3º termo

Cálculo Integral e Diferencial II

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Integrais

$f(x) = 3x^2$

$f'(x) = 6x$

$\int \rightarrow$ sinal de integração

$\int f'(x) = \int 6x = 3x^2$

$dx = 3x^2$

$\downarrow$ diferencial que indica qual variável que vai integrar.

Propriedades

① Uma constante pode ser coloc
da antes do sinal de integraç

$$\int 6x\, dx = 6 \int x\, dx$$

② A integral de uma soma é igual à soma das integrai
das parcelas.

$$\int (3x^2 + 7x + 2)\, dx =$$

$$\int 3x^2 dx + \int 7x\, dx + \int 2\, dx =$$

$$3 \int x^2 dx + 7 \int x\, dx + 2 \int dx$$

Integral Indefinida - Constante de integração.

$f(x) = 6x^2 \rightarrow f'(x) = 12x\, dx$

$f(x) = 6x^2 + 8 \rightarrow f'(x) = 12x\, dx$

$f(x) = 6x^2 + b \rightarrow f'(x) = 12x\, dx$

Estas funções diferem apenas por uma constante, então a derivação é a mesma.

Integrando, teremos:

$$\int 12x = 6x^2 + c$$

$c \rightarrow$ constante de integração

$\frac{dy}{dx} = 7x^2 \rightarrow$ forma derivada

$dy = 7x^2 dx \rightarrow$ forma diferencial

$y = 8x^3 \rightarrow$ função

$\frac{dy}{dx} = 24x^2 \rightarrow$ derivada

$dy = 24x^2 dx \rightarrow$ diferencial

a partir da diferencial, faz-se a integração.

Artifício de cálculo

Multiplicando-se ou dividindo-se uma integral por um nº diferente de zero, a integral não se altera.

$$3 \int x^4 dx = \frac{3}{\cancel{3}} \int \cancel{3} x^4 dx = \int 3x^4 dx$$

Regras Práticas

① $\boxed{\int (dy + dv - dw) = \int dy + \int dv - \int dw}$

② $\boxed{\int a\,dv = a \int dv}$

③ $\boxed{\int dx = x + c}$

④ $\boxed{\int v^n dv = \frac{v^{n+1}}{n+1} + c}$

⑤ $\boxed{\int \frac{dv}{v} = \ln v + c = \ln cv}$

Exercícios de Aplicação

① $\frac{d}{dx}(3x^4) = 4 \cdot 3x^{4-1} = 12x^3$

$\int 12x^3 dx = \frac{12x^{3+1}}{3+1} + c = \boxed{\frac{12x^4}{4} + c}$

② $\int x^4 dx$

aplicando a regra da potência:

$x^4 dx = \boxed{\frac{x^5}{5} + c}$

③ $\int \frac{dx}{x^2}$

$\int x^{-2} dx = \frac{x^{-1}}{-1} + c = \boxed{-\frac{1}{x} + c}$

④ $\int x^{\frac{2}{3}} dx$

$\int x^{\frac{2}{3}} dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c = \boxed{\frac{3x^{\frac{5}{3}}}{5}}$

como se faz a transformação:

$$x^{\frac{5}{3}} \div \frac{5}{3} = x^{\frac{5}{3}} \times \frac{3}{5} = \boxed{\frac{3x^{\frac{5}{3}}}{5}}$$

⑤ $\int \frac{dx}{\sqrt{x}}$

$$\int \frac{dx}{x^{\frac{1}{2}}} = \int x^{-\frac{1}{2}} dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \boxed{2\sqrt{x} + c}$$

⑥ $\int \frac{dx}{\sqrt[3]{x}}$

$$\int \frac{dx}{x^{\frac{1}{3}}} = \int x^{-\frac{1}{3}} dx = \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c = \boxed{\frac{3x^{\frac{2}{3}}}{2} + c}$$

⑦ $\int 3ay^2 dy$ 3a → constante

$$3a \int y^2 dy = \cancel{3}a \left[\frac{y^3}{\cancel{3}}\right] + c = \boxed{ay^3 + c}$$

⑧ $\int \frac{2dt}{t^2}$

$$2\int t^{-2} dt = 2\left[\frac{t^{-1}}{-1}\right] + c = \boxed{-\frac{2}{t} + c}$$

⑨ $\int \sqrt{ax}\, dx$

$$a^{\frac{1}{2}} \int x^{\frac{1}{2}} dx = a^{\frac{1}{2}} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$$

$$\frac{2x\sqrt{ax}}{3} + c \qquad x^{\frac{3}{2}} = \boxed{\sqrt{x^3} = x\sqrt{x}}$$

<u>transformação</u>:

$$\int \sqrt{ax}\, dx = (ax)^{\frac{1}{2}} = a^{\frac{1}{2}} = x^{\frac{1}{2}}$$

elimina-se o radical e aplica-se a integral de uma vez.

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⑩ $\int \frac{dx}{\sqrt{2x}}$

$$\int \frac{dx}{(2x)^{\frac{1}{2}}} = \int (2x)^{-\frac{1}{2}} \cdot 2\, dx =$$

$$\cancel{\tfrac{1}{2}} \left[\left(\frac{2x}{\cancel{\frac{1}{2}}}\right)^{\frac{1}{2}} + c = \sqrt{2x} + c\right.$$

$$\frac{dx}{\sqrt{2x}} = \int \frac{dx}{2^{\frac{1}{2}} \cdot x^{\frac{1}{2}}} = \frac{1}{2^{\frac{1}{2}}} \int \frac{dx}{x^{\frac{1}{2}}} =$$

$$\int x^{-\frac{1}{2}} \cdot dx = \frac{1}{\sqrt{2}} \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right] + c =$$

$$\frac{2\sqrt{x}}{\sqrt{2}} + c = \frac{2\sqrt{2x}}{2} + c = \boxed{\sqrt{2x} + c}$$

(11) $\int \sqrt[3]{3t}\, dt$

$$\int 3^{\frac{1}{3}} \cdot t^{\frac{1}{3}} dt =$$

$$3^{\frac{1}{3}} \int t^{\frac{1}{3}} dt =$$

$$3^{\frac{1}{3}} \cdot \left[\frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + c =$$

$$\frac{3^{\frac{4}{3}} \cdot t^{\frac{4}{3}}}{4} + c = \boxed{\frac{(3t)^{\frac{4}{3}}}{4} + c}$$

(12) $\int (x^{\frac{3}{2}} - 2x^{\frac{2}{3}} + 5\sqrt{x} - 3)\, dx =$

$$\int x^{\frac{3}{2}} dx - \int 2x^{\frac{2}{3}} dx + \int 5\sqrt{x}\, dx - \int 3\, dx =$$

$$\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{2 \cdot x^{\frac{5}{3}}}{\frac{5}{3}} + 5 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 3x =$$

$$\boxed{\frac{2x^{\frac{5}{2}}}{5} - \frac{6x^{\frac{5}{3}}}{5} + \frac{10x^{\frac{3}{2}}}{3} - 3x + c}$$

(13) $\int \frac{4x^2 - 2\sqrt{x}}{x} \cdot dx$

$$\int \left(\frac{4x^{\cancel{2}}}{\cancel{x}} - \frac{2\sqrt{x}}{x} \right) dx$$

?

$$4 \int x\, dx - \int \frac{2}{x^{\frac{1}{2}}} =$$

$$4 \int x\, dx - 2 \int x^{-\frac{1}{2}} =$$

$$\cancel{4}^{2} \left(\frac{x^2}{\cancel{2}} \right) - 2 \left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right) + c =$$

$$\boxed{2x^2 - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c} = \boxed{2x^2 - 4\sqrt{x} + c}$$

(14) $\int \left(\frac{x^2}{2} - \frac{2}{x^2}\right) dx$

$\int \frac{x^2}{2} dx - 2\int x^{-2} dx =$

$\frac{1}{2}\int x^2 dx - 2\int x^{-2} dx =$

$\frac{1}{2}\left[\frac{x^3}{3}\right] dx - 2\left[\frac{x^{-1}}{-1}\right] + c$

$\boxed{\frac{x^3}{6} + \frac{2}{x} + c}$

(15) $\int \sqrt{x}\cdot(3x-2)\, dx$

$\int (3\sqrt{x^3} - 2\sqrt{x})\, dx = \int 3\sqrt{x^3}\, dx - \int 2\sqrt{x}\, dx =$

$3\int x^{\frac{3}{2}} dx - 2\int x^{\frac{1}{2}} dx = 3\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) - 2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)$

$\boxed{\frac{6x^{\frac{5}{2}}}{5} - \frac{4x^{\frac{3}{2}}}{3}}$

(16) $\int \frac{x^3 - 6x + 5}{x} dx$

$\left(\frac{dv}{v}\right) = \ln v + c$

$\int \left(\frac{x^{\cancel{3}2}}{x} - \frac{6\cancel{x}}{\cancel{x}} + \frac{5}{x}\right) dx =$

$\int x^2 \cdot x - 6\int dx + 5\int \frac{dx}{x} =$

$\boxed{\frac{x^3}{3} - 6x + 5\ln x + c}$

(17) $\int \sqrt{a+bx}\, dx$

$\int (a+bx)^{\frac{1}{2}} \cdot dx =$

$\frac{1}{b}\int (a+bx)^{\frac{1}{2}} (b) dx = \frac{1}{b}\left[\frac{(a+bx)^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$

$\boxed{\frac{2(a+bx)^{\frac{3}{2}}}{3b} + c}$

(18) $\int \frac{dy}{\sqrt{a-by}}$

$\int \frac{dy}{(a-by)^{\frac{1}{2}}} = \int (a-by)^{-\frac{1}{2}} dy =$

22 $\int x(2x+1)^2\,dx$

$\int (2x+1)^2 x\,dx$

$\int (4x^2+4x+1)\,x\,dx =$

$\int (4x^3+4x^2+x)\,dx =$

$4\int x^3\,dx + 4\int x^2\,dx + \int x\,dx =$

$4.\dfrac{x^4}{4} + 4\dfrac{x^3}{3} + \dfrac{x^2}{2} + c$

$\boxed{x^4 + \dfrac{4x^3}{3} + \dfrac{x^2}{2} + c}$

23 $\int \dfrac{4x^2\,dx}{\sqrt{x^2+8}}$

$\int \dfrac{4x^2\,dx}{(x^3+8)^{\frac{1}{2}}} = \int 4x^2(x^3+8)^{-\frac{1}{2}}\,dx =$

$4\int (x^3+8)^{-\frac{1}{2}}.\,x^2\,dx =$

$\times 3$
$\div 3$ } o q. falta p/ compl. a der.

$\dfrac{4}{3}\int (x^3+8)^{-\frac{1}{2}}.\,3x^2\,dx =$

$\dfrac{4}{3}\left[\dfrac{(x^3+8)^{\frac{1}{2}}}{\frac{1}{2}}\right] + c =$

$\dfrac{8}{3}(x^3+8)^{\frac{1}{2}} + c = \boxed{\dfrac{8\sqrt{x^3+8}}{3} + c}$

24 $\int \dfrac{6z\,dz}{(5-3z^2)^2}$

$-\int (5-3z^2)^{-2} - 6z\,dz$

$-\dfrac{(5-3z^2)^{-1}}{-1} + c =$

$\boxed{\dfrac{1}{5-3z^2} + c}$

25 $\int (\sqrt{a} - \sqrt{x})^2\,dx$ $\qquad \int dx = x$

desenvolve 1º o quadrado da diferença.

$(\sqrt{a^2} - 2.\sqrt{a}.\sqrt{x} + \sqrt{x^2})\,dx =$

$\int a\,dx - 2\int \sqrt{a}\,\sqrt{x} + \int x\,dx =$

$a\int dx - 2\sqrt{a}\int x^{\frac{1}{2}} + \int x + c$

$ax - 2\sqrt{a}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^2}{2} + c$

$ax - \frac{4x\sqrt{ax}}{3} + \frac{x^2}{2} + c$

(26) $\int \frac{(\sqrt{a} - \sqrt{x})^2}{\sqrt{x}}\,dx$

der. de $\sqrt{x}$

$-2\int(\sqrt{a} - \sqrt{x})^2 - \frac{1}{2\sqrt{x}}\,dx =$

$-2\left[\frac{(\sqrt{a} - \sqrt{x})^3}{3} + c =\right.$

$$\boxed{-2\,\frac{(\sqrt{a} - \sqrt{x})^3}{3} + c}$$

OBS: der. de $\sqrt{a} - \sqrt{x}$: $-\sqrt{x} = -(x)^{\frac{1}{2}}$

$\frac{d}{dx} - (x)^{\frac{1}{2}} = -\frac{1}{2}x^{-\frac{1}{2}} = -\frac{1}{2\sqrt{x}}$

(27) $\int \sqrt{x}\,.\,(\sqrt{a} - \sqrt{x})^2\,dx$

quadrado da soma

$\int (a - 2\sqrt{a}\sqrt{x} + x)\,\sqrt{x}\,dx$

desenvolvendo a potência:

$\int(a\sqrt{x} - 2\sqrt{a}x + \sqrt{x^3})\,dx =$

$a\int\sqrt{x}\,dx - 2\sqrt{a}\int x\,dx + \int x^{\frac{3}{2}}\,dx =$

$a\int x^{\frac{1}{2}}\,dx - 2\sqrt{a}\int x\,dx + \int x^{\frac{3}{2}}\,dx$

$a.\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2\sqrt{a}\,\frac{x^2}{2} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$$\boxed{\frac{2ax^{\frac{3}{2}}}{3} - x^2\sqrt{a} + \frac{2x^{\frac{5}{2}}}{5} + c}$$

(28) $\int \frac{t^3\,dt}{\sqrt{a^4 + t^4}}$

$\int (a^4 + t^4)^{-\frac{1}{2}}\,t^3\,dt =$

$$\int (a^4+t^4)^{-\frac{1}{2}}\ 4t^3 dt =$$

$$\frac{1}{4}\int (a^4+t^4)^{-\frac{1}{2}}$$

$$\frac{1}{4}\left[\frac{(a^4+t^4)^{\frac{1}{2}}}{\frac{1}{2}}\right] + c =$$

$$\boxed{\frac{(a^4+t^4)^{\frac{1}{2}}}{2} + c}$$

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(33) $$\int \frac{dy}{(a+by)^3}$$

$$\int (a+by)^{-3}\, dy =$$

$$\frac{1}{b}\int (a+by)^{-3}\, b\, dy$$

$$\frac{1}{b}\left[\frac{(a+by)^{-2}}{-2} + c\right] =$$

$$\boxed{-\frac{1}{2b(a+by)^2} + c}$$

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(34) $$\int \frac{(2x+3)\,dx}{\sqrt{x^2+3x}}$$

$$\int (x^2+3x)^{-\frac{1}{2}}\cdot(2x+3)\,dx$$

$$\frac{(x^2+3x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$\frac{(x^2+3x)^{\frac{1}{2}}}{\frac{1}{2}} + c = \boxed{2\sqrt{x^2+3x} + c}$$

(35) $$\int \frac{(x^2+1)\,dx}{\sqrt{x^3+3x}} =$$

$$\int (x^3+3x)^{-\frac{1}{2}}\cdot(x^2+1)\,dx$$

$$\frac{1}{3}\,\frac{(x^3+3x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\cdot 3(x^2+1)\,dx$$

$$\frac{1}{3}\,\frac{(x^3+3x)^{\frac{1}{2}}}{\frac{1}{2}} + c = \boxed{\frac{2\sqrt{x^3+3x}}{3} + c}$$

12/2/73 $\int v^N dv = \frac{v^{N+1}}{N+1} + c$

Integral de uma constante

$\int ax\, dv = a \int x\, dv$

Integral de uma soma

$\int (a + bx^2)\, dx = \int ax\, dx + \int b x^2\, dx$

$\int (x^a)^N \cdot a x^{a-1}\, dx = \int v^N dv$

Continuação das fórmulas

$\boxed{\frac{d}{dx}(\text{sen}\, v) = \cos v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\cos v) = -\text{sen}\, v\, \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(tg\, v) = \sec^2 v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\text{cotg}\, v) = -\text{cossec}^2 v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\sec v) = \sec v \cdot tg\, v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\text{cossec}\, v) = -\text{cossec}\, v \cdot \text{cotg} \cdot v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\text{vers}\, v) = +\text{sen}\, v \cdot \frac{dv}{dx}}$

Exercícios:

30) $\int \frac{x\, dx}{(a+bx^2)^3} = \int (a+bx^2)^{-3} x\, dx =$

$\frac{1}{2b} \int (a^v + b x^2)^{\overset{N}{-3}} \cdot 2 b x\, dx$

desenvolver

$\frac{1}{2b} \frac{[a + bx^2]^{-2}}{-2} + C =$

$\frac{(a+bx^2)^{-2}}{-4b} + C = -\frac{1}{4b(a+bx^2)^2} + C$

12/2/73. $\int v^N dv = \frac{v^{N+1}}{N+1} + c$

Integral de uma constante

$\int ax\, dv = a \int x\, dv$

Integral de uma soma

$\int (a+bx^2)\, dx = \int ax\, dx + \int bx^2\, dx$

$\int (x^a)^N \cdot a x^{a-1}\, dx = \int v^N dv$

Continuação das fórmulas

$\boxed{\frac{d}{dx}(\operatorname{sen} v) = \cos v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\cos v) = -\operatorname{sen} v \, \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\operatorname{tg} v) = \sec^2 v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\operatorname{cotg} v) = -\operatorname{cossec}^2 v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\sec v) = \sec v \cdot \operatorname{tg} v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\operatorname{cossec} v) = -\operatorname{cossec} v \cdot \operatorname{cotg} v \cdot \frac{dv}{dx}}$

$\boxed{\frac{d}{dx}(\operatorname{vers} v) = +\operatorname{sen} v \cdot \frac{dv}{dx}}$

Exercícios:

(30) $\int \frac{x\, dx}{(a+bx^2)^3} = \int (a+bx^2)^{-3} x\, dx =$

$\frac{1}{2b} \int (a+bx^2)^{-3} \cdot 2bx\, dx$

desenvolver

$\frac{1}{2b} \left[\frac{(a+bx^2)^{-2}}{-2} \right] + C =$

$\frac{(a+bx^2)^{-2}}{-4b} + C = -\frac{1}{4b(a+bx^2)^2} + C$

derivada do seno

(38) $\int sen\, ax \cdot cos\, ax\, dx = (sen ax)' = a.cos ax$

$\frac{1}{a}\int (sen\, ax)^1 \cdot a\, cos\, ax\, dx$

$\frac{1}{a}\left(\frac{sen\, ax}{2}\right)^2 + c = \frac{sen^2 ax}{2a} + c$

(39) $\int sen\, 2x\, cos^2 2x\, dx =$

$\int (cos\, 2x)^2 \cdot sen\, 2x\, dx =$

?

$-\frac{1}{2}\int (cos\, 2dx)^2 \cdot (-2\ sen\, x)dx =$ -2 ?

$\frac{1}{2}\left(\frac{cos\, 2x}{3}\right)^3 + c = -\frac{cos^3 2x}{6} + c$

(40) $\int tg\, \frac{x}{2}\ sec^2 \frac{x}{2}\, dx =$ N+1

$2\int \left(tg\, \frac{x}{2}\right)^1 \frac{1}{2}\ sec^2 \frac{x}{2} \cdot dx =$

$\cancel{2}\frac{tg^2 \frac{x}{2}}{\cancel{2}} + c = tg^2 \frac{x}{2} + c$

(41) $\int \frac{cos\, ax\, dx}{\sqrt{b + sen\, ax}} =$

$\int (b + sen\, ax)^{-\frac{1}{2}} cos\, ax\, dx$

$\frac{1}{a}\frac{(b + sen\, ax)^{\frac{1}{2}}}{\frac{1}{2}} + c =$

$\frac{2\sqrt{b + sen\, ax}}{a} + c$

(42) $\int \left(\frac{sec\, x}{1 + tg x}\right)^2 dx =$

$\int \frac{sec^2 x\, dx}{(1 + tg\, x)^2} = \int (1 + tg\, x)^{-2} \cdot sec^2 x$

$\frac{(1 + tg x)^{-1}}{-1} + c = -\frac{1}{1 + tg x} + c$

(43) $\int \frac{dx}{2 + 3x} =$

$\frac{1}{3}\int \frac{3\, dx}{2 + 3x} = \frac{1}{3} ln v + c \rightarrow \frac{1}{3} ln(2+3x$ dv

$\int \frac{dv}{v} = \ln v + c =$

$\frac{1}{3} \ln (2 + 3x) + c$

(44) $\int \frac{x^2 dx}{2+x^3} =$

$\int u^n du = \frac{u^{N+1}}{N+1} + c \rightarrow n \neq -1$

$\ln u + c \rightarrow N = -1$

$\frac{1}{3} \int \frac{3x^2 dx}{2+x^3} = \frac{1}{3} \ln v + c =$

$\frac{1}{3} \ln (2 + x^3) + c$

(45) $\int \frac{t\,dt}{(a+bt^2)^{+1}}$

$\ln v + c \rightarrow n-1$

$\frac{1}{2b} \int \frac{2\,bt\,dt}{a+bt^2} = \frac{1}{2b} \ln v + c$

$\frac{1}{2b} \ln (a + bt^2) + c$

(46) $\int \frac{(2x+3)\,dx}{x^2+3x}$

$\ln (x^2+3x) + c$

(47) $\int \frac{(y+2)\,dy}{y^2+4y}$

$\frac{1}{2} \int \frac{2(y+2)\,dy}{y^2+4y} = \frac{\ln (y^2+4y)}{2}$

(48) $\int \frac{e^\theta d\theta}{a+be^\theta} =$

$\frac{1}{b} \int \frac{be^\theta d\theta}{a+be^\theta} = \frac{1}{b} \ln (a+be^\theta) + c$

$\int \frac{dv}{v} = \ln v + c =$

$\frac{1}{3} \ln(2+3x) + c$

(44) $\int \frac{x^2\,dx}{2+x^3} =$

$\int u^n\,du = \frac{u^{n+1}}{n+1} + c \rightarrow n \neq -1$

$\ln u + c \rightarrow n = -1$

$\frac{1}{3}\int \frac{3x^2\,dx}{2+x^3} = \frac{1}{3}\ln v + c =$

$\frac{1}{3}\ln(2+x^3) + c$

(45) $\int \frac{t\,dt}{(a+bt^2)^{+1}}$

$\ln v + c \rightarrow n-1$

$\frac{1}{2b}\int \frac{2bt\,dt}{a+bt^2} = \frac{1}{2b}\ln v + c$

$\frac{1}{2b}\ln(a+bt^2) + c$

(46) $\int \frac{(2x+3)\,dx}{x^2+3x}$

$\ln(x^2+3x) + c$

(47) $\int \frac{(y+2)\,dy}{y^2+4y}$

$\frac{1}{2}\int \frac{2(y+2)\,dy}{y^2+4y} = \frac{\ln(y^2+4y)}{2} + c$

(48) $\int \frac{e^\theta\,d\theta}{a+be^\theta} =$

$\frac{1}{b}\int \frac{be^\theta\,d\theta}{a+be^\theta} = \frac{1}{b}\ln(a+be^\theta) + c$

(49) $\int \frac{\operatorname{sen} x \, dx}{1-\cos x} = \ln(1-\cos x) + c$

(50) $\int \frac{\sec^2 y \, dy}{a + b\,tg\,y} =$

$\frac{1}{b}\int \frac{b \sec^2 y \, dy}{a + b\,tg\,y} = \frac{1}{b} \ln(a + b\,tg\,y) + c$

(51) $\int \frac{(2x+3)\,dx}{x+2}$

$\int \left(2 - \frac{1}{x+2}\right) dx$

$\int 2\,dx - \int \frac{dx}{x+2} = 2x - \ln(x+2) + c$

Aplica-se a divisão:

$$\begin{array}{l|l} 2x+3 & x+2 \\ \hline -2x-4 & 2 - \frac{1}{x+2} \\ \hline -1 & \\ +1 & \\ \hline 0 & \end{array}$$

$$\begin{array}{l|l} x^2+0x+2 & x+1 \\ \hline -x^2-x & x - 1 + \frac{3}{x+1} \\ \hline -\quad -x+2 & \\ +x+1 & \\ \hline -\quad +3 & \end{array}$$

(52) $\int \frac{(x^2+2)}{x+1} dx$

$\int \left(x - 1 + \frac{3}{x+1}\right) dx$

$\int x\,dx - \int dx + 3\int \frac{1}{x+1} dx =$

$\frac{x^2}{2} - x + 3\ln(x+1) + c$

(53) $\int \frac{(x+4)\,dx}{(2x+3)^1}$

$$\begin{array}{l|l} x+4 & 2x+3 \\ \hline -x-\frac{3}{2} & \frac{1}{2} + \frac{5/2}{2x+3} \\ \hline \quad \frac{5}{2} & \\ -\frac{5}{2} & \\ \hline 0 & \end{array}$$

$\int \left(\frac{1}{2} + \frac{5}{2(x+3)}\right) dx$

$\downarrow \times 2$

$\frac{1}{2}\int dx + \frac{5}{4}\int \frac{2}{2x+3} dx =$ 1×2

$\frac{x}{2} + \frac{5}{4} \cdot \ln(2x+3) + c$

(54) $\int \frac{e^{2s}\,ds}{e^{2s}+1} = \frac{1}{2}\int \frac{2e^{2s}\,ds}{e^{2s}+1} = \frac{1}{2}\ln(e^{2s}+1)$

(68) $\int \left(\frac{x^2-4}{x^4}\right) dx$

$\int (x)^{-4} \cdot (x^2-4)\, dx$

$\frac{1}{2} \int (x)^{-4} \cdot 2(x^2-4)\, dx =$

$\frac{1}{2} \cdot \frac{x^{-3}}{-3} + c = \frac{x^{-3}}{6} = -\frac{1}{6x^3} + c$

16/02 **Fórmulas**

6) $\boxed{\int a^v dv = \frac{a^v}{\ln a} + c}$ → qdo a base é uma constante

7) $\boxed{\int e^v dv = e^v + c}$ $\ln e = 1$

Exercícios

(1) $\int 6e^{3x} \cdot dx = 6 \int e^{3x} dx$ (e) é constante

$\frac{6}{3} \int e^{\overset{v}{3x}} \cdot \underbrace{3\,dx}_{dv} = 2 \cdot e^{3x} + c$ $\frac{d}{dx}(3x) = 3$

$d(3x) = 3dx$

(2) $\int e^4 dx = e^4 x$

$e^4 \int dx$

$\boxed{e^4 \cdot x + c}$

(3) $\int e^{\frac{x}{N}} dx$

dx não é dif. de $\frac{x}{N}$ $(dx \neq d\,\frac{x}{N})$

(68) $\int \frac{(x^2-4)}{x^4}\,dx$

$$\int (x)^{-4} \cdot (x^2-4)\,dx$$

$$\frac{1}{2}\int (x)^{-4} \cdot 2(x^2-4)\,dx =$$

$$\frac{1}{2} \cdot \frac{x^{-3}}{-3} + C = \frac{x^{-3}}{6} = -\frac{1}{6x^3} + C$$

16/02 **Fórmulas**

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Exercícios

(1) $\int 6e^{3x} \cdot dx = 6\int e^{3x}\,dx$ (e) é constante

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$d(3x) = 3dx$

(2) $\int e^4\,dx = e^4 x$

$e^4 \int dx$

$\boxed{e^4 \cdot x + C}$

(3) $\int e^{\frac{x}{N}}\,dx$

dx não é dif. de $\frac{x}{N}$ $\left(dx \neq d\,\frac{x}{N}\right)$

a derivada de $\frac{x}{N}$:

$\frac{d}{dx}\left(\frac{x}{N}\right) = \frac{1}{N}$ → falta antes de dx:

então, temos:

$$\int e^{\frac{x}{N}} dx = N \int e^{\frac{x}{N}} \cdot \frac{1}{N} dx = \boxed{N e^{\frac{x}{N}} + C}$$

④ $\int \frac{dx}{e^x}$

$$\int e^{-x} dx = - \int e^{-x} (-dx) = \boxed{-\frac{1}{e^x} + c}$$

⑤ $\int 10^x dx$

aplica-se a fórmula: $\int a^v dv = \frac{a^v}{\ln a} + c$
porque 10 é uma constante.

$$\int 10^x dx = \boxed{\frac{10^x}{\ln 10} + c}$$

⑥ $\int a^{ny} dy$

$d(ny) = n\,dy$ então $\int a^{ny} dy = \frac{1}{N} \int a^{ny} \cdot n\,dy$ →

$$\boxed{\frac{a^{NY}}{N \ln a} + c}$$

⑦ $\int \frac{e^{\sqrt{x}} dx}{\sqrt{x}}$

$$\int e^{x^{\frac{1}{2}}} \cdot x^{-\frac{1}{2}} dx$$

$$2 \int e^{\sqrt{x}} \cdot \frac{1}{2} x^{-\frac{1}{2}} dx = \boxed{2 e^{\sqrt{x}} + c}$$

$$\frac{d}{dx}(x^{\frac{1}{2}}) = \frac{1}{2} x^{-\frac{1}{2}} \to \frac{1}{2\sqrt{x}}$$

⑧ $\int \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) dx$

$$\int e^{\frac{x}{a}} dx + \int e^{-\frac{x}{a}} dx =$$

$$a \int e^{\frac{x}{a}} \cdot \frac{1}{a} dx + (-a) \int e^{-\frac{x}{a}} \cdot (-\tfrac{1}{a}) dx =$$

$$\boxed{a \left(e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right) + c}$$

⑨ $\int \left(e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right)^2 \cdot dx$

aplica-se primeiramente o quadrado da diferença.

$$\int \left(e^{\frac{2x}{a}} - 2 \cdot e^{\frac{x}{a}} \cdot e^{-\frac{x}{a}}\right) dx = $$

$$\int e^{\frac{2x}{a}} dx + \int e^{\frac{-2x}{a}} dx - \int 2\, dx$$

Obs: $d\left(\frac{2x}{a}\right) = \frac{2}{a} dx$

$d\left(-\frac{2x}{a}\right) = -\frac{2}{a} dx$

$$= \frac{a}{2} \int e^{\frac{2x}{a}} \cdot \frac{2}{a} dx - \frac{a}{2} \int e^{-\frac{2x}{a}} \left(-\frac{2}{a}\right) dx - 2\int dx$$

$$= \frac{a}{2} \cdot e^{\frac{2x}{a}} - \frac{a}{2} \cdot e^{\frac{-2x}{a}} - 2x + c$$

$$\boxed{= \frac{a}{2}\left(e^{\frac{2x}{a}} - e^{\frac{-2x}{a}}\right) - 2x + c}$$

$$\int \sqrt{e^t}\, dt$$

$$\int (e^t)^{\frac{1}{2}} dt \quad 2\int e^{\frac{t}{2}} \cdot \frac{t}{2\sqrt{t}}\, dt = 2e^{\frac{t}{2}} + C = \boxed{2\sqrt{e^t} + C}$$

$$\int e^{\frac{t}{2}} dt = \boxed{\int e^{t/2} dt}$$

17/02

Exercícios

⑨ $\int x e^{x^2} dx$

$$\int e^{\overset{v}{x^2}} \cdot \underbrace{x\, dx}_{dv} = d(x^2) = 2x\, dx = \frac{1}{2}\int e^{x^2} \cdot 2x\, dx$$

$$\boxed{\frac{e^{x^2}}{2} + c}$$

⑩ $\int e^{\operatorname{sen} x} \cos x\, dx$

$v = e^{\operatorname{sen} x}$ $\quad dv = \cos x\, dx$

solução: imediata $\rightarrow$ $\boxed{e^{\operatorname{sen} x} + c}$

⑪ $\int e^{tg\,\theta} \sec^2\theta\, d\theta$

$v = e^{tg\,\theta}$ $\quad dv = \sec^2\theta$

solução: imediata $\rightarrow$ $\boxed{e^{tg\,\theta} + c}$

(12) $\int \sqrt{e^t}\, dt$

$\int e^{\frac{t}{2}}\, dt = 2\int e^{\frac{t}{2}} \cdot \frac{1}{2}\, dt = \boxed{2\sqrt{e^t} + c}$

(13) $\int a^x e^x\, dx$

$\int (ae)^x\, dx = \dfrac{a^x e^x}{\ln(ae)} + c = \boxed{\dfrac{a^x e^x}{1+\ln a} + c}$

(14) $\int a^{2x}\, dx$

$\dfrac{\frac{1}{2}\int a^{2x} \cdot 2\, dx}{\ln a} = \boxed{\dfrac{a^{2x}}{2\ln a} + c}$

(15) $\int (e^{5x} + a^{5x})\, dx =$

$\int e^{5x} dx + \int a^{5x} dx = \frac{1}{5}\int e^{5x} \cdot 5\, dx +$

$\frac{1}{5}\int a^{5x} \cdot 5\, dx = \frac{1}{5} e^{5x} + \frac{1}{5}\dfrac{a^{5x}}{\ln a} + c =$

$\boxed{\frac{1}{5}\left(e^{5x} + \dfrac{a^{5x}}{\ln a}\right) + c}$

(16) $\int 5 e^{ax}\, dx$

$5\int e^{ax} dx = \frac{5}{a}\int e^{ax} \cdot a\, dx = \boxed{\frac{5}{a} e^{ax} + c}$

(17) $\int \dfrac{3\, dx}{e^x} =$

$3\int e^{-x} (-dx) = -3 \cdot \dfrac{1}{e^x} + c = \boxed{-\dfrac{3}{e^x} + c}$

(18) $\int \dfrac{4\, dt}{\sqrt{e^t}} =$

$\int e^{-\frac{t}{2}}\, 4\, dt = 4\int e^{-\frac{t}{2}}\, dt =$

$-\frac{1}{2}$ é a derivada do expoente:

$-2 \cdot 4\int e^{-\frac{t}{2}} \cdot \left(-\frac{1}{2}\right) dt = -8 \cdot e^{-\frac{t}{2}} + c =$

$\boxed{\dfrac{-8}{\sqrt{e^t}} + c}$

(19) $\int e^{ax}\,dx =$

$$\frac{1}{a}\int e^{ax}\cdot a\,dx = \frac{e^{ax}}{a} + c$$

(20) $\int \frac{dx}{4^{2x}} =$

$$\int 4^{-2x}\,dx =$$

$$-\frac{1}{2}\int 4^{-2x}(-2)\,dx = \boxed{-\frac{4^{-2x}}{2\ln 4} + c}$$

Pode ser escrito também assim:

$$\boxed{-\frac{1}{4^{2x}\cdot 2\ln 4} + c}$$

Fórmulas

(8) $\boxed{\int \operatorname{sen} v\cdot dv = -\cos v + c}$

(9) $\boxed{\int \cos v\cdot dv = \operatorname{sen} v + c}$

(10) $\boxed{\int \sec^2 v\,dv = \operatorname{tg} v + c}$

(11) $\boxed{\int \operatorname{cossec}^2 v\cdot dv = -\operatorname{cotg} v + c}$

(12) $\boxed{\int \sec v\cdot \operatorname{tg} v\cdot dv = \sec v + c}$

(13) $\boxed{\int \operatorname{cossec} v\ \operatorname{cotg} v\,dv = -\operatorname{cossec} v + c}$

(14) $\boxed{\int \operatorname{tg} v\,dv = -\ln \cos v + c = \ln \sec v + c}$

(15) $\boxed{\int \operatorname{cotg} v\,dv = \ln \operatorname{sen} v + c}$

(16) $\boxed{\int \sec v\,dv = \ln(\sec v + \operatorname{tg} v) + c}$

(17) $\boxed{\int \operatorname{cossec} v\,dv = \ln(\operatorname{cossec} v - \operatorname{cotg} v) + c}$

Exercícios

① $\int \underbrace{\cos m x}_{v} \, \underbrace{dx}_{dv} =$

$\frac{1}{m} \int \cos mx \, . \, m\, dx =$

$\boxed{\frac{1}{m} . \operatorname{sen} mx + c}$

② $\int tg \underbrace{bx}_{v} \, \underbrace{dx}_{dv} =$

$\frac{1}{b} \int tg\, bx \, . \, b\, dx =$

$\boxed{-\frac{1}{b} . \ln \cos bx + c}$

③ $\int \sec ax \, dx =$

$\frac{1}{a} \int \sec ax \, . \, a\, dx =$

$\boxed{\frac{1}{a} . \ln (\sec ax + tg\, ax) + c}$

④ $\int \operatorname{cossec} v . dv =$

$\boxed{\ln (\operatorname{cossec} v - \operatorname{cotg} v) + c}$

⑤ $\int \sec \underbrace{3t}_{v} . tg \underbrace{3t}_{v} . dt =$

$= \frac{1}{3} \int \sec 3t \; tg\, 3t . 3\, dt$

$\boxed{= \frac{1}{3} . \sec . 3t + c}$

⑥ $\int \operatorname{cossec} . ay . \operatorname{cotg} dy =$

$\frac{1}{a} \int \operatorname{cossec} . ay . \operatorname{cotg} ay \; a\, dy =$

$\boxed{\frac{1}{a} . (-\operatorname{cossec} ay) + c}$

⑦ $\int \operatorname{cossec}^2 3x \, dx$

$\frac{1}{3} \int \operatorname{cossec}^2 . 3x . 3\, dx =$

$\boxed{-\frac{1}{3} \operatorname{cotg} 3x + c}$

(8) $\int \text{tg}\, \frac{x}{2}\, dx$

$2 \int \text{cotg}\, \frac{x}{2} \cdot \frac{1}{2}\, dx =$

$\boxed{2 \ln \text{sen}\, \frac{x}{2} + c}$

(9) $\int x^2 \cdot \sec^2 x^3\, dx =$

$\frac{1}{3} \int \sec^2 x^3\; 3x^2\, dx =$

$\boxed{\frac{1}{3}\, \text{tg}\, x^3 + c}$

(10) $\int \frac{dx}{\text{sen}^2 x} =$

inverso do seno

$\int \text{cossec}^2 x\, dx = \boxed{-\text{cotg}\, x + c}$

(13) → trabalho

19/02 (11) $\int \frac{ds}{\cos^2 s} =$

$\int \sec^2 s\, ds = \boxed{\text{tg}\, s + c}$

(12) $\int (\text{tg}\,\theta + \text{cotg}\,\theta)^2\, d\theta$

$\int (\text{tg}^2\theta + \underbrace{2\, \text{tg}\,\theta \cdot \text{cotg}\,\theta}_{2 \cdot \frac{1}{\text{ctg}} \cdot \text{ctg} = 2}) + \text{cotg}^2\theta)\, d\theta =$

$\int (\text{tg}^2\theta + \underbrace{2}_{1+1} + \text{cotg}^2\theta)\, d\theta =$

$\int (\underbrace{\text{tg}^2\theta + 1}_{\sec^2\theta} + \underbrace{\text{cotg}^2\theta + 1}_{\text{cossec}^2\theta})\, d\theta =$

$\int (\sec^2\theta + \text{cossec}^2\theta)\, d\theta =$

$\int \sec^2\theta\, d\theta + \int \text{cossec}^2\theta\, d\theta =$

$\boxed{\text{tg}\,\theta - \text{cotg}\,\theta + c}$

(14) $\int \frac{dx}{1+\cos x}$ (rac.)

$\frac{dx}{1+\cos x} \cdot \frac{(1-\cos x)}{(1-\cos x)} = \int \frac{1-\cos x}{1-\cos^2 x}\, dx =$ → $\text{sen}^2 x$

$$\int \frac{(1-\cos x)\,dx}{\operatorname{sen}^2 x} = \int \frac{1}{\operatorname{sen}^2 x}\,dx - \int \frac{\cos x}{\operatorname{sen}^2 x}$$

$$\int \underbrace{\operatorname{cossec}^2 x\,dx}_{-\operatorname{cotg} x} - \int \underbrace{\operatorname{cotg} x \cdot \operatorname{cossec} x}_{\operatorname{cossec} x}\,dx =$$

$$\boxed{-\operatorname{cotg} x + \operatorname{cossec} x + c}$$

(15) $\int \frac{dx}{1+\operatorname{sen} x} =$

$$\int \frac{dx}{1+\operatorname{sen} x} \cdot \frac{1-\operatorname{sen} x}{1-\operatorname{sen} x} = \int \frac{(1-\operatorname{sen} x)\,dx}{(1-\operatorname{sen}^2 x)_{\to \cos^2 x}} =$$

$$\int \frac{(1-\operatorname{sen} x)\,dx}{\cos^2 x} = \int \frac{1}{\cos^2 x} - \int \frac{\operatorname{sen} x}{\cos^2 x} =$$

$$\frac{\operatorname{sen} x}{\cos x} \cdot \frac{1}{\cos x}$$

$$\int \sec^2 x - \int \frac{\operatorname{sen} x}{\cos x} \cdot \frac{1}{\cos x} =$$

$$\int \sec^2 x\,dx - \int \operatorname{tg} x \cdot \sec x\,dx =$$

$$\boxed{\operatorname{tg} x - \sec x + c}$$

(16) $-\int \frac{-\operatorname{sen} v\,dv}{1+\cos v} =$ fórmula: $\int \frac{dv}{v}$

$$\boxed{-\ln(1+\cos v) + c}$$

(17) $\int \frac{\sec^2 dx}{1+\operatorname{tg} x} =$ fórmula: $\int \frac{dv}{v} = \ln v + c$

$$\boxed{\ln(1+\operatorname{tg} x) + c}$$

(18) $\int v \cos v^2\,dv =$

$$\frac{1}{2} \int \cos v^2\, 2v\,dv = \frac{1}{2}(+\operatorname{sen} v^2) =$$

$$\boxed{\frac{1}{2} \cdot \operatorname{sen} v^2 + c}$$

(19) $\int (x + \operatorname{sen} 2x)\,dx =$

$$\int x\,dx + \int \underbrace{\operatorname{sen} 2x\,dx}_{\int \operatorname{sen} v\,dv \to \text{fórmula}} =$$

$\frac{x^2}{2} + \frac{1}{2} \int sen\ 2x \cdot 2\,dx =$

$\frac{x^2}{2} - \frac{1}{2} \cdot \cos 2x + c$ ou

$\boxed{\frac{1}{2} \cdot (x^2 - \cos 2x) + c}$

(20) $\int \frac{sen\ x\ dx}{\sqrt{4 - \cos x}} =$

$\int (4 - \cos x)^{-\frac{1}{2}} \cdot sen\ x\ dx$ $\int v\,dv$

$\frac{(4 - \cos x)^{\frac{1}{2}}}{\frac{1}{2}} + c = \boxed{2\sqrt{4 - \cos x} + c}$

(21) $\int \frac{(1 + \cos x)\ dx}{x + sen\ x} =$

$\boxed{\ln (x + sen\ x) + c}$

(22) $\int \frac{\sec^2 \theta\ d\theta}{\sqrt{1 + 2tg\theta}} =$

$\int (1 + 2tg\theta)^{-\frac{1}{2}} \cdot \int \sec^2 \theta\ d\theta =$

$\frac{1}{2} \int (1 + 2tg\theta)^{-\frac{1}{2}} \cdot 2 \sec^2\theta\ d\theta =$

$\frac{1}{\cancel{2}} \cdot \left(\frac{1 + 2tg\theta}{\frac{1}{\cancel{2}}} \right) + c =$

$\boxed{\sqrt{1 + 2tg\theta} + c}$

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Fórmulas:

(18) $\boxed{\int \frac{dv}{v^2 + a^2} = \frac{1}{a} \cdot arc\ tg\ \frac{v}{a} + c}$

(19) $\boxed{\int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \cdot \ln \frac{v - a}{v + a} + c}$

(20) $\boxed{\int \frac{dv}{a^2 - v^2} = \frac{1}{2a} \cdot \ln \frac{a + v}{a - v} + c}$

(21) $$\boxed{\int \frac{dv}{\sqrt{a^2-v^2}} = \text{arc sen}\,\frac{v}{a} + c}$$

(22) $$\boxed{\int \frac{dv}{\sqrt{v^2 \pm a^2}} = \ln\left(v + \sqrt{v^2 \pm a^2}\right) + c}$$

(23) $$\boxed{\int \sqrt{a^2-v^2}\,dv = \frac{v}{2}\sqrt{a^2-v^2} + \frac{a^2}{2}\,\text{arc sen}\,\frac{v}{a} + c}$$

(24) $$\boxed{\int \sqrt{v^2 \pm a^2}\,dv = \frac{v}{2}\sqrt{v^2 \pm a^2} \pm \frac{a^2}{2}\ln\left(v + \sqrt{v^2 \pm a^2}\right) + c}$$

Aplicação

(1) $$\int \frac{dx}{x^2+9} = \int \frac{dx}{(x)^2+(3)^2} = \boxed{\frac{1}{3}\,\text{arc tg}\,\frac{x}{3} + c}$$

v a

(2) $$\int \frac{dx}{x^2-4} = \int \frac{dx}{(x)^2-(2)^2} = \boxed{\frac{1}{4}\cdot\ln\frac{x-2}{x-2} + c}$$

(3) $$\int \frac{dy}{\sqrt{25-y^2}} = \int \frac{dy}{\sqrt{(5)^2-(y)^2}} = \boxed{\text{arc sen}\,\frac{y}{5} + c}$$

(4) $$\int \frac{dv}{\sqrt{v^2-16}} = \int \frac{dv}{\sqrt{(v)^2-(4)^2}} =$$

$$\boxed{\ln\left(v + \sqrt{v^2-16}\right) + c}$$

(5) $$\int \frac{dx}{9x^2-4} = \frac{1}{3}\int \frac{3\,dx}{(3x)^2-2^2} =$$

$$\frac{1}{3}\cdot\frac{1}{4}\ln\frac{3x-\cancel{a}2}{3x+a} + c = \boxed{\frac{1}{12}\ln\frac{3x-2}{3x+2} + c}$$

(6) $$\int \frac{dx}{\sqrt{16-9x^2}} = \frac{1}{3}\int \frac{3\,dx}{\sqrt{4^2-(3x)^2}} =$$

$$\boxed{\frac{1}{3}\cdot\text{arc sen}\,\frac{3x}{4} + c}$$

(7) $$\int \frac{dx}{9x^2-1} = \frac{1}{3}\int \frac{3\,dx}{(3x)^2-1} =$$

$$\frac{1}{3}\cdot\frac{1}{2}\cdot\ln\frac{3x-1}{3x+1} + c = \boxed{\frac{1}{6}\cdot\ln\frac{3x-1}{3x+1} + }$$

8) $$\int \frac{dt}{4-9t^2} = \frac{1}{3}\int \frac{3\,dt}{\underset{a}{2^2}-\underset{v}{(3t)^2}} =$$

$$\frac{1}{3}\cdot\frac{1}{4}\ \ln \frac{2+3t}{2-3t} + c = \boxed{\frac{1}{12}\cdot \ln \frac{2+3t}{2-3t} + c}$$

9) $$\int \frac{e^x\,dx}{1+e^{2x}} = \int \frac{e^x\,dx}{e^{2x}+1} = \int \frac{e^x\,dx}{\underset{v}{(e^x)^2}+\underset{a}{1}}$$

$$\frac{1}{1}\cdot \text{arc tg}\ \frac{e^x}{1} + c = \boxed{\text{arc tg}\ e^x + c}$$

10) $$\int \frac{\cos\theta\, d\theta}{4-\text{sen}^2\theta} = \int \frac{\cos\theta\, d\theta}{\underset{a}{2^2}-\underset{v}{(\text{sen}\,\theta)^2}} =$$

$$\boxed{\frac{1}{4}\cdot \ln \frac{(2+\text{sen}\,\theta)}{2-\text{sen}\,\theta} + c}$$

11) $$\int \frac{bx}{\underset{v}{a^2x^2}-\underset{a}{c^2}} = \frac{b}{a}\int \frac{a\,dx}{(ax)^2-(c)^2} =$$

$$\frac{b}{a}\cdot\frac{1}{2x}\cdot \ln \frac{ax-c}{ax+c} + c = \boxed{\frac{b}{2ac}\cdot \ln \frac{ax-c}{ax+c} + c}$$

12) $$\int \frac{5x\,dx}{\sqrt{\underset{a}{1}-\underset{v}{x^2}}} = \frac{5}{2}\int \frac{2x\,dx}{\sqrt{1-(x^2)^2}} =$$

$$\boxed{\frac{5}{2}\cdot \text{arc sen}\ \frac{x^2}{1} + c}$$

13) $$\int \frac{ax\,dx}{\underset{v}{x^4}+\underset{a}{b^4}} = \frac{a}{2}\int \frac{2x\,dx}{(x^2)^2+(b^2)^2} =$$

$$\frac{a}{2}\cdot\frac{1}{b^2}\ \text{arc tg}\ \frac{x^2}{b^2} + c =$$

$$\boxed{\frac{a}{2b^2}\ \text{arc tg}\ \frac{x^2}{b^2} + c}$$

14) $$\int \frac{dt}{(t-2)^2+9} = \int \frac{dt}{(t-2)^2+(3)^2} = \frac{1}{3}\cdot\text{arc tg}\ \frac{(t-2)}{3} + c$$

15) $$\int \frac{dy}{\sqrt{1+a^2y^2}} = \frac{1}{a}\int \frac{a\,dy}{\sqrt{1+(ay)^2}} =$$

$$\boxed{\frac{1}{a}\cdot \ln\left(ay+\sqrt{ay^2+1}\right) + c}$$

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(16) $\int \frac{du}{\sqrt{4-(u+3)^2}} = \int \frac{du}{\sqrt{2^2-(u+3)^2}} =$

$\boxed{\text{arc sen}\left(\frac{u+3}{2}\right) + c}$

(17) $\int \frac{dx}{\sqrt{9-16x^2}} = \frac{1}{4}\int \frac{4\,dx}{\sqrt{3^2-(4x)^2}} =$

$\boxed{\frac{1}{4}\cdot \text{arc sen}\ \frac{4x}{3} + c}$

(18) $\int \frac{dy}{\sqrt{9y^2+4}} = \frac{1}{3}\int \frac{3\,dy}{\sqrt{(3y)^2+2^2}} =$

$\boxed{\frac{1}{3}\cdot \ln\left(3y + \sqrt{(3y)^2+2^2}\right) + c}$

(19) $\int \frac{dt}{4\cancel{dt}^2+25} = \frac{1}{2}\int \frac{2\,dt}{(2t)^2+(5)^2} =$

$\frac{1}{2}\cdot\frac{1}{5}\ \text{arc tg}\ \frac{2t}{5} + c = \boxed{\frac{1}{10}\cdot \text{arc tg}\ \frac{2t}{5} + c}$

(20) $\int \frac{dx}{25x^2-4} = \frac{1}{5}\int \frac{5\,dx}{(5x)^2-(2)^2}$

$\frac{1}{5}\cdot\frac{1}{4}\ln\ \frac{(5x-a)}{5x+a} + c =$

$\boxed{\frac{1}{20}\cdot \ln\ \frac{(5x-a)}{5x+a} + c}$

$(\sqrt{3})^2 = 3$

(21) $\int \frac{7\,dx}{3+7x^2} = \frac{7}{\sqrt{7}}\int \frac{\sqrt{7}\,dx}{(\sqrt{3})^2+(\sqrt{7}x)^2}$

$a \qquad v$

$\frac{7}{\sqrt{7}}\cdot\frac{1}{\sqrt{3}}\cdot \text{arc tg}\left(\frac{\sqrt{7}x}{\sqrt{3}}\right) + c$

$\frac{7}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \frac{7\sqrt{7}}{7} \qquad \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \frac{7\sqrt{7}}{7}\cdot\frac{\sqrt{3}}{3} = \frac{\cancel{7}\sqrt{21}}{\cancel{21}\cdot 3} = \frac{\sqrt{21}}{3}$

$\boxed{\frac{\cancel{7}\sqrt{21}}{\cancel{2}1\cdot 3}\ \text{arc tg}\ \frac{\sqrt{21}x}{3} + c}$

(22) $\int \frac{3\,dy}{9y^2-16} = \int \frac{3\,dy}{(3y)^2-(4)^2} =$

$\boxed{\frac{1}{8}\cdot \ln\ \frac{(3y-4)}{(3y+4)} + c}$

(23) $\int \frac{dv}{\sqrt{4v^2+5}} =$

$\frac{1}{2}\int \frac{2\,dv}{\sqrt{(2v)^2+(\sqrt{5})^2}} =$

$\boxed{\frac{1}{2} \cdot \ln\left(2v + \sqrt{(4v)^2+5}\right) + C}$

(24) $\int \frac{t\,dt}{\sqrt{t^4-4}} =$

$\boxed{\frac{1}{2} \ln\left(t^2 + \sqrt{t^4-4}\right) + C}$

(25) $\int \frac{x\,dx}{\sqrt{5x^4+3}} =$

$\frac{1}{2\sqrt{5}} \int \frac{2\sqrt{5}x\,dx}{\sqrt{(\sqrt{5}x^2)^2+(\sqrt{3})^2}} =$

$\boxed{\frac{1}{2\sqrt{5}} \cdot \ln\left(\sqrt{5}x^2 + \sqrt{5x^4+3}\right) + C}$

(26) $\int \frac{2e^x\,dx}{\sqrt{1-e^{2x}}} =$

$2\int \frac{e^x\,dx}{\sqrt{1-(e^x)^2}} = \boxed{2 \arcsin e^x + C}$

23/02/

Calcular as integrais:

(1) $\int \frac{dx}{x^2+4x+3}$

fórmula: $\int \frac{dv}{v^2-a^2} = \frac{1}{2a} \cdot \ln\left(\frac{v-a}{v+a}\right) + C$

Tornando o denominador um quadrado perfeito: $x^2+4x+3 = (x^2+4x+3+1)-1 = (x^2+4x+4)-1 = (x+2)^2-1$.

Assim, podemos aplicar a fórmula:

$\int \frac{dx}{x^2+4x+3} = \int \frac{dx}{(x+2)^2-1} =$

$\frac{1}{2} \cdot \ln\left(\frac{x+2-1}{x+2+1}\right) + C = \boxed{\frac{1}{2} \ln \frac{(x+1)}{x+3} + C}$

② $\int \frac{dx}{2x-x^2-10}$:

fórmula: $\frac{dx}{v^2+a^2} = \frac{1}{a} \cdot \text{arc tg } \frac{v}{a} + c$

$-\int \frac{dx}{x^2-2x+10} = -\int \frac{dx}{\underbrace{(x-1)^2}_{v} + \underbrace{3^2}_{a}} =$

$\boxed{-\frac{1}{3} \cdot \text{arc tg } \frac{x-1}{3} + c}$

desenvolvimento:
$x^2-2x+10 = (x^2-2x+1)+9 = (x-1)^2+9$

③ $\int \frac{3\,dx}{x^2-8x+25} =$

desenvolvimento: $x^2-8x+25 = (x^2-8x+16)+9 =$
$(x-4)^2+9$

$3\int \frac{dx}{\underbrace{(x-4)^2}_{v} + \underbrace{3^2}_{a}} = \frac{\cancel{3}}{\cancel{3}} \cdot \text{arc tg } \frac{(x-4)}{3} + c =$

$\boxed{\text{arc tg } \frac{x-4}{3} + c}$

④ $\frac{dx}{\sqrt{3x-x^2-2}} =$

fórmula: $\frac{dx}{\sqrt{a^2-v^2}} = \text{arc.sen } \frac{v}{a} + c$

desenvolvimento:
$3x-x^2-2 = -(x^2-3x+\frac{9}{4})-2+\frac{9}{4} =$

$\downarrow (\frac{3}{2})^2$

$\underbrace{-(x-\frac{3}{2})^2}_{} + \frac{1}{4}$

$-v^2 + a^2 = a^2-v^2$

$(x-\frac{3}{2}) \cdot \frac{2}{1} = 2x - \frac{3 \cdot \cancel{2}}{\cancel{2}} = 2x-3$

$\int \frac{dx}{\sqrt{\frac{1}{4}-(x-\frac{3}{2})^2}} = \text{arc sen } \frac{x-3/2}{1/2} + c =$

$\boxed{\text{arc sen } (2x-3)+c}$

⑤ $\int \frac{dv}{v^2-6v+5}$:

fórmula: $\frac{dv}{v^2-a^2}$

② $\int \frac{dx}{2x - x^2 - 10} =$

fórmula: $\frac{dx}{v^2 + a^2} = \frac{1}{a} \cdot \text{arc tg } \frac{v}{a} + c$

$$-\int \frac{dx}{x^2 - 2x + 10} = -\int \frac{dx}{\underbrace{(x-1)^2}_{v} + \underbrace{3^2}_{a}} =$$

$$\boxed{-\frac{1}{3} \cdot \text{arc tg } \frac{x-1}{3} + c}$$

desenvolvimento:
$x^2 - 2x + 10 = (x^2 - 2x + 1) + 9 = (x-1)^2 + 9$

③ $\int \frac{3\,dx}{x^2 - 8x + 25} =$

desenvolvimento: $x^2 - 8x + 25 = (x^2 - 8x + 16) + 9 =$
$(x-4)^2 + 9$

$$3 \int \frac{dx}{\underbrace{(\cancel{x}-4)^2}_{v} + \underbrace{3^2}_{a}} = \frac{\cancel{3}}{\cancel{3}} \cdot \text{arc tg } \frac{(\cancel{x}-4)}{3} + c =$$

$$\boxed{\text{arc tg } \frac{\cancel{x}-4}{3} + c}$$

④ $\frac{dx}{\sqrt{3x - x^2 - 2}} =$

fórmula: $\frac{dx}{\sqrt{a^2 - v^2}} = \text{arc . sen } \frac{v}{a} + c$

desenvolvimento:
$3x - x^2 - 2 = -(x^2 - 3x + \frac{9}{4}) - 2 + \frac{9}{4} =$
$\downarrow$
$(\frac{3}{2})^2$

$$\underbrace{-(x - \frac{3}{2})^2}_{-v^2} + \frac{1}{4}$$

$-v^2 + a^2 = a^2 - v^2$

? $(x - \frac{3}{2}) \cdot \frac{2}{1} = 2x - \frac{3 \cdot \cancel{2}}{\cancel{2}} = 2x - 3$

$$\int \frac{dx}{\sqrt{\frac{1}{4} - (x - \frac{3}{2})^2}} = \text{arc sen } \frac{x - 3/2}{1/2} + c =$$

$$\boxed{\text{arc sen } (2x - 3) + c}$$

⑤ $\int \frac{dv}{v^2 - 6v + 5} =$

fórmula: $\frac{dv}{v^2 - a^2}$

② $\int \frac{dx}{2x-x^2-10}$:

fórmula: $\frac{dx}{v^2+a^2} = \frac{1}{a} \cdot \text{arc tg } \frac{v}{a} + c$

$-\int \frac{dx}{x^2-2x+10} = -\int \frac{dx}{\underbrace{(x-1)^2}_{v}+\underbrace{3^2}_{a}} =$

$$\boxed{-\frac{1}{3} \cdot \text{arc tg } \frac{x-1}{3} + c}$$

desenvolvimento:

$x^2-2x+10 = (x^2-2x+1)+9 = (x-1)^2+9$

③ $\int \frac{3\,dx}{x^2-8x+25} =$

desenvolvimento: $x^2-8x+25 = (x^2-8x+16)+9 =$
$(x-4)^2+9$

$3\int \frac{dx}{\underbrace{(x-4)^2}_{v}+\underbrace{3^2}_{a}} = \frac{\cancel{3}}{\cancel{3}} \cdot \text{arc tg } \frac{(x-4)}{3} + c =$

$$\boxed{\text{arc tg } \frac{x-4}{3} + c}$$

④ $\frac{dx}{\sqrt{3x-x^2-2}} =$

fórmula. $\frac{dx}{\sqrt{a^2-v^2}} = \text{arc. sen } \frac{v}{a} + c$

desenvolvimento:

$3x-x^2-2 = -(x^2-3x+\frac{9}{4})-2+\frac{9}{4} =$

$\frac{9}{4} \to (\frac{3}{2})^2$

$\underbrace{-(x-\frac{3}{2})^2}_{-v^2} + \underbrace{\frac{1}{4}}_{+a^2} = a^2 - v^2$

$(x-\frac{3}{2}) \cdot \frac{2}{1} = 2x - \frac{3 \cdot \cancel{2}}{\cancel{2}} = 2x-3$

$\int \frac{dx}{\sqrt{\frac{1}{4}-(x-\frac{3}{2})^2}} = \text{arc sen } \frac{x-3/2}{1/2} + c =$

$$\boxed{\text{arc sen } (2x-3) + c}$$

⑤ $\int \frac{dv}{v^2-6v+5}$:

fórmula: $\frac{dv}{v^2-a^2}$

desenvolvimento:
$(v^2 - 6v + 9) - 9 + 5 = (v-3)^2 - 4$

$$\int \frac{dv}{(v-3)^2 - 4} = \boxed{\frac{1}{4} \ln \left(\frac{v-3-2}{v-3+2}\right) + c}$$

⑥ $$\int \frac{dx}{2x^2 - 2x + 1} =$$

fórmula: $\frac{dv}{v^2 + a^2}$

desenvolvimento: $2\left[(x^2 - x) + \frac{1}{2}\right] =$

$$2\left[\left(x^2 - x + \frac{1}{4}\right) + \frac{1}{2} - \frac{1}{4}\right] = 2\left[\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}\right]$$

$$\int \frac{dx}{2x^2 - 2x + 1} = \frac{1}{2} \int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}} =$$

$$\frac{1}{\cancel{2}} \cdot \cancel{2} \text{ arc tg } \frac{\left(x - \frac{1}{2}\right)}{\frac{1}{2}} + c =$$

$$\boxed{\text{arc tg } (2x - 1) + c}$$

⑦ $$\int \frac{dx}{\sqrt{15 + 2x - x^2}} =$$

desenvolvimento:
$-x^2 + 2x + 15 = -(x^2 - 2x) + 15 =$
$-(x^2 - 2x + 1) + 15 + 1 = -(x-1)^2 + 16$

$$\int \frac{dx}{\sqrt{16 - (x-1)^2}} = \boxed{\text{arc. sen } \frac{(x-1)}{4} + c}$$

⑧ $$\int \frac{dx}{x^2 + 2x} =$$

desenvolvimento: $\underbrace{(x^2 + 2x + 1)}_{v^2} \underbrace{- 1}_{-a^2} = (x+1)^2 - 1$

$$\int \frac{dx}{(x+1)^2 - 1} = \boxed{\frac{1}{2} \ln \left(\frac{x}{x+2}\right) + c}$$

⑨ $$\int \frac{dx}{4x - x^2} =$$

desenvolvimento: $-(x^2 - 4x) = -(x^2 - 4x + 4) + 4 =$
$-(x-2)^2 + 4$

$$\int \frac{dx}{4-(x-2)^2} = \frac{1}{4} \cdot \ln \cdot \frac{2+x-2}{2-x+2} + c =$$

$$\boxed{\frac{1}{4} \cdot \ln \frac{x}{4-x} + c}$$

(10) $$\int \frac{(1+2x)\,dx}{1+x^2} = \int \frac{1\,dx}{1+x^2} + \int \frac{2x\,dx}{1+x^2}$$

$$\boxed{\operatorname{arc\,tg} x + \ln(1+x^2) + c}$$

(2) $$\int \frac{(2x+1)\,dx}{\sqrt{x^2-1}} = \int \frac{2x\,dx}{\sqrt{x^2-1}} + \int \frac{dx}{\sqrt{x^2-1}}$$

$$\boxed{\int (x^2-1)^{-1/2}\, 2x\,dx + \int \frac{dx}{\sqrt{x^2-1}}}$$

24/2/13

$$\int \frac{dx}{3x^2-2x+2} =$$

$$3\left(x^2 - \frac{2x}{3} + \frac{2}{3}\right) = \frac{1}{3} \int \frac{dx}{x^2 - \frac{2x}{3} + \frac{2}{3}} =$$

$$\frac{1}{3} \int \frac{dx}{(x-\frac{1}{3})^2 + \frac{\sqrt{5}}{3}} = \left(x^2 - \frac{2x}{3} + \frac{1}{9}\right) \cdot \frac{1}{9} + \frac{2}{3} =$$

$$\boxed{\frac{1}{3} \cdot \frac{1}{\sqrt{5}/3} \cdot \operatorname{arc\,tg} \frac{x - 1/3}{\sqrt{5}/3} + c}$$

(4) $$\int \frac{(3x-1)\,dx}{x^2+9} =$$

$$\int \frac{3x\,dx}{(x)^2+9} - \int \frac{1 \cdot dx}{x^2+9} =$$

$$\frac{3}{2} \int \frac{2x\,dx}{x^2+9} - \int \frac{dx}{x^2+9} =$$

$$\boxed{\frac{3}{2} \ln(x^2+9) - \frac{1}{3} \cdot \operatorname{arc\,tg} \frac{x}{3} + c}$$

(5) $$\int \frac{(3x-2)\,dx}{\sqrt{9-x^2}} =$$

$$\int \frac{3x\,dx}{\sqrt{9-x^2}} - \int \frac{2\,dx}{\sqrt{9-x^2}} =$$

$$3\int \frac{x\,dx}{\sqrt{9-x^2}} - 2\int \frac{dx}{\sqrt{9-x^2}} =$$

$$3\int (9-x^2)^{-\frac{1}{2}}\, x\,dx - 2\int \frac{dx}{\sqrt{9-x^2}} =$$

$$-\frac{3}{\cancel{2}} \cdot \frac{(9-x^2)^{\frac{1}{2}}}{\frac{1}{\cancel{2}}} - 2 \text{ arc sen } \frac{x}{3} + c =$$

$$\boxed{-3\sqrt{9-x^2} - 2 \text{ arc sen } \frac{x}{3} + c}$$

⑥ $$\int \frac{(x+3)\,dx}{\sqrt{x^2+4}} = \int \frac{x\,dx}{\sqrt{x^2+4}} + 3\int \frac{dx}{\sqrt{x^2+4}} =$$

$$\frac{1}{2}\int (x^2+4)^{-\frac{1}{2}}\, 2x\,dx + 3\int \frac{dx}{\sqrt{x^2+4}} =$$

$$\frac{1}{\cancel{2}} \cdot \frac{(x^2+4)^{\frac{1}{2}}}{\frac{1}{\cancel{2}}} + 3 \ln\left(x + \sqrt{x^2+4}\right) + C =$$

$$\boxed{\sqrt{x^2+4} + 3\ln\left(x+\sqrt{x^2+4}\right) + c}$$

26/2/73

Fórmulas:

$$\boxed{\int \sqrt{a^2-v^2}\,dv = \frac{v}{2}\sqrt{a^2-v^2} + \frac{a^2}{2} \text{ arc sen } \frac{v}{a} + c}$$

$$\boxed{\int \sqrt{v^2 \pm a^2}\,dv = \frac{v}{2}\sqrt{v^2 \pm a^2} \pm \frac{a^2}{2}\ln\left(v + \sqrt{v^2 \pm a^2}\right) + c}$$

Aplicação

① $$\frac{1}{2}\int \sqrt{1-4x^2}\,dx =$$

$$\frac{1}{2} \cdot \frac{\cancel{2}x}{\cancel{2}}\sqrt{1-4x^2} + \frac{1}{2} \cdot \frac{1}{2} \text{ arc sen } 2x + c =$$

$$\boxed{\frac{1}{2}\sqrt{1-4x^2} + \frac{1}{4} \text{ arc sen } 2x + c}$$

② $$\int \frac{1}{3}\sqrt{1+9x^2}\;3\,dx =$$

$$\frac{1}{\cancel{3}} \cdot \frac{\cancel{3}x}{2} + \frac{1}{3} \cdot \frac{1}{2} \ln\left(3x + \sqrt{9x^2+1}\right) + c =$$

$$\boxed{\frac{x}{2} + \frac{1}{6} \ln \left(3x + \sqrt{9x^2+1}\right) + c}$$

③ $\int \sqrt{\frac{x^2}{4} - 1}\, dx =$

$\int \sqrt{\frac{x^2-4}{4}} = \int \frac{\sqrt{x^2-4}}{2} =$

$\frac{1}{2} \int \sqrt{x^2-4} = \frac{1}{2} \cdot \frac{x}{2} \sqrt{x^2-4} - \frac{1}{2} \cdot \frac{4}{2} \ln \left|\right.$

$\left(x \sqrt{x^2-4}\right) + c =$

$$\boxed{\frac{x}{4} \sqrt{x^2-4} - \ln \left(x \sqrt{x^2-4}\right) + c}$$

④ $\int \sqrt{25-9x^2}\, dx =$

$\frac{1}{3} \int \sqrt{25-9x^2} \cdot 3\, dx =$

$\frac{1}{3} \cdot \frac{3x}{2} \int \sqrt{25-9x^2} + \frac{1}{3} \cdot \frac{25}{2} \text{ arc sen } \frac{3x}{5} + c$

$$\boxed{\frac{x}{2} \sqrt{25-9x^2} + \frac{25}{6} \text{ arc sen } \frac{3x}{5} + c}$$

⑤ $\int \sqrt{4x^2+9}\, dx =$

$\frac{1}{\cancel{2}} \cdot \frac{2x}{2} \sqrt{4x^2+9} + \frac{1}{2} \cdot \frac{9}{2} \ln \left(2x + \sqrt{4x^2+9}\right) + c =$

$$\boxed{\frac{x}{2} \sqrt{4x^2+9} + \frac{9}{4} \ln \left(2x + \sqrt{4x^2+9}\right) + c}$$

⑥ $\int \sqrt{5-3x^2}\, dx = \frac{1}{\sqrt{3}} \int \sqrt{5-3x^2}\, dx =$

$\frac{1}{\cancel{\sqrt{3}}} \cdot \frac{\sqrt{3}\,x}{2} \sqrt{5-3x^2} + \frac{1}{\sqrt{3}} \cdot \frac{(\sqrt{5})^2}{2} \text{ arc. sen } \frac{\sqrt{3}\,x}{\sqrt{5}} + c =$

$$\boxed{\frac{x}{2} \cdot \sqrt{5-3x^2} + \frac{5}{2\sqrt{3}} \text{ arc sen } x \frac{\sqrt{3}}{\sqrt{5}} + c}$$

⑦ $\int \sqrt{3-2x-x^2}\, dx =$

$3 - 2x - x^2 = -(x^2+2x+1) + 3 + 1 =$

$-(x+1)^2 + 4 =$

$-(x+1)^2 + (2)^2$

$\int \sqrt{-(x^2+2x)} + 3\, dx = \int \sqrt{-(x^2+2x+1)} + 3 + 1 =$

$\int \sqrt{4-(x+1)^2}\, dx = \sqrt{-(x+1)^2} + 4 =$

$\left(\frac{x+1}{2}\right) \cdot \sqrt{4-(x+1)^2} + \frac{4}{2}^2 \text{ arc sen } \left(\frac{x+1}{2}\right) + c.$

$$\boxed{\left(\frac{x+1}{2}\right) \cdot \sqrt{4-(x+1)^2} + 2 \text{ arc sen } \left(\frac{x+1}{2}\right) + c}$$

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Diferenciais Trigonométricas

$$\boxed{\int \text{sen}^m . u . \cos^n u \, du.}$$

Se m é ímpar, posso torná-lo par.

$$\boxed{\int \text{sen}^{m-1} . u . \text{sen } u . \cos^n . u \, du.}$$

$\text{sen}^4 u = (\text{sen}^2 u)^2 = (1 - \cos^2 u)^2$

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$\int \text{sen } x = \cos x$

Aplicações

① $\int \text{sen}^3 x \, dx =$

$\int \text{sen}^2 x . \text{sen } x . dx =$

$\int (1 - \cos^2 x) . \text{sen } x \, dx =$

$\int \text{sen } x \, dx - \int \cos^2 x . \text{sen } x \, dx =$

$\int \text{sen } x \, dx + \int (\cos x)^2 (-) \text{sen } x \, dx =$

$$\boxed{-\cos x + \frac{\cos^3 x}{3} + c}$$

② $\int \text{sen}^2 \theta \cos \theta \, d\theta =$

$\int (\text{sen } \theta)^2 . \cos \theta \, d\theta = \frac{\text{sen}^3 \theta}{3} + c =$

$$\boxed{\frac{1}{3} \text{ sen}^3 \theta + c}$$

$\left(\frac{x+1}{2}\right) \cdot \sqrt{4-(x+1)^2} + \frac{4^2}{2} \text{ arc sen } \left(\frac{x+1}{2}\right) + c.$

$$\boxed{\left(\frac{x+1}{2}\right) \cdot \sqrt{4-(x+1)^2} + 2 \text{ arc sen } \left(\frac{x+1}{2}\right) + c}$$

27/02/73

Diferenciais Trigonométricas

$$\boxed{\int \text{sen}^m . u . \cos^n u \, du.}$$

Se m é ímpar, posso torná-lo par.

$$\boxed{\int \text{sen}^{m-1} . u . \text{sen } u . \cos^n . u \, du.}$$

$\text{sen}^4 u = (\text{sen}^2 u)^2 = (1 - \cos^2 u)^2$

Fórmulas → pág. 263

$\int \text{sen } x = \cos x$

Aplicações

① $\int \text{sen}^3 x \, dx =$

$\int \text{sen}^2 x . \text{sen } x . dx =$

$\int (1 - \cos^2 x) . \text{sen } x \, dx =$

$\int \text{sen } x \, dx - \int \cos^2 x . \text{sen } x \, dx =$

$\int \text{sen } x \, dx + \int (\cos x)^2 (-) \text{sen } x \, dx =$

$$\boxed{-\cos x + \frac{\cos^3 x}{3} + c}$$

② $\int \text{sen}^2 \theta \cos \theta \, d\theta =$

$\int (\text{sen } \theta)^2 . \cos \theta \, d\theta = \frac{\text{sen}^3 \theta}{3} + c =$

$$\boxed{\frac{1}{3} \text{sen}^3 \theta + c}$$

③ $\int \cos^2\theta \cdot \text{sen}\,\theta = \int (\cos\theta)^2 \cdot \text{sen}\,\theta \, d\theta =$

$\boxed{\dfrac{-\cos^3\theta}{3} + c}$ ou $\boxed{-\dfrac{1}{3}\cos^3\theta + c}$

④ $\frac{1}{6}\int \text{sen}^3 6x \cdot \cos 6x \, dx =$

$\frac{1}{6}\cdot\frac{1}{4}\,\text{sen}^4 6x + c = \boxed{\dfrac{1}{24}\,\text{sen}^4 6x + c}$

⑤ $\frac{1}{2}\int \cos^3 2\theta \cdot \text{sen}\, 2\theta \, d\theta =$

$\frac{1}{2}\cdot\frac{1}{4}\cdot \cos^4 2\theta + c = \boxed{-\dfrac{1}{8}\cos^4 2\theta + c}$

⑥ $\int \dfrac{\cos^3 x}{\text{sen}^4 x}\,dx = \int (\text{sen}\,x)^{-4}\cdot\cos^3 x \, dx =$

$\int (\text{sen}\,x)^{-4}\cdot\cos^2 x \cdot \cos x \, dx =$

$\int (\text{sen}\,x)^{-4}\cdot(1-\text{sen}^2 x)\cdot \cos x \, dx =$

$\int (\text{sen}\,x)^{-4}\cdot(\cos x - \text{sen}^2 x\cdot\cos x)\,dx =$

$\int (\text{sen}\,x)^{-4}\cdot\cos x \, dx - \int (\text{sen}\,x)^{-2}\cos x\, dx =$

$\boxed{\dfrac{\text{sen}^{-3} x}{-3} - \dfrac{(\text{sen}\,x)^{-1}}{-1} + c} \rightarrow$

$+\,\text{cossec}\,x - \frac{1}{3}\,\text{cossec}^3 x$

⑦ $\int \dfrac{\text{sen}^3\theta \, d\theta}{\cos^2\theta} =$

$\text{sen}^3\phi = \text{sen}^2\phi \cdot \text{sen}\,\phi = (1-\cos^2\phi)\,\text{sen}\,\phi =$
$\text{sen}\,\phi - \cos^2\phi \cdot \text{sen}\,\phi$

$\int (\cos\phi)^{-2}\,(\text{sen}\,\phi - \text{sen}\,\phi\cdot\cos^2\phi)\,d\phi =$

$(-)\int (\cos\phi)^{-2}\cdot \text{sen}\,\phi \, d\phi - \int \text{sen}\,\phi \, d\phi =$

$-\dfrac{(\cos\phi)^{-1}}{-1} + \cos\phi + c =$

$\boxed{\sec\phi + \cos\phi + c}$

$$\int (\text{sen}\, x)^{-4} \cdot (\cos x - \text{sen}^2 x \cdot \cos x)\, dx =$$

$$\int (\text{sen}\, x)^{-4} \cdot \cos x\, dx - \int (\text{sen}\, x)^{-2} \cos x\, dx =$$

$$\boxed{\frac{\text{sen}^{-3} x}{-3} - \frac{(\text{sen}\, x)^{-1}}{-1} + c} \rightarrow$$

$$+ \text{cossec}\, x - \frac{1}{3} \text{cossec}^3 x + c$$

⑦ $\displaystyle\int \frac{\text{sen}^3 \phi \, d\phi}{\cos^2 \phi} =$

$$\text{sen}^3 \phi = \text{sen}^2 \phi \cdot \text{sen}\, \phi = (1 - \cos^2 \phi)\, \text{sen}\, \phi =$$

$$\text{sen}\, \phi - \cos^2 \phi \cdot \text{sen}\, \phi$$

$$\int (\cos \phi)^{-2} \, (\text{sen}\, \phi - \text{sen}\, \phi \cdot \cos^2 \phi)\, d\phi =$$

$$(-) \int (\cos \phi)^{-2} \cdot \underset{(-)}{\text{sen}\, \phi}\, d\phi - \int \text{sen}\, \phi \, d\phi =$$

$$- \frac{(\cos \phi)^{-1}}{-1} + \cos \phi + c =$$

$$\boxed{\sec \phi + \cos \phi + c}$$

③ $\displaystyle\int \cos^2 \theta \, \text{sen}\, \theta = \int (\cos \theta)^2 \cdot \text{sen}\, \theta \, d\theta =$

$$\boxed{\frac{-\cos^3 \theta}{3} + c} \quad \text{ou} \quad \boxed{-\frac{1}{3} \cos^3 \theta + c}$$

④ $\displaystyle\frac{1}{6} \int \text{sen}^3 6x \, \underbrace{\cos 6x \, dx}_{dv} =$

$$\frac{1}{6} \cdot \frac{1}{4} \text{sen}^4 6x + c = \boxed{\frac{1}{24} \text{sen}^4 6x + c}$$

⑤ $\displaystyle\frac{1}{2} \int \cos^3 2\theta \, (-) \text{sen}\, 2\theta \, d\theta =$

$$\frac{1}{2} \cdot \frac{1}{4} \cdot \cos^4 2\theta + c = \boxed{-\frac{1}{8} \cos^4 2\theta + c}$$

⑥ $\displaystyle\int \frac{\cos^3 x}{\text{sen}^4 x} dx = \int (\text{sen}\, x)^{-4} \cdot \cos^3 x \, dx =$

$$\int (\text{sen}\, x)^{-4} \cdot \cos^2 x \cdot \cos x \, dx =$$

$$\int (\text{sen}\, x)^{-4} \cdot (1 - \text{sen}^2 x) \cdot \cos x \, dx =$$

③ $\int \cos^2\theta \, sen\,\theta = \int (\cos\theta)^2 \cdot sen\,\theta \, d\theta =$

$\boxed{\dfrac{-\cos^3\theta}{3} + c}$ ou $\boxed{-\dfrac{1}{3}\cos^3\theta + c}$

④ $\frac{1}{6}\int sen^3 6x \cos 6x \, dx =$

$\frac{1}{6}\, sen^4 6x + c$

$\frac{1}{6}\cdot\frac{1}{4}\, sen^4 6x + c = \boxed{\frac{1}{24}\, sen^4 6x + c}$

⑤ $\frac{1}{2}\int \cos^3 2\theta \, sen\, 2\theta \, d\theta =$

$\frac{1}{2}\cdot\frac{1}{4}\cdot\cos^4 2\theta + c = \boxed{-\frac{1}{8}\cos^4 2\theta + c}$

⑥ $\int \dfrac{\cos^3 x}{sen^4 x}\, dx = \int (sen\,x)^{-4}\cos^3 x \, dx =$

$\int (sen\,x)^{-4}\cdot\cos^2 x \cdot \cos x \, dx =$

$\int (sen\,x)^{-4}\cdot(1 - sen^2 x)\cdot \cos x \, dx =$

$\int (sen\,x)^{-4}\cdot(\cos x - sen^2 x \cdot \cos x)\, dx =$

$\int (sen\,x)^{-4}\cdot \cos x \, dx - \int (sen\,x)^{-2}\cos x \, dx$

$\boxed{\dfrac{sen^{-3} x}{-3} - \dfrac{(sen\,x)^{-1}}{-1} + c} \rightarrow$

$+ \operatorname{cossec} x - \frac{1}{3}\operatorname{cossec}^3 x + c$

⑦ $\int \dfrac{sen^3\phi \, d\phi}{\cos^2\phi} =$

$sen^3\phi = sen^2\phi \cdot sen\,\phi = (1-\cos^2\phi)\, sen\,\phi =$
$sen\,\phi - \cos^2\phi \cdot sen\,\phi$

$\int (\cos\phi)^{-2}(sen\,\phi - sen\,\phi \cdot \cos^2\phi)\, d\phi =$

$(-)\int (\cos\phi)^{-2}\cdot sen\,\phi \, d\phi - \int sen\,\phi \, d\phi =$
$(-)$

$-\dfrac{(\cos\phi)^{-1}}{-1} + \cos\phi + c =$

$\boxed{\sec\phi + \cos\phi + c}$

$\int tg^N u\,du$ e $\int cotg^N u\,du$:

Nota: desmembrar desde que esteja na fórmula $\int v^N dv$

Exemplos:

$\int tg^4 x\,dx = \int tg^2 x \; tg^2 x$

$tg^2 x + 1 = sec^2 x.$

$\int tg^2 x (sec^2 x - 1) dx =$

$\int tg^2 x . sec^2 x\,dx - \int tg^2 x\,dx =$

$\int tg^2 dx = - \int (-1 + sec^2 x) dx = + \int dx - \int sec^2 x\,dx$

$\int (tg\,x)^2 sec^2 x\,dx + \int dx - \int sec^2 x\,dx =$

$$\boxed{\frac{tg^3 x}{3} + x - tg\,x + c}$$

Obs: $tg^2 x = sec^2_x - 1$

$cotg^2 x = cossec^2 x - 1$

① $\int tg^3 x\,dx = \int (tg\,x)^3 . dx = \int tg^2 x\; tg\,x\,dx =$

$\int (sec^2 - 1) . tg\,x\,dx = \int (sec^2)\, tg\,x\,dx - \int tg\,x\,dx =$

$$\boxed{\frac{tg\,x^2}{2} + ln\,cos\,x + c = \frac{1}{2} tg\,x^2 + ln\,cos\,x + c}$$

② $\int cotg^3 \frac{x}{3}\,dx = \int cotg^2 \frac{x}{3} . cotg \frac{x}{3}\,dx =$

$\int (cossec^2 \frac{x}{3} - 1) . cotg \frac{x}{3}\,dx =$

$-3 \int cotg \frac{x}{3(-\frac{1}{3})} . cossec^2 \frac{x}{3}\,dx - 3 \int cotg \frac{x}{3}\,dx =$

$$\boxed{-\frac{3}{2} cotg^2 \frac{x}{3} - ln\,sen \frac{x}{3} + c}$$

Resumo:

$$\boxed{\int sen^m u . cos^n u\,du}$$

$$\boxed{\int tg^n u\,du} \qquad \boxed{\int sec^n u\,du}$$

Nota: $\int tg^N u\, du$ e $\int cotg^N u\, du$: desmembrar desde que esteja na fórmula $\int v^N dv$

Exemplos:

$\int tg^4 x\, dx = \int tg^2 x\ tg^2 x$

$tg^2 x + 1 = sec^2 x$.

$\int tg^2 x (sec^2 x - 1) dx =$

$\int tg^2 x . sec^2 x\, dx - \int tg^2 x\, dx =$

$\int tg^2 dx = - \int (-1 + sec^2 x) dx = + \int dx - \int sec^2 x\, dx$

$\int (tg x)^2 sec^2 x\, dx + \int dx - \int sec^2 x\, dx =$

$\boxed{\frac{tg^3 x}{3} + x - tg\, x + c}$

Obs: $tg^2 x = sec^2_x - 1$

$cotg^2 x = cossec^2 x - 1$

① $\int tg^3 x\, dx = \int (tg\, x)^3 . dx = \int tg^2 x\ tg\, x\, dx =$

$\int (sec^2 - 1) . tg\, x\, dx = \int (sec^2)\, tg\, x\, dx - \int tg\, x\, dx =$

$\boxed{\frac{tg\, x^2}{2} + \ln \cos x + c = \frac{1}{2} tg\, x^2 + \ln \cos x + c}$

② $\int cotg^3 \frac{x}{3} dx = \int cotg^2 \frac{x}{3} . cotg \frac{x}{3} dx =$

$\int (cossec^2 \frac{x}{3} - 1) . cotg \frac{x}{3}\ dx =$

$-3 \int cotg \frac{x}{3} . cossec^2 \frac{x}{3} dx - 3 \int cotg \frac{x}{3} dx =$ $(-\frac{1}{3})$

$\boxed{-\frac{3}{2} cotg^2 \frac{x}{3} - \ln sen \frac{x}{3} + c}$

Resumo:

$\boxed{\int sen^m u . cos^n u\, du}$

$\boxed{\int tg^n u\, du}$ $\boxed{\int sec^n u\, du}$

$\boxed{\int \operatorname{cotg}^N u \, du}$ $\boxed{\int \operatorname{cossec}^N u \, du}$

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(3) $\int \operatorname{cotg}^3 2x \operatorname{cossec} 2x \, dx =$

$\int \operatorname{cotg}^2 2x \operatorname{ctg} 2x \cdot \operatorname{cossec} 2x \, dx :$

$\int (\operatorname{cossec}^2 2x - 1) \operatorname{ctg} 2x \cdot \operatorname{cossec} 2x \, dx - \frac{1}{2} \int \operatorname{cotg} 2x \cdot \operatorname{cossec} 2x \, dx$

$-\frac{1}{2} \int (\operatorname{cossec} 2x)^2 2 \operatorname{cotg} 2x \cdot \operatorname{cossec} 2x \, dx - \frac{1}{2} \int \operatorname{ctg} 2x \operatorname{cossec} 2x \, dx =$

$-\frac{1}{2} \frac{\operatorname{cossec}^3 2x}{3} + \frac{1}{2} \operatorname{cossec} 2x =$

$\boxed{\frac{1}{2} \cdot \operatorname{cossec} 2x - \frac{1}{6} \operatorname{cossec}^3 2x}$

(4) $\int \left(\operatorname{cossec}^4 \frac{x}{4}\right) dx :$

$\int \left(\operatorname{cossec}^2 \frac{x}{4}\right)^2 dx$

$\int \left(\operatorname{cotg}^2 \frac{x}{4} + 1\right) \operatorname{cossec}^2 \frac{x}{4} \, dx :$

$-4 \int \left(\operatorname{cotg}^2 \frac{x}{4}\right)^2 \frac{1}{4} \operatorname{cossec}^2 \frac{x}{4} \, dx + 4 \operatorname{cossec}^2 \frac{x}{4} \, dx =$

$\boxed{-4 \frac{\operatorname{cotg}^3 \frac{x}{4}}{3} - 4 \operatorname{cotg} \frac{x}{4} + c}$

(5) $\int \operatorname{tg}^5 3\theta \, d\theta =$

$\int (\operatorname{tg}^2 3\theta) \operatorname{tg}^3 3\theta \, d\theta = \int (\sec^2 3\theta - 1) \operatorname{tg}^3 3\theta \, d\theta :$

$\frac{1}{3} \int (\operatorname{tg} 3\theta)^3 \cdot 3 \sec^2 3\theta \cdot d\theta - \int \operatorname{tg}^3 3\theta \, d\theta \, * :$

faz-se separado

$* \; - \int \operatorname{tg}^2 3\theta \cdot \operatorname{tg} 3\theta \cdot d\theta =$

$- \int (\sec^2 3\theta - 1) \cdot \operatorname{tg} 3\theta \, d\theta =$

$- \int \operatorname{tg} 3\theta \cdot \sec^2 3\theta \, d\theta + \int \operatorname{tg} 3\theta \cdot d\theta .$

$\frac{1}{3} \int (\operatorname{tg} 3\theta)^3 \cdot 3 \sec^2 3\theta \, d\theta - \frac{1}{3} \int 3\theta - 3 \sec^2 3\theta \, d\theta + \int \operatorname{tg} 3\theta \, d\theta =$

$\boxed{\frac{1}{3} \frac{\operatorname{tg}^4 3\theta}{4} - \frac{1}{3} \frac{\operatorname{tg}^2 3\theta}{2} + \frac{1}{3} \ln \sec 3\theta + c}$

(6) $\int \frac{sen^2 \phi \, d\phi}{\cos^4 \phi} =$

$\int \frac{sen^2 \phi}{\cos^2 \phi} \cdot \frac{1}{\cos^2 \phi} \cdot d\phi =$

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$\int (tg\, \phi)^2 \cdot \sec^2 \phi \cdot d\phi$

$\frac{tg^3 \phi}{3} + c = \boxed{\frac{1}{3} tg^3 \phi + c}$

(7) $\int \frac{dx}{sen^2 2x \cdot \cos^4 2x} =$

Resumo:

(1) $\sqrt{a^2-u^2} \rightarrow a \, sen\, z \qquad (a \cos z)$

(2) $\sqrt{a^2+u^2} \rightarrow a\, tg\, z \qquad (a \sec z)$

(3) $\sqrt{u^2-a^2} \rightarrow a \sec z \qquad (a\, tg\, z)$

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Integrais da Expressão contendo $\sqrt{a^2-u^2}$ ou $\sqrt{u^2 \pm a^2}$ por substituição trigonométrica.

(1) $\sqrt{a^2-u^2} \rightarrow$ põe-se: $u = a \, sen\, z$

(2) $\sqrt{u^2+a^2} \rightarrow$ põe-se: $u = a\, tg\, z$

(3) $\sqrt{u^2-a^2} \rightarrow$ põe-se: $u = a \sec z$.

Exemplos dedutivos das fórmulas

$u = a \, sen\, z$

$\sqrt{a^2-u^2}$

(1) $\sqrt{\underbrace{a^2}_{a^2} - \underbrace{a^2 sen^2 z}_{u^2}}$

$\sqrt{a^2(1-sen^2 z)}$ (substituindo & colocando em evidência)

$a\sqrt{1-sen^2 z}$

$a\sqrt{\cos^2 z} = \boxed{a \cos z}$

(2) $\sqrt{u^2+a^2}$

$\sqrt{a^2 tg^2 z + a^2}$

$\sqrt{a^2(1+tg^2 z)}$

$a\sqrt{1+tg^2 z}$

$a\sqrt{sec^2 z}$

$\boxed{a\ sec\ z}$

③ $\sqrt{u^2-a^2}$

$\sqrt{a^2 \cdot sec^2 z - a^2}$

$\sqrt{a^2(sec^2 z - 1)}$

$a\sqrt{tg^2 z}$

$\boxed{a \cdot tg\ z}$

Aplicações:

① $\int \frac{du}{(a^2-u^2)^{3/2}}$ du

$a^2-u^2 \rightarrow a\cos z\, dz$

$u = a\ sen\ z$

$du = a.\cos z.dz$

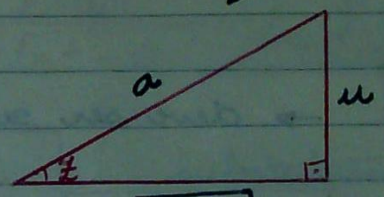
$\int \frac{\cancel{a \cos z\, dz}}{(a.\cos z)^{3/2}}$ → du

o expoente ③ vem do denominador. $(\sqrt{a^2-u^2})^3$

$\int \frac{dz}{a^2\cos^2 z} = \frac{1}{a^2}\int \frac{dz}{\cos^2 z} =$

O inverso de $\cos^2 z$ é $sec^2 z$

$\frac{1}{a^2}\int sec^2 z.dz = \boxed{\frac{1}{a^2}\, tg\, z + c}$



u → cateto oposto

a → hipotenusa

$sen\ z = \frac{u}{a}$

$\sqrt{a^2-u^2}$ → cateto adjacente

cont.

$\frac{1}{a^2}\, tg^2 + c$ e substituindo, temos:

$\frac{1}{a^2} \cdot \frac{u}{\sqrt{a^2-u^2}} + c = \boxed{\frac{u}{a^2\sqrt{a^2-u^2}} + c}$

cateto oposto

cateto adjacente

② $\int \frac{dx}{(x^2+2)^{3/2}}$

constante

$a = \sqrt{2}$ $\quad du = dx$

$u = x$

$u = a.tg\ z$

$du = a. sec^2 z$

derivada da tg z.

$\int \frac{du}{(u^2+a^2)^{3/2}} =$

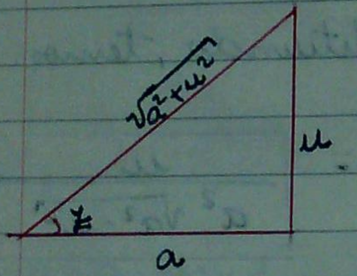
$$\int \frac{a \cdot \sec^2 z}{(a \sec z)^{3/2}} \rightarrow \int \frac{\cancel{a} \cdot \sec^2 z \, dz}{a^{3\,2} \sec^{3} z} =$$

$$\frac{1}{a^2} \int \cos z \, dz =$$ inverso do sec z

$$\boxed{\frac{1}{a^2} \operatorname{sen} z + c}$$ → deve ser substituíd

$\boxed{u = a \, tg \, z}$ → parte daqui

$$tg \, z = \frac{u}{a}$$



$$tg \, z = \frac{\text{cat. oposto}}{\text{cat. adjacente}}$$

a → cateto adjacente
u → cateto oposto
$\sqrt{a^2+u^2}$ → hipotenusa

$$\operatorname{sen} z = \frac{u}{\sqrt{a^2+u^2}}$$

Substituindo:

$$\frac{1}{a^2} \cdot \frac{u}{\sqrt{u^2+a^2}} + c$$ e voltando às variáveis do problema:

$$\boxed{\frac{1}{2} : \frac{x}{\sqrt{x^2+2}} + c}$$

③ $$\int \frac{x^2 \, dx}{\sqrt{x^2-6}} = \int \frac{u^2 \, du}{\sqrt{u^2-a^2}}.$$

$u = x$
$du = dx$
$a = \sqrt{6}$

substituindo o u por $a \sec z$

$$\boxed{u = a \sec z}$$

$$\boxed{du = a \sec z \, . \, tg \, z \, dz}$$

$$\int \frac{a^2 \sec^2 z \, . \, \cancel{a \sec z} \, . \, \cancel{tg \, z} \, . \, dz}{\cancel{a \, tg \, z}} =$$ $(\sqrt{u^2-a^2})$

$$a^2 \int \sec^3 z \, dz$$ (→ pa integrar faz-se a decomposição

$$a^2 \int \sec^2 z \, . \, \sec z \, dz$$

sendo : $\sec^2 z = tg^2 + 1$, temos:

$$a^2 \int (tg^2 z + 1) \sec z \, . \, dz$$

multiplicando e separando

$$a^2 \int tg^2 z \cdot \sec z \, dz + a^2 \int \sec z \, dz =$$

(incompleta {ainda não encontrada solução})

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$$\int \frac{dx}{(5-x^2)^{3/2}} =$$

fórmula: $\sqrt{a^2-u^2} = a x$ → a cos

substituindo:

$\boxed{u = x \quad du = dx \quad a = \sqrt{5}}$

1º) $\int \frac{du}{(a^2-u^2)^{3/2}}$

2º) $u = a \operatorname{sen} z$

$du = a \cdot \cos z \cdot dz$

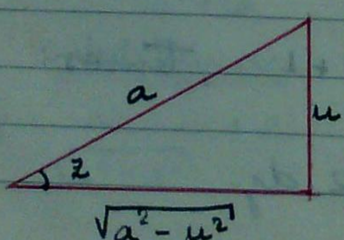
continuando:

a é constante, pois vem antes de $\int$ inverso de $a \cos z \cdot \sec^2 z$

$$\int \frac{\cancel{a \cos z} \, dz}{(a \cos)^{3/2}} = \frac{1}{a^2} \int \sec^2 z \, dz =$$

$$\boxed{\frac{1}{a^2} \cdot tg\, z + c}$$

$\boxed{u = a \operatorname{sen} z}$



$\operatorname{sen} u = \frac{u}{a}$

$tg\, z = \frac{u}{\sqrt{a^2-u^2}}$ → $\frac{\text{cateto oposto}}{\text{cateto adjacente}}$

então: $\frac{1}{a^2} \cdot tg\, z = \frac{1}{a^2} \cdot \frac{u}{\sqrt{a^2-u^2}} + c = \frac{x}{5\sqrt{5-x^2}} + c$

ou $\boxed{\frac{x}{5\sqrt{5-x^2}} + c}$

$$\int \frac{t^2 \, dt}{\sqrt{4-t^2}} =$$

fórmula: $\sqrt{a^2-u^2} = a \operatorname{sen} z = a \cos z$

$u = t$

$du = \sqrt{4} \Rightarrow 2$

$a = dt$

$u = a \operatorname{sen} z$

$du = a \cos z \, dz$

$$\int \frac{a^2 \operatorname{sen}^2 z \cdot \cancel{a \cos z} \, dz}{\cancel{a \cos z}} =$$

$$a^2 \int \operatorname{sen}^2 z \, dz = a^2 \int \operatorname{sen}^2 z \, dz =$$