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Orbit Sensitivity Analysis for Catarina Constellation

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Abstract

The Brazilian Data Collection System (SBCD) currently relies on geostationary satellites to gather environmental data and provide climate and defense analyses in Brazilian territory. However, due to the high costs associated with large satellite missions, the development of CubeSats - small, cost-effective satellites - has become a more attractive option for both companies and universities. As a result, the Catarina Constellation project was established to create a constellation of nanosatellites that would significantly improve remote sensing capabilities, stimulate the growth of the space industry in Santa Catarina, and foster collaboration between industry, government, and universities. To ensure the success of the Catarina Constellation project and due to launch uncertainties, a sensitivity analysis was conducted on the inclination, altitude, and right ascension of the ascending node of the orbit. This analysis aimed to determine the impact of these variations on specific mission parameters, including the number of times communication is established, the time the communication between the nanosatellite and the ground station, and the evaluation of days when communication is established. Using an orbital propagation code developed by the authors, a series of simulations were conducted to perturb the orbital parameters within a margin of error of $+/-$ 25 km ($\pm 0.4\%$) for the semi-major axis, $+/-$ 5 degrees ($\pm 5\%$) for the inclination, and $+/-$ 10 degrees ($\pm 5\%$) for the right ascension of the ascending node. From this data the results in a variation of 25% in the contact time and the average communication time, and a variation of 20% in the number of passes.

1 Introduction

The miniaturization of digital circuits has promoted the popularization of nanosatellites, mainly due to the lower costs of developing, launching, and maintaining these devices [1]. These advances have made nanosatellites more accessible to private companies and universities, allowing space missions and research to be conducted at a lower cost [2, 3, 4, 5].

Along with miniaturization, satellite constellations consisting of multiple satellites working together have become an essential tool for a wide range of technological and scientific applications. These systems offer enhanced communication [6], navigation [7, 8, 9], environmental monitoring [10] and Earth observation capabilities [11, 12].

In this context, the Catarina Constellation mission was born with the aim of launching a constellation of nanosatellites with missions ranging from data collection to Earth imaging. The Catarina Constellation, specifically Fleet A, consists of three CubeSats and its main mission is data collection. It is being developed by the Federal University of Santa Catarina (UFSC) and the SENAI Institute for Innovation in Embedded Systems (ISI-SE), with the support of the Brazilian Space Agency (AEB) and the National Institute of Space Research (INPE). The overall objective of this constellation is to promote the aerospace industry in the state of Santa Catarina and to train qualified human resources, taking advantage of the strategic collaboration between AEB, UFSC and ISI-SENAI [13]. In addition, Catarina Constellation seeks to provide services that contribute to the Brazilian Data Collection System (SBCD).

For a comprehensive analysis of the potential launch of the Catarina Constellation mission, with orbital parameters defined by a semi-major axis of 6896 km, an inclination of 98^o, and three satellites, distributed in a triangular formation, with Right Ascension of Ascending Node (RAAN) angles of 10^o, 130^o, and 250^o, according to [14], the initial optimization for this mission showed that the angular separation between the satellites should be at least 40^o to avoid interference during communication with the ground station.

The sensitivity analysis is a fundamental tool in various fields, including engineering and economics. It allows to assess how variations in input parameters influence the outcomes of a model, this process is crucial for understanding the robustness and reliability of predictions and decisions. In this context, the Latin Hypercube Sampling method was chosen because of its ability to provide robust and efficient results [15, 16].

In this article, a sensitivity analysis was performed on the initial parameters of semi-major axis, inclination, and longitude of the ascending node predicted for the launch of the Catarina Constellation mission.

2 Sensitivity Analysis Formulation

In order to calculate the orbital propagation of the CubeSat, a Python code was developed which makes use of the Gauss equations as outlined by Curtis in its book [17].

$$\frac{dh}{dt} = rp_s \tag{1}$$

$$\frac{de}{dt} = \frac{h}{\mu}\sin\theta p_r + \frac{1}{\mu h}[(h^2+\mu r)\cos\theta + \mu e r]p_r \tag{2}$$

$$\frac{d\theta}{dt} = \frac{h}{r^2} + \frac{1}{eh}\left[\frac{h^2}{\mu}\cos\theta p_r - \left(r + \frac{h^2}{\mu}\right)\sin\theta p_s\right] \tag{3}$$

$$\frac{d\Omega}{dt} = \frac{r}{h\sin i}\sin(\omega+\theta)p_\omega \tag{4}$$

$$\frac{di}{dt} = \frac{r}{h}\cos(\omega+\theta)p_\omega \tag{5}$$

$$\begin{aligned}\frac{d\omega}{dt} &= -\frac{1}{eh}\left[\frac{h^2}{\mu}\cos\theta p_r - \left(r + \frac{h^2}{\mu}\right)\sin\theta p_s\right] \\ &- \frac{r\sin(\omega+\theta)}{h\tan i}p_\omega\end{aligned} \tag{6}$$

where $r = \frac{h^2}{[\mu(1+e\cos\theta)]}$ and p_s, p_r and p_ω are the perturbations. The system of ordinary differential equations can be integrated based on the given equations and the initial orbital parameters of the mission, utilizing a package called ODEINT, which is based on a FORTRAN library.

In order to generate the communication criteria, that is to say, the antenna sighting, trigonometric relationships were employed based on the vectors of Figure 1. They are the vector between the center of the Earth and the satellite, the center of the Earth and the ground station, and, to complete the triangle, between the satellite and the ground station (V_{TS}, V_{TG} and V_{SG}, respectively). With regard to the angles, there is the minimum elevation angle of the ground station antenna for effective communication, defined in the manual, and the angle (γ), which is defined as the communication criteria and is defined by the equation 7.

In the case of the antenna under consideration, situated at the Natal Multi-Mission Station with coordinates -5.8717778^o N, -35.206864^o W, the minimum elevation is 15^o.

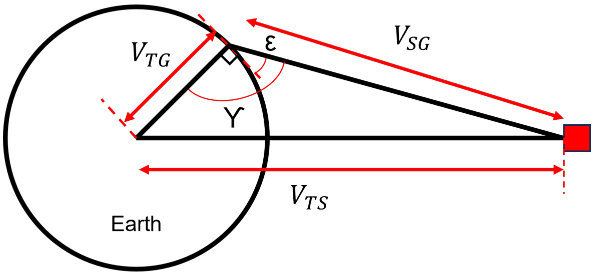

Figure 1: Relationship between the angles of the satellite, target, and center of the Earth [14].

$$\gamma = \pi - \arccos\frac{\vec{V_{TS}} \cdot \vec{V_{SG}}}{|\vec{V_{TS}}||\vec{V_{GS}}|} - \arccos\frac{\vec{V_{TG}} \cdot \vec{V_{TS}}}{|\vec{V_{TG}}||\vec{V_{TS}}|} \geq 105^o \quad (7)$$

In accordance with the established communication criterion, the remaining communication metrics are delineated, including: contact time, which represents the duration of satellite communication with the ground station; average contact time, which is the mean value of all contact intervals; and the number of passes, which is the frequency of satellite passages through the antenna's field of view.

To evaluate the influence of discrepancies between the planned initial condition and the reference mission, the Latin Hypercube scattering method was employed, resulting in the generation of 1000 samples. The generated samples represent potential variations in the launch, thus enabling a comprehensive analysis of the impact of these uncertainties on the satellite's communication parameters.

In the generated samples, variations in the values of three primary parameters were considered: the semi-major axis, inclination, and longitude of the ascending node for each of the three satellites. The introduced variations were in accordance with the specifications outlined in Table 1, with an variation of ± 25 km for the semi-major axis, $\pm 5^o$ for the inclination, and $\pm 10^o$ for the longitude of the ascending node.

To conduct the sensitivity analysis, two Intel® Xeon® CPU E5-2640 v4 @ 2.40GHz processors were utilized, comprising a total of 20 cores and 125 GB of RAM. This configuration was provided by the Laboratório de Computação Científica (LCC) at the Technological Center of Joinville from the Federal University of Santa Catarina (CTJ-UFSC).

3 Result and Discussions

To facilitate the graphical representation of the data, the case with a semi-major axis of $6896km$, an inclination of 98^o and the longi-

Table 1: Upper and Lower Bounds from the Sensitivity Analysis

Satellite	*Variables*	*Lower Bound*	*Upper Bound*
1	SMA[a]	6871 km	6921 km
1	I[b]	93^o	103^o
1	RAAN[c]	0^o	20^o
2	SMA[a]	6871 km	6921 km
2	I[b]	93^o	103^o
2	RAAN[c]	120^o	140^o
3	SMA[a]	6871 km	6921 km
3	I[b]	93^o	103^o
3	RAAN[c]	240^o	260^o

[a]Semi-Major Axis.
[b]Inclination.
[c]Longitude of the Ascending Node.

tude of the ascending node of $10^o, 130^o$ and 250^o for satellites 1, 2 and 3, respectively, was selected as a reference point. All subsequent cases were then normalized relative to this reference.

In Figure 2, the dispersion of the data, conducted via the Latin Hypercube sampling method, is depicted in this analysis. It is evident that, despite the presence of some regions that have been less extensively investigated, the dispersion is effective across the entire domain of variables. This efficiency is demonstrated by the representative distribution of the sample points, which are dispersed uniformly throughout the space of the variables.

It is imperative that the distribution of sample points be uniform for a robust analysis, as this ensures that all regions of the domain are properly explored. This characteristic is of particular importance in ensuring that the analysis incorporates a comprehensive and accurate representation of all potential variations within the domain under study. Furthermore, the histograms displayed on the main diagonal of the dispersion matrix indicate that there is no significant concentration of values in any specific region of the range. This suggests that the distribution of points is uniform, thereby reinforcing the effectiveness of the method in comprehensively covering the space of the variables.

The results of the orbital propagation simulations were used to generate a dispersion matrix, which is illustrated in Figure 3. In this matrix, each satellite is represented by a distinct color: black for satellite one, red for satellite two, and green for satellite three.

The data analysis indicates that there is no evident trend in the inclination and longitude of the ascending node parameters. This is reflected in the distribution of results relating to the number of passes with communication, contact time, and average communication, which are scattered throughout the

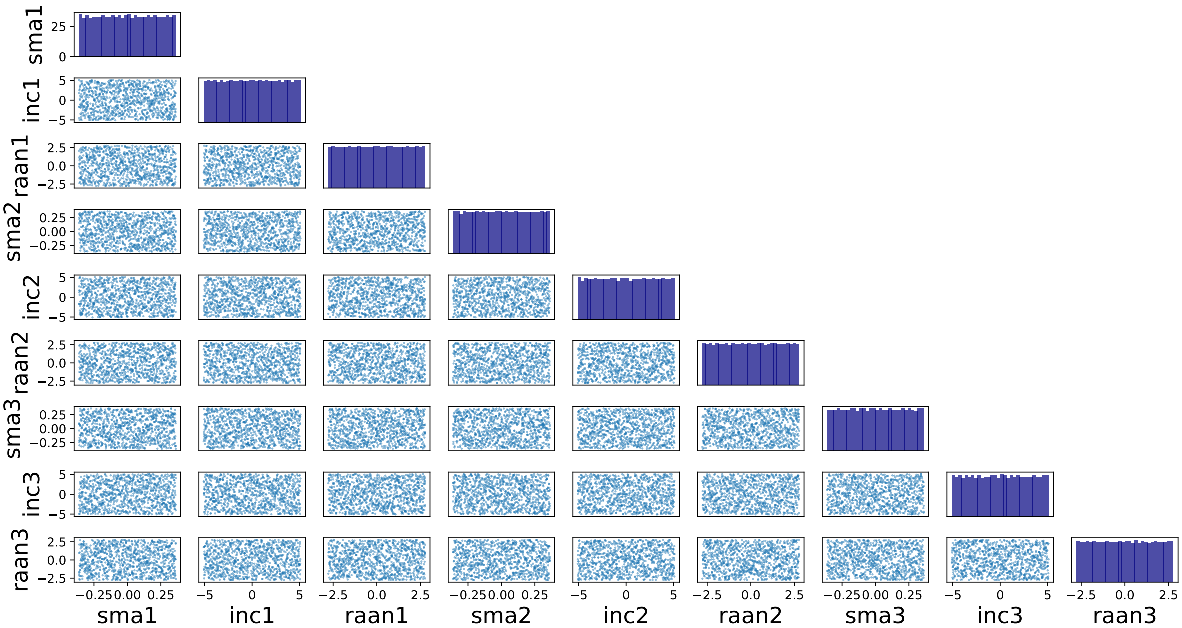

Figure 2: Initial dispersion scatter matrix.

domain, in other words, the average trend is flat, indicating the absence of a discernible trend.

With regard to the semi-major axis, a notable trend emerges with respect to the parameters of communication time and average communication. Minor fluctuations in the semi-major axis, amounting to $\pm 0.4\%$, give rise to variations of up to $\pm 10\%$ in these parameters, thereby underscoring the considerable sensitivity of communication performance with respect to this variable.

Furthermore, the inclination and length of the ascending node exert a minor influence on the communication parameters. The observed variations in these parameters result in a change of approximately $\pm 2.5\%$, which has the effect of preventing the formation of a single, clear line in the data set under analysis. This dispersion indicates that, while the slope and length of the ascending node are not the primary determinants of the results, they nevertheless contribute to the observed variability.

With regard to the number of passes, a similar trend was also discernible, exhibiting a comparatively weaker effect, with variations of approximately $\pm 5\%$. This indicates that, although they are affected, the inclination and longitude of the ascending node have a comparatively minor impact on the number of passages in comparison to the semi-major axis.

In order to evaluate the total contact time of the constellation, a dispersion matrix was generated, relating the semi-major axis, inclination, and longitude of the ascending node of each satellite to the total contact time of the constellation. The results indicate that the concentration of data occurs predominantly in the central region of the matrix, suggesting that the influence of these parameters on the total contact time of the constellation is not

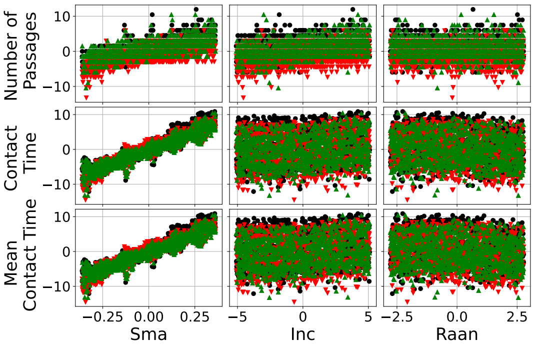

Figure 3: Communication metrics scatter matrix.

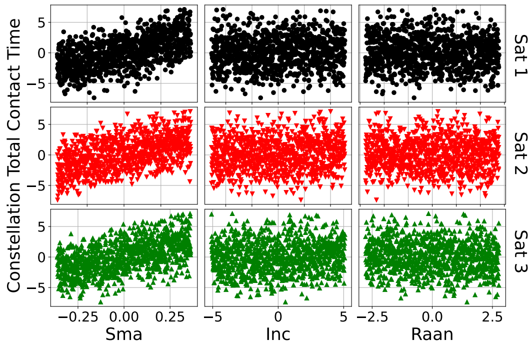

Figure 4: Constellation total contact time scatter matrix.

particularly significant.

Figure 4 shows the relationship between the variation of the semi-major axis, inclination and longitude of the ascending node and the total contact time of the satellite constellation for each of the three satellites. The satellites are arranged in a linear sequence, with Satellite 1 situated in the first row, Satellite 2 in the second row, and Satellite 3 in the third row, the arrangement is illustrated on the right.The semi-major axis exhibits a somewhat more discernible trend in comparison to the other parameters. In particular, a change of $\pm 0.4\%$ in the semi-major axis is associated with a change of approximately $\pm 6\%$ in the total contact time. This trend, although more defined, still indicates that the semi-major axis exerts a moderate influence on the total contact time of the constellation, in contrast to the even more subtle influence of the inclination and longitude of the ascending node.

To identify the relationships between communication parameters, the parallel coordinates strategy was used, which provides an effective visualization for exploring multidimensional relationships between different variables and identifying patterns. This approach optimizes data interpretation by focusing on the variables that are most relevant to the study of relationships between communication parameters. Each of the three satellites was analyzed separately to improve the clarity of the visualization.

The parallel coordinate axes, which have identical limits, represent the variations in the semi-major axis (SMA), inclination (Inc), longitude of the ascending node (RAAN), number of passes (NP), and mean communication time (MCT), respectively. Furthermore, a color scheme was employed to illustrate the variation in the total contact time of the constellation, thereby facilitating a more intuitive examination of the relationships between the variables.

The axis representing the communication time of the satellites was excluded from the analysis, as its direct correlation with the average communication time would not contribute any additional insights.

Figure 5 shows the analysis of the first satellite, where it can first be seen that higher values of Semi-Major Axis (SMA) result in a longer total contact time. In addition, there is a clear correlation between SMA and inclination (Inc), with higher SMA values tending to be associated with lower inclination values, indicating an inverse relationship between these two variables.

The data also show a significant dispersion between the axes representing the number of passes and the average contact time. This dispersion suggests that there is no strong correlation between these parameters, indicating a variability in communication results that is not directly related to SMA or tilt.

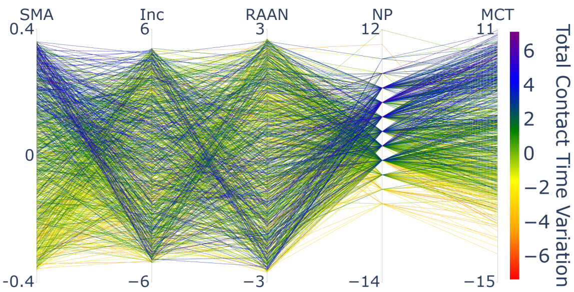

Figure 5: Satellite 1 parallel coordinates.

Satellite two, shown in Figure 6, shows a very similar pattern to satellite one. However, it can be seen that although satellite two tends to have a lower number of passes, the average contact time is not affected to the same extent. This suggests that despite the differences in the number of passes, the performance in terms of contact time remains relatively stable, highlighting a robustness in the communication parameters of this particular satellite.

Satellite three, shown in Figure 7, shows a distribution very close to that observed for satellite one. It can be seen that all three satellites show a general consistency in their patterns, indicating similar behavior in terms of communication parameters.

In addition, the distribution of the variation

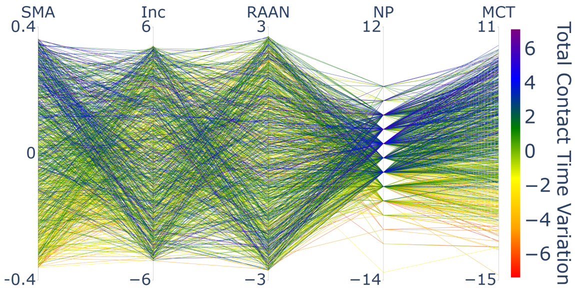

Figure 6: Satellite 2 parallel coordinates.

in the total contact time of the constellation shows a remarkable consistency, reflecting a stability in the communication characteristics between the satellites. This uniformity suggests that the satellites, despite their individual differences, maintain a relatively homogeneous performance in terms of the total contact time of the constellation.

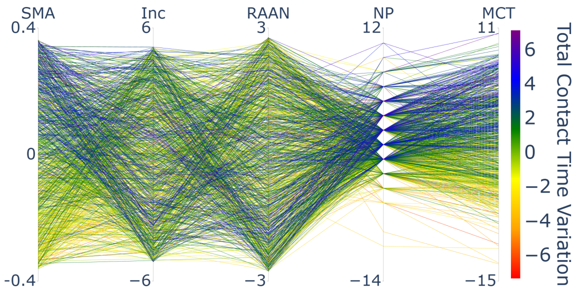

Figure 7: Satellite 3 parallel coordinates.

The parallel coordinate plots proved to be a valuable tool for gaining insight into the communication parameters of the three satellites in the Catarina constellation. The analysis yielded a correlation between semi-major axis (SMA) and inclination (Inc), whereby higher SMA values were found to be associated with lower inclination values.

Furthermore, the data exhibited considerable dispersion along the axes representing the number of passes and average contact time, indicating significant variability in the communication characteristics between the satellites. Furthermore, an examination of the colors utilized in the graphs indicated that the variation in the total contact time of the constellation is predominantly concentrated in values near zero and four, which are associated with the colors green and blue, respectively.

These findings provide a robust foundation for comprehending the behavior of the satellites with regard to communication and offer invaluable insights for future optimization of the constellation.

4 Conclusions

A sensitivity analysis was conducted to ascertain the impact of variations in orbital parameters, particularly the semi-major axis, inclination, and longitude of the ascending node, on the communication parameters of the satellites. The perturbations were carried out using the Latin Hypercube method, with variations of $\pm$25 kilometers for the semi-major axis, $\pm$5 degrees for the inclination, and $\pm$10 degrees for the longitude of the ascending node. These perturbations correspond to $\pm 0.4\%$, $\pm 5\%$, and $\pm 3\%$, respectively.

The results demonstrated that the number of passages, contact time, and average communication time exhibited absolute variations of up to 20%, 25%, and 25%, respectively. It was observed that the semi-major axis exerts a more pronounced influence on communication parameters than the inclination and length of the ascending node, which did not exhibit discernible trends in communication parameters.

The findings of this study have significant implications for the planning of future space missions. Given the advanced technology and high costs associated with the space field, it is imperative to conduct simulations of various scenarios to guarantee the accomplishment of mission objectives and to substantiate the efficacy of the development process.

Acknowledgment

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