{2\sqrt{(4-x)^3}}{3}}$$

pela fórmula: $\int u\, dv = vu - \int v\, du$

$$A = \left[-\frac{2\sqrt{(4-x)^3}}{3} . x\right]_0^4 - \int_0^4 -\frac{2\sqrt{(4-x)^3}}{3} . dx$$

$$\Delta$$

$$A = \Delta \cancel{(-)} + \frac{2}{3}\int_0^4 (4-x)^{\frac{3}{2}}(-)\, dx$$

$$A = \Delta - \frac{2}{3}\left[\frac{(4-x)^{5/2}}{5/2}\right]_0^4 =$$

$$A = -\frac{2}{3}\left[x\overset{\Delta}{\sqrt{(4-x)^3}}\right]_0^4 - \frac{4}{15}\left[\sqrt{(4-x)^5}\right]_0^4$$

limite superior = 0

$$A = \left\{-\frac{2}{3}\left[4.\sqrt{(4-4)^3}\right] - \frac{4}{15}\left[\sqrt{(4-4)^5}\right]\right\} -$$

limite inf. 1ª parte = 0

$$\left\{0 - \frac{4}{15} . \sqrt{(4-0)^5}\right\} =$$

$$A = +\frac{4}{15} . \sqrt{2^{10}}$$

$$A = \frac{4}{15} . 2^5 = \boxed{\frac{128}{15}}$$

$$A = \frac{1}{3}\left[\sqrt{(2^2-1)^3}\right] - \frac{1}{3}\left[\underbrace{\sqrt{(1-1)^3}}_{=0}\right]$$

$$A = \frac{1}{3}\cdot\sqrt{3^3} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = \boxed{\sqrt{3}}$$ livro $2\sqrt{3}$?

Achar a área compreendida pelo laço da curva de equação

$$y^2 = x^2(9-x)$$

$\underline{y=0} \rightarrow x^2(9-x)=0 \rightarrow x^2=0 \therefore \underline{x=0}$ l. inf.

$9-x=0 \rightarrow -x=-9 \therefore \underline{x=9}$ limite sup.

isolando y:

$$y^2=x^2(9-x) \rightarrow y=\sqrt{x^2(9-x)} \rightarrow \boxed{y = x\sqrt{9-x}}$$

$$A = \int_0^9 x\sqrt{9-x}\,dx$$ (int. por partes)

$$\int u\,dv = vu - \int v\,du \quad \begin{cases} u = x & dv = \sqrt{9-x} \\ du = dx & v = -\dfrac{2(9-x)^{1/2}}{3} \end{cases}$$

det. v: $\int (9-x)^{1/2}dx = -\int (9-x)^{1/2}(-)dx =$

$$-\frac{(9-x)^{3/2}}{3/2} = \boxed{-\frac{2\sqrt{(9-x)^3}}{3}} \rightarrow v$$

$$A = \left[-\frac{2\sqrt{(9-x)^3}}{3}\cdot x\right]_0^9 - \int_0^9 -\frac{2\sqrt{(9-x)^3}}{3}dx =$$

$$A = \Delta + \frac{2}{3}\int_0^9 (9-x)^{3/2}\cdot(-)dx \rightarrow \Delta - \frac{2}{3}\left[\frac{(9-x)^{5/2}}{5/2}\right]_0^9$$

$$A = \Delta - \frac{4}{15}\left[\sqrt{(9-x)^5}\right]_0^9 =$$

$$A = \left[-\frac{2\sqrt{(9-x)^3}}{3}\cdot x\right]_0^9 - \frac{4}{15}\left[\sqrt{(9-x)^5}\right]_0^9 =$$

$$A = \left\{\left[-\frac{2\sqrt{(9-9)^3}}{3}\cdot 9\right] - \frac{4}{15}\left[\sqrt{(9-9)^5}\right]\right\} - \left\{\left[-\frac{2\sqrt{(9-0)^3}}{3}\cdot 0\right] - \frac{4}{15}\left[\sqrt{(9-0)^5}\right]\right\} =$$

$$A = \left\{\left[-\frac{2\sqrt{0^3}}{3}\cdot 9\right] - \frac{4}{15}\left[\sqrt{0^5}\right]\right\} - \left\{\left[-\frac{2\cdot\sqrt{9^3}\cdot 0}{3}\right] - \frac{4}{15}\cdot\sqrt{9^5}$$

$$A = \{\,0\,\} - \left\{\left[-\frac{2\cdot 9\cdot 3\cdot 0}{3}\right] - \frac{4}{15}\left[9\cdot 9\cdot 3\right]\right\} =$$

$$A = 0 - \left[0 - \frac{4}{15}\cdot 243\right] =$$

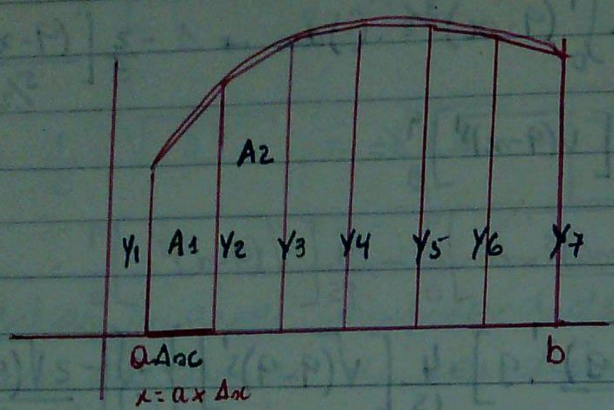
$$A = +\frac{4}{15}\cdot 243 =$$

$$A = \frac{972}{15} \therefore \boxed{A = \frac{324}{5}}$$ no livro $\frac{648}{15}$?

Cálculo Aproximado da Área

$\frac{13}{4}$

$\frac{4}{73}$



$$A_1 = \frac{1}{2}(y_1 + y_2)\Delta x$$

$$A_2 = \frac{1}{2}(y_2 + y_3)\Delta x$$

- - - - - - - - - - - - - - -

$$A_n = \frac{1}{2}(y_{n-1} + y_n)\Delta x$$

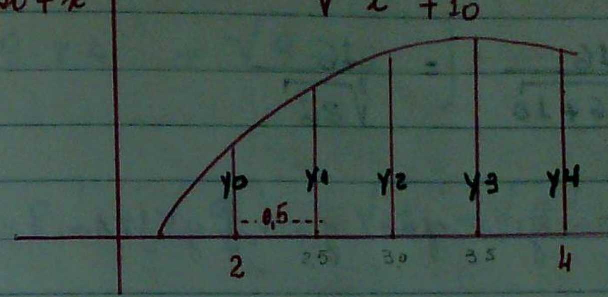
$$A = \left(\frac{1}{2}y + \frac{1}{2}y_2 + \frac{1}{2}y_2 + \frac{1}{2}y_3 + \dots + \frac{1}{2}y_{n-1} + \frac{1}{2}y_n\right)\Delta x$$

$$\boxed{A = \left(\frac{1}{2}y + y_2 + y_3 + \dots + y_{n-1} + \frac{1}{2}y_n\right)\Delta x}$$

$\Delta x = \frac{b-a}{\text{nº de partes}}$ $\quad \Delta x = \frac{b-a}{N}$

Método p/a calcular áreas aproximada

$$\int_2^4 \frac{x^2\,dx}{\sqrt[3]{10+x^2}} \Rightarrow y = \frac{x^2}{\sqrt[3]{x^2+10}}$$



$$\Delta x = \frac{4-2}{4} = 0,5$$

$y_0 = \frac{4}{\sqrt[3]{4+10}}$ extraindo a r. cúbica por logaritmo:

$$\log y_0 = \log 4 - \frac{1}{3}\log 14$$

$$y_1 = \frac{6,25}{\sqrt[3]{6,25+10}} = \frac{6,25}{\sqrt[3]{16,25}}$$

$$y_2 = \frac{9}{\sqrt[3]{9+10}} = \frac{9}{\sqrt[3]{19}}$$

$$y^3 = \frac{12,25}{\sqrt[3]{12,25+10}} = \frac{12,25}{\sqrt[3]{22,25}}$$

$$y^4 = \frac{16}{\sqrt[3]{16+10}} = \frac{16}{\sqrt[3]{26}}$$

$$A = \left\{\frac{1}{2}y_0 + y_1 + y_2 + y_3 + \frac{1}{2}y_4\right\}.\Delta x$$

$$A = \left\{\frac{2}{\sqrt[3]{14}} + \frac{6,25}{\sqrt[3]{16,25}} + \frac{9}{\sqrt{19}} + \frac{12,25}{\sqrt[3]{22,25}} + \frac{8}{\sqrt[3]{26}}\right\}.0,5 =$$

Para resolver, aplica-se logaritmação.

Exercícios do trabalho p/a crédito

① $\displaystyle\int_0^4 \frac{dx}{\sqrt{9-2x}}$

$$\int_0^4 (9-2x)^{-1/2}\,dx =$$

$$-\frac{1}{2}\int_0^4 (9-2x)^{-\frac{1}{2}}.(-2)\,dx =$$

$$-\frac{1}{\cancel{2}}\left[\frac{(9-2x)^{\frac{1}{2}}}{\frac{1}{\cancel{2}}}\right]_0^4 = -\left[\sqrt{9-2x}\right]_0^4 =$$

$$-\left[\sqrt{9-2.4} + \sqrt{9-2.0}\right] =$$

$$-\sqrt{9-8} + \sqrt{9} = -\sqrt{1} + \sqrt{9} = -1+3 = 2$$

② $\displaystyle\int_0^3 \frac{t\,dt}{\sqrt{t^2+16}}$

$$\int_0^3 (t^2+16)^{-1/2}\,t\,dt =$$

$$\frac{1}{2}\int_0^3 (t^2+16)^{-1/2}.(2)\,t\,dt =$$

$$\frac{1}{\cancel{2}}\left[\frac{(t^2+16)^{1/2}}{\frac{1}{\cancel{2}}}\right]_0^3 = \left[\sqrt{t^2+16}\right]_0^3 =$$

$$\left[\sqrt{3^2+16}\right] - \left[\sqrt{0^2+16}\right] =$$

$$\sqrt{9+16} - \sqrt{16} = \sqrt{25} - \sqrt{16} = 5-4 = 1$$

③ $\int_0^a \sqrt{a^2-x^2}\,dx =$

$$\int_0^a \sqrt{(a)^2-(x)^2}\,dx =$$

fórmula:

$$\boxed{\int \sqrt{a^2-v^2}\,dv = \frac{v}{2}\sqrt{a^2-v^2} + \frac{a^2}{2}\operatorname{arc\,sen}\frac{v}{a} + C}$$

$$\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\operatorname{arc\,sen}\frac{x}{a}\right]_0^a =$$

$$\left[\frac{a}{2}\sqrt{a^2-a^2} + \frac{a^2}{2}\operatorname{arc\,sen}\frac{a}{a}\right] - \left[\frac{0}{2}\sqrt{a^2-0^2} + \frac{0}{2}\operatorname{arc\,sen}\frac{0}{a}\right] =$$

$$\left[\frac{a}{2}\cdot 0 + \frac{a^2}{2}\operatorname{arc\,sen} 1\right] - \left[\frac{0}{2}\cdot a + \frac{0}{2}\operatorname{arc\,sen} 0\right] =$$

$$\left[0 + \frac{a^2}{2}\operatorname{arc\,sen} 1\right] - \left[0\right] =$$

$$\frac{a^2}{2}\operatorname{arc\,sen} 1 = \frac{a^2}{2}\cdot\frac{\pi}{2} = \boxed{\frac{a^2\pi}{4}}$$

④ Achar a área da curva limitada pela hipérbole:

$x^2 - y^2 = a^2$ reta $= 2a$

$\underline{y=0}$ $x^2 - 0 = a^2$

$x^2 = a^2$

$x = \sqrt{a^2}$ ∴ $\underline{x = a}$

limites $\begin{cases} x = 2a \rightarrow \text{superior} \\ x = a \rightarrow \text{inferior.} \end{cases}$

isolando y:

$$x^2 - y^2 = a^2 \Rightarrow -y^2 = a^2 - x^2 \Rightarrow y^2 = -a^2 + x^2$$

$y = \sqrt{-a^2+x^2} \therefore y = \sqrt{x^2-a^2} \Rightarrow$

$$\boxed{y = \sqrt{x^2-a^2}}$$

fórmula: $\int \sqrt{v^2 \pm a^2}\, dv =$

$\frac{v}{2}\sqrt{v^2 \pm a^2} \pm \frac{a^2}{2} \ln\left(v + \sqrt{v^2 \pm a^2}\right) + C$

$A = \int_a^{2a} \sqrt{x^2 - a^2}\, dx =$

$A = \int_a^{2a} \sqrt{(x)^2 - (a)^2}\, dx =$

$A = \left[\frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left(x + \sqrt{x^2-a^2}\right)\right]_a^{2a} =$

$A = \left[\frac{\cancel{2}a}{\cancel{2}}\sqrt{(2a)^2 - a^2} - \frac{a^2}{2}\ln\left(2a + \sqrt{(2a)^2 - a^2}\right)\right] - \left[\frac{a}{2}\sqrt{a^2 - a^2}\right.$

$\left. - \frac{a^2}{2}\ln\left(a + \sqrt{a^2 - a^2}\right)\right] =$

$A = \left[a\sqrt{3a^2} - \frac{a^2}{2}\ln\left(2a + \sqrt{3a^2}\right)\right] - \left[\frac{a}{2} \cdot 0 - \frac{a^2}{2}\ln(a+0)\right.$

$a^2\sqrt{3} - \frac{a^2}{2}\ln(2a^2 + \sqrt{3}) + \frac{a^2}{2}\ln a =$

$a^2\sqrt{3} - \frac{a^2}{2}\ln(2a^2 + \sqrt{3}) + \frac{a^2}{2} \cdot 0 =$

$a^2\sqrt{3} - \frac{a^2}{2}\ln(2a^2 + \sqrt{3}) =$

$a^2\left[\sqrt{3} - \frac{1}{2}\ln(2 + \sqrt{3})\right] =$

$$\boxed{a^2\left[2\sqrt{3} - \ln(2 + \sqrt{3})\right]}$$

16/4/73.

$\int_3^{10} \frac{dx}{x}$ $\qquad n = 7$

$y_0 \to y_7$

$A = \Delta x\left(\frac{1}{2}y_0 + y_1 + y_2 \ldots y_{n-1} + \frac{1}{2}y_n\right)$

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{10-3}{7} = 1 \qquad y = \frac{1}{x}$

$y = \sqrt{-a^2+x^2} \quad \therefore \quad y = \sqrt{x^2-a^2} \Rightarrow$

$$\boxed{y = \sqrt{x^2 - a^2}}$$

__fórmula:__ $\int \sqrt{v^2 \pm a^2}\, dv =$

$\frac{v}{2}\sqrt{v^2 \pm a^2} \pm \frac{a^2}{2} \ln\left(v + \sqrt{v^2 \pm a^2}\right) + C$

$A = \int_a^{2a} \sqrt{x^2 - a^2}\, dx =$

$A = \int_a^{2a} \sqrt{(x)^2 - (a)^2}\, dx =$

$A = \left[\frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left(x + \sqrt{x^2-a^2}\right)\right]_a^{2a} =$

$A = \left[\frac{\cancel{2}a}{\cancel{2}}\sqrt{(2a)^2 - a^2} - \frac{a^2}{2}\ln\left(2a + \sqrt{(2a)^2 - a^2}\right)\right] - \left[\frac{a}{2}\sqrt{a^2 - a^2}\right.$

$\left. - \frac{a^2}{2}\ln\left(a + \sqrt{a^2 - a^2}\right)\right] =$

$A = \left[a\sqrt{3a^2} - \frac{a^2}{2}\ln\left(2a + \sqrt{3a^2}\right)\right] - \left[\frac{a}{2} \cdot 0 - \frac{a^2}{2}\ln(a + 0)\right.$

$a^2\sqrt{3} - \frac{a^2}{2}\ln\left(2a^2 + \sqrt{3}\right) + \frac{a^2}{2}\ln a =$

$a^2\sqrt{3} - \frac{a^2}{2}\ln\left(2a^2 + \sqrt{3}\right) + \frac{a^2}{2} \cdot 0 =$

$a^2\sqrt{3} - \frac{a^2}{2}\ln\left(2a^2 + \sqrt{3}\right) =$

$a^2\left[\sqrt{3} - \frac{1}{2}\ln\left(2 + \sqrt{3}\right)\right] =$

$$\boxed{a^2\left[2\sqrt{3} - \ln\left(2 + \sqrt{3}\right)\right]}$$

16/4/73.

$\int_3^{10} \frac{dx}{x}$

$n = 7$

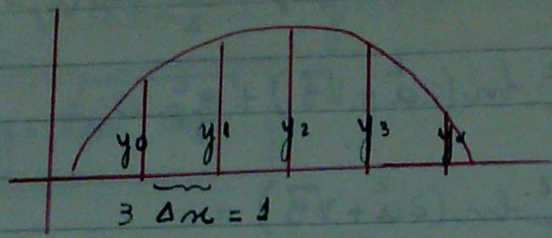
$y_0 \rightarrow y_7$

$A = \Delta x\left(\frac{1}{2}y_0 + y_1 + y_2 \ldots y_{n-1} + \frac{1}{2}y_n\right)$

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{10-3}{7} = 1$

$y = \frac{1}{x}$



$3 \ \Delta x = 1$

$y_0 = \frac{1}{3}$

$y_1 = \frac{1}{4}$

$y_2 = \frac{1}{5}$

$y_3 = \frac{1}{6}$

$y_4 = \frac{1}{7}$

$y_5 = \frac{1}{8}$

$y_6 = \frac{1}{9}$

$y_7 = \frac{1}{10}$

$$A = 1\left(\frac{1}{6} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{20}\right)$$

m.m.c = 2.520

$$A = \frac{420 + 630 + 504 + 420 + 360 + 315 + 280 + h}{2.520}$$

$$A = \frac{3055}{2520}$$

$$\boxed{A = 1{,}21}$$

$$\int_0^5 x\sqrt{25 - x^2}\, dx \qquad n = 10$$

$$y = x\sqrt{25 - x^2}$$

$$\Delta x = \frac{b - a}{x}$$

$$y_0 = 0\sqrt{25 - 0} = 0$$

$$y_{10} = 5\sqrt{25 - 25} = 0$$

$$\Delta x = \frac{5 - 0}{10} = \frac{5}{10} \therefore \Delta x = 0{,}5$$

$$y_1 = 0{,}5\sqrt{25 - 0{,}25}$$

$$y_2 = 0{,}5\sqrt{24{,}75}$$

$$\boxed{y_1 = 2{,}45}$$

$$y_2 = 1 \cdot \sqrt{25 - 1} = \sqrt{24}$$

$$\boxed{y_2 = 4{,}8}$$

$y_3 = 1{,}5\sqrt{25-(1{,}5)^2}$

$y_3 =$

$y_4 = 2\sqrt{25-4}$
$y_4 = 2\sqrt{21}$

$y_5 = 2{,}5\sqrt{25-6{,}25}$
$y_5 = 2{,}5\sqrt{31{,}25}$

$y_6 = 3\sqrt{25-9}$
$y_6 = 3.\sqrt{16} = 3.4 = 12$

$y_7 = 3{,}5\sqrt{25-(3{,}5)^2}$
$y_7 = 3{,}5\sqrt{25.}$

$y_8 = 4\sqrt{25-16}$
$y_8 = 4.\sqrt{9}$
$y_8 = 12$

$y_9 = 4.5\sqrt{25-(4{,}5)^2}$

$y_{10} = 0$

$A = 0{,}5\,(0 + 2{,}45 + 4{,}8 + \ldots\ldots + 0)$.

$\int_0^4 \frac{dx}{\sqrt{4+x^3}}$ $n = 4$

$y = \frac{1}{\sqrt{4-x^3}}$ $\Delta x = \frac{b-a}{x}$ $\Delta x = \frac{4-0}{4} = 1$

$y_0 = \frac{1}{\sqrt{4+0^3}} = \frac{1}{2}$

$y_0 = \frac{1}{\sqrt{4+0^3}} = \frac{1}{2}$
$y_1 = \frac{1}{\sqrt{4+1}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} = 0{,}44$

$y_2 = \frac{1}{\sqrt{4+8}} = \frac{\sqrt{12}}{12} = 0{,}28$

$y_3 = \frac{1}{\sqrt{4+27}} = \frac{\sqrt{31}}{31} = \frac{5{,}5}{31} = 0{,}17$

$y_4 = \frac{1}{\sqrt{4+64}} = \frac{1}{\sqrt{68}} = 0{,}12$

$A = 1\left(\frac{1}{4} + 0{,}44 + 0{,}28 + 0{,}17 + 0{,}06\right)$