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POWERTRAIN OPTIMIZATION OF HYBRID AND ELECTRIC VEHICLES

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POWERTRAIN OPTIMIZATION OF HYBRID AND ELECTRIC VEHICLES

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Toute réussite déguise une abdication
(Simone de Beauvoir)

RESUMO

Uma demanda crescente por tecnologias sustentáveis e regulamentações governamentais mais rígidas motivam o desenvolvimento de tecnologias inovadoras para trens de força elétricos e híbridos, visando reduzir as emissões e o consumo de combustível. Os veículos elétricos híbridos (HEV, do inglês *hybrid electric vehicle*) apresentam um sistema de trem de força híbrido, que usa um único ou vários trens de engrenagens planetárias (PGT, do inglês *planetary gear train*) como dispositivo de divisão de potência. A maioria dos trabalhos sobre consumo de HEVs não aborda o efeito da eficiência mecânica dos PGTs no desempenho do trem de força. Porém, investigar a eficiência dos PGTs tem potencial para reduzir o uso de combustível e a emissão de poluentes. Esta tese introduz um novo método para calcular consumo e emissões de HEVs que integra a eficiência do motor à combustão interna, do motor elétrico e do gerador elétrico à variabilidade da eficiência da transmissão por PGTs, de acordo com a velocidade do veículo. A tese apresenta três estudos de caso: um robô de inspeção para linhas aéreas, representando um veículo elétrico; o Toyota Prius 1ª geração; e um HEV semelhante ao Prius, mas com uma transmissão composta por engrenagens cônicas chamada de sistema híbrido baseado em diferencial (DHS, do inglês *differential-based hybrid system*). O consumo de energia do robô de inspeção é quantificado por meio do estado de carga das baterias, calculado para um perfil de velocidade que emula o robô se deslocando sobre um condutor. O impacto da eficiência do PGT que atua como redutor do motor de tração sobre o estado de carga das baterias é analisado, ao otimizar sua eficiência por meio de um algoritmo genético, alterando o módulo das engrenagens e o número de dentes. O consumo de combustível do Toyota Prius é estimado para o ciclo de condução japonês modos 10-15 e o consumo composto de combustível é estimado para os ciclos FTP75 e HWFET, considerando os mapas de eficiência das fontes de propulsão. Os resultados de consumo são comparados com os valores oficiais fornecidos pela Toyota: o consumo calculado pelo método da tese para o ciclo modos 10-15 apresentou uma diferença de 9,7% do valor oficial, enquanto o consumo composto de combustível difere apenas 1,7% do valor oficial. A variação mínima em relação aos valores oficiais comprova que o método da tese é robusto e tem potencial para prever consumo e emissões de forma confiável na fase de projeto de um novo HEV. Por fim, o último estudo de caso compreende a estimativa de consumo de combustível de um HEV equipado com o DHS. O HEV é semelhante ao Prius, mas com o DHS gerenciando a potência das fontes de propulsão. O consumo composto de combustível do HEV é calculado para os ciclos FTP75 e HWFET, resultando em um valor ligeiramente superior ao obtido para o Prius, pois a eficiência do DHS é menor que a eficiência do PGT do Prius original. O consumo composto de combustível do HEV equipado com o DHS é otimizado alterando as dimensões das engrenagens do DHS e a relação de transmissão do redutor que conecta o DHS ao gerador, levando a uma redução de 2,2% no consumo. As principais contribuições originais da tese são a inclusão da variabilidade da eficiência da transmissão e o cálculo simultâneo da eficiência das fontes de propulsão e da transmissão para determinar o consumo e as emissões de um HEV.

Palavras-chave: Veículos híbridos e elétricos. Consumo de combustível. Eficiência de trem de potência. Eficiência de trens de engrenagens. Eficiência de engrenagens cônicas.

RESUMO EXPANDIDO

Introdução

O mercado de veículos elétricos (EV, do inglês *electric vehicle*) experimenta um ambiente fértil para a inovação desde os anos 2010. Uma demanda crescente por tecnologias sustentáveis e regulamentações governamentais cada vez mais rigorosas motivam o desenvolvimento de tecnologias inovadoras de trens de força elétricos e híbridos visando diminuir a emissão de poluentes e aumentar a economia de combustível. Veículos elétricos à bateria (BEV, do inglês *battery electric vehicle*) têm vantagem de não emitir poluentes, mas têm autonomia limitada e baterias com vida útil curta. Veículos híbridos elétricos (HEV, do inglês *hybrid electric vehicle*) possuem um sistema de armazenamento híbrido de energia, por exemplo, um tanque de combustível associado a baterias, que permite superar as desvantagens dos BEVs. A tecnologia híbrida permite dissociar a operação do motor à combustão interna (ICE, do inglês *internal combustion engine*) da condição de tráfego, de modo que o consumo de combustível possa ser otimizado, e permite que a bateria seja recarregada pelo ICE durante o funcionamento do veículo. Além de um sistema híbrido de armazenamento de energia, a transmissão automotiva é outro elemento que pode ajudar a aumentar a autonomia dos EVs. Um interesse especial reside nos trens de força híbridos com distribuição de potência, que utilizam um único ou vários trens de engrenagens planetárias (PGT, do inglês *planetary gear train*) como dispositivo de divisão de energia para criar um conjunto de configurações através de diversas combinações de fontes de potência e elementos dos PGTs conectados por embreagens e freios. Investigar a eficiência mecânica dos PGTs tem potencial para reduzir o consumo de combustível e a emissão de poluentes devido a diversos fatores. A eficiência da transmissão é necessária para desenvolver o sistema de controle do veículo, pois permite obter a potência desejada tendo um trem de força tão eficiente quanto possível (considerando a eficiência do ICE, motor elétrico (M, do inglês *motor*), gerador elétrico (G) e sistema de transmissão). Além disso, garantir o funcionamento do ICE no seu ponto ótimo não é uma condição suficiente para minimizar o consumo de combustível, uma vez que pode haver perdas significativas de potência na transmissão. Portanto, a eficiência dos PGT é um parâmetro que deve ser considerado nas simulações de consumo de veículos híbridos. Esta tese introduz um novo método para simular o consumo e as emissões de EV e HEV que integra as eficiências das fontes de propulsão à variabilidade de eficiência de transmissões compostas de PGTs de acordo com a velocidade do veículo. As simulações da tese compreendem EV e HEV com sistemas de transmissão compostos de PGTs, apresentando relações de transmissão constantes ou continuamente variáveis. A eficiência da transmissão e a sua variação de acordo com o perfil de velocidade do veículo são levadas em conta no cálculo do consumo quando se utiliza o método Davies para calcular a eficiência dos PGT. O método de Davies permite calcular a eficiência de PGTs com qualquer arquitetura, incluindo várias fontes de perdas, tais como o atrito entre os dentes das engrenagens e o atrito nos rolamentos de suporte dos eixos, principais fontes de perdas em PGTs. A utilização do método Davies para incluir a variabilidade da eficiência do sistema de transmissão nas simulações de consumo gera resultados mais realistas e permite realizar uma otimização do consumo tendo as dimensões das engrenagens como variáveis de projeto.

Objetivos

O principal objetivo da tese é desenvolver um novo método para simular o consumo de combustível e as emissões de EV e HEV, integrando a eficiência de ICE, M e G com a variabilidade — de acordo com a velocidade do veículo — da eficiência mecânica de transmissões compostas de PGTs. Objetiva-se que o método apresentado nessa tese sirva como ferramenta para o projeto de novas transmissões automotivas para veículos híbridos.

Metodologia

O método desenvolvido nesta tese integra as eficiências de ICE, M, G com a variação da eficiência da transmissão para calcular o consumo de combustível e as emissões de HEVs. A variação da eficiência desses quatro elementos do trem de força durante o funcionamento do HEV é levada em conta por meio do uso de mapas de eficiência das três fontes de propulsão e do cálculo da eficiência da transmissão pelo Método de Davies. Este trabalho usa equações clássicas de dinâmica veicular para calcular o consumo de combustível de um HEV e o estado de carga (SOC, do inglês *state-of-charge*) de sua bateria. A partir de perfis de velocidade padrão, a força trativa necessária para o HEV se deslocar e a potência requerida pelo veículo são determinadas. O perfil de velocidade do veículo permite identificar a rotação e o torque de ICE, M e G, com os quais se navega pelos mapas de consumo, emissões e de eficiência, essenciais para determinar a autonomia do veículo. A eficiência mecânica da transmissão é necessária para quantificar a potência requerida pelo veículo e ela é determinada pelo Método de Davies. No Método de Davies, as teorias de grafos e helicoides são associadas para realizar as análises cinemática e estática de sistemas multicorpos, por meio de uma analogia com as leis de Kirchhoff. O Método de Davies permite incluir modelos de atrito para simular diversas fontes de perdas de potência, logo, a eficiência de PGTs é calculada de forma realista para cada instante de um ciclo de condução padrão. As principais contribuições da tese são: (a) a inclusão da variabilidade da eficiência da transmissão, em função da velocidade do veículo, e (b) o cálculo simultâneo da eficiência das fontes de propulsão e da transmissão, ambos para determinar o consumo e as emissões de um HEV.

Resultados e Discussão

O método desenvolvido nesta tese é aplicado em três estudos de caso: um robô de inspeção para redes elétricas de distribuição, emulando um veículo puramente elétrico, um HEV comercial e um HEV com transmissão por engrenagens cônicas. No primeiro estudo de caso, o consumo de energia do robô de inspeção é quantificado por meio do estado de carga das baterias, o qual é calculado para um perfil de velocidade que emula o robô se deslocando sobre um condutor ligando dois postes consecutivos. O impacto da eficiência do redutor do motor de tração do robô sobre o estado de carga das baterias é analisado. Um problema de otimização é definido para maximizar a eficiência do PGT através da alteração do módulo e do número de dentes das engrenagens. O problema de otimização é solucionado por meio de um algoritmo genético. O veículo comercial escolhido como segundo estudo de caso é o Toyota Prius 1^a Geração. O

cálculo do consumo de combustível do Toyota Prius integra as eficiências do ICE, dos motor-geradores MG1 e MG2 e da transmissão, composta por um PGT chamado de *power-split device* (PSD), utilizando mapas de consumo de combustível e eficiência e o Método de Davies para calcular a eficiência do PSD. O consumo de combustível é estimado para ciclo de condução japonês padrão chamado modos 10-15. Já o consumo composto de combustível é estimado para os ciclos FTP75 e HWFET, que simulam condições de tráfego urbano e de autoestrada, respectivamente. Os resultados de consumo de combustível são comparados com os valores oficiais divulgados pela Toyota: o consumo de combustível calculado através do método desta tese para o ciclo modos 10-15 apresentou uma diferença de 9,7% em relação ao valor oficial. Já o consumo composto de combustível calculado por meio do método desta tese difere apenas 1,7% da valor oficial. As diferenças entre os valores de consumo de combustível obtidos através do método desenvolvido nesta tese e os valores oficiais resultam das hipóteses adotadas ao longo do desenvolvimento do método e de algumas suposições feitas devido à falta de informações oficiais. Apesar disso, a variação dos valores oficiais de consumo de combustível é mínima, provando que o método da tese é robusto e tem potencial para prever o consumo e as emissões de uma forma confiável na fase inicial de projeto de um novo HEV. Finalmente, o último estudo de caso trata da estimativa do consumo de combustível de um HEV equipado com uma transmissão composta por engrenagens cônicas, chamada em inglês de *differential-based hybrid system* (DHS). O HEV é semelhante ao Toyota Prius 1ª Geração, mas em vez de utilizar um PGT como dispositivo de distribuição de potência, a potência das fontes de propulsão é gerida pelo DHS. O consumo composto de combustível do HEV é calculado utilizando o método desenvolvido nesta tese e os ciclos de condução padrão FTP75 e HWFET, resultando em um valor ligeiramente superior ao obtido para o Toyota Prius. O HEV equipado com o DHS consome mais combustível do que o Toyota Prius porque a eficiência do DHS é em média menor do que a eficiência do PSD durante os ciclos de condução. Após o cálculo da autonomia do HEV equipado com o DHS, cria-se um problema de otimização para minimizar o consumo de combustível. O consumo é minimizado ao maximizar a eficiência do DHS, variando o módulo e o número de dentes das engrenagens e a relação de transmissão do redutor conectando o gerador ao DHS. Um algoritmo genético é utilizado para resolver o problema de otimização, encontrando os valores de módulo, número de dentes e relação de transmissão que maximizem a eficiência do DHS. O consumo composto de combustível obtido por meio da otimização é 2,2% menor do que o original.

Considerações Finais

Os resultados de consumo obtidos por meio do método desenvolvido na tese para o Toyota Prius 1ª Geração são muito próximos dos valores oficiais, demonstrando a robustez do método. Logo, o método pode servir de ferramenta de projeto de novos HEVs. Em relação ao último estudo de caso, existem diversos fatores que podem ter contribuído para a redução do consumo composto. Entre eles, um possível ligeiro aumento de eficiência do DHS e a mudança do ponto de operações das fontes de propulsão. O aumento da eficiência mecânica da transmissão, associado ao desenvolvimento de motores mais eficientes, pode fazer a diferença no momento de validação de um novo HEV a ser lançado no mercado. Portanto, mais estudos são necessários para entender a interação entre as eficiências das fontes de propulsão e

da transmissão e como o consumo de combustível é afetada por elas.

Palavras-chave: Veículos híbridos e elétricos. Consumo de combustível. Eficiência de trem de potência. Eficiência de trens de engrenagens. Eficiência de engrenagens cônicas.

ABSTRACT

A rising customer demand for sustainable technologies and ever-tightening government regulations motivate the development of innovative electric and hybrid powertrain technology to decrease harmful emissions and increase fuel economy. Hybrid electric vehicles (HEV) present a power-split hybrid powertrain system, which use a single or multiple planetary gear trains (PGT) as power-split device. Most of the works on HEV fuel economy do not address the effect of the PGTs mechanical efficiency on the powertrain performance. Nonetheless, investigating the PGTs efficiency is a potential approach to reduce fuel usage and pollutants. This Thesis presents a new method to determine consumption and emissions of HEVs which integrates the efficiency of the internal combustion engine, motor, generator with the variability of transmission efficiency by PGTs, according to vehicle speed. The Thesis presents three case studies: an inspection robot for overhead power lines, representing an electric vehicle; the Toyota Prius 1st Generation; and a HEV similar to Prius, but equipped with a power-split device composed of bevel gears called differential-based hybrid system (DHS). The inspection robot power consumption is quantified through the batteries state-of-charge (SOC), computed for a speed profile emulating the robot travelling on a conductor. The impact of the efficiency of the PGT reducer of the robot's traction motor on the SOC is analyzed. The PGT efficiency is maximized through a genetic algorithm, by altering the gears module and number of teeth. The fuel economy of the Toyota Prius is estimated for the Japanese 10-15 mode cycle and its composite fuel economy is estimated for the FTP75 and HWFET cycles, considering the propulsion sources efficiency maps. The fuel economy results are compared to Toyota official values: the fuel economy calculated through the Thesis method for the 10-15 mode cycle presented a difference of 9.7% from the official value, while the composite fuel economy differs only 1.7% from the official value. The minimal variation from official fuel economy values proves that this Thesis method is robust and has potential to predict consumption and emissions in a reliable way on the design phase of a new HEV. Finally, the last case study comprises the fuel consumption estimation of an HEV equipped with the DHS. The HEV is similar to the Prius, but with the DHS managing the power of the propulsion sources. The composite fuel economy of the HEV is calculated for the FTP75 and HWFET cycles, resulting in a slightly higher value than the one obtained for the Prius, because the efficiency of the DHS is lower than the PGT efficiency of the original Prius. The composite fuel economy of the HEV equipped with the DHS is optimized by changing the gear dimensions of the DHS and the gear ratio of the reducer connecting the DHS to the generator, leading to a 2.2% reduction in consumption. The main original contributions of the Thesis are the inclusion of the variability of the transmission efficiency and the simultaneous calculation of the efficiency of the propulsion sources and the transmission efficiency to determine the consumption and emissions of a HEV.

Keywords: Hybrid and electric vehicles. Fuel consumption. Powertrain efficiency. Gear train efficiency. Bevel gears efficiency.

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LIST OF ABBREVIATIONS AND ACRONYMS

AMT	Automated manual transmission
ANEEL	Agência Nacional de Energia Elétrica
BEV	Battery electric vehicle
Celesc	Centrais Elétricas de Santa Catarina S.A.
CM	Center of mass
CO	Carbon monoxide
CVT	Continuously variable transmission
DFD	Data flow diagram
DHS	Differential-based Hybrid System
DHT	Differential Hybrid Transmission
DOF	Degree-of-freedom
EMS	Energy management strategy
EPA	US Environmental Protection Agency
EV	Electric vehicle
G	Generator
GA	Genetic algorithm
HC	Unburned hydrocarbons
HEV	Hybrid electric vehicle
ICE	Internal combustion engine
ISA	Instantaneous screw axis
LAR	Laboratory of Applied Robotics of Federal University of Santa Catarina
M	Electric motor
MG	Motor-generator
NO _x	Nitrogen oxides
OOL	Optimum operation line
PGT	Planetary gear train
PSD	Power-split device
SOC	State-of-charge

LIST OF SYMBOLS

F_t	Vehicle's tractive effort
F_r	Tire rolling resistance
M_{veh}	Vehicle mass
g	Gravity acceleration
f_r	Rolling resistance coefficient
α	Road angle
v	Vehicle linear speed
ρ_a	Air density
C_D	Aerodynamic drag coefficient
A_f	Vehicle frontal area
τ	Time
δ	Rotational inertia factor
$\mathcal{P}_{dem}$	Vehicle power demand
η_t	Transmission efficiency
b_e	ICE's specific fuel consumption
N_w	Drive wheel angular speed
r_d	Effective radius of the wheel
$N_{t_{out}}$	Transmission output speed
i_0	Gear ratio of the final drive
N_{ICE}	ICE speed
N_M	M speed
N_G	G speed
η_{ICE}	ICE efficiency
Q	Fuel specific energy
$\dot{m}_{fuel}$	Fuel consumption rate
$\mathcal{P}_{ICE}$	ICE power
$FC_{L/100km}$	Fuel economy in L/100 km
ρ_{fuel}	Fuel density
S_{cyc}	Distance travelled by the vehicle in a complete drive cycle
$FC_{km/L}$	Fuel economy in km/L
FC_C	Composite fuel economy
FC_{FTP}	Fuel economy in the FTP75 cycle
FC_{HW}	Fuel economy in the HWFET cycle
E_0	Battery pack initial energy capacity
$\mathcal{P}_{bat}$	Real-time charging or discharging power of the battery pack
η_{bat}	Battery pack's charging/discharging effectiveness
c_{bat}	Battery pack nominal capacity
V_{bat}	Battery pack nominal voltage

$\mathcal{P}_{M_{bat}}$	Battery power consumption of M
$\mathcal{P}_{G_{bat}}$	Battery power consumption of G
$\mathcal{P}_{aux}$	Power consumption for high-voltage electrical appliances
$\mathcal{P}_M$	M power
$\mathcal{P}_G$	G power
η_M	M efficiency
η_G	G efficiency
v	Number of fundamental circuits
e	Number of couplings of a mechanism
n	Number of links of a mechanism
κ	Number of fundamental cutsets
F	Gross degree of freedom
f	Independent unitary motion allowed by a coupling
C	Gross degree of constraint
c	Independent unitary constraint of a coupling
$[B_M]$	Motion graph fundamental circuit matrix
$[Q_A]$	Action graph fundamental cutset matrix
λ	Order of the screw system
$\$$	Screw
h	Screw pitch
$\$^m$	Twist
$\$^a$	Wrench
ϱ	Screw magnitude
$\hat{\$}$	Normalised screw
$[\hat{M}_D]$	Unit motion matrix for direct couplings
$\{\vec{\psi}\}$	Twists' magnitudes vector
$[\hat{M}_N]$	Network unit motion matrix
F_N	Net degree of freedom
$[M]$	Motion matrix
$[\hat{A}_D]$	Unit action matrix for direct couplings
$\{\vec{\psi}\}$	Wrenches' magnitudes vector
$[\hat{A}_N]$	Network unit action matrix
C_N	Net degree of constraint
t	Angular speed magnitude in z direction
U	Force magnitude in x direction
T	Torque magnitude in z direction
s	Angular speed magnitude in y direction
S	Torque magnitude in y direction
$\mathcal{P}$	Power

η	Efficiency
$[M_i]$	Absolute motion matrix
$[I_i]$	Reduced incidence matrix
ζ	Coulomb friction factor
ξ	Viscous friction coefficient
ϱ_L	Friction torque magnitude
μ	Dimensionless friction coefficient
r_{num}	Gear pitch radius
η_{ext}	Ordinary efficiency of an external meshing gear pair
η_{int}	Ordinary efficiency of an internal meshing gear pair
m	Gear module
z	Gear number of teeth
i_{PGT}	PGT speed ratio
d	Gear pitch diameter
V_{ext}	Volume of an external gear
V_{int}	Volume of an internal gear
b	Tooth's face width
d_v	Diameter of the internal gear's external circle
V''	Clearance volume
m_b	Backup ratio
h_t	Whole depth of a gear tooth
d_f	Root diameter of the internal gear
s_a	Addendum circle thickness
c^{*l}	Clearance height
V_{PGT}	PGT volume
k	PGT number of planets
σ_b	Bending stress at the root of the tooth
σ_c	Contact stress at the tooth surface or pitting resistance
W_t	Tangential force acting on the tooth flank
J	Bending strength geometry factor
K_a	Application factor
K_v	Dynamic factor
K_m	Load distribution factor
K_S	Size factor
K_B	Rim thickness factor
K_I	Idler factor
Y	Form factor
K_f	Stress concentration factor
S_{fb}	Corrected allowable bending stress

K_L	Life factor
K_T	Temperature factor
K_R	Reliability factor
S'_{fb}	Published AGMA bending-fatigue strength
SF_b	Safety factor against bending failure
I	Surface geometry factor
C_p	Elastic coefficient
C_f	Surface finish factor
S_{fc}	Corrected allowable contact stress
C_L	Surface-life factor
C_H	Hardness ratio factor for pitting resistance
S'_{fc}	Published AGMA surface-fatigue strength
SF_c	Safety factor against surface failure
s	Catenary sag
p	Cable unit weight
S	Cable length
$\mathcal{P}_t$	Traction power
SOC_{ICE}	SOC threshold for starting ICE
v_{eb}	HEV speed threshold for starting ICE
$\dot{m}_{pol}$	pollutant emission rate
m_{pol}	Accumulated pollutant mass
η_{cat}	Conversion efficiency of the catalytic converter
$m_{pol}^{g/dist}$	Pollutant mass per distance unit
SOC_t	Top of SOC operating range
N_{ICE}^{idle}	ICE idle speed
N_{MG1}^{max}	MG1 maximum speed
T_{MG1}^{max}	MG1 maximum torque for a given speed
V_{DHS}	DHS volume
Γ	Pitch cone angle
K_A	Overload factor for straight bevel gears
Y_x	Size factor for bending strength of straight bevel gears
$K_{H\beta}$	Load distribution factor for straight bevel gears
Y_β	Lengthwise curvature factor for bending strength of straight bevel gears
Y_J	Bending strength geometry factor for straight bevel gears
Y_{NT}	Stress-cycle factor for bending strength of straight bevel gears
Y_Z	Reliability factor for bending strength of straight bevel gears
Z_I	Pitting resistance geometry factor for straight bevel gears
Z_x	Size factor for pitting resistance of straight bevel gears
Z_{xc}	Crowning factor for pitting resistance of straight bevel gears

Z_{NT}	Stress cycle factor for pitting resistance of straight bevel gears
Z_W	Hardness ratio factor for pitting resistance of straight bevel gears
Z_z	Reliability factor for pitting resistance of straight bevel gears
SOC_0	Initial SOC
$\mathcal{P}_{MG2}$	MG2 power
T_{MG2}	MG2 torque
T_{MG2}^{max}	MG2 maximum torque for a given speed
η_{MG1}	MG1 efficiency
η_{MG2}	MG2 efficiency
FC_{Prius}	Prius fuel economy
i_{ab}	Ratio between ICE speed and MG1 speed

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1 INTRODUCTION

The market of electric vehicles (EV) experiences a fertile environment for innovation worldwide since the 2010s. Ever-tightening government regulations and rising customer demand for alternative sustainable technology motivate the development of innovative electric and hybrid powertrain technology to decrease harmful emissions and increase fuel economy (HAYES; GOODARZI, 2018). United States, European Union, China, Japan and Brazil are adopting standards with stricter greenhouse gas emissions limits. For example, new carbon-dioxide regulations established in December 2018 by the European Union set a target of 37.5% reduction in car emissions by 2030, with respect to 2021 (CONTINENTAL, 2019).

Battery electric vehicles (BEV) present the advantage of not emitting pollutants, but have the downside of limited driving range and battery lifespan. Hybrid electric vehicles (HEV) present a hybrid energy storage system, e.g., an association of a battery and a fuel tank, that allows to overcome the BEVs' drawbacks (YANG et al., 2020). Hybrid technology enables to decouple the internal combustion engine (ICE) use from the traffic condition, such that the fuel consumption can be optimized. Also, it enables the recharge of the battery by the engine during the vehicle operation (HAYES; GOODARZI, 2018).

Besides a hybrid energy storage system, a multi-speed transmission is another component that boosts the EVs driving range. A special interest lies on power-split hybrid powertrain systems, which use a single or multiple planetary gear trains (PGT) as power-split device to create a collection of configurations through diverse combinations of power source and PGT elements connected by shift actuators, i.e., clutches and brakes (GU et al., 2020).

The versatile power distribution among engine, electric motor (M) and generator (G) in HEVs produces a lot of research opportunities. Most of the works found in literature focuses on minimizing fuel consumption and greenhouse gases emission by means of the optimization of an energy management strategy (EMS) and gear shifting. Li, Chen and Evangelou (2020) investigate the optimal design of a multi-speed transmission of a HEV, through a two steps optimization. The energy management and the gear shifting are optimized to minimize the fuel consumption, assuming that the mechanical efficiency of the gear train is constant. Zhao et al. (2018) develop an optimal control for a four-wheel-drive HEV with a dual clutch transmission, focused on the transition from pure electrical rear drive to front wheel drive with engine or hybrid drive. The control development does not assess transmission efficiency. Lin et al. (2019) present a novel powertrain architecture for a hybrid electric bus based on dual planetary gear with a rule-based power management strategy, whose effectiveness is evaluated through a dynamic programming algorithm. The mechanical efficiency

of the dual planetary gear is not taken into account in the fuel economy evaluation. Losses in the PGT unit are ignored in the real-time optimization strategy for HEVs developed by Zhang et al. (2020). Zhou, Qin, and Hu (2017) carry out a configuration-sizing-control integrated multi-objective optimization to optimize acceleration capacity and fuel and electric energy consumption. Their work concentrates on motor efficiency, not on transmission performance. Chen et al. (2018) use a dynamic particle swarm optimization to develop optimal strategies for EMS and gear shifting in HEVs. The transmission of their vehicle model is an automated manual transmission (AMT), with constant efficiency. An AMT is also used by Guo et al. (2018) in their EMS optimization, where gear train efficiency is not addressed. It must be highlighted that HEVs with PGTs composing their powertrains, not AMTs, have dominant market shares (ZHUANG et al., 2016).

Some researches focus on powertrain design to enhance HEV performance. A multi-objective optimization seeking to find a trade-off between acceleration capacity and fuel economy is performed by Zhou et al. (2020). In their work, the architecture, component parameters and control strategy are simultaneously optimized to obtain the Pareto optimal design schemes of HEV powertrains. The design does not address gear train features. Gu et al. (2020) propose some systematic methods to develop, analyze and evaluate power-split hybrid powertrain systems with two PGTs. Although their screening is very complete, the PGT performance is not considered in the process. Peng et al. (2020) propose a method to synthesize and analyze HEV powertrain configurations with one motor, using new types of adjacency matrices. Their method generates all the powertrain configurations with PGTs and next analyzes the drive modes. In their work, the transmission efficiency is included in the design process.

These contributions above are quite complete; however, most of them do not address the effect of the gear trains mechanical efficiency on the powertrain performance. Considering the mechanical efficiency of gear trains to be constant or equal to 1 is a common assumption, although in reality it varies according to the torque and speed of the propulsion sources, which are defined from the vehicle speed. Nonetheless, investigating the gear trains mechanical efficiency is another potential approach to reduce fuel usage and pollutants (MOSKALIK et al., 2016). Knowing the mechanical efficiency of PGTs and its variation throughout the vehicle's operation is essential to minimize consumption and the emission of pollutants due to several factors. Transmission efficiency is necessary to develop the vehicle control system, as it allows to obtain the desired power output having a powertrain as efficient as possible (considering the efficiency of ICE, M, G and transmission system) (ROTELLA; CAMMALLERI, 2018). Furthermore, guaranteeing the operation of the ICE at its optimal point is not a sufficient condition to minimize fuel consumption as there can be significant power losses in the transmission (AHN et al., 2006).

Therefore, the efficiency of PGTs is a parameter that needs to be considered in consumption simulations of hybrid vehicles. This Thesis introduces a novel method to simulate consumption and emissions of EVs and HEVs which integrates the efficiencies of ICE, M and G with the variability of the mechanical efficiency of transmissions composed of PGTs. The Thesis simulations comprise EVs and HEVs with transmission systems composed of PGTs, presenting constant and continuously variable transmission ratios. The transmission efficiency and its variation according to the vehicle speed profile are taken into account in the consumption calculation when using the Davies method to calculate the efficiency of PGTs (LAUS; SIMAS; MARTINS, 2012). Davies' method allows calculating the efficiency of PGTs of any architecture, including power losses due to various causes, such as friction between gear teeth surfaces and friction in shaft support bearings, main causes of loss in gear trains (RADZIMOVSKY, 1956). Up to the best of the author's knowledge, the method proposed in this Thesis is the first method for computing fuel consumption which includes the variability of the transmission efficiency according to the vehicle speed.

The use of the Davies method to include the variability of the transmission system efficiency in the consumption simulations generates more realistic results and allows to carry out an optimization of consumption having the dimensions of the gears as design variables.

1.1 STATE OF ART OF PGTS EFFICIENCY CALCULATION

Despite of most of the works on optimizing HEVs' consumption do not pay so much attention to the transmission efficiency, there are plenty of methods on determining PGTs efficiency focusing on automotive applications, each method with its own limitation.

Among the limitations of the methods found on literature, two stand out: neglecting multiple causes of loss and not being applicable to bevel gears. In spite of losses in PGTs having different origins, such as bearing friction, seal friction and gear meshing friction, most of the methods only consider meshing friction in the efficiency computation and/or their description does not make clear if other causes of loss can be included. The limitations of the analyses done by Esmail, Pennestrì and Juber (2018) and Chen and Chen (2015) fall into this category: only meshing losses are taken into account.

Pennestrì et al. (2012) model and analyze power split transmissions, emphasizing PGTs efficiency. In addition to neglecting power losses due to churning of lubricant and friction in the clutches and bearings, their investigation is limited to gear trains which: (a) obey the Gruebler-Kutzbach degree-of-freedom equation (GOGU, 2005), (b) are represented by planar graphs with topological features listed by Freudenstein (1971) and, (c) whose number of independent circuits is equal to the number of gear

pairs (BUCHSBAUM; FREUDENSTEIN, 1970).

Esmail (2016) develops a systematic methodology for the efficiency analysis of one and two degrees-of-freedom (DOF) PGTs, relating the PGTs efficiency to the efficiency of a conventional gear pair. Thus, power losses not related to gear pairs, such as bearing friction, are not considered. Besides, his methodology is not applicable to transmissions with more than two DOF, which is the case of hybrid transmissions with shifting actuators. Hybrid transmissions composed of PGTs with clutches and brakes usually have at least four DOF: the ICE input, the M or G input and the engagement/disengagement of the shifting actuators.

The method developed by Du and Yang (2020) designs optimal topology of transmission systems based on PGTs, including the presence of clutches and brakes. Their method uses meshing efficiency to compute the transmission performance, but it is not explicitly said if other causes of loss, unrelated to the gear pairs, can be taken into consideration.

The second main limitation of the methods found on literature is that most of them cannot be applied to bevel gears. This limitation comes from the fact that most of the methods proposed are only suitable for PGTs with parallel axes since they rely on the Willis' equation. The Willis' equation is a scalar equation, what prevents its use to determine the efficiency of PGTs composed of bevel gears, since the relationship between the absolute angular speeds of bevel gears is not scalar (NELSON; CIPRA, 2005; DEL PIO; PENNESTRÌ; VALENTINI, 2013). The methods developed by Ettore Pennestrì et al. (2012), Gupta and Ramanarayanan (2013), Yang, Feng and Zhang (2015), Rotella and Cammalleri (2018) and Cammalleri and Castellano (2021) present this limitation.

The method developed by Yang, Feng and Zhang (2015) does not compute the statics and the kinematics independently, but it is one of the few methods that considers other power losses besides gear meshing friction: the losses on the planet bearings are included in their analysis.

Rotella and Cammalleri (2018) present a simplified method to compute power losses in power-split continuously variable transmissions (CVT) using "blocks" of PGTs units. A block is a PGT composed of a sun gear, a carrier, a ring gear and planet gears and has always one fixed link. The simplified method has other limitations besides not being applicable to bevel gears: it is not suitable for transmissions with Willis's ratio — the ratio between the ring gear speed over the sun gear speed when the carrier is locked (ROCHA et al., 2021) — greater than 0.5 and lesser than 1.5; and it does not consider other power losses besides meshing friction. Cammalleri and Castellano (2021) expand the method developed by Rotella and Cammalleri (2018) and apply it to compute the power losses of a commercial HEV.

Works on the efficiency of bevel gear trains are scarce and are limited to some

specific configurations. Chen (2013) obtains an analytical efficiency expression of a compound PGT, containing bevel gears, through a virtual power approach. However, no generic method is detailed to derive the expression. Besides, only power losses due to the gear-mesh sliding friction are taken into account. The efficiency optimization of a single bevel gear pair is one of the goals in the work developed by Padmanabhan et al. (2011), where an equation to calculate the power loss of a gear pair is used as cost function. Ding et al. (2022) propose a new efficiency prediction model for aero-engines bevel gear power transmission, using thermal elastohydrodynamic lubrication analysis. An equation for efficiency of helical bevel geared motors is experimentally obtained by Carnogurska, Prihoda, and Sirillova (2014).

Wan and Yao (2019) propose a novel double-arc bevel gear nutation transmission system and model its efficiency by transforming the nutation transmission of double-arc bevel gears into a spur planetary gear transmission mechanism with few teeth difference. The transformation is possible when the nutation angle is equal to zero, limiting the efficiency model to this configuration. Besides, just gear meshing friction is considered in the efficiency calculation. Gear meshing friction is also the exclusive cause of loss considered in the work of Fanghella et al. (2016), where the efficiency of a PGT based on nutating bevel gears is evaluated via elementary efficiencies. A simplified method to determine the efficiency of bevel gear trains with 1 DOF is proposed by Fanghella (2009), but it requires that all losses except gear meshing losses are neglected.

Nelson and Cipra (2005) analyze the efficiency of bevel PGTs also assuming that gear meshing is the single cause of power loss. Moreover, they limit their work to PGTs whose input and output axes are collinear.

Laus, Simas, and Martins (2012) developed a method which overcomes the limitations of most of the works listed herein. Their method allows to compute PGTs efficiency with no topological, geometrical or mobility constraints, and enables modelling any cause of loss. The method can be applied both in PGTs with parallel and non-parallel axes, as it is based on the Davies method. The Davies method associates graph and screw theories with the Kirchhoff's laws to compute the kinematics and the statics of a mechanism independently (DAVIES, 2006), without restrictions.

The novel method proposed in this Thesis uses the work of Laus, Simas, and Martins (2012) to compute the efficiency of HEVs transmissions. Therefore, this Thesis method does not present the limitations of the methods listed in this section. Tab. 1 summarizes the the main limitations of the methods reviewed in this section, comparing them with the method developed in this work. The features analyzed in Tab. 1 are the inclusion of other loss causes besides gear meshing friction; if the method does not have any topological constraint; and if the method can be applied to bevel gears.

In Tab. 1, the ✗ symbol indicates that a given method does not have the feature defined by the column. The symbol ✓ indicates the opposite. When the work presenting

Table 1 – Comparison of methods for determining PGTs efficiency

	Multiple causes of loss	No topological constraints	Applicable to bevel gears
Esmail, Pennestri and Juber (2018)	✗	✗	
Chen and Chen (2015)	✗		
Pennestri et al. (2012)	✗	✗	✗
Esmail (2016)	✗		
Du and Yang (2020)	✗		
Gupta and Ramanarayanan (2013)	✗	✗	✗
Yang, Feng and Zhang (2015)	✓		✗
Rotella and Cammalleri (2018)	✗		✗
Cammalleri and Castellano (2021)	✗		✗
Chen (2013)	✗	✗	✓
Padmanabhan et al. (2011)		✗	✓
Ding et al. (2022)	✓	✗	✓
Carnogurska, Prihoda, and Sirilova (2014)		✗	✓
Wan and Yao (2019)	✗	✗	✓
Fanghella et al. (2016)	✗	✗	✓
Fanghella (2009)	✗	✗	✓
Nelson and Cipra (2005)	✗		✓
Method proposed in the Thesis	✓	✓	✓

Source: From the author.

the method does not explicitly state or does not give any indication that the method has or does not have a given feature, the cell is empty.

1.2 THESIS OBJECTIVES

The main objective of the Thesis is to develop a novel and complete method to simulate fuel consumption and emissions of EVs and HEVs, integrating the efficiency of ICE, M and G with the variability of the mechanical efficiency of transmissions composed of PGTs.

The following specific objectives are defined in order to achieve the main objective:

- Conduct an extensive literature review to investigate how HEV consumption and emissions are currently estimated in academia;
- Research on the methods to calculate the efficiency of PGTs existing in the literature, to highlight the differential of the Davies method when applied for this purpose;
- Propose a novel method to calculate fuel consumption and emissions, which considers the variation of the transmission efficiency in addition to the propulsion sources' efficiency;
- Use an electric vehicle (EV) as a first case study of the method;
- Use a commercial HEV as a second case study;

- Compare the consumption and emissions obtained with the method with official values of the commercial vehicle;
- Apply the proposed method in a transmission with complex architecture, composed of bevel gears, as a third case study;
- Optimize the fuel consumption of the HEV whose transmission has complex geometry, using the gears' dimensions as design variables and the proposed method to obtain the objective function.

1.3 THESIS CONTRIBUTIONS

This Thesis contributes to the field of vehicles' consumption simulation, using the Davies method to compute the transmission efficiency each second of the vehicle's operation. This way, the variability of the transmission efficiency due to the combination of the power provided by ICE, M and G is considered in the fuel consumption estimation. The Davies method also allows to integrate the propulsion sources efficiency with the transmission efficiency, enabling to evaluate the impact of the whole powertrain performance on the HEV fuel economy.

To the best of the author's knowledge, the method introduced in this Thesis is the first one to consider the variability of the transmission efficiency in fuel economy computation. The main contributions of the Thesis are summarized as follows:

- Development of a new method to compute fuel economy which includes the dependence of the transmission efficiency on the vehicle speed;
- Development of a tool to size automotive gearboxes focusing on minimizing fuel consumption;
- Inclusion of different causes of loss besides gear meshing friction to calculate the efficiency of automotive transmissions, by using the Davies method adapted by Laus (2011);
- Simultaneous computation of the efficiency variation of the PGTs composing the transmission and the efficiency of the propulsion sources (ICE, M and G) in the HEV's consumption calculation;
- Presentation of an equation to compute the efficiency of PGTs with multiple inputs and multiple outputs, using the Davies method. The equation is shown in Chapter 6, Eq. (98);
- Computation of the efficiency of a HEV's transmission composed of bevel gears;
- Generation of an analytical expression to compute the ordinary efficiency and the Coulomb friction factor of bevel gears with perpendicular axes (see Appendix A).

1.4 THESIS OUTLINE

The Thesis is organized in eight chapters and nine appendices.

In Chapter 1, the importance of considering the mechanical efficiency of the transmission in the consumption estimation of EVs and HEVs is discussed and a brief bibliographic review on methods to compute HEVs' fuel consumption and PGT efficiency, focused on automotive applications, is made. The Thesis objectives and contributions are also presented in this Chapter.

In Chapter 2, a brief review on how fuel and electric energy consumption is calculated for HEVs is made. Chapter 3 reviews the Davies method and explains its use to compute the efficiency of cylindrical and bevel gear trains. The method proposed in this Thesis is presented in Chapter 4.

The first case study of the Thesis is presented in Chapter 5, where the driving range of a purely electric machine is estimated through the battery pack state-of-charge (SOC). The efficiency of the traction motor's reduction, composed of a PGT, is calculated through the Davies method and the result is applied in the SOC estimation. An optimization of the PGT efficiency is made to verify its impact on the batteries SOC.

The second case study is introduced in Chapter 6, where the method proposed in this Thesis is effectively fully applied. The fuel economy of a commercial HEV is estimated through the method and compared with official values provided by the manufacturer.

Chapter 7 presents the fuel economy calculation of a HEV with a transmission of complex geometry, composed exclusively of bevel gears. Next, an optimization problem is defined so that the fuel consumption is minimized by altering the gears dimensions.

Chapter 8 presents some final considerations about this work and some suggestions on how it can be continued.

Appendix A contains a detailed demonstration of the generation of equations to compute the ordinary efficiency of a simple bevel gear train.

Appendices B and H contain detailed explanations on determining some factors to estimate bending and contact stresses on spur and straight bevel gears.

Appendices C and F show the matrices generated through Davies method to compute the efficiency of the transmissions studied in Chapters 6 and 7.

Appendix D contains the algorithm of the method introduced in Chapter 4 applied to the commercial HEV described in Chapter 6.

Appendices E and G present some graphics generated during the fuel economy calculation of Chapters 6 and 7.

Finally, Appendix I contains a brief analysis on the causes of the fuel economy reduction obtained by means of the optimization of Chapter 7.

2 A BRIEF REVIEW ON VEHICLES' CONSUMPTION

This Chapter presents some concepts on how to calculate fuel consumption, electric energy consumption and batteries' state-of-charge (SOC) of EVs and HEVs.

2.1 VEHICLE'S POWER DEMAND

An automotive powertrain consists of a power plant, a drivetrain and the drive wheels. The power plant of an EV is an electric motor (M), while the power plant of a HEV is composed of more than one propulsion source: the internal combustion engine (ICE), M and the generator (G). The power plant's traction power is transmitted to the drive wheels through the drivetrain, which includes the transmission and the final drive (EHSANI et al., 2018).

A vehicle's powertrain must supply power to the vehicle move. While a vehicle is moving, there are some resistive forces that try to stop its movement. The movement resistance includes tire rolling resistance, aerodynamic drag, grading resistance and inertial resistance. The powertrain's traction power depends on the tractive force F_t that the powertrain provides to propel the vehicle forward, which must balance the resistive forces (EHSANI et al., 2018; NAUNHEIMER et al., 2011).

The magnitude of the tractive force F_t is defined according to the magnitudes of the resistive forces. The tire rolling resistance F_r is estimated through Eq. (1):

$$F_r = M_{veh} g f_r \cos(\alpha) \quad (1)$$

where M_{veh} is the vehicle mass, g is the gravity acceleration, f_r is the rolling resistance coefficient and α is the road angle (EHSANI et al., 2018).

The rolling resistance coefficient f_r depends on the tire material, tire structure, tire temperature, tire inflation pressure, tread geometry, road roughness, road material, and presence of liquids on the road. f_r also varies with the vehicle speed and, for performance calculations, this variation of f_r can be assumed as a linear function of speed. For the most common range of inflation pressure, Eq. (2) can be used to estimate f_r for a passenger car on a concrete road:

$$f_r = 0.01 \left(1 + \frac{v}{160} \right) \quad (2)$$

where v is the vehicle speed and must be in km/h when applied in Eq. (2). Eq. (2) predicts the rolling resistance coefficient with acceptable accuracy for speeds up to 128 km/h (EHSANI et al., 2018).

The magnitude of the aerodynamic drag is defined as:

$$F_w = \frac{1}{2} \rho_a C_D A_f v^2 \quad (3)$$

where ρ_a is the air density, C_D is the aerodynamic drag coefficient and A_f is the vehicle frontal area (EHSANI et al., 2018).

When a vehicle goes up or down a slope, its weight can oppose the forward motion (grade climbing) or helps the forward motion (grade descending). This phenomenon is the grading resistance and its magnitude is calculated as:

$$F_g = M_{veh}g \sin \alpha \quad (4)$$

The acceleration resistance comprises inertial forces that occur when the vehicle accelerates and brakes. The acceleration resistance magnitude is computed as:

$$F_a = M_{veh}\delta \frac{dv}{d\tau} \quad (5)$$

where τ represents the time and δ is the rotational inertia factor, that converts the rotational inertias of rotating components into translational mass (NAUNHEIMER et al., 2011).

In a short period of time, the vehicle speed can be assumed to be linear with the time, so the acceleration $\frac{dv}{d\tau}$ can be obtained by:

$$\frac{dv}{d\tau} = \frac{v_{k+1} - v_k}{\tau_{k+1} - \tau_k}, \text{ for } k = 1, 2, \dots, n \quad (6)$$

where n is the total number of points of a vehicle speed profile.

Finally, the tractive force F_t that the powertrain must provide so the vehicle moves is the sum of Eqs. (1), (3), (4) and (5):

$$F_t = M_{veh}gf_r \cos(\alpha) + \frac{1}{2}\rho_a C_D A_f v^2 + M_{veh}g \sin \alpha + M_{veh}\delta \frac{dv}{d\tau} \quad (7)$$

The traction power supplied by the powertrain must be equal to the vehicle's power demand. The power demand $\mathcal{P}_{dem}$ related to vehicle kinetics is directly related to F_t and is calculated through Eq. (8):

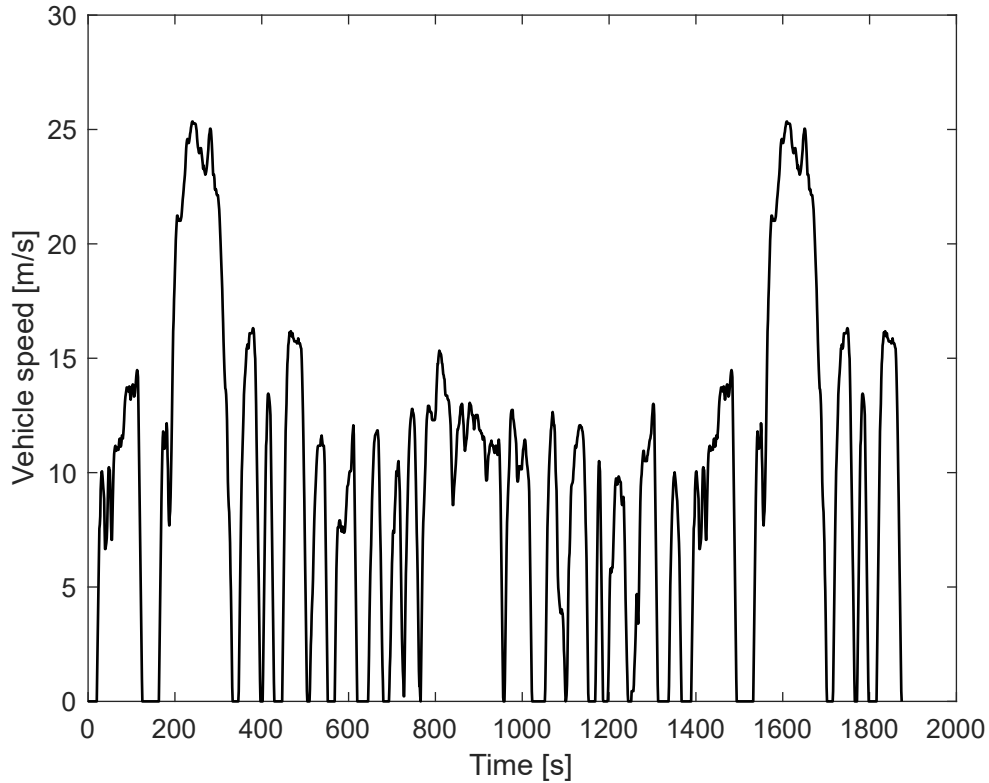
$$\mathcal{P}_{dem} = \frac{F_t v}{\eta_t} \quad (8)$$

where η_t is the transmission efficiency. Thus, the traction power provided by the powertrain must also compensate the transmission losses. In this Thesis, η_t is calculated using Davies' method, which is explained in Chapter 3.

2.2 FUEL ECONOMY

Fuel economy and emissions are estimated using standard drive cycles that simulate real driving conditions. Multiple drive cycles have been developed over the last decades, such as the FTP75, which emulates urban driving conditions, shown in Fig. 1.

Figure 1 – FTP75 drive cycle.



Source: Adapted from EPA (2021).

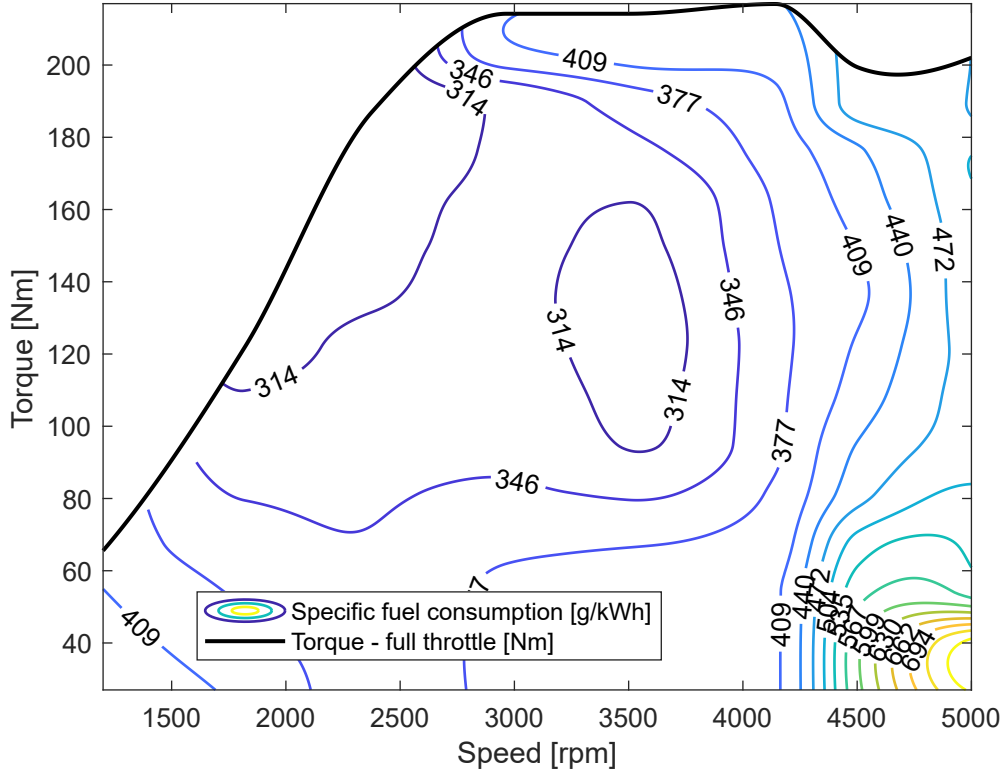
Drive cycles are usually represented by the vehicle speed along with the driven time, as illustrated by Fig. 1. The vehicle power demand $\mathcal{P}_{dem}$ and the speed and torque of the power plant components (e.g., ICE, M and G) can be obtained from drive cycles (DE CARLO; MANTRIOTA, 2020).

Determining the power plant's speed and torque is essential to compute fuel economy. In HEVs, one of the members of the power plant is the ICE, whose consumption is estimated from a fuel consumption map as the one shown in Fig. 2.

A typical fuel consumption map has as abscissa the ICE speed, in rpm, and the ICE torque, in Nm, as ordinate. The fuel economy characteristic of the ICE is defined by the specific fuel consumption b_e , in g/kWh, represented by the contour lines in the fuel consumption map. The full line curve is the maximum torque the ICE can provide when in full throttle, in function of speed.

The ICE speed and torque in each instant of a drive cycle must be known to obtain b_e for each instant, for then, with some mathematical manipulation, computing the fuel economy for the whole drive cycle. That is the reason why it is important to determine the speed and torque of the power sources composing the power plant.

Figure 2 – Fuel consumption map of a Dodge Caravan engine, a 3.0 L, 6-cyl., 136 hp, 1991 model year



Source: Adapted from ADVISOR (MARKEL et al., 2002).

2.2.1 Power plant speed

The speed of the power plant propulsion sources is determined through the gear ratio of the transmission system, the gear ratio of the final drive and the drive wheels' speed.

The drive wheel angular speed N_w is given by:

$$N_w = \frac{v}{r_d} \quad (9)$$

where r_d is the effective radius of the wheel (EHSANI et al., 2018).

The transmission output speed $N_{t_{out}}$ is defined as:

$$N_{t_{out}} = N_w i_0 \quad (10)$$

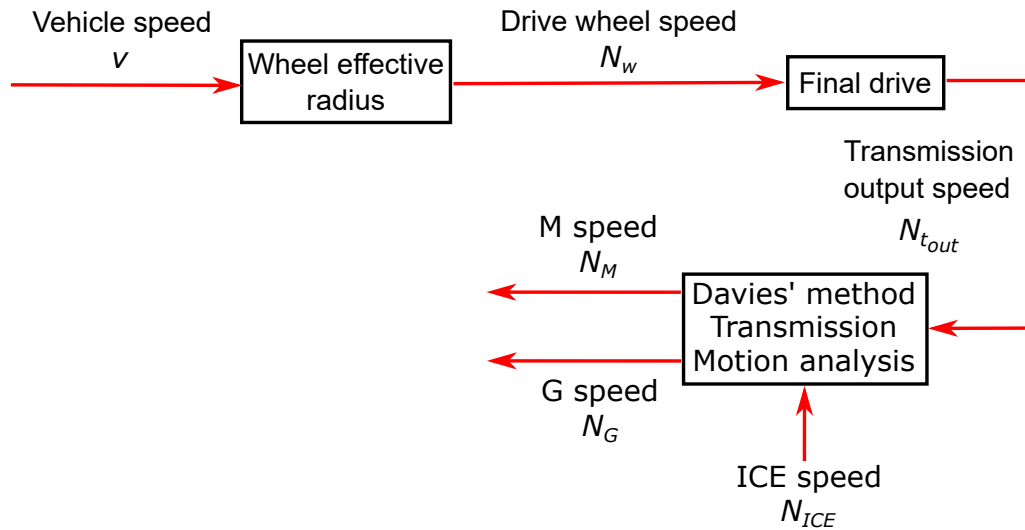
where i_0 is the gear ratio of the final drive (EHSANI et al., 2018).

As it will be seen in Chapters 6 and 7, hybrid transmissions without shifting actuators usually have 2 DOF, meaning that two speeds must be known to solve the system's motion analysis. In HEVs, the known speeds are the ICE speed, N_{ICE} , and

the transmission output speed, since $N_{t_{out}}$ can be calculated using Eqs. (9) and (10). The vehicle speed v comes from the drive cycles. N_{ICE} is imposed so that the ICE fuel consumption is minimal (for example, considering the map shown in Fig. 2, N_{ICE} would be around 3500 rpm when solving the motion analysis, as it corresponds to the region where b_e is minimal).

The motion analysis solution provides the M speed, N_M , and the G speed, N_G . This way, the speed that M and G must have so the ICE operates with minimum consumption is defined. Besides, the M and G speeds are necessary to navigate in the efficiency maps of M and G, as it will be seen in Sec. 2.3.1. In this Thesis, the motion analysis is solved using Davies' method, detailed in Chapter 3. The diagram shown in Fig. 3 summarizes the algorithm to compute the power plant speed for HEVs.

Figure 3 – Algorithm to compute the power plant speed for HEVs



Source: From the author.

2.2.2 Power plant torque

In Chapters 6 and 7, it will be seen that hybrid transmissions without shifting actuators usually require only one known torque to determine the torque of the power plant propulsion sources and the transmission output torque. In HEVs, the ICE torque is used as input in the action analysis of the transmission to determine the remaining torques.

Since the lowest specific fuel consumption is sought, the ICE optimum operation line (OOL) is used to determine which value the ICE torque assumes. The OOL is the correspondent torque value for each speed where the ICE efficiency presents a maximum (TRINDADE; TOLEDO FLEURY; VOGELAAR, 2014). The ICE efficiency

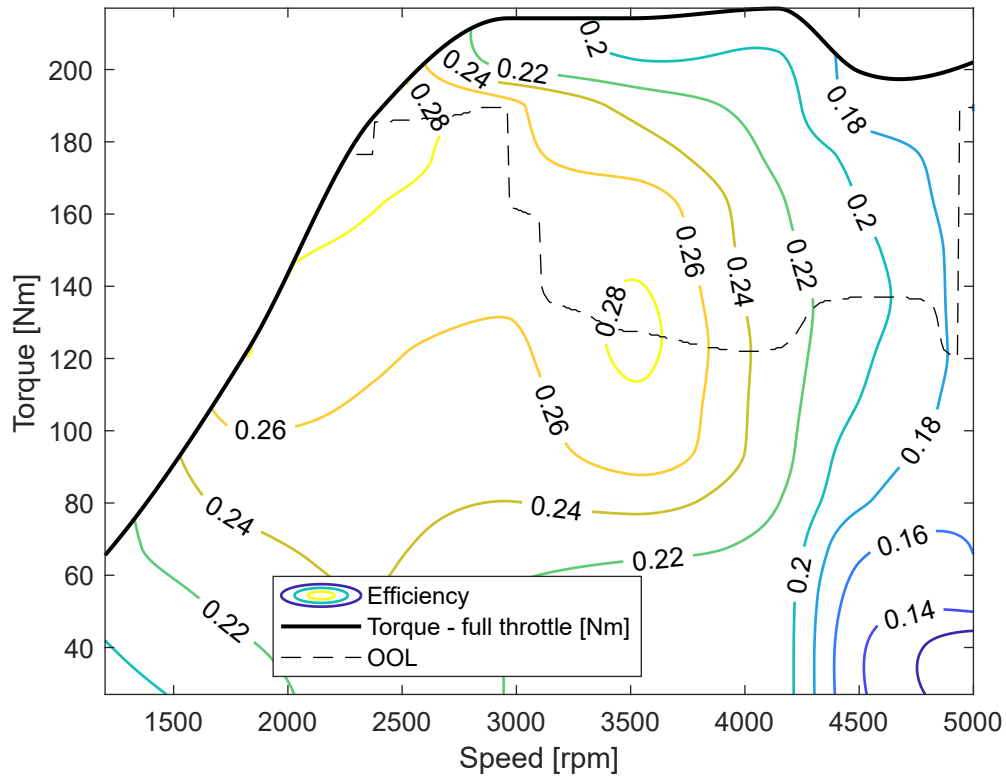
η_{ICE} is inversely proportional to the specific fuel consumption and can be calculated according to (HAYES; GOODARZI, 2018):

$$\eta_{ICE} = \frac{3600}{Qb_e} \quad (11)$$

where Q is the fuel specific energy. In this Thesis, the fuel adopted is gasoline, which has $Q = 42.6$ kJ/g (HAYES; GOODARZI, 2018).

Applying Eq. (11) in the values of specific fuel consumption of the consumption map shown in Fig. 2, the efficiency map shown in Fig. 4 is generated.

Figure 4 – Efficiency map of a Dodge Caravan engine, a 3.0 L, 6-cyl., 136 hp, 1991 model year



Source: Adapted from ADVISOR (MARKEL et al., 2002).

The points presenting the maximum efficiency on the map shown in Fig. 4 are connected by a dashed line, resulting in the OOL. Thus, in the action analysis of the transmission of a HEV whose ICE has the efficiency map of Fig. 4, the ICE torque chosen as input comes from the dashed line. The optimum operation point of the ICE for the efficiency map of Fig. 4 would be in the region where the ICE torque is around 120 Nm and $N_{ICE} = 3500$ rpm, since it presents the highest efficiency on the map (around 28%).

The transmission action analysis gives M and G torques for a given value of ICE torque. When using the ICE torque which results in the ICE maximum efficiency (or the minimum specific fuel consumption), the M and G torques that assure that the ICE operates at its optimum point are obtained. In this Thesis, the transmission action analysis is performed through the Davies method, explained in Chapter 3.

The M and G torques are also needed to navigate in the efficiency maps of M and G, as it will be seen in Sec. 2.3.1.

2.2.3 Fuel economy for a whole drive cycle

After determining the specific fuel consumption b_e , the fuel consumption rate $\dot{m}_{fuel}$, in g/s, can be calculated through Eq. (12):

$$\dot{m}_{fuel} = \frac{b_e \mathcal{P}_{ICE}}{3600} \quad (12)$$

where $\mathcal{P}_{ICE}$ is the ICE power, in kW (EHSANI et al., 2018).

To calculate the fuel economy in a drive cycle, the total fuel consumption can be obtained through the summation of $\dot{m}_{fuel}$ in each time interval $\Delta\tau_i$, assumed as $\Delta\tau_i = 1$ s in this Thesis:

$$m_{fuel} = \sum_{i=0}^n \dot{m}_{fuel_i} \Delta\tau_i \quad (13)$$

where n is the total number of points of the drive cycle.

The result of Eq. (13) is a mass in grams. Usually, emission standards impose fuel economy limits in km/L or L/100 km (NAUNHEIMER et al., 2011). So, it is more convenient to convert the result of Eq. (13) into these units.

To convert the result of Eq. (12) into a fuel economy value $FC_{L/100km}$ in L/100 km, the following equation should be used (NAUNHEIMER et al., 2011):

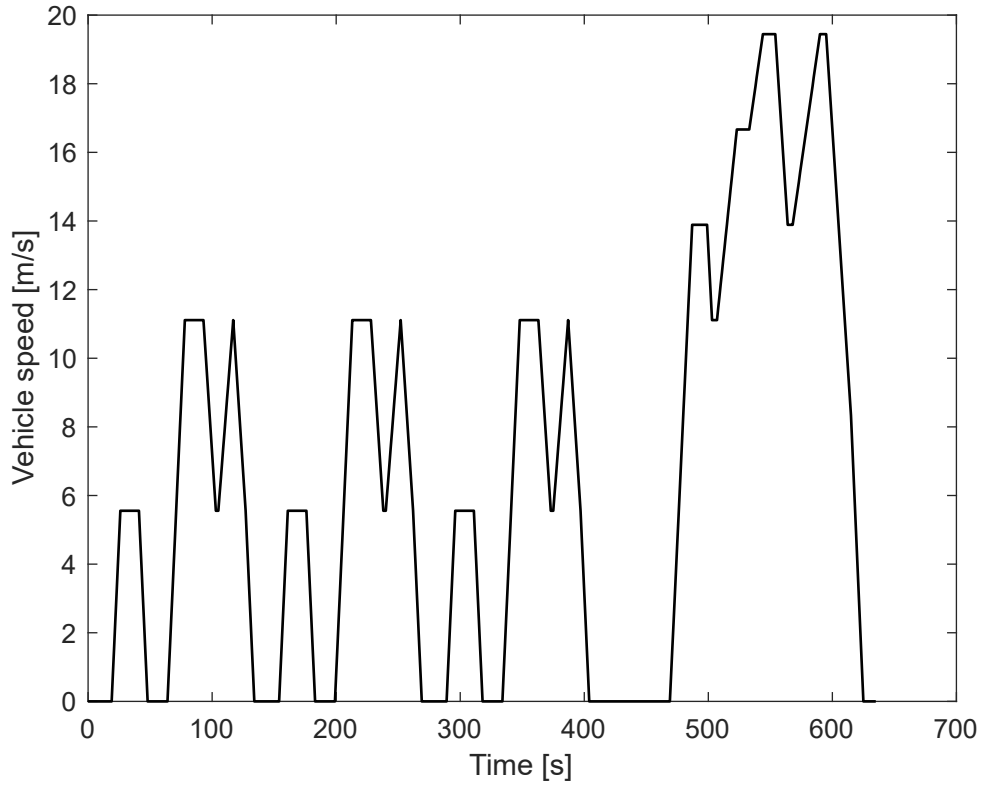
$$FC_{L/100km} = \frac{m_{fuel}}{\rho_{fuel} S_{cyc}} 100 \quad (14)$$

where ρ_{fuel} is the fuel density, assumed as $\rho_{fuel} = 749$ g/L (MI; MASRUR, 2017) in this Thesis, corresponding to the gasoline density. S_{cyc} is the distance travelled by the vehicle when the drive cycle is complete and is obtained by integrating the speed profile of the drive cycle along the time. In this Thesis, the integrations are done using numerical trapezoidal integration, as it presents low distortion and absolute stability (MI; MASRUR, 2017).

To convert the result of Eq. (12) into a fuel economy value $FC_{km/L}$ in km/L, the following equation should be used (NAUNHEIMER et al., 2011):

$$FC_{km/L} = \frac{S_{cyc} \rho_{fuel}}{m_{fuel}} \quad (15)$$

Figure 5 – 10-15 mode drive cycle.



Source: Adapted from EPA (2021).

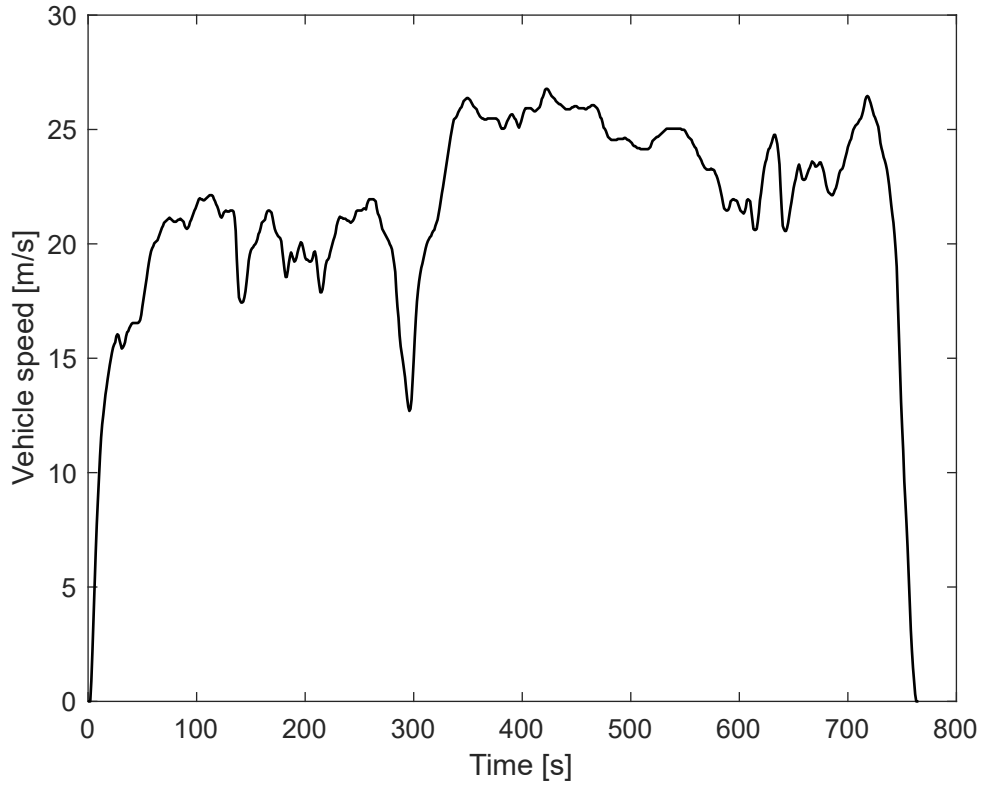
It can be noted that the fuel economy depends on the distance travelled, that varies with the drive cycle. In this Thesis, three drive cycles are adopted:

1. The 10-15 mode cycle, adopted to measure emissions in Japan along the 1990's (DIESELNET, 2015; VAN BERKEL et al., 2011), shown in Fig. 5. The unit of the fuel economy obtained from the 10-15 mode cycle is km/L;
2. The FTP75, shown in Fig. 1, simulating urban driving conditions;
3. The HWFET, shown in Fig. 6, simulating highway driving conditions.

The FTP75 and the HWFET cycles were developed by the US Environmental Protection Agency (EPA) and are closer to real-world driving style behaviour (GEBISA et al., 2021). The EPA imposes that the fuel economy value for certification of passenger cars must be calculated as a weighted composite value of the fuel economies obtained in the FTP75 and HWFET cycles. The FTP75's weighting is 55%, while the HWFET's weighting is 45% (GEBISA et al., 2021; MI; MASRUR, 2017):

$$FC_C = \frac{1}{\frac{0.55}{FC_{FTP}} + \frac{0.45}{FC_{HW}}} \quad (16)$$

Figure 6 – HWFET drive cycle.



Source: Adapted from EPA (2021).

where FC_C is the composite fuel economy, FC_{FTP} is the fuel economy calculated for the FTP75 cycle and FC_{HW} is the fuel economy obtained for the HWFET cycle.

2.3 ELECTRIC ENERGY CONSUMPTION

One way to quantify the electric energy consumption of a HEV is through the state-of-charge (SOC) of the battery pack. The state-of-charge can be defined as the amount of charge a battery has at a particular moment, compared to its rated charge (MI; MASRUR, 2017).

For efficiency reasons, the optimal operating range of chemical batteries SOC goes from 0.4 to 0.7, so HEVs control is designed so that the battery pack SOC fluctuates between these two values (EHSANI et al., 2018).

For EVs and HEVs, the battery pack SOC in the instant τ , $SOC(\tau)$, is estimated based on the initial energy capacity E_0 according to Eq. (17):

$$SOC(\tau) = SOC(\tau_0) - \frac{1}{\eta_{bat} \text{sign}(\mathcal{P}_{bat})} \frac{1}{E_0} \int_0^\tau \mathcal{P}_{bat}(\tau) d\tau \quad (17)$$

where $\mathcal{P}_{bat}$ is the real-time charging or discharging power of the battery pack (KULARATNA, 2014) and η_{bat} is the battery pack's charging/discharging effectiveness (ZHANG, X. et al., 2021).

The initial energy capacity E_0 is calculated as the product between the battery pack nominal capacity c_{bat} , in Ah, and the nominal voltage V_{bat} , in V (MI; MASRUR, 2017):

$$E_0 = 3600c_{bat}V_{bat} \quad (18)$$

where the factor 3600 converts the unit Wh to J.

The battery pack power $\mathcal{P}_{bat}$ is calculated using Eq. (19) (GU et al., 2020):

$$\mathcal{P}_{bat} = \mathcal{P}_{M_{bat}} + \mathcal{P}_{G_{bat}} + \mathcal{P}_{aux} \quad (19)$$

where $\mathcal{P}_{M_{bat}}$ is the power transferred from the battery to the traction motor (or vice versa), $\mathcal{P}_{G_{bat}}$ is the power transferred from the battery to the generator (or vice versa) and $\mathcal{P}_{aux}$ is the power consumption for high-voltage electrical appliances present in the vehicle.

When $\mathcal{P}_{bat} > 0$, the battery pack is discharged and its SOC decreases. Otherwise, if $\mathcal{P}_{bat} < 0$, the battery pack is charged by M and/or G, when they are in generating mode, and its SOC increases (KULARATNA, 2014).

Usually $\mathcal{P}_{aux}$ is assumed as constant and depends on the vehicle class, while $\mathcal{P}_{M_{bat}}$ and $\mathcal{P}_{G_{bat}}$ depends on the electric machines' power and efficiency, as stated by Eqs. (20) and (21) (EHSANI et al., 2018):

$$\mathcal{P}_{M_{bat}} = \frac{\mathcal{P}_M}{\eta_M \text{sign}(\mathcal{P}_M)} \quad (20)$$

$$\mathcal{P}_{G_{bat}} = \frac{\mathcal{P}_G}{\eta_G \text{sign}(\mathcal{P}_G)} \quad (21)$$

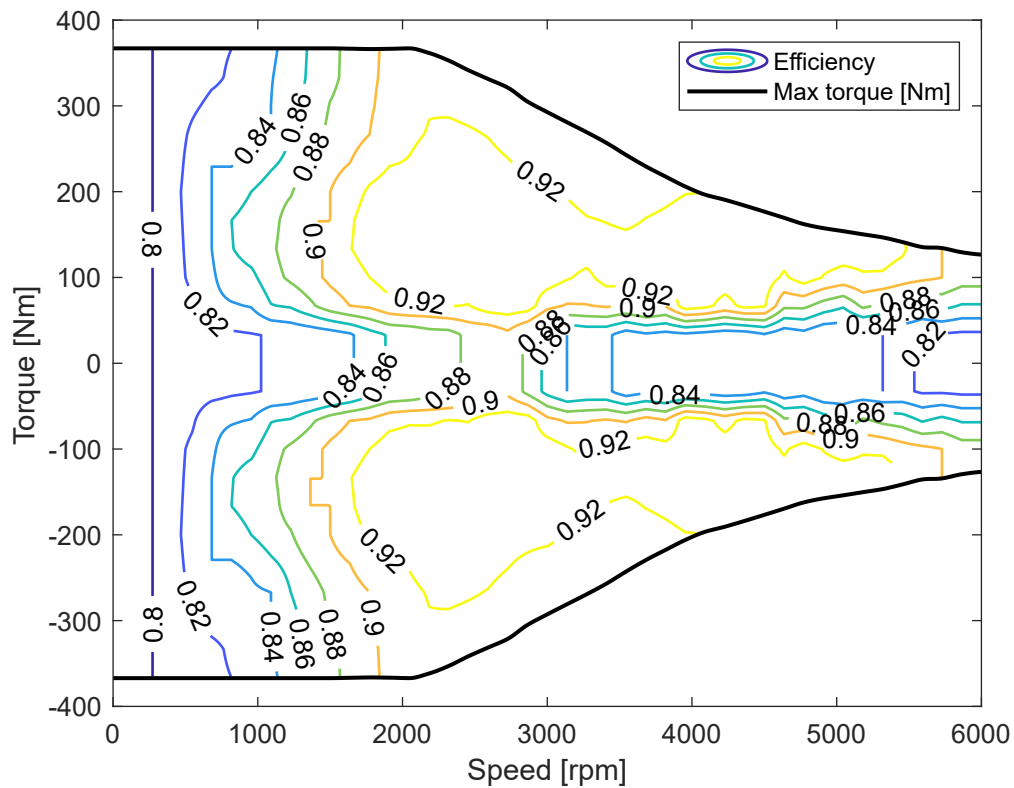
where $\mathcal{P}_M$ and $\mathcal{P}_G$ are the traction motor (M) and generator (G) power, respectively. M and G's efficiency are indicated by η_M and η_G , respectively, and are defined through efficiency maps.

2.3.1 M and G efficiency maps

Fig. 7 contains an example of a typical efficiency map of a traction motor. Generators' efficiency maps have the same style, so Fig. 7 exemplifies an efficiency map for both electric machines.

A traction motor efficiency map has as abscissa the M speed, in rpm, and the M torque, in Nm, as ordinate. The efficiency is given by the contour lines in the map and

Figure 7 – Efficiency map of a 58 kW, permanent magnet motor



Source: Adapted from ADVISOR (MARKEL et al., 2002).

the maximum torque that M can provide, in function of speed, is given by the full line curve.

Secs. 2.2.1 and 2.2.2 described how to obtain M and G 's speed and torque to navigate in their efficiency maps. Calculating the speed and torque of the electric machines for a given instant of a drive cycle enables to define their efficiency at this instant and then using Eqs. (20) and (21) to determine $\mathcal{P}_{bat}$ through Eq. (19). For example, if the HEV analyzed has the traction motor whose efficiency map is the one shown in Fig. 7, when M rotates at a speed of around 3000 rpm and supplies a torque of approximately 200 Nm, $\eta_M = 92\%$.

The speed and torque of the electrical machines are calculated via Davies' method, which is introduced in Chapter 3.

3 REVIEW ON CALCULATING PGT EFFICIENCY VIA DAVIES' METHOD

This Chapter briefly explains how the Davies method is applied to compute the efficiency of PGTs. Readers who want to delve deeper into the subject are invited to read the Thesis of Laus (2011).

3.1 DAVIES' METHOD

Davies (1981) developed an equivalent of the Kirchhoff's voltage and current laws for a network of links and couplings, based on graph and screw theories. The Laboratory of Applied Robotics (LAR) from the Federal University of Santa Catarina (UFSC) has developed the Davies method based on Davies work and has named the method in his honor.

In the adaptation of the Kirchhoff's laws proposed by Davies (1981), a multibody system is represented by a graph, whose nodes represent the links and its edges, the couplings. The edge variables model the motions allowed by the couplings and the actions transmitted by the couplings. The edge variables modelling the actions are analogous to the electric current and the edge variables modelling the motions are analogous to the voltage. Through these analogies, motion and action analyses of a mechanism are solved independently.

In this Thesis, the notation introduced by Davies (2006) is adopted. From graph theory, v is the number of fundamental circuits of a graph and is determined by $v = e - n + 1$, where e is the number of couplings of the mechanism and n is the number of links. The number of fundamental cutsets κ of a graph is the number of links diminished by one ($\kappa = n - 1$). The fundamental circuits and the fundamental cutsets of a graph can be mathematically represented by the *fundamental circuit matrix* $[B]_{v \times e}$ and *fundamental cutset matrix* $[Q]_{\kappa \times e}$, respectively. F is the *gross degree of freedom* of a mechanism, which is the sum of all independent motions f allowed by each coupling. C is the *gross degree of constraint*, determined by the sum of all independent constraints c of each coupling and the constraints representing external torques and forces acting on the mechanism. The fundamental circuit matrix the matrix $[B]_{v \times e}$ is converted to $[B_M]_{v \times F}$ by replicating f_j times the column j . Similarly, the fundamental cutset matrix $[Q]_{\kappa \times e}$ is converted to the matrix $[Q_A]_{\kappa \times C}$ by replicating c_j times the column j .

The number of independent motions and the number of independent constraints are dual terms that define the minimum order of the screw system λ . The order of the screw system λ is the dimension of the space where the coupling is represented and is given by $\lambda = f + c$. The order of the screw system ranges from 1 to 6 ($1 \leq \lambda \leq 6$), for example, planar mechanisms are represented in a planar space with $\lambda = 3$, while spatial mechanisms are represented in a space with $\lambda = 6$.

A screw \$ is a geometrical element that can represent mechanical quantities.

$\$$ is defined by a straight line and an associated pitch h , which has unit of length. A motion screw $\$^m$ is called twist and is composed of two vectors: an angular velocity vector $\{r, s, t\}^T$ and a linear velocity vector $\{u, v, w\}^T$ of a point on a link instantaneously coincident with the reference frame origin. Similarly, an action screw $\$^a$ is called a wrench and consists of two vectors: a force vector $\{U, V, W\}^T$ and a moment vector $\{R, S, T\}^T$ at the origin. The motion screw $\$^m = \{r, s, t; u, v, w\}^T$ is usually written in ray-coordinate order and the action screw $\$^a = \{R, S, T; U, V, W\}^T$ in axis-coordinate order. The screw pitch is defined by the relationship between the first and second parts of the screw. A pure angular velocity and a pure force have $h = 0$. A pure linear velocity and a pure torque have $h \rightarrow \infty$.

Tab. 2 contains the Davies' notation for the elements of the twists and wrenches and their respective physical meaning, considering a reference coordinate system O_{xyz} . Each element of the screws is parallel to an axis composing the O_{xyz} frame, indicated in the last column of Tab. 2.

Table 2 – Physical meaning of twists and wrenches' elements, considering the Davies' notation

Twist		Wrench		O_{xyz} axis
r	Angular speed	R	Torque	x
s	Angular speed	S	Torque	y
t	Angular speed	T	Torque	z
u	Linear speed	U	Force	x
v	Linear speed	V	Force	y
w	Linear speed	W	Force	z

Source: From the author.

A screw can be expressed by the product between a magnitude ϱ and a normalised screw $\hat{\$}$: $\$ = \varrho \hat{\$}$. A *normalised screw* $\hat{\$}$ has a unitary vector standing for its defining straight line. If the screw pitch is finite, the direction of the *instantaneous screw axis* (ISA) is defined by the unit vector of $\{r, s, t\}^T$ and ϱ is an angular speed magnitude for twists. For wrenches, the ISA direction is given by the unit vector of $\{U, V, W\}^T$ and ϱ is a force magnitude. If $h \rightarrow \infty$, the twist direction is given by the unit vector of $\{u, v, w\}^T$ and ϱ is a translational speed magnitude. For wrenches, the direction is defined by $\{R, S, T\}^T$ and ϱ is a torque magnitude.

By disposing side by side the normalised twists, the unit motion matrix $[\hat{M}_D]_{\lambda \times F}$ is generated. Relations among the motions allowed by the couplings belonging to the same circuit are established using an analogy with the Kirchhoff's Voltage Law: the algebraic sum of the twists around any circuit is zero. This statement can be mathematically translated by the homogeneous linear system presented in Eq. (22).

$$[\hat{M}_N]_{\lambda \times F} \{\vec{\psi}\}_{F \times 1} = \{\vec{0}\}_{\lambda \times 1} \quad (22)$$

where the vector $\{\vec{\psi}\}_{F \times 1}$ contains the twists magnitudes and $[\hat{M}_N]_{\lambda v \times F}$ is the network unit motion matrix. $[\hat{M}_N]_{\lambda v \times F}$ is obtained by multiplying $[\hat{M}_D]_{\lambda \times F}$ v times by a diagonal matrix composed of a row of the circuit matrix:

$$[\hat{M}_N]_{\lambda v \times F} = \begin{bmatrix} [\hat{M}_D]_{\lambda \times F} \text{diag}([B_M]_1) \\ [\hat{M}_D]_{\lambda \times F} \text{diag}([B_M]_2) \\ \vdots \\ [\hat{M}_D]_{\lambda \times F} \text{diag}([B_M]_v) \end{bmatrix} \quad (23)$$

where $\text{diag}([B_M]_i)$ is a diagonal matrix whose main diagonal is the i^{th} row of $[B_M]_{v \times F}$.

The net degree of freedom F_N is the difference between the gross degree of freedom F and $\text{rank}([\hat{M}_N]_{\lambda v \times F})$ and establishes the number of variables that must be imposed to solve Eq. (22). These variables are called primary variables. For automotive transmissions, usually the primary variables are given by the power sources speed and the clutches/brakes engagement.

After solving Eq. (22), the motion matrix $[M]_{\lambda \times F}$ can be computed through the product between $[\hat{M}_D]_{\lambda \times F}$ and a diagonal matrix built from the vector $\{\vec{\psi}\}_{F \times 1}$ containing the solution of Eq. (22):

$$[M]_{\lambda \times F} = [\hat{M}_D]_{\lambda \times F} \text{diag}(\{\vec{\psi}\}_{F \times 1}) \quad (24)$$

Similarly, by disposing side by side the normalised wrenches, the unit action matrix $[\hat{A}_D]_{\lambda \times C}$ is generated. Relations among the actions transmitted by the couplings crossing the same cutset are established using an analogy with the Kirchhoff's Current Law: the algebraic sum of the wrenches entering and exiting a cutset must be equal to zero. This statement can be mathematically translated by the homogeneous linear system presented in Eq. (25):

$$[\hat{A}_N]_{\lambda k \times C} \{\vec{\psi}\}_{C \times 1} = \{\vec{0}\}_{\lambda k \times 1} \quad (25)$$

where the vector $\{\vec{\psi}\}_{C \times 1}$ contains the wrenches magnitudes and $[\hat{A}_N]_{\lambda k \times C}$ is the network unit action matrix. $[\hat{A}_N]_{\lambda k \times C}$ is obtained by multiplying $[\hat{A}_D]_{\lambda \times C}$ k times by a diagonal matrix composed of a row of the cutset matrix:

$$[\hat{A}_N]_{\lambda k \times C} = \begin{bmatrix} [\hat{A}_D]_{\lambda \times C} \text{diag}([Q_A]_1) \\ [\hat{A}_D]_{\lambda \times C} \text{diag}([Q_A]_2) \\ \vdots \\ [\hat{A}_D]_{\lambda \times C} \text{diag}([Q_A]_k) \end{bmatrix} \quad (26)$$

where $\text{diag}([Q_A]_i)$ is a diagonal matrix whose main diagonal is the i^{th} row of $[Q_A]_{k \times C}$.

The net degree of constraint C_N is the difference between the gross degree of constraint C and $\text{rank}([\hat{A}_N]_{\lambda k \times C})$ and establishes the number of primary variables that

must be imposed to solve Eq. (25). For automotive transmissions, usually the primary variables are given by the power sources torque and the clutches/brakes engagement.

After solving Eq. (25), the action matrix $[A]_{\lambda \times C}$ can be computed through the product between $[\hat{A}_D]_{\lambda \times C}$ and a diagonal matrix built from the vector $\{\vec{\psi}\}_{C \times 1}$ containing the solution of Eq. (25):

$$[A]_{\lambda \times C} = [\hat{A}_D]_{\lambda \times C} \text{diag}(\{\vec{\psi}\}_{C \times 1}) \quad (27)$$

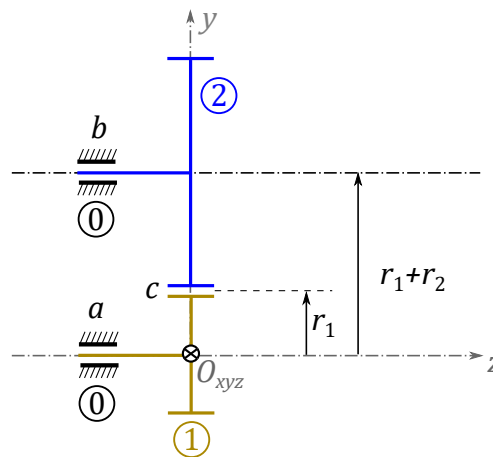
3.2 SCREW SYSTEM FOR MOTION AND ACTION ANALYSES OF PGTS

Some hypotheses are taken into consideration in the motion and action analyses of PGTS so that the order of the screw system can be reduced, simplifying the solution of the homogeneous linear systems shown in Eqs. (22) and (25). More details on these simplifying hypotheses can be found in the paper written by Laus, Simas, and Martins (2012).

3.2.1 Cylindrical gears

For PGTS composed exclusively of cylindrical spur gears, the order of the screw-system can be adopted as $\lambda = 2$ (DAVIES, 2000). Consider the simple spur gear train shown in Fig. 8 to follow the assumptions that make this adoption possible:

Figure 8 – Simple spur gear train.



Source: From the author.

A reference frame O_{xyz} is attached to gear 1, with the z -axis parallel to the gear shafts. r_1 and r_2 stand for the pitch radii of gears 1 and 2, respectively. The following simplifying hypotheses should be adopted to use $\lambda = 2$ when applying the Davies method to the simple spur gear train of Fig. 8:

- The PGT occupies negligible axial length, leading to all forces to lie in the plane $z = 0$ and to be parallel to the x -axis;
- All gears are thin spur gears whose tooth contacts lie on the y -axis;
- The pressure angle is assumed as zero.

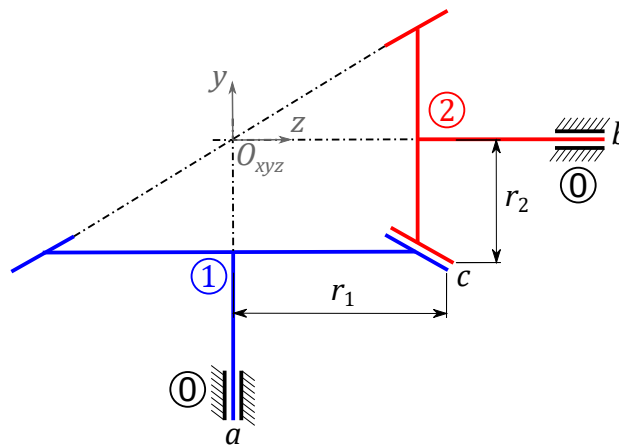
When these hypotheses are adopted, the relative motions on the spur gear train are angular velocities about axes parallel to the z -axis in the plane $x = 0$. So, the 2nd special 2-system of screws is used in the motion analysis. In Davies' notation, the twists can be spanned by the 2-system of screws $\{t, u\}$ (LAUS; SIMAS; MARTINS, 2012; HUNT, 1978). The relative motions are represented by a twist with $h = 0$, ISA parallel to z -axis and angular speed magnitude t .

Another consequence of adopting the simplifying hypotheses is that the only actions that can exist in the spur gear train are forces parallel to x -axis in the plane $z = 0$ and torques parallel to z -axis. Thus, the screw system used in the action analysis of cylindrical spur gears is the 2nd special 2-system of screws, represented by $\{T, U\}$ in Davies' notation. The forces are represented by a wrench with $h = 0$, ISA parallel to x -axis and magnitude U , while the torques are represented by wrenches with $h \rightarrow \infty$, ISA parallel to the z -axis and magnitude T .

3.2.2 Bevel gears

Figure 9 is used to illustrate the simplifying assumptions for bevel gear trains and shows a simple bevel gear train with perpendicular axes, as the most common angle between the shafts of two meshing bevel gears is 90° (NORTON, 2010). A reference frame O_{xyz} is attached to the intersection point of the gears' axis. r_1 and r_2 stand for the pitch radii of gears 1 and 2, respectively.

Figure 9 – Simple bevel gear train.



Source: From the author.

The following hypotheses are taken into consideration in the motion and action analyses of bevel PGTs through the Davies method:

1. The gear train has negligible axial length so that all forces are parallel to x -axis.
2. All gears are thin straight bevel gears making tooth contacts at points on the plane $x = 0$.
3. The pressure angle is assumed as zero.

When these simplifying hypotheses are adopted, the relative motions of the simple bevel gear train are rotations whose ISAs passes through the origin of O_{xyz} in the plane $x = 0$ (LAUS; SIMAS; MARTINS, 2012), what makes all twists belong to a 1st special 2-system of screws, according to Hunt's classification (HUNT, 1978). Following the Davies' notation (DAVIES, 2006), the twists can be spanned by the 2-system of screws $\{s, t\}$. Thus, the relative motions are given by twists with $h = 0$. When the twist's ISA is parallel to the y -axis, its angular speed magnitude is indicated by s . When the ISA is parallel to the z -axis, the angular speed magnitude is indicated by t .

The simplifying hypotheses make that the actions transmitted by the couplings are forces parallel to the x -axis and torques parallel to the plane $x = 0$ (LAUS; SIMAS; MARTINS, 2012). Thus, the wrenches belong to a 5th special 3-system of screws (HUNT, 1978) and can be spanned by the 3-system of screws $\{S, T; U\}$. The forces are represented by a wrench with $h = 0$, ISA parallel to x -axis and magnitude U , while the torques are represented by wrenches with $h \rightarrow \infty$. When the torque wrench's ISA is parallel to the z -axis, its magnitude is given by T . When the torque wrench's ISA is parallel to the y -axis, the magnitude is indicated by S .

It is desirable that both action and motion systems of screws have the same dimension λ and that the twists and wrenches spanning the screw systems are not reciprocal, so that power can be computed through Eq. (28). Thus, the motion system of screws is expanded from $\{s, t\}$ to $\{s, t, u\}$, the 4th special 3-system of screws (HUNT, 1978). The last coordinate is null for all the twists (LAUS; SIMAS; MARTINS, 2012). Consequently, the dimension of the action and motion systems of screws for bevel gear trains is $\lambda = 3$.

Although simple gear trains, shown in Figs. 8 and 9, illustrate the simplifying assumptions to define the screw system, the systems of screws described in this section can be adopted in more complex architectures.

3.3 POWER FLOW AND EFFICIENCY

The amount of power $\mathcal{P}$ which enters or leaves the network through a coupling p is computed pairing the twist and the wrench of p , coming from the columns of $[M]_{\lambda \times F}$ and $[A]_{\lambda \times C}$, as expressed by Eq. (28):

$$\mathcal{P}_p = \$p^a \cdot \$p^m = rR + sS + tT + uU + vV + wW \quad (28)$$

In the sign convention adopted in this work, if $\mathcal{P}_p < 0$, power leaves the network through coupling p . Otherwise, if $\mathcal{P}_p > 0$, power enters the network through coupling p (LAUS, L.; SIMAS; MARTINS, 2020).

The overall efficiency η of a gear train is the ratio between the output and input power:

$$\eta = \frac{\mathcal{P}_{out}}{\mathcal{P}_{in}} \quad (29)$$

For automotive transmissions, power enters through couplings connecting the power sources to the PGTs, while power leaves the transmission through the coupling connecting the housing to the output shaft.

3.4 VIRTUAL POWER FLOW AND ORDINARY EFFICIENCY

The overall efficiency of a PGT can be given by the ordinary efficiency of each gear coupling composing the PGT. The ordinary efficiency of a simple gear pair and the gears roles (driver and follower gear) in the pair are determined using the concept of *virtual power flow*. The virtual power flow is the power flow measured in respect to a frame that is not common to the input and output links. If the reference frame is common to the input and the output links, the power flow is the *real power flow* (LAUS; SIMAS; MARTINS, 2012).

For gears, the carrier i is used as reference to convert the motion matrix $[M]_{\lambda \times e}$ in the absolute motion matrix $[M_i]_{\lambda \times n-1}$:

$$[M_i]_{\lambda \times n-1} = -[M]_{\lambda \times e} [I_i^\dagger]_{e \times n-1} \quad (30)$$

where n is the number of links, e is the number of edges of the coupling graph and $[I_i^\dagger]_{e \times n-1}$ is the Moore-Penrose pseudoinverse of $[I_i]_{e \times n-1}$, the reduced incidence matrix of the graph representing the gear train. $[I_i]_{e \times n-1}$ is obtained by removing the row of the incidence matrix corresponding to the carrier i .

The pseudo-inverse $[I_i^\dagger]_{j \times n-1}$ is defined by:

$$[I_i^\dagger]_{j \times n-1} = ([I_i]_{n-1 \times j})^T \left([I_i]_{n-1 \times j} ([I_i]_{n-1 \times j})^T \right)^{-1} \quad (31)$$

The virtual power flow crossing a gear coupling gp connecting gears 1 and 2 is calculated through the inner product between the column of M_i corresponding to gp and the respective wrench $\$_{gp}^a$ in $[A]_{\lambda \times C}$. If the result is positive, power flows in the same direction of the edge connecting gears 1 and 2 in the digraph representing the gear train. Otherwise, power flows in the opposite direction. The virtual power flow direction defines the driver/follower roles of gears 1 and 2. The ordinary efficiency of the gear

pair is computed as the ratio between the virtual power flow that enters the driven gear and the virtual power that leaves the driver gear (LAUS; SIMAS; MARTINS, 2012).

3.5 POWER LOSSES AND FRICTION TORQUES IN PGTS

Losses in gear trains, coming from seal friction and gear meshing friction, for example, can be included in the Davies method by considering their effect equivalent to a pure torque. A pure torque can be assumed when the gears shaft intersect (bevel gears) or are parallel (cylindrical gears) (LAUS; SIMAS; MARTINS, 2012). The losses are taken into account by appending to $[\hat{A}_N]_{\lambda \times C}$ constitutive equations that relate the action transmitted through a coupling with a loss action due to friction or other power loss.

Available friction models in the literature include Coulomb (ARMSTRONG-HÉLOUVRY; DUPONT; DE WIT, 1994) and viscous friction and can be easily added in the action analysis (LAUS; SIMAS; MARTINS, 2012). Tab. 3 contains two friction models and a general form to compute the torque magnitude.

Table 3 – Torque magnitudes according to some friction models

Friction model	Torque magnitude
Coulomb	$- e_i^a \text{sign}(e_i^m) \zeta_i$
Viscous	$-\xi_i e_i^m $
General	$-g(e_i^a , e_i^m)$

Source: Laus, Simas, and Martins (2012).

In Tab. 3, e_i^a is the magnitude of the action transmitted by the coupling i ; e_i^m is the magnitude of the motion allowed by coupling i ; ζ_i is the Coulomb friction factor, with unit of length, which quantifies the losses at constant speed or that are independent on speed magnitude; and ξ_i is the viscous friction coefficient. A general function, g , represents the main advantage of calculating efficiency through Davies' method: any friction model can be incorporated. The friction model g can be as function of the magnitude e_i^a of the action transmitted and/or the magnitude e_i^m of the motion allowed by the coupling. For example, g could be a function modelling the losses in hydrodynamic coupling where lubricant flows.

Two causes of loss, independent of speed, are considered in this work: meshing friction on the gear pairs and load-dependent bushing/bearing losses at the revolute pairs. Other origins of loss are neglected as the main causes of loss in gear trains are friction between gear teeth surfaces and friction in shaft support bearings (RADZIMOVSKY, 1956).

In this Thesis, the losses due to meshing friction on gear pairs are quantified by the Coulomb friction model. The factor ζ_i is introduced by Laus (2011) to simplify the equations always when the friction model does not depend on the speed such as in

Coulomb friction or viscous friction at constant speed. The numeric value of ζ_i can be computed in many different ways, from integration very similarly to the classic text on gear efficiency of Buckingham (1963) or numeric simulation, see details in the Thesis of Laus (2011). But the main advantage of using ζ_i is that it can be expressed in term of the ordinary efficiency as Eqs. (33) and (34). The ordinary efficiency on its turn can be determined experimentally or expressed in terms of coefficient of friction (FRAZER, Robert C.; FISH; PALMER, 2010; LITVIN; FUENTES, 2004; BUCKINGHAM, 1963), or even estimated with reasonable accuracy for most cases.

Assuming that power losses are not high enough to induce changes in the power flow direction, the absolute value operator and the sign function of the Coulomb friction model can be omitted in the equation shown in Tab. 3. Thus, the magnitude $\varrho_{L_{gp}}$ of the Coulomb friction torque in a gear pair gp is calculated by the product (SOUZA et al., 2019):

$$\varrho_{L_{gp}} = \zeta_{gp} U_{gp} \quad (32)$$

where $\varrho_{gp}^a = U_{gp}$ is the force transmitted by the gear pair gp and ζ_{gp} is the loss factor, in unit of length, which converts U_{gp} into a friction torque. This simplification helps the efficiency calculation of complex PGTs composed of multiple gear pairs, such as automotive gearboxes.

The coefficient ζ_{gp} is defined in terms of gears pitch radii (r_d and r_f , the pitch radii of the driver gear and the follower gear, respectively) and the efficiency η_{gp} of the gear pair. For cylindrical spur gears, ζ_{gp} is calculated according to Eq. (33) (SOUZA et al., 2017):

$$\zeta_{gp} = \frac{r_d r_f (1 - \eta_{gp})}{r_d \pm \eta_{gp} r_f} \quad (33)$$

The sign in the denominator of Eq. (33) depends on the meshing type: the plus sign is adopted for external meshing and the minus sign, for internal meshing. In this work, when calculating the efficiency of PGTs composed of spur gears, $\eta_{gp} = 98\%$ for external meshing and $\eta_{gp} = 99\%$ for internal meshing (GLOVER, 1965).

For straight bevel gears with perpendicular axes and external meshing, ζ_{gp} is calculated according to Eq. (34):

$$\zeta_{gp} = \frac{r_d r_f \sqrt{r_d^2 + r_f^2} (1 - \eta_{gp})}{r_d^2 + \eta_{gp} r_f^2} \quad (34)$$

The steps to obtain Eq. (34) are presented in Appendix A. For straight bevel gears with external meshing, η_{gp} can be assumed as 98% (PATHAK; JAIN; PALANI, 2016).

After calculating ζ_{gp} , the equality $\text{sign}(\zeta_{gp}) = -\text{sign}(U_{gp}) \text{sign}(\varrho_{gp}^m)$ must be checked, where ϱ_{gp}^m is the magnitude of the angular speed of gear pair gp . If the equality is not respected, ζ_{gp} must be recalculated using η_{gp}^{-1} instead of η_{gp} in Eq. (33) or Eq. (34). This verification is done to confirm if the roles of driver/follower on the gear pair are correct.

The second cause of loss considered in this Thesis is bearing friction, which are quantified by a frictional torque. Manufacturers provide a formula to define the frictional torque, which depends on the bearing bore, the radial load and a dimensionless friction coefficient (NTN, 2018; INC, 2020). The dimensionless friction coefficient varies with some factors such as speed and lubrication, but can assume an average constant value, provided by manufacturers' catalogs.

In order to model the bearing/bushing losses in a revolute pair j , the equation shown in Eq. (35) is adopted (NTN, 2018):

$$\varrho_{Lj} = -\mu r_j \text{sign}(\varrho_j^m) |U_j| = -\mu r_j \text{sign}(\varrho_j^m) \text{sign}(U_j) U_j \quad (35)$$

where μ is the dimensionless friction coefficient, r_j is the internal radius of the bearing, ϱ_j^m is the angular speed of the revolute coupling j and U_j is the force transmitted by j . The sign function is used to avoid the absolute value operator.

In this Thesis, the pitch radius of a gear is indicated by r_{num} , i.e. the letter r with a numeric subscript designating the gear. For example, in Figs. 8 and 9, the pitch radius of gear 1 is designated by r_1 . When r has a letter as a subscript, it indicates the internal radius of a bearing at a revolute coupling. For example, in Figs. 8 and 9, the internal radius of the bearing on coupling a is designated by r_a .

The first case study of this Thesis illustrates how the Davies method is used to compute efficiency of PGTs and how the resulting efficiency can impact the driving range of an EV.

4 NEW METHOD FOR CALCULATING FUEL ECONOMY

This Chapter describes the novel method proposed in this Thesis. The method calculates fuel consumption and emissions of EVs and HEVs, integrating the efficiency of the propulsion sources — internal combustion engine (ICE), electric motor (M) and electric generator (G) — with the variability of the transmission efficiency. The transmission is composed of planetary gear trains (PGT). The innovation of the method consists of considering the variation of the transmission efficiency according to the vehicle speed in the fuel economy determination. Besides, the method calculates simultaneously the efficiency of the ICE, M and G with the efficiency of the transmission, for each instant of the vehicle operation.

The main steps of the method are listed below:

1. **Definition of the ICE optimum operation point:** the ICE optimum operation point, i.e. the ICE speed N_{ICE}^{opt} and the ICE torque T_{ICE}^{opt} which give the highest ICE efficiency, is defined from the ICE efficiency map, like the one shown in Fig. 4;
2. **Selection of the vehicle speed profile:** the method computes the fuel economy for a vehicle speed profile. The speed profile consists of the vehicle speed along with the driven time, such as the standard drive cycles presented in Sec. 2.2;
3. **Computation of the vehicle tractive force:** from the vehicle speed profile and the vehicle parameters, the tractive force F_t that the powertrain must provide so the vehicle moves forward can be calculated via Eq. (7). F_t is calculated for each second of the vehicle speed profile;
4. **Computation of the transmission output speed:** from the vehicle speed profile and the vehicle parameters, the transmission output speed $N_{t_{out}}$ can be calculated via Eqs. (9) and (10). $N_{t_{out}}$ is calculated for each second of the vehicle speed profile;
5. **Analyzing the transmission via Davies' method:** the ICE optimum speed N_{ICE}^{opt} and the transmission output speed $N_{t_{out}}$ are the primary variables of the transmission's motion analysis, done via Davies' method, see Sec. 2.2.1. The ICE optimum torque T_{ICE}^{opt} is the primary variable of the transmission's action analysis, also done via Davies' method. The following items are given by the modelling of the transmission by means of the Davies method:
 - a) Transmission mechanical efficiency η_t ;
 - b) Generator speed N_G , generator torque T_G and the power $\mathcal{P}_G$ that G provides to or absorbs from the transmission;
 - c) Transmission output torque $T_{t_{out}}$ and output power $\mathcal{P}_{t_{out}}$.

The Davies method gives the items above for each second of the vehicle speed profile. This way, the speed and torque that G must provide so that the ICE operates with maximum efficiency are determined. If N_G or T_G surpass the maximum speed/maximum torque of G, ICE speed N_{ICE} is changed so N_G decrease; and ICE torque T_{ICE} is redefined from the ICE optimum operation line (OOL), so that N_G or T_G respect the G limits. Thus, the Davies method is applied again to obtain new values for η_t , N_G , T_G , $\mathcal{P}_G$, $T_{t_{out}}$ and $\mathcal{P}_{t_{out}}$.

6. **Computation of the vehicle power demand:** the current vehicle speed, F_t and η_t are applied to Eq. (8) to obtain the vehicle power demand $\mathcal{P}_{dem}$. $\mathcal{P}_{dem}$ is calculated for each second of the vehicle speed profile;
7. **Computation of the torque and power of the traction motor:** the traction motor power $\mathcal{P}_M$ is calculated by subtracting $\mathcal{P}_{t_{out}}$ from $\mathcal{P}_{dem}$. Since the traction motor connects the PGT output to the final drive, the motor speed N_M is equal to $N_{t_{out}}$. Then, the motor torque T_M is defined by dividing $\mathcal{P}_M$ by N_M ;
8. **Definition of ICE consumption and emissions:** the specific fuel consumption b_e and the pollutant emission rate $\dot{m}_{pol}$ for a given second of the vehicle speed profile are obtained using N_{ICE} and T_{ICE} to navigate in the ICE consumption map, as the one shown in Fig. 2, and the ICE emission maps. b_e and the ICE power $\mathcal{P}_{ICE}$ are applied to Eq. (12) to compute the fuel consumption rate $\dot{m}_{fuel}$ for a given second of the vehicle speed profile;
9. **Definition of M efficiency:** the motor efficiency η_M for a given second of the vehicle speed profile is obtained using N_M and T_M to navigate in the motor efficiency map, like the one shown in Fig. 7;
10. **Computation of the power transferred from the battery pack to M (or vice versa):** $\mathcal{P}_M$ and η_M are applied to Eq. (20) to compute the power $\mathcal{P}_{M_{bat}}$ transferred from the battery pack to M (or vice versa, if M operates as generator). $\mathcal{P}_{M_{bat}}$ is calculated for each second of the vehicle speed profile;
11. **Definition of G efficiency:** the generator efficiency η_G for a given second of the vehicle speed profile is obtained using N_G and T_G to navigate in the generator efficiency map;
12. **Computation of the power transferred from G to the battery pack (or vice versa):** $\mathcal{P}_G$ and η_G are applied to Eq. (21) to compute the power $\mathcal{P}_{G_{bat}}$ transferred from G to the battery pack (or vice versa, if G operates as motor). $\mathcal{P}_{G_{bat}}$ is calculated for each second of the vehicle speed profile;

13. **Computation of the real-time charging or discharging power of the battery pack:** $\mathcal{P}_{M_{bat}}$ and $\mathcal{P}_{G_{bat}}$ are applied to Eq. (19) to calculate the real-time charging or discharging power of the battery pack, $\mathcal{P}_{bat}$. $\mathcal{P}_{bat}$ is calculated for each second of the vehicle speed profile;
14. **Computation of the battery pack current state-of-charge (SOC):** $\mathcal{P}_{bat}$ is used in Eq. (17) to obtain the current SOC of the battery pack. The battery pack SOC defines when the ICE turns on or off, i.e. when the HEV is driven exclusively by traction motor. Therefore, the batteries SOC is crucial for determining the HEV fuel consumption;
15. **Computation of the total mass of pollutants emitted:** the total mass of pollutant m_{pol} emitted during the vehicle speed profile is obtained by adding up each $\dot{m}_{pol}$ obtained for each second of the vehicle speed profile. The total mass can be divided by the distance travelled by the vehicle for the whole speed profile to obtain an average;
16. **Computation of the fuel economy:** the fuel economy FC is calculated through Eq. (13) and Eq. (14) or (15), depending on the unit desired.

The method steps described in this Chapter are aimed to HEVs whose traction motor connects the PGT output to the final drive, as the case studies of Chapters 6 and 7. However, nothing prevents that the method is adapted to HEVs whose hybrid transmissions contain the traction motor, such as the Dedicated Hybrid Transmissions (DHT). DHTs have an electric motor and a generator fully integrated in the transmission system (GOPPELT, 2016; EHSANI et al., 2018).

A flowchart summarizing the steps listed above is shown in Fig. 10, with the steps indicated by a red bold text and highlighted by a dashed line rectangle when necessary.

Step 5 is the differential of the method proposed in this Thesis. In step 5, the Davies method is applied to compute η_t , which is calculated for each vehicle speed value of the speed profile. η_t is calculated for each instant of the speed profile because depends on the propulsion sources torque and speed, which vary with the vehicle power demand. Therefore, the value of η_t fluctuates along the speed profile and is not constant, as many works described in Chapter 1 assume when simulating fuel economy.

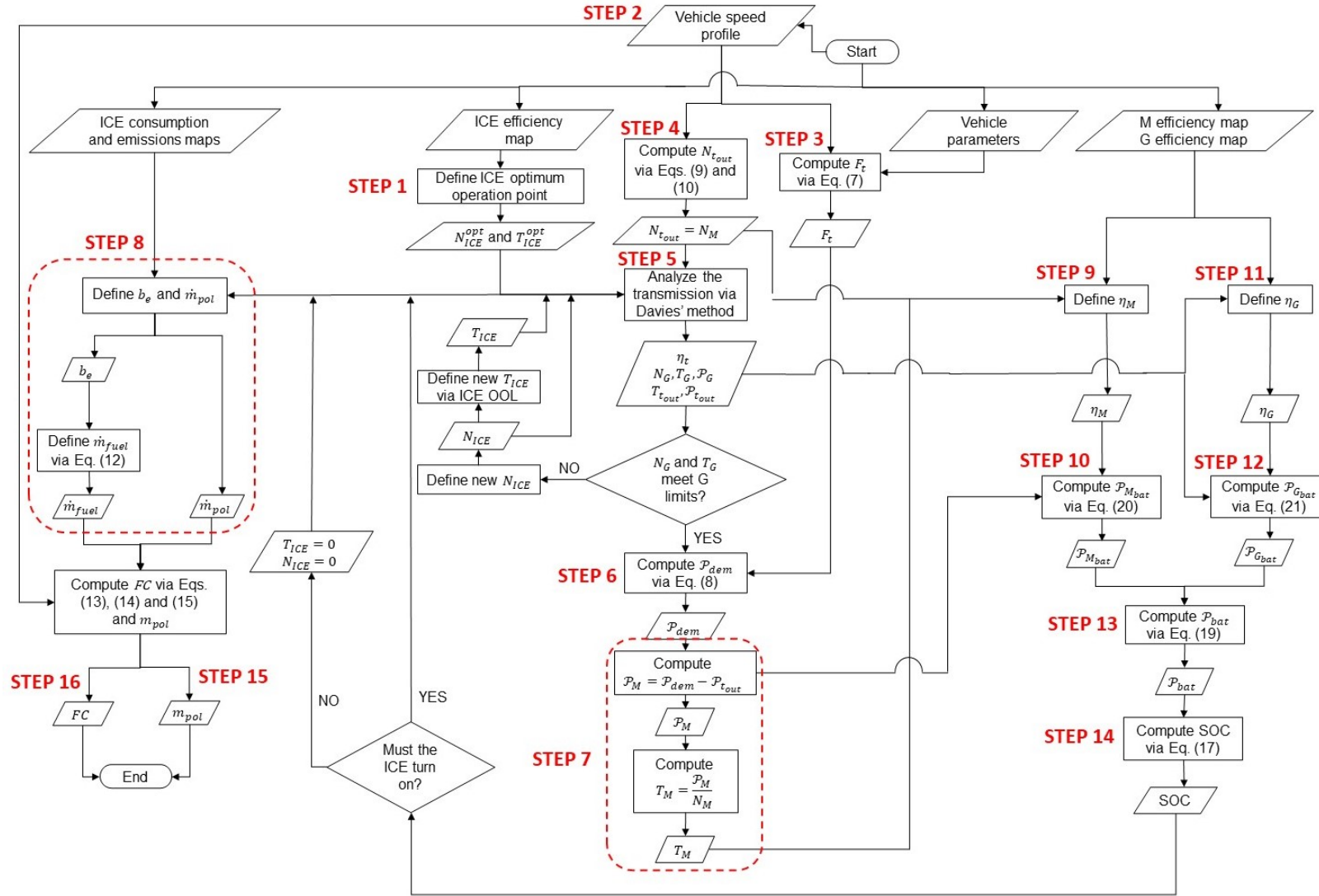
Another feature of the method proposed in this Thesis comes from step 5: the method can be part of a sizing tool which finds the gears dimensions that minimize fuel consumption. The Davies method allows to define an optimization problem where the objective function is the fuel economy and the design variables are the gears dimensions. This is possible because the Davies method allows to derive an analytical expression for η_t as function of the gears' pitch radii. When the expression of η_t is

applied to Eq. (8), $\mathcal{P}_{dem}$ depends on the gears' pitch radii, consequently, the fuel economy becomes as function of the gear dimensions too.

Two examples of the sizing tool are provided in this Thesis, in Chapters 5 and 7. However, in Chapter 5, the goal is maximizing η_t instead of minimizing the energy consumption, since the gear reducer of the purely electric machine has constant efficiency as it has 1 DOF.

Chapter 5 presents a simplified application of the method described in this Chapter. The transmission of the purely electric machine of Chapter 5 has 1 DOF, so its efficiency does not vary with the speed profile. Besides, the purely electric machine has a single propulsion source, a traction motor, whose efficiency is also constant. So only steps 2, 3, 5, 6, 13 and 14 are applied in Chapter 5, with some modification when needed.

Figure 10 – Flowchart of the method proposed in this Thesis.



Source: From the author.

5 CONSUMPTION OF A PURELY ELECTRIC MACHINE

In this Chapter, the consumption of a purely electric machine is quantified and then optimized. Instead of simulating a BEV driving range, the purely electric machine chosen to illustrate the calculation process is an inspection robot for distribution power lines, due to the participation of the author in the R&D project entitled "Development of a Robotized System for Inspection of Electricity Distribution Lines" (05697-0317 / 2017), regulated by ANEEL (Agência Nacional de Energia Elétrica) and financed by Celesc (Centrais Elétricas de Santa Catarina S.A.), the electrical utility company of the state of Santa Catarina, in the Southern part of Brazil. The process to compute and optimize the consumption of the inspection robot can be easily adapted to a BEV.

The inspection robot was developed by the Laboratory of Applied Robotics (LAR), from the Federal University of Santa Catarina, in a partnership with Celesc. The robotic system is remotely operated and moves along the distribution lines, rolling over the electrical elements present on the line.

5.1 CONTEXTUALIZATION

The interruption in power supply represents financial losses for electrical utility companies and for businesses and industries, which rely on energy to operate. Blackouts may also represent risks for citizens, as hospitals, transport, illumination and other public systems are dependent on energy. Furthermore, continuous energy supply is an issue of national security. For this reason, it is essential to periodically inspect distribution lines to identify and repair defects in their electrical components.

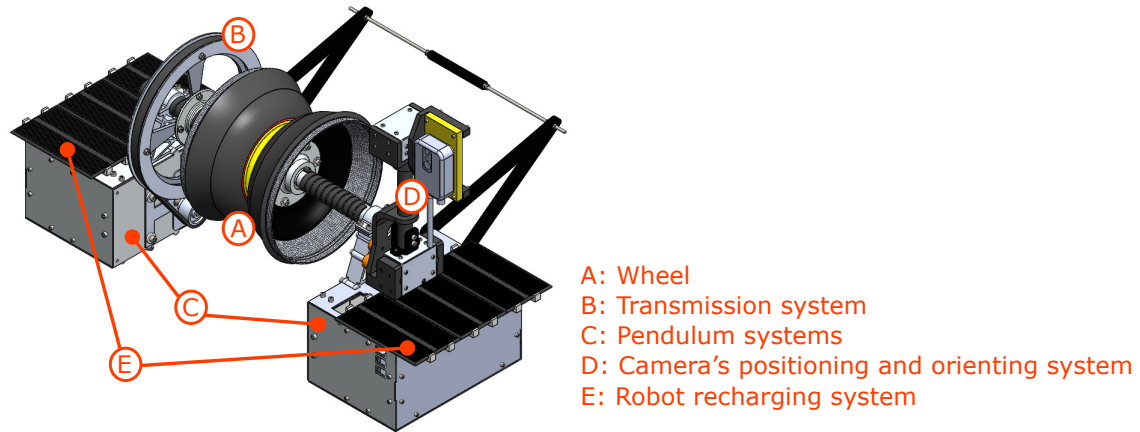
Currently, power distribution companies employ ground patrols to conduct visual inspections of their lines by taking thermal pictures of the network elements. After that, technicians assess the measured temperatures of the network elements and other energy supply conditions at the day and time the thermograms have been taken to determine if there are problems in the network. However, due to the extent of the Brazilian distribution network, conducting regular inspections of all lines is not practical. To automate the inspection process and make it safer, more reliable and efficient, the LAR and Celesc carried out a R&D project to develop an overhead power distribution lines robotic inspection system. The R&D project designed a functional prototype of the system, which was tested and validated.

5.2 INSPECTION ROBOT DESCRIPTION

As represented in Fig. 11, the power line inspection robot assessed in this study is composed of five systems, indicated by letters A to E. Letter A identifies the *wheel*, which makes the contact between the robot and the energy grid (conductors

and obstacles), allowing the robot to move along the power line.

Figure 11 – CAD model of the overhead power distribution inspection robot.



Source: From the author.

Letter B indicates the *transmission system* responsible for providing motion from an electric DC motor to the wheel. The transmission system is composed of three different subsystems. First, there is the robot's motor, which is a 24V electric DC motor with torque of 0.1729 Nm and speed of 279 rpm at rated load (MOTRON, 2019). After the motor, there is the reducer gearbox, to be optimized in this work. The reducer is connected to two external pulleys and a belt to transfer the motion from the DC motor to the wheel.

Letter C in Fig. 11 shows the *pendulum systems*, designed to store the electronic components, batteries and DC motor. Letter D identifies the *camera's positioning and orienting system*, composed of two actuated servo motors that provide the pitch and yaw adjustments to the images the robot captures from the electrical network. Finally, letter E indicates the *robot recharging system* which consists of a set of photovoltaic cells.

The inspection robot shown in Fig. 11 is subjected to several design constraints to be able to efficiently operate. The robot's mass cannot exceed 15 kg, otherwise, the robot will break the conductors. The robot is also limited in terms of volume to prevent crashing against the distribution line during obstacle transposition. Consequently, the number of components and batteries that can be added to the robot is restricted. As the robot rolls over the conductors, it has to have sufficient friction and torque to climb both the conductor inclinations and the energy grid obstacles. Additionally, the robot's center of mass (CM) must be below the contact point between the robot and the distribution line, as lower as possible, to guarantee operation stability and transposition capability.

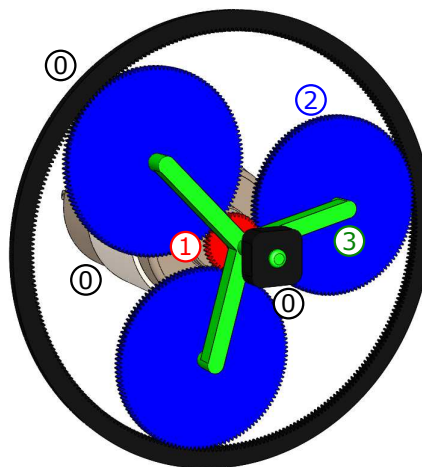
The robot's DC motor weights 1.3 kg and was specified as the lightest motor available in the Brazilian market that meets the torque and speed requirements for this

application. Once the motor alone corresponds to almost 10% of robot's total mass, to lower the CM, the motor was placed at one of the pendulum systems, below the robot's wheel rotation axis. Therefore, to meet the requirements previously described, the design of the robot's transmission system must be divided into two steps: (1) selection of the external pulleys and belt, required to transfer the motion to the wheel and (2) design of the reducer planetary gear train, to guarantee the extra reduction necessary for achieving the robot's required torque. A gear train could not be used instead of the pulleys and a belt because they would increase the robot's mass, reducing the robot's driving range and compromising its operation.

L series timing belts and pulleys were chosen as they have the highest efficiency regarding other belt transmission systems, from 97 to 99%, and are recommended for precision driving (BUDYNAS; NISBETT, 2020). The pulleys and the belt were specified according to the distance between the rotation axes of both the robot wheel and motor, as well as the required reduction ratio. The belt was chosen with 0.50 pol width and 300 grooves. The pulleys connected to the motor and wheel axes were specified with the same 0.50 pol width and 12 and 72 teeth, respectively, accounting for a 1:6 reduction ratio.

Since the pulleys and belts are standardized and cannot be changed, the only option for improving the robot's transmission system efficiency is through the optimization of its reducer gearbox, which was chosen as a PGT (Fig. 12) due to its compactness. The PGT is illustrated in Fig. 12, which has fixed links indicated in black and by the number 0. The DC motor is also fixed. The output link is indicated by the the green carrier numbered as 3. The ring (also fixed), sun and planet gears are indicated by black, red and blue colours and by the numbers 0, 1 and 2, respectively.

Figure 12 – Planetary gear train representation.



Source: From the author.

Considering the design constraints detailed above, a prototype of the inspection robot was built for field testing, as shown in Fig. 13.

Figure 13 – Inspection robot prototype on a field test.



Source: From the author.

5.3 USING THE NEW METHOD TO CALCULATE ELECTRIC ENERGY CONSUMPTION

The method detailed in Chapter 4 is adapted to compute the energy consumed by the inspection robot. In the adaptation, not all steps of the Thesis method are applied. For example, since the robot is driven by a single electric motor whose efficiency is assumed as constant, the first step of the method ("*1. Definition of the ICE optimum operation point*") does not apply to this case study. Another implication of the robot having a single electric propulsion source is that the steps related to the ICE in the Thesis method (steps 8, 15 and 16) and the steps related to the generator (steps 11 and 12) are not applied in this Chapter.

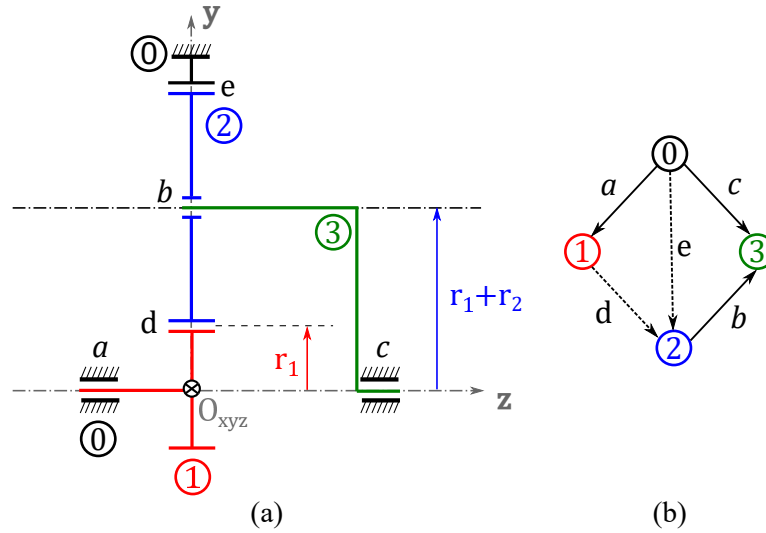
Steps 2 and 3 will be seen in Sec. 5.3.3. Step 4 ("*Computation of the transmission output speed*") is not considered in this case study neither, as the PGT composing the robot transmission has 1 DOF, so its motion analysis requires a single primary variable to be solved and the primary variable chosen is the DC motor speed. Moreover, steps 7 and 10 are unnecessary in this Chapter due to the simplicity of the robot's power plant.

Therefore, the case study starts with the step 5: "*Analyzing the transmission via Davies' method*". However, since the inspection robot has a single propulsion source, the fifth step is simpler than described in Chapter 4. In this case study, the Davies method provides only the reducer efficiency η_{PGT} .

5.3.1 DC motor reducer efficiency

This Section corresponds to the fifth step of the Thesis method: "5. *Analyzing the transmission via Davies' method*". The PGT of Fig. 12 has four links: the sun gear 1, the planet gear 2, the carrier 3 and the ring gear and frame (both fixed) correspond to link 0. The schematic representation and the graph representation of the PGT are shown in Fig. 14, where r_1 is the pitch radius of the sun gear and r_2 is the pitch radius of the planet gears. In Fig. 14, five couplings are identified: three revolute couplings (a , b and c) and two gear pairs (d and e). The revolute couplings contain bushings, which are assumed to be lubricated with grease. The DC motor is connected to coupling a and coupling c is the reducer output, which provides speed and torque to the robot displacement.

Figure 14 – (a) Schematic representation and (b) graph representation of the PGT.



Source: From the author.

A reference frame O_{xyz} is attached to the center of gear 1 in Fig. 14(a), giving the position vectors of the couplings: $[0; 0; 0]^T$ for a and c , $[0; r_1 + r_2; 0]^T$ for coupling b , $[0; r_1; 0]^T$ for d and $[0; r_1 + 2r_2; 0]^T$ for coupling e .

Since the robot's PGT is composed exclusively of spur gears, the order of the screw system for both motion and action analyses is $\lambda = 2$, according to Sec. 3.2.1.

Motion analysis

Considering that the five couplings have one freedom each, the motion graph of the PGT is equivalent to the coupling graph shown in Fig. 14(b). So the matrix $[B_M]_{2 \times 5}$ is equal to the fundamental circuit matrix $[B]_{2 \times 5}$, shown in Eq. (36), using edges d and e as chords:

$$[B]_{2 \times 5} = [B_M]_{2 \times 5} = \begin{bmatrix} 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} a & b & c & d & e \\ d \\ e \end{matrix} \quad (36)$$

The single motion allowed by the couplings is a rotation around z-axis. So the magnitude of the twist representing this motion is designated by t_i , where the subscript i identifies the coupling, and the twist's ISA corresponds to a unit vector on z direction. Tab. 4 summarizes the parameters needed to solve the motion analysis. Tab. 4 contains only the y coordinate of the couplings due to the hypotheses listed in Sec. 3.2.1.

Table 4 – Twists' parameters of the robot's PGT

Coupling	y coordinate	ISA direction	Twist's magnitude
a	0	$\{0; 0; 1\}^T$	t_a
b	$r_1 + r_2$	$\{0; 0; 1\}^T$	t_b
c	0	$\{0; 0; 1\}^T$	t_c
d	r_1	$\{0; 0; 1\}^T$	t_d
e	$r_1 + 2r_2$	$\{0; 0; 1\}^T$	t_e

Source: From the author.

The twists representing the motions of the PGT are built from the parameters of Tab. 4. The unit motion matrix $[\hat{M}_D]_{2 \times 5}$ is generated by placing the normalized twists side by side:

$$[\hat{M}_D]_{2 \times 5} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & r_1 + r_2 & 0 & r_1 & r_1 + 2r_2 \end{bmatrix} \begin{matrix} t_a & t_b & t_c & t_d & t_e \\ t \\ u \end{matrix} \quad (37)$$

The row labels t and u on the right side of $[\hat{M}_D]_{2 \times 5}$ indicate the twist coordinate, while column labels on the top of the matrix indicate the normalized twist.

The network unit motion matrix $[\hat{M}_N]_{4 \times 5}$ is obtained by applying Eqs. (36) and (37) in o Eq. (23):

$$[\hat{M}_N]_{4 \times 5} = \begin{bmatrix} 1 & 1 & -1 & 1 & 0 \\ 0 & r_1 + r_2 & 0 & r_1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & r_1 + r_2 & 0 & 0 & r_1 + 2r_2 \end{bmatrix} \begin{matrix} t_a & t_b & t_c & t_d & t_e \\ t \\ u \\ t \\ u \end{matrix} \quad (38)$$

Column labels on the top of $[\hat{M}_N]_{4 \times 5}$ indicate the twist to which the column is related.

Equation (38) is the coefficient matrix of the homogeneous linear system shown in Eq. (22), so the equation system has 6 equations and 5 unknowns. The homogeneous linear system has $F_N = 1$, as expected: since the ring gear (link 0) is fixed, the PGT is a mechanism of 1 DOF. Thus, only one primary variable is used to solve Eq. (22) for the PGT. Once the motor is connected to coupling a , t_a is equal to the motor speed and can be taken as the primary variable of the system, resulting in the vector $\{\vec{\psi}\}_{5 \times 1}$, which contains the twists' magnitudes:

$$\{\vec{\psi}\}_{5 \times 1} = \begin{Bmatrix} t_a \\ \frac{r_1 t_a (r_1 + 2r_2)}{2r_2 (r_1 + r_2)} \\ \frac{r_1 t_a}{2(r_1 + r_2)} \\ -\frac{t_a (r_1 + 2r_2)}{2r_2} \\ -\frac{r_1 t_a}{2r_2} \end{Bmatrix} \begin{matrix} t_a \\ t_b \\ t_c \\ t_d \\ t_e \end{matrix} \quad (39)$$

where the row labels on the right of $\{\vec{\psi}\}_{5 \times 1}$ indicate the magnitude to which the vector element corresponds.

The motion matrix $[M]_{2 \times 5}$ is composed of the twists allowed by the reducer and is obtained by applying Eqs. (37) and (39) in Eq.(24):

$$[M]_{2 \times 5} = \frac{t_a}{2r_2} \begin{bmatrix} \overset{\$a^m}{2r_2} & \overset{\$b^m}{\frac{r_0 r_1}{r_1 + r_2}} & \overset{\$c^m}{\frac{2r_1 r_2}{2(r_1 + r_2)}} & \overset{\$d^m}{-r_0} & \overset{\$e^m}{-r_1} \\ 0 & r_0 r_1 & 0 & -r_0 r_1 & -r_0 r_1 \end{bmatrix} \begin{matrix} t \\ u \end{matrix} \quad (40)$$

where $r_0 = r_1 + 2r_2$ and corresponds to the pitch radius of the ring gear.

In this Thesis, $t_a = 279$ rpm is adopted to calculate the efficiency through the Davies method, corresponding to the motor speed at rated load. From Eq. (39), we have that $t_b = \frac{r_0 r_1}{2r_2(r_1 + r_2)} t_a$ and $t_c = \frac{r_1}{2(r_1 + r_2)} t_a$, so $\text{sign}(t_b) = \text{sign}(t_c) = \text{sign}(t_a) = 1$.

Action analysis ignoring power losses

According to Sec. 3.2.1, the passive constraints are forces parallel to the x-axis, indicated by U_i . There are also two external torques acting on the PGT: the torque provided by the motor, T_a , acting on coupling a , and the output torque, T_c , acting on the output coupling c . To model power losses, additional five external torques are added to represent the friction at the couplings, following the friction models presented in

Sec. 3.5. Power leaves through revolute couplings a , b and c due to bushing friction and through gear pairs d and e due to meshing friction. The friction torque magnitudes are indicated by T_{iL} , where the subscript i designate the coupling and L indicates that it is a friction torque.

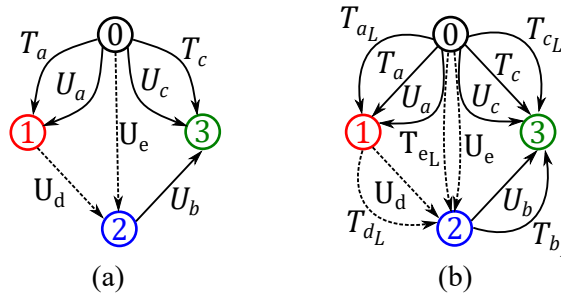
Tab. 5 contains the data that is extracted from the screw-system definition and Fig. 14, such as the couplings' position and the wrenches' ISAs direction:

Table 5 – Wrenches' parameters of the robot's PGT

Coupling	y coordinate	Action	ISA direction	Twist's magnitude
a	0	Torque	$\{0; 0; 1\}^T$	T_a
		Torque	$\{0; 0; 1\}^T$	T_{aL}
		Force	$\{1; 0; 0\}^T$	U_a
b	$r_1 + r_2$	Torque	$\{0; 0; 1\}^T$	T_{bL}
		Force	$\{1; 0; 0\}^T$	U_b
		Torque	$\{0; 0; 1\}^T$	T_c
c	0	Torque	$\{0; 0; 1\}^T$	T_{cL}
		Force	$\{1; 0; 0\}^T$	U_c
		Torque	$\{0; 0; 1\}^T$	T_{dL}
d	r_1	Torque	$\{0; 0; 1\}^T$	T_d
		Force	$\{1; 0; 0\}^T$	U_d
		Torque	$\{0; 0; 1\}^T$	T_{eL}
e	$r_1 + 2r_2$	Force	$\{1; 0; 0\}^T$	U_e

Source: From the author.

Figure 15 – PGT actions graphs (a) ignoring power losses and (b) including power losses.



Source: From the author.

Ignoring power losses, the resulting action graph of the PGT is shown in Fig. 15(a). The action graph shown in Fig. 15(a) is used to build the fundamental cutset matrix $[Q_A]_{3 \times 7}$ shown in Eq. (41), considering edges T_a , U_b and T_c as branches of the spanning tree:

$$[Q_A]_{3 \times 7} = \begin{array}{cccccc} \begin{array}{cccccc} a & a & b & c & c & d & e \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} T_a \\ U_b \\ T_c \end{array} \quad (41)$$

Column labels on the top of $[Q_A]_{3 \times 7}$ indicate the action to which the column is related.

The unit action matrix $[\hat{A}_D]_{2 \times 7}$, ignoring power losses, is generated from Tab. 5:

$$[\hat{A}_D]_{2 \times 7} = \begin{array}{cccccc} \begin{array}{cccccc} T_a & U_a & U_b & T_c & U_c & U_d & U_e \end{array} \\ \begin{bmatrix} 1 & 0 & -r_1 - r_2 & 1 & 0 & -r_1 & -r_1 - 2r_2 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} T \\ U \end{array} \quad (42)$$

where the row labels T and U on the right side of $[\hat{A}_D]_{2 \times 7}$ indicate the wrench coordinate, while the column labels on the top of the matrix indicate the normalized wrench.

By associating Eqs. (42) and (41), the network unit action matrix $[\hat{A}_N]_{6 \times 7}$ is generated according to Eq. (26):

$$[\hat{A}_N]_{6 \times 7} = \begin{array}{cccccc} \begin{array}{cccccc} T_a & U_a & U_b & T_c & U_c & U_d & U_e \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & r_1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -r_1 - r_2 & 0 & 0 & r_1 & r_1 + 2r_2 \\ 0 & 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & -r_1 & -r_1 - 2r_2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} T \\ U \\ T \\ U \\ T \\ U \end{array} \quad (43)$$

Equation (43) is the coefficient matrix of Eq. (25), resulting in a homogeneous linear system with $C_N = 1$. Thus, one primary variable is necessary to solve the action analysis. The homogeneous linear system is solved by taking T_a , i.e. the motor torque, as primary variable, giving the vector $\{\vec{\psi}\}_{7 \times 1}$, which contains the wrenches magnitudes:

$$\{\vec{\psi}\}_{7 \times 1} = \begin{Bmatrix} T_a \\ -\frac{T_a}{r_1} \\ -\frac{2T_a}{r_1} \\ -\frac{2T_a(r_1 + r_2)}{r_1} \\ \frac{2T_a}{r_1} \\ -\frac{T_a}{r_1} \\ -\frac{T_a}{r_1} \end{Bmatrix} \begin{matrix} T_a \\ U_a \\ U_b \\ T_c \\ U_c \\ U_d \\ U_e \end{matrix} \quad (44)$$

where the row labels on the right of $\{\vec{\psi}\}_{7 \times 1}$ indicate the magnitude to which the vector element corresponds.

In this Thesis, $T_a = 0.1729$ Nm is adopted to calculate the efficiency through the Davies method, corresponding to the motor nominal torque.

The solution of the action analysis neglecting power losses helps defining the sign function of Eq. (35), since the power losses are not high enough to change the power flow direction (LAUS; SIMAS; MARTINS, 2012). From Eq. (44), we have that $U_a = -\frac{T_a}{r_1}$, $U_b = -\frac{2T_a}{r_1}$ and $U_c = \frac{2T_a}{r_1}$, so $\text{sign}(U_a) = \text{sign}(U_b) = -\text{sign}(T_a) = -1$ and $\text{sign}(U_c) = \text{sign}(T_a) = 1$, since $T_a > 0$ is assumed.

Applying Eq. (44) and Eq. (42) in Eq. (27) leads to the action matrix $[A]_{2 \times 7}$, when power losses are neglected:

$$[A]_{2 \times 7} = T_a \begin{bmatrix} \overset{\$T_a^a}{1} & \overset{\$U_a^a}{0} & \overset{\$U_b^a}{\frac{2(r_1+r_2)}{r_1}} & \overset{\$T_c^a}{-\frac{2(r_1+r_2)}{r_1}} & \overset{\$U_c^a}{0} & \overset{\$U_d^a}{1} & \overset{\$U_e^a}{\frac{(r_1+2r_2)}{r_1}} \\ 0 & -\frac{1}{r_1} & -\frac{2}{r_1} & 0 & \frac{2}{r_1} & -\frac{1}{r_1} & -\frac{1}{r_1} \end{bmatrix} \begin{matrix} T \\ U \end{matrix} \quad (45)$$

The wrenches composing Eq. (45) are used to determine the power flow direction of the ideal reducer.

Action analysis considering power losses

To consider power losses, the action graph shown in Fig. 15(b) is used to generate a new fundamental cutset matrix $[Q_A]_{3 \times 12}$, shown in Eq. (46)

$$[Q_A]_{3 \times 12} = \begin{bmatrix} \textcolor{red}{a} & \textcolor{red}{a} & \textcolor{red}{a} & \textcolor{red}{b} & \textcolor{red}{b} & \textcolor{red}{c} & \textcolor{red}{c} & \textcolor{red}{c} & \textcolor{red}{d} & \textcolor{red}{d} & \textcolor{red}{e} & \textcolor{red}{e} \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} \textcolor{red}{T}_a \\ \textcolor{red}{U}_b \\ \textcolor{red}{T}_c \end{matrix} \quad (46)$$

A new unit action matrix $[\hat{A}_D]_{2 \times 12}$ is generated including the friction torques from Tab. 5:

$$\begin{aligned} [\hat{A}_D]_{2 \times 12} &= \\ &= \begin{bmatrix} \textcolor{red}{T}_a & \textcolor{red}{T}_{a_L} & \textcolor{red}{U}_a & \textcolor{red}{T}_{b_L} & \textcolor{red}{U}_b & \textcolor{red}{T}_c & \textcolor{red}{T}_{c_L} & \textcolor{red}{U}_c & \textcolor{red}{T}_{d_L} & \textcolor{red}{U}_d & \textcolor{red}{T}_{e_L} & \textcolor{red}{U}_e \\ 1 & 1 & 0 & 1 & -r_1 - r_2 & 1 & 1 & 0 & 1 & -r_1 & 1 & -r_1 - 2r_2 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{matrix} \textcolor{red}{T} \\ \textcolor{red}{U} \end{matrix} \end{aligned} \quad (47)$$

Constitutive equations

Eqs. (46) and (47) are applied to Eq. (26), resulting in a new network action matrix $[\hat{A}_N]_{6 \times 12}$, which has $C_N = 6$. This C_N value implies that five constitutive equations defining the magnitudes of the friction torques should be appended.

The magnitude of the bushings' frictional torques on couplings a , b and c are defined by the friction model given by Eq. (35):

$$T_{a_L} = \mu r_a U_a \quad (48)$$

$$T_{b_L} = \mu r_b U_b \quad (49)$$

$$T_{c_L} = -\mu r_c U_c \quad (50)$$

where r_a , r_b and r_c are the internal radius of the bushings at couplings a , b and c , respectively. The sign equalities $\text{sign}(t_b) = \text{sign}(t_c) = \text{sign}(t_a) = 1$, $\text{sign}(U_a) = \text{sign}(U_b) = -\text{sign}(T_a) = -1$ and $\text{sign}(U_c) = \text{sign}(T_a) = 1$ were already applied in Eqs. (48), (49) and (50). The reducer's bushings are lubricated with grease, leading to a friction coefficient of $\mu = 0.15$ (THECNOLUB, 2017).

The Coulomb friction model, defined by Eq. (32), is used to calculate T_{d_L} and T_{e_L} :

$$T_{d_L} = \zeta_d U_d \quad (51)$$

$$T_{eL} = \zeta_e U_e \quad (52)$$

Eqs. (48) to (52) are appended into the new matrix $[\hat{A}_N]_{6 \times 12}$ in matrix form, generating the augmented action matrix $[D_A]_{11 \times 12}$:

$$[D_A]_{11 \times 12} =$$

	T_a	T_{aL}	U_a	T_{bL}	U_b	T_c	T_{cL}	U_c	T_{dL}	U_d	T_{eL}	U_e	
=	1	1	0	0	0	0	0	0	-1	r_1	0	0	T
	0	0	1	0	0	0	0	0	0	-1	0	0	U
	0	0	0	1	$-r_1 - r_2$	0	0	0	-1	r_1	-1	$r_1 + 2r_2$	T
	0	0	0	0	1	0	0	0	0	-1	0	-1	U
	0	0	0	0	0	1	1	0	1	$-r_1$	1	$-r_1 - 2r_2$	T
	0	0	0	0	0	0	0	1	0	1	0	1	U
	0	1	$-\mu r_a$	0	0	0	0	0	0	0	0	0	c.e.
	0	0	0	1	$-\mu r_b$	0	0	0	0	0	0	0	c.e.
	0	0	0	0	0	0	1	μr_c	0	0	0	0	c.e.
	0	0	0	0	0	0	0	0	1	$-\zeta_d$	0	0	c.e.
	0	0	0	0	0	0	0	0	0	0	1	$-\zeta_e$	c.e.

(53)

where the row label c.e. indicates that the row comes from a constitutive equation.

The matrix of Eq. (53) has $C_N = 1$ and is the coefficient matrix of the linear system of Eq. (25), which is also solved by taking T_a as primary variable, leading to a new action matrix:

$$[A]_{2 \times 12} =$$

	$\$T_a^a$	$\$T_{aL}^a$	$\$U_a^a$	$\$T_{bL}^a$	$\$U_b^a$	$\$T_c^a$	
=	T_a	$-\mu\gamma_1 r_a$	0	$-\mu\gamma_1\gamma_2\gamma_3 r_b$	$\gamma_1\gamma_2\gamma_3 (r_1 + r_2)$	$-\gamma_1\gamma_2\gamma_3 (r_1 + r_2 - \mu r_b - \mu r_c)$	...
	0	0	$-\gamma_1$	0	$-\gamma_1\gamma_2\gamma_3$	0	
...	$-\mu\gamma_1\gamma_2\gamma_3 r_c$	0	$-\gamma_1\zeta_d$	$\gamma_1 r_1$	$-\gamma_1\gamma_2\zeta_e (r_2 + \zeta_d - \mu r_b)$	$\gamma_1\gamma_2 r_0 (r_2 + \zeta_d - \mu r_b)$	T
	0	$\gamma_1\gamma_2\gamma_3$	0	$-\gamma_1$	0	$-\gamma_1\gamma_2 (r_2 + \zeta_d - \mu r_b)$	U

(54)

where $\gamma_1 = \frac{T_a}{r_1 - \zeta_d + \mu r_a}$, $\gamma_2 = \frac{1}{r_2 - \zeta_e + \mu r_b}$ and $\gamma_3 = 2r_2 + \zeta_d - \zeta_e$.

Virtual power flow and Coulomb friction factors

As seen in Sec. 3.4, the virtual power flow defines the roles of the gears in a gear pair. Determining the gears roles is essential to compute the Coulomb friction factors ζ_d and ζ_e .

The carrier of the PGT shown in Fig. 14(a) is link 3. The twists relative to the carrier 3 can be obtained through the reduced incidence matrix $[l_3]_{3 \times 5}$, which is determined by inspecting the coupling graph of Fig. 14(b):

$$[l_3]_{3 \times 5} = \begin{array}{ccccc} & a & b & c & d & e \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \end{bmatrix} & 0 & 1 & 2 \end{array} \quad (55)$$

where the element $i_{i,j}$ of l_3 is 1 if node i is the starting node of the edge e_j , -1 if node i is the arrival node of the edge e_j and 0 if edge e_j is not incident to node i . The pseudoinverse of $[l_3]_{3 \times 5}$ is calculated through Eq. (31):

$$[l_3^\dagger]_{3 \times 5} = \begin{array}{ccc} & 0 & 1 & 2 \\ \begin{bmatrix} 1/8 & -1/2 & -1/8 \\ 3/8 & 1/2 & 5/8 \\ 5/8 & 1/2 & 3/8 \\ 1/8 & 1/2 & -1/8 \\ 1/4 & 0 & -1/4 \end{bmatrix} & a & b & c & d & e \end{array} \quad (56)$$

The absolute motion matrix $[M_3]_{2 \times 3}$ is obtained by applying Eqs. (40) and (56) in Eq. (30):

$$[M_3]_{2 \times 3} = t_a \begin{array}{ccc} & {}^3\$_0^m & {}^3\$_1^m & {}^3\$_2^m \\ \begin{bmatrix} -\frac{r_1}{2(r_1+r_2)} & \frac{r_0}{2(r_1+r_2)} & -\frac{r_1 r_0}{2r_2(r_1+r_2)} \\ 0 & 0 & -\frac{r_1 r_0}{2r_2} \end{bmatrix} & t & u \end{array} \quad (57)$$

where the superscript 3 on the columns labels indicates that the twists refer to link 3, the PGT carrier.

Power losses do not change the power flow direction, so they can be neglected in the driver/follower relationship inspection (LAUS; SIMAS; MARTINS, 2012). Neglecting power losses in the power flow direction analysis generates simpler expressions, easing the turning conditions evaluation.

The virtual power flowing through coupling d , i.e. the power exchanged by gears 1 and 2, is calculated applying the sixth column of $[A]_{2 \times 7}$ shown in Eq. (45) and the second column of $[M_3]_{2 \times 3}$, shown in Eq. (57), in Eq. (28), resulting in:

$$\mathcal{P}_{12} = \$_{U_d}^a \cdot {}^3\$_1^m = T_a t_a \frac{r_0}{2(r_1 + r_2)} \quad (58)$$

Note that $\mathcal{P}_{12} > 0$, as $T_a t_a > 0$, meaning that the virtual power flows in the same direction of the edge d connecting links 1 and 2 in the coupling graph shown in Fig. 14(b), i.e. virtual power flows from gear 1 to gear 2, thus gear 1 drives gear 2. Therefore, Eq. (33) becomes Eq. (59) for calculating ζ_d :

$$\zeta_d = \frac{r_1 r_2 (1 - \eta_{ext})}{r_1 + \eta_{ext} r_2} \quad (59)$$

where η_{ext} is the ordinary efficiency for external meshing.

The virtual power flowing through coupling e , i.e. the power exchanged by gears 0 and 2, is calculated applying the seventh column of $[A]_{2 \times 7}$ shown in Eq. (45) and the third column of $[M_3]_{2 \times 3}$, shown in Eq. (57), in Eq. (28), resulting in:

$$\mathcal{P}_{02} = \$_{U_e}^a \cdot {}^3\$_2^m = -T_a t_a \frac{r_0}{2(r_1 + r_2)} \quad (60)$$

The virtual power flow $\mathcal{P}_{02}$ is always negative, since $T_a t_a > 0$. As $\mathcal{P}_{02} < 0$, the virtual power flows in the opposite direction of the edge e connecting links 0 and 2 in the coupling graph shown in Fig. 14(b). So power flows from gear 2 to ring gear 0, meaning that gear 2 drives gear 0. Thus, ζ_e is calculated according to Eq. (61), adapted from Eq. (33):

$$\zeta_e = \frac{r_2 r_0 (1 - \eta_{int})}{r_2 - \eta_{int} r_0} \quad (61)$$

where η_{int} is the ordinary efficiency for internal meshing.

After defining ζ_d and ζ_e , the bevel gear train efficiency can be calculated.

Efficiency

To compute the PGT efficiency, the power provided by the motor, which enters through coupling a , and the power that leaves through coupling c , and is effectively consumed by the robot, are calculated using Eq. (28). The twist $\$_a^m$ and the wrench $\$_{T_a}^a$ which quantifies the power supplied by the motor are the first columns of $[M]_{2 \times 5}$ (Eq. (40)) and $[A]_{2 \times 12}$ (Eq. (54)), respectively. The twist $\$_c^m$ and the wrench $\$_{T_c}^a$ of the output coupling c are the third column of $[M]_{2 \times 5}$ and the sixth column of $[A]_{2 \times 12}$, respectively. So, according to Eq. (29), the efficiency of the PGT is given by:

$$\eta_{PGT} = \frac{\mathcal{P}_c}{\mathcal{P}_a} = -\frac{\$^m_c \cdot \$^a_{T_c}}{\$^m_a \cdot \$^a_{T_a}} = \frac{r_1 (2r_2 + \zeta_d - \zeta_e) (r_1 + r_2 - \mu r_b - \mu r_c)}{(r_0 + r_1) (r_1 - \zeta_d + \mu r_a) (r_2 - \zeta_e + \mu r_b)} \quad (62)$$

Note that η_{PGT} depends only on the bushings' friction coefficient and gears' dimensions and the DC motor speed and torque do not impact the reducer efficiency. Besides, the coefficients μ , ζ_d and ζ_e are speed-independent. Therefore, η_{PGT} is constant during the robot's travel and does not depend on the robot speed. This feature is due to the fact that the PGT has only 1 DOF and it is fed by a single propulsion source, the traction motor.

Optimization of efficiency, energy consumption or fuel economy is not a step composing the method developed in the Thesis. However, the Thesis method can be used to build optimization problems. The efficiency of Eq. (62) is the cost function of an optimization problem which aims to determine the gears dimensions that make the gear train the most efficient as possible.

5.3.2 Reducer Efficiency Optimization

Engineering problems can be translated into optimization problems, where a descriptive statement of the problem is converted into a well defined mathematical statement used to find an optimal solution. An optimization problem is mainly composed of the following elements (ARORA, 2004; FERNANDES, 2020):

- *Design variables*: a set of variables that describe the problem. The design variables are the parameters which are changed to produce different outputs in the analysis. Each design variable is independent from the others.
- *Cost function or objective function*: mathematical formulation representing a criterion to be minimized or maximized. The cost function enables the comparison among different sets of design variables to select the set that gives the optimum design.
- *Constraint functions*: engineering problems usually have some constraints which must be respected during the search of the optimum design. For example, the design of a structure that must not fail under defined operating loads. The constraints are expressed by the constraint functions, which are as function of the design variables, as these parameters vary in the optimization.

In this Thesis, the optimization problem is solved through a genetic algorithm (GA) belonging to the optimization tool of MATLAB, developed by MathWorks. The GA is based on a natural selection process that mimics biological evolution. The algorithm continually modifies a population of individual solutions. At each step, individuals from the current population are randomly selected by the GA to parent the children for the next generation. This way, over successive generations, the population evolves towards an optimal design (MATHWORKS, 2004).

In this Chapter, the cost function of the optimization problem is the reducer efficiency, given by Eq. (62). The design variables are the gears' module m and z_1 and z_2 , the number of teeth of the sun gear and the planet gear, respectively. The design variables do not explicitly compose the cost function, but η_{PGT} in Eq. (62) implicitly contains m , z_1 and z_2 , as $r_1 = mz_1/2$, $r_2 = mz_2/2$ and $r_0 = r_1 + 2r_2$. The module m is a discrete design variable that assumes values between 0.1 mm and 2 mm, belonging to a standard set according to Metric Gearing Standards ISO (ISO, 1996). It is imposed that z_1 and z_2 are integer design variables.

The lower and upper bounds of the number of teeth for the PGT optimization are 18 and 300, for both z_1 and z_2 . The minimum number of teeth is set as 18 to avoid undercutting against a standard rack with a pressure angle of 20° (NORTON, 2010). The choice of 300 for the upper bound is due to the fact the CAD software used to design the reducer imposes a limit of 300 teeth to automatically generate standard gears.

Originally, the gears composing the PGT have $m = 0.35$ mm and the sun gear has $z_1 = 22$ teeth, while the planet gear has $z_2 = 88$ teeth, leading to a speed ratio of $i_{PGT}^{original} = 10$. Applying these parameters in Eq. (62) gives an efficiency of $\eta_{PGT}^{original} = 78.81\%$. The optimization will provide the PGT dimensions that maximizes the efficiency, considering the constraints functions to which the cost function is subject. The constraint functions are related to the speed ratio, the size of the reducer and the maximum stresses the teeth can withstand.

The speed ratio i_{PGT} of the PGT must belong to the interval $10 \leq i_{PGT} \leq 14$. The size constraints impose that the pitch radii of the gears are greater than or equal to their shafts radii and that the reducer fits inside the housing. The DC motor output shaft has a diameter of 7.68 mm, so the pitch diameter d_1 of the PGT's sun gear must be $d_1 \geq 7.68$ mm. Finally, the shaft of the planet gears composing the PGT has a diameter of 4 mm, meaning that the planet gears' pitch diameter d_2 must be $d_2 \geq 4$ mm. The shafts' diameters were measured from the prototype's motor (MOTRON, 2019).

The housing is assumed as a parallelepiped with a square face of sides $H = 150$ mm and a base with $e = 30$ mm of depth, setting a maximum volume of $V_{max} = H^2 e$ for the reducer. The reducer volume is calculated by adding the volume of each individual gear. The volume of an external gear with solid structure V_{ext} and the volume of an internal gear V_{int} are defined according to Eqs. (63) and (64), respectively (WANG, C.; SHI, 2015; WANG, C., 2019):

$$V_{ext} = \frac{\pi b}{4}(mz)^2 - \frac{\pi b}{4}d_{zh}^2 - V'' \quad (63)$$

$$V_{int} = \frac{\pi b}{4}[d_v^2 - (mz)^2] - V'' \quad (64)$$

where b is the tooth's face width, m is the module, z is the number of teeth, d_v is the diameter of the internal gear's external circle, V'' is the clearance volume, and d_{zh} is the diameter of the gear shaft (assumed as $d_{zh} = 0$, since it does not interfere in the reducer fitting in the housing). The nominal value for the face width is $b = 12m$ (NORTON, 2010).

The diameter of the internal gear's external circle d_v is computed as (SDP, 2019; NORTON, 2010):

$$d_v = 2m_b h_t + d_f \quad (65)$$

where m_b is the backup ratio, assumed as $m_b = 1.2$ (NORTON, 2010), h_t is the whole depth of the teeth and d_f is the root diameter of the internal gear.

The clearance volume V'' is calculated as:

$$V'' = s_a c^{*'} b z \quad (66)$$

where s_a is the addendum circle thickness and $c^{*'}$ is the clearance height (WANG, C.; SHI, 2015).

The PGT volume V_{PGT} is computed by adding the results of Eq. (63) and (64):

$$V_{PGT} = V_{int} + V_{ext}^s + k V_{ext}^p \quad (67)$$

where V_{int} is the volume of the ring gear, V_{ext}^s is the volume of the sun gear, V_{ext}^p is the volume of the planet gear and k is the number of planet gears composing the PGT (assumed here as $k = 3$) (WANG, C., 2019).

The remaining size constraint imposes that the diameter of the ring gear's external circle is lesser than or equal to the housing height H :

$$d_v \leq H \quad (68)$$

where d_v is calculated through Eq. (65).

The third constraint category is related to the stresses to which the gears teeth are submitted. In this Thesis, two kinds of stress are considered: bending stress at the root of the tooth (σ_b) and contact stress at the tooth surface or pitting resistance (σ_c). The AGMA bending stress equation for spur gears is given by (NORTON, 2010; AGMA, 2004):

$$\sigma_b = \frac{W_t}{bmJ} \frac{K_a K_m}{K_v} K_S K_B K_I \quad (69)$$

where W_t is the tangential force acting on the tooth flank, J is the bending strength geometry factor, K_a is the application factor, K_v is the dynamic factor, K_m is the load distribution factor, K_S is the size factor, K_B is the rim thickness factor and K_I is the idler factor.

Equation (69) is valid only for external gears and AGMA does not have an official standard to calculate bending stresses in internal gears. An adaptation of the AGMA method for internal gears is adopted in this work (LISLE; SHAW; FRAZER, Robert C, 2019):

$$\sigma_b^{int} = \frac{W_t K_f}{b Y} \quad (70)$$

where Y is the form factor and K_f is the stress concentration factor.

The peak of the bending stress σ_b must be lesser than the corrected allowable bending stress S_{fb} , calculated as (NORTON, 2010):

$$S_{fb} = \frac{K_L}{K_T K_R} S'_{fb} \quad (71)$$

where K_L is the life factor, K_T is the temperature factor, K_R is the reliability factor and S'_{fb} is the published AGMA bending-fatigue strength.

The safety factor (SF_b) against bending failure is given by the ratio (NORTON, 2010):

$$SF_b = \frac{S_{fb}}{\sigma_b} \quad (72)$$

The AGMA pitting resistance formula is given by (AGMA, 2004; NORTON, 2010):

$$\sigma_c = C_p \sqrt{\frac{W_t K_a K_m}{b l d} \frac{K_S C_f}{K_v}} \quad (73)$$

where d is the pitch diameter of the smaller of the two gears in mesh, l is a dimensionless surface geometry factor, C_p is an elastic coefficient and C_f is the surface finish factor.

Similarly to the bending stress, the pitting resistance σ_c must be lesser than the corrected allowable contact stress S_{fc} :

$$S_{fc} = \frac{C_L C_H}{K_T K_R} S'_{fc} \quad (74)$$

where C_L is the surface-life factor, C_H is the hardness ratio factor for pitting resistance and S'_{fc} is the published AGMA surface-fatigue strength.

The safety factor (SF_c) against surface failure is given by the ratio (NORTON, 2010):

$$SF_c = \left(\frac{S_{fc}}{\sigma_c} \right)^2 \quad (75)$$

The stress constraint in the optimization problem is defined by the safety factors SF_b and SF_c , which must be greater than or equal to 1.6 (NAUNHEIMER et al., 2011).

The gears are made of carburized and case hardened steel, so $S'_{fb} = 520$ MPa and $S'_{fc} = 1300$ MPa (NORTON, 2010). How the gear design factors are defined is found in Appendix B. A more detailed explanation can be found in the books of Norton (2010) and Budynas and Nisbett (2020). Tab. 6 summarizes the factor values adopted in this work which are constant.

Table 6 – Constant gear design factors

K_a	K_m	K_S	K_B	K_I	C_p [MPa ^{0.5}]	C_f	K_L	K_T	K_R	C_L	C_H
1	1.6	1	1	1	191	1	1	1	1	1	1

Source: From the author.

The dynamic factor K_V depends on the pitch-line velocity, which depends on the gears pitch radii (AGMA, 2004). Since the gears size change during the optimization, K_V is recalculated at each evaluation of the objective function, so it is not included in Tab. 6. That is the same reason why the bending strength geometry factor J , the form factor Y , the stress concentration factor K_f and the surface geometry factor I are not included neither. J and Y depend on the number of teeth, which corresponds to two design variables of the optimization, while K_f is related to the module, the remaining design variable (BUDYNAS; NISBETT, 2020). I is computed from the radii of the curvature of the gear teeth, which also depend on the gears size (AGMA, 2004).

Equation (76) summarizes the optimization problem of this Chapter:

Maximize $\eta_{PGT}(m, z_1, z_2) \rightarrow \text{Eq. (62)}$

Subject to

$$\left\{ \begin{array}{l} m \in \text{standard ISO module set} \\ 18 \leq z_1 \leq 300 \\ 18 \leq z_2 \leq 300 \\ 10 \leq i_{PGT} \leq 14 \\ d_1 \geq 7.68 \text{ mm} \\ d_2 \geq 4 \text{ mm} \\ V_{PGT} \leq V_{max} \\ d_v \leq H \\ SF_b \geq 1.6 \\ SF_c \geq 1.6 \end{array} \right.$$

(76)

5.3.3 Inspection robot's power consumption

In this Section, the remaining steps of the method presented in Chapter 4 are applied to calculate the robot's power consumption. Since the inspection robot is a

purely electric machine, its consumption is quantified by its batteries' SOC, given by the 14th step of the Thesis method.

The power demanded to displace the robot on the conductor depends on the traction force required to execute the motion. The required traction force so that the robot moves along the cable is computed through Eq. (77), obtained from Eq. (7) by neglecting aerodynamic drag:

$$F_t = M_r g [f_r \cos(\alpha) + \sin(\alpha)] + M_r \delta \frac{dv}{d\tau} \quad (77)$$

where F_t is the traction force, M_r is the robot mass, g is the gravity acceleration, f_r is the rolling resistance coefficient, α is the slope angle, δ is the rotational inertia factor, v is the robot linear speed and $dv/d\tau$ stands for the robot linear acceleration. The robot mass is $M_r = 12.8$ kg and corresponds to the mass of the final prototype. The gravity acceleration is taken as $g = 9.81$ m/s². The value of the rolling resistance coefficient is $f_r = 0.074$ and can be found in (SOUZA et al., 2021a) and (SOUZA et al., 2021b). The rotational inertia factor is assumed as $\delta = 1$, supposing that the increase of equivalent mass due to the angular moments of the rotating components is negligible (EHSANI et al., 2018). The speed variation $dv/d\tau$ is assumed as linear with time, with a time step of 1 s (EHSANI et al., 2018).

The calculation of the traction force via Eq. (77) corresponds to the third step of the Thesis method: "3. Computation of the vehicle tractive force".

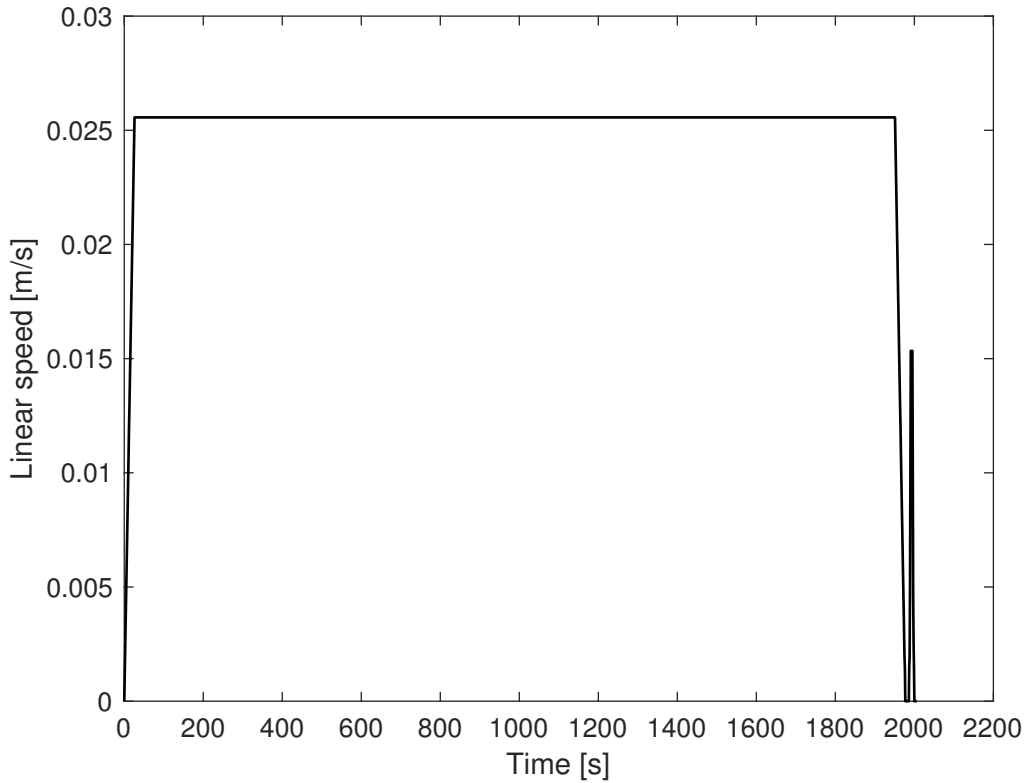
The robot's linear speed is given by:

$$v = \frac{\omega r_{disc}}{i_0 i_{PGT}} \quad (78)$$

where ω is the motor speed in rad/s, r_{disc} is the radius of the robot's inner disc in meters, i_0 is the reduction ratio of the pulley/belt and i_{PGT} is the speed ratio of the motor reducer. The prototype's inner disc radius is $r_{disc} = 0.0525$ m and the pulley/belt transmission provides a reduction ratio of $i_0 = 6$.

Fig. 16 shows the speed profile adopted in this work, representing the displacement rate of the inspection robot while it travels along a cable connecting two consecutive poles with the same height. In the first 25 s, the robot departs from a pole and accelerates until reaching its cruising speed, corresponding to the motor nominal speed (279 rpm). The robot's speed is kept constant until the robot is a few seconds away from the next pole. Then, the robot decelerates, stops for a few seconds to cease oscillations provoked by the displacement and accelerates again, to reach the nearest pole and overcome the vertical insulator on the top of the pole. The robot's speed to overcome an obstacle corresponds to 60% of the motor nominal speed. After surpassing the insulator, the robot decelerates again until it stops. The speed profile of Fig. 16 can be repeated as many times as necessary to simulate the robot displacement over more consecutive poles.

Figure 16 – Robot's linear speed profile



Source: From the author.

The use of the speed profile shown in Fig. 16 to estimate the robot's consumption corresponds to the second step of the Thesis method: "*2. Selection of the vehicle speed profile*".

The slope angle α depends on the cable length and the robot's position. The cable length can be defined using two catenary equations: firstly, the catenary sag is computed and then the cable length is estimated. Eq. (79) determines the catenary sag s (LABEGALINI et al., 1992):

$$s = \frac{pL^2}{8T_0} \quad (79)$$

where p is the cable unit weight, L is the distance between two consecutive poles and T_0 is the conductor design tension.

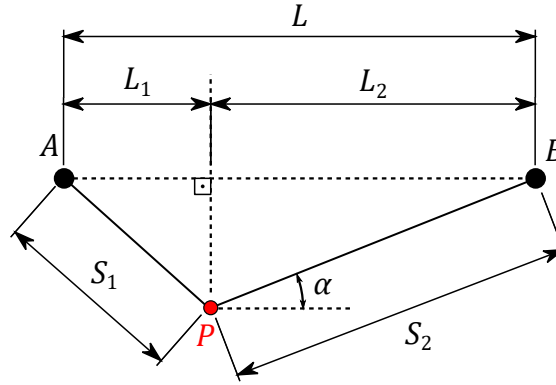
The cable length S is defined according to Eq. (80):

$$S = L + \frac{8s^2}{3L} \quad (80)$$

The parameters to calculate s and S are $p = 0.296$ daN/m, $L = 50$ m and $T_0 = 348$ daN. p and T_0 come from the manual developed for Celesc (ENGENHARIA

E NORMAS - DVEN, 2014), while L is the average distance between consecutive poles in urban areas covered by Celesc.

Figure 17 – Simplified geometry of the conductor when the robot moves along the cable.



Source: From the author.

The robot's position on the conductor is defined through a simplified geometric representation of the cable state when the robot is placed on it, shown in Fig. 17. The simplified geometry enables the use of trigonometric relations to determine the slope angle α according to the robot's position. In Fig. 17, points A and B are two consecutive poles and the robot's position is represented by the point P. The point P divides the distance L between the poles into L_1 and L_2 and the cable length S into S_1 and S_2 . Thus:

$$L = L_1 + L_2 \quad (81)$$

$$S = S_1 + S_2 \quad (82)$$

Assuming that the robot departs from pole A, the distance already travelled by the robot is S_1 and is given by:

$$S_1(\tau) = \int_0^\tau v d\tau \quad (83)$$

where v is the robot linear speed and τ is the instant when the robot is in the P position. S_2 is determined by replacing the solution of Eq. (83) in Eq. (82).

The distance L_1 is defined through trigonometry, resulting in Eq. (84):

$$L_1(\tau) = \frac{2SS_1(\tau) - S^2 + L^2}{2L} \quad (84)$$

The distance L_2 is found by substituting Eq. (84) in Eq. (81). The angle α between the conductor and the horizontal is given by the ratio between L_2 and S_2 and is updated during the robot displacement:

$$\alpha(\tau) = \arccos \left(\frac{L_2(\tau)}{S_2(\tau)} \right) \quad (85)$$

The traction force multiplied by the robot linear speed gives the power needed to the robot displacement, i.e. the traction power $\mathcal{P}_t$. The robot power sources, the batteries and the photovoltaic plates, should provide more power than the power required by the displacement, to compensate for the motor and transmission losses. The additional power is computed by considering the belt transmission efficiency η_{belt} , the reducer efficiency η_{PGT} and the motor efficiency η_m in $\mathcal{P}_t$ calculation, resulting in Eq. (86)

$$\mathcal{P}_t(\tau) = \frac{F_t(\tau)v(\tau)}{\eta_{belt}\eta_{PGT}\eta_m} \quad (86)$$

The motor efficiency is considered constant as $\eta_m = 0.68$, according to the motor's datasheet (MOTRON, 2019), and the timing belt transmission efficiency is $\eta_{belt} = 0.97$ (BUDYNAS; NISBETT, 2020). Defining $\eta_m = 0.68$ is a simplification of the step "9. Definition of M efficiency" detailed in Chapter 4.

Besides the traction power $\mathcal{P}_t$, the power demanded by the robot ($\mathcal{P}_{dem}$) include some auxiliary loads regarding the electronic components used during robot operation. In this work, three sets of components are taken into consideration and their power demand is supposed to be constant. They are the microprocessor, the sensors and the half-bridge, whose power is denoted by $\mathcal{P}_{mic}$, $\mathcal{P}_{sens}$ and $\mathcal{P}_{hb}$, respectively. Thus, the power demanded during the robot operation is calculated through Eq. (8):

$$\mathcal{P}_{dem}(\tau) = \mathcal{P}_t(\tau) + \mathcal{P}_{mic} + \mathcal{P}_{sens} + \mathcal{P}_{hb} \quad (87)$$

The values adopted for $\mathcal{P}_{mic}$, $\mathcal{P}_{sens}$ and $\mathcal{P}_{hb}$ are 10 W, 10 W and 6 W, respectively, following the maximum power magnitudes assumed by Coelho et al. (2021) in the simulation of the inspection robot's power flow control.

Using Eq. (87) to compute the power demanded by the robot corresponds to the sixth step of the Thesis method: "6. Computation of the vehicle power demand".

The batteries and the photovoltaic plates provide power to meet the power demand $\mathcal{P}_{dem}$, leading to the power balance equation given by Eq. (88) (COELHO et al., 2021):

$$\mathcal{P}_{dem}(\tau) = \mathcal{P}_{PV} \pm \mathcal{P}_{bat}(\tau) \quad (88)$$

where $\mathcal{P}_{PV}$ is the power supplied by the photovoltaic plates and $\mathcal{P}_{bat}(\tau)$ is the power provided or absorbed by the batteries. The robot has six photovoltaic plates, each one producing at most 4.4 W of power. In this work, it is assumed that the photovoltaic plates operate on their maximum power during the whole trajectory, supplying a constant power of $\mathcal{P}_{PV} = 26.4$ W.

In Eq. (88), the sign of $\mathcal{P}_{bat}(\tau)$ depends on the $\mathcal{P}_{dem}(\tau)$ intensity and indicates if the battery is being charged or discharged, influencing the batteries SOC.

Determining $\mathcal{P}_{bat}(\tau)$ through Eq. (88) corresponds to step 13 of the Thesis method: "*Computation of the real-time charging or discharging power of the battery pack*".

The current SOC is determined from the batteries power $\mathcal{P}_{bat}(\tau)$ throughout the robot operation. Based on SOC estimation of electric vehicles, the battery SOC at each instant τ of the robot's speed profile is given by Eq. (17). The SOC calculation corresponds to step 14 of the Thesis method: "*Computation of the battery pack current state-of-charge (SOC)*".

The initial energy capacity E_0 for the inspection robot is calculated using Eq. (18), but considering the amount of batteries installed:

$$E_0 = 3600 n_{bat} c_{bat} V_{bat} \quad (89)$$

where n_{bat} is the number of batteries, c_{bat} is the battery capacity and V_{bat} is the battery voltage. The inspection robot has $n_{bat} = 8$ batteries, each one with $c_{bat} = 2.2$ Ah and $V_{bat} = 11.1$ V.

5.3.3.1 Batteries SOC

According to the power balance of Eq. (88), if the robot power demand is higher than the power produced by the photovoltaic plates, i.e. $\mathcal{P}_{dem}(\tau) > \mathcal{P}_{PV}$, the batteries must provide the remaining power that fulfills the power demand. Consequently, $\mathcal{P}_{bat}(\tau)$ is positive and the batteries are discharged, decreasing their SOC, according to Eq. (17).

On the other hand, if the power demand is lower than the power provided by the photovoltaic plates, i.e. $\mathcal{P}_{dem}(\tau) < \mathcal{P}_{PV}$, the batteries do not need to supply any power and can be even recharged, as the photovoltaic plates provide all the power required and even more. If the SOC level is above a threshold SOC_b and the photovoltaic plates provide enough power to move the robot, the batteries do not provide any power, so $\mathcal{P}_{bat}(\tau) = 0$ and the SOC remains unchanged. In contrast, when the SOC level is below SOC_b , the batteries are charged and the SOC increases, meaning that $\mathcal{P}_{bat}(\tau)$ is negative. In this Chapter, it is assumed $SOC_b = 0.4$.

The pseudo-code to compute the batteries SOC is shown in Algorithm 1. In the pseudo code, τ_f is the time to travel a span of 50 m between consecutive poles A and B and $\Delta\tau$ is the time interval when the transition between consecutive spans occurs. It is assumed that $\Delta\tau = 18$ s, according to the tests made with the prototype.

5.3.4 Results and Discussion

The PGT efficiency is optimized following the procedure described in Sec. 5.3.2. After 59 generations and the cost function being evaluated 2401 times, the GA

Algorithm 1 Pseudo-code to compute the batteries SOC

```

1: Compute the catenary sag  $s$  and the cable length  $S$  using Eq. (79) and (80)
2: for each instant  $\tau$  of the speed profile do
3:   if the robot position  $P$  is between poles  $A$  and  $B$ , i.e.  $A \leq r \leq B$  and  $\tau < \tau_f - (\Delta\tau + 2)$ 
   then
4:     Compute  $\alpha$  using Eqs. (81) to (85)
5:     else if the robot is approaching pole  $B$  and  $\tau_f - (\Delta\tau + 2) \leq \tau \leq \tau_f - \Delta\tau$  then
6:        $\alpha(\tau) \leftarrow \max(\vec{\alpha}), \vec{\alpha} = [\alpha(0), \alpha(1), \dots, \alpha(\tau - 1)]^T$ 
7:     else if the robot is moving away from pole  $B$  and  $\tau_f - \Delta\tau \leq \tau \leq \tau_f$  then
8:        $S_1(\tau) \leftarrow 0$ 
9:        $inc \leftarrow \max(\vec{\alpha})/(\Delta\tau + 1)$ 
10:       $\alpha(\tau) \leftarrow \alpha(\tau - 1) - inc$ 
11:    end if
12:    Compute  $F_t(\tau)$  using Eq. (77) and  $\mathcal{P}_t(\tau)$  using Eq. (86)
13:    Compute  $\mathcal{P}_{dem}(\tau)$  using Eq. (87)
14:    if  $\mathcal{P}_{dem}(\tau) > \mathcal{P}_{PV}$  then
15:       $\mathcal{P}_{bat}(\tau) \leftarrow \mathcal{P}_{dem}(\tau) - \mathcal{P}_{PV}$ 
16:      Compute the integration of  $\mathcal{P}_{bat}(\tau)$  from 0 to  $\tau$  s
17:      Compute  $SOC(\tau)$  using Eq. (17)
18:       $SOC_0 \leftarrow SOC(\tau)$ 
19:    else
20:       $SOC(\tau) \leftarrow SOC(\tau - 1)$ 
21:      if  $SOC(\tau) < SOC_b$  then
22:         $SOC(\tau) \leftarrow SOC(\tau) + (\mathcal{P}_{PV} - \mathcal{P}_{dem}(\tau))d\tau/E_0$   $\triangleright d\tau = 1s$ 
23:        if  $SOC(\tau) > 1$  then
24:           $SOC(\tau) \leftarrow 1$ 
25:        end if
26:      end if
27:    end if
28: end for

```

converged to the local minimum $m = 0.5$ mm, $z_1 = 32$ and $z_2 = 128$. These values of module and number of teeth give a PGT with a speed ratio of $i_{PGT}^{opt} = 10$ and an efficiency of $\eta_{PGT}^{opt} = 87.75\%$, respecting the constraint functions listed in Sec. 5.3.2. Tab. 7 presents the design variables, the speed ratio and the efficiency of the original and the optimized motor reducer.

Table 7 – Comparison between the original PGT and the local minimum of the PGT optimization

	m [mm]	z_1	z_2	i_{PGT}	η_{PGT}
Original	0.35	22	88	10	78.81%
Optimized	0.5	32	128	10	87.75%

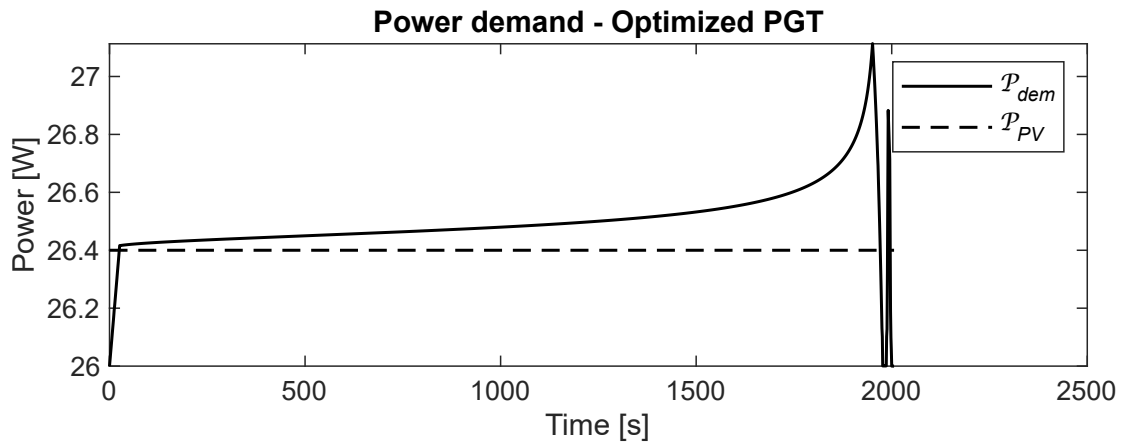
Source: From the author.

Note that $i_{PGT}^{opt} = i_{PGT}^{original} = 10$. This way, the inspection robot follows the same

speed profile of Fig. 16 with both the original and the optimized configurations. So, the batteries' SOC at the end of the route can be directly compared, as the only parameter distinguishing both configurations in the SOC calculation is the reducer efficiency η_{PGT} in Eq. (86), which calculates the traction power.

The difference in efficiency between the original PGT and the optimized PGT is 8.94 percentage points, which compensates for the belt losses. The impact of this difference on the robot consumption is evaluated by simulating the robot travelling along a cable connecting two poles, following the speed profile of Fig. 16, and comparing the batteries SOC at the end of the route for both configurations.

Figure 18 – Robot power demand with the optimized PGT reducer.



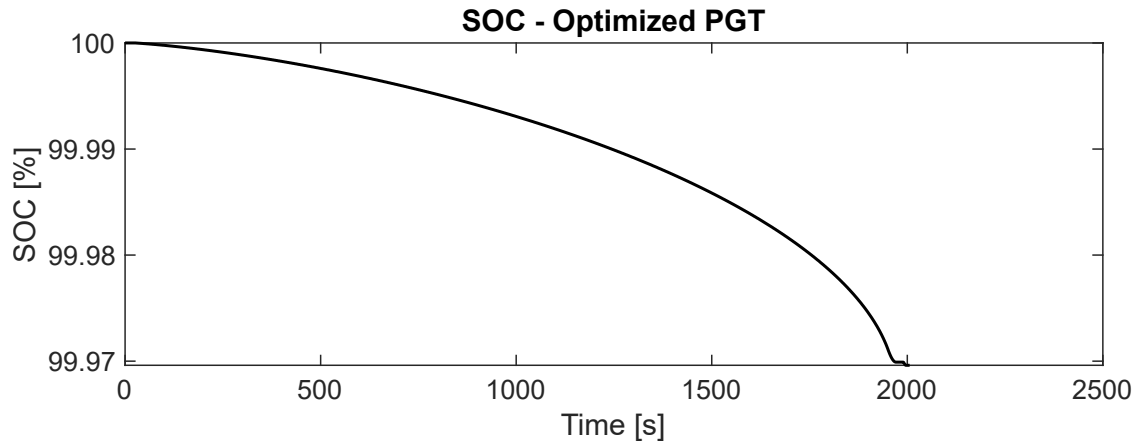
Source: From the author.

Fig. 18 shows the robot's power demand with the optimized PGT reducer, as a function of time, when it is traveling between two consecutive poles. The curve in full line stands for the power demand and the curve in dashed line represents the power provided by the photovoltaic plates. The power demand curve for the original PGT is practically the same since the only difference in the power demand computation between both cases is the reducer efficiency η_{PGT} . That is the reason why only the consumption plot of the optimized PGT is shown in this Chapter.

The similarity between the power demand curves indicates that the difference in reducer efficiency does not have a significant impact on the robot's consumption. To confirm this statement, the batteries' SOC after concluding the route, i.e. the final SOC, is compared for both cases. Fig. 19 shows the batteries' SOC during the robot's travel along a cable connecting two consecutive poles, as a function of time, when the motor reducer is the optimized PGT.

Similarly to the power demand curves, the SOC curves for the original and optimized PGTs are nearly identical, so only the curve related to the optimized PGT is shown in this Chapter, in Fig. 19. The final SOC for both cases is practically the same: when the reducer of the motor is the original PGT, the batteries' final SOC is 99.95%. If

Figure 19 – Batteries SOC when with the optimized PGT reducer.

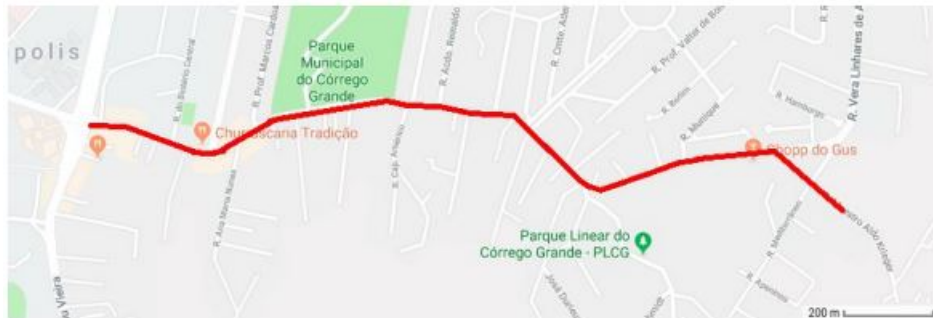


Source: From the author.

the reducer is the optimized PGT, the final SOC goes to 99.97%, an almost insignificant increase.

The influence of the reducer efficiency increases with the distance travelled by the robot. This effect can be seen by simulating the robot travelling along the pilot network adopted in the project, shown in Fig. 20. The pilot network is composed of 58 poles and connects UFSC to the Trindade substation, located in the Córrego Grande neighborhood in the city of Florianópolis, Brazil.

Figure 20 – Pilot network adopted in the project.



Source: From the author.

If the robot travels along 58 consecutive poles, the final SOC with the original PGT reducer is 97.35%, while the final SOC with the optimized PGT reducer is 98.27%, a difference of 0.92 percentage points.

The higher efficiency of the PGT does not bring a considerable enhancement in the robot's consumption under the conditions adopted in this Chapter. The main reason behind the low impact of η_{PGT} on the final SOC is the power supplied by the photovoltaic plates, as they provide a constant power that is high enough to meet almost all the power demanded by the robot. However, this is an ideal case, where the solar irradiation ensure that the photovoltaic plates operate on their maximum power.

Moreover, it is sought that the inspection robot remains installed in the electric network as long as possible, operating continuously without changing the batteries. Under this circumstance, considering a distance much greater than the length of a network composed of 58 poles, the SOC gain coming from the η_{PGT} optimization may be more prominent.

Therefore, the impact of the η_{PGT} maximization on the robot consumption may be more significant when the robot operates continuously under not ideal conditions.

6 FUEL ECONOMY CALCULATION OF A COMMERCIAL HEV

In Chapter 5, the impact of the powertrain efficiency on the consumption of a purely electric machine is analyzed. The inspection robot has a single propulsion source, an electric motor, and is analogue to an EV. The motor and reduction efficiencies are constant.

In this Chapter, the powertrain efficiency of a commercial HEV is studied and its fuel economy is calculated using the Davies method to integrate the ICE, M, G and transmission efficiencies, which vary with the HEV speed. At the end, the fuel economy obtained is compared to official values to prove the robustness of the method proposed in this Thesis.

The commercial HEV chosen as case study is the 1st generation Prius, shown in Fig. 21, launched by Toyota in 1997 as the world's first mass-produced hybrid passenger vehicle (MI; MASRUR, 2017). Toyota Prius is a well studied HEV; therefore, enough data is available in technical reports and scientific papers.

Figure 21 – 1st generation Toyota Prius.



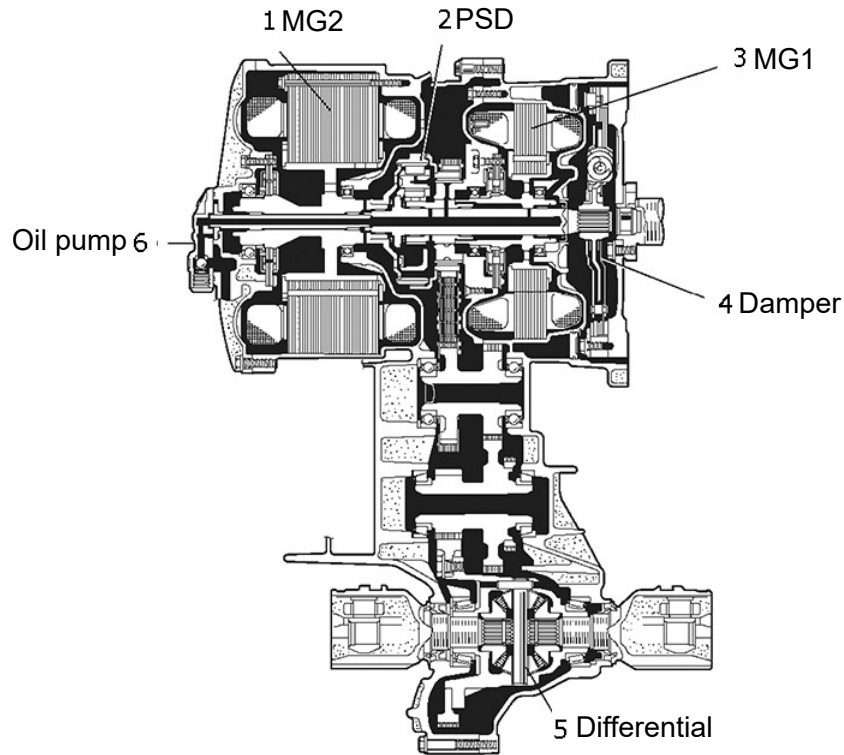
Source: Toyota (n.d.).

The transmission line-up model of the 1st generation Prius is labelled as P110 (TOYOTA-CLUB, 2021). No graphic representation of P110 was found, so the next transmission model, P111, shown in Fig. 22, is used herein to illustrate how the Prius transmission works.

In Fig. 22, MG stands for motor-generator. MG2 acts mainly as traction motor and MG1, as a generator, but both of them can operate as motor or generator, depending on the vehicle demand. PSD stands for power-split device and corresponds to a PGT, which realizes continuous variable transmission.

Fig. 23(a) shows a schematic representation of the PSD. Link 0 is the transmission housing (fixed link), link 1 is the PSD carrier, link 2 is the PSD sun gear, link 3 is the PSD planet gear and link 4 is the PSD ring gear. The ICE is connected to

Figure 22 – P111 transmission line-up model.



Source: Adapted from Toyota-Club (2021).

the carrier, while MG1 is connected to the sun gear. The ring gear is coupled to MG2 and to the final drive (MI; MASRUR, 2017).

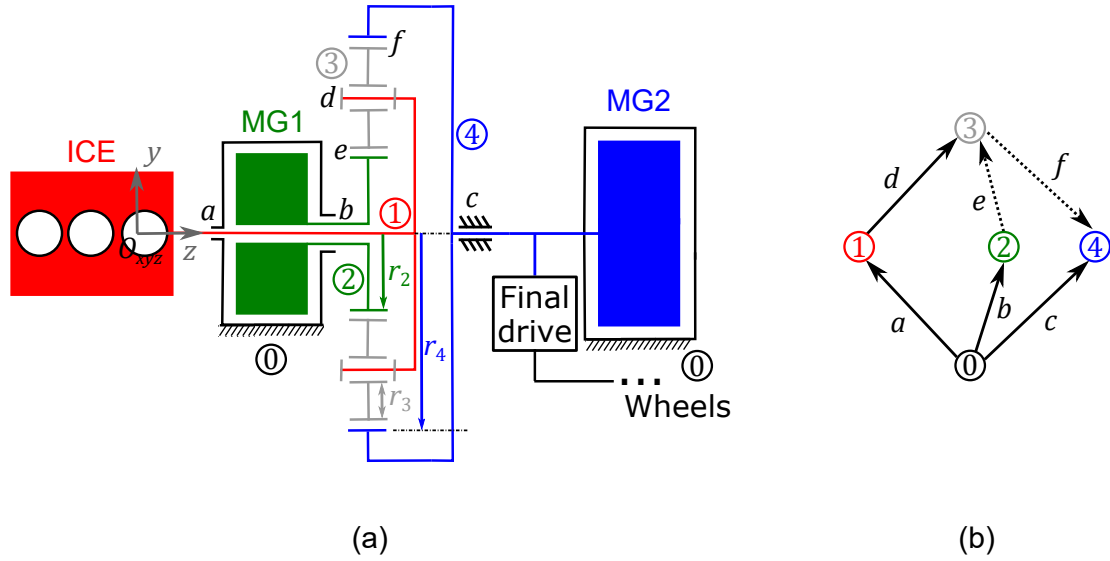
There is no clutch in the transmission, so the PSD is always running when the Prius is moving. MG2 speed is directly proportional to the vehicle speed and determines the ring gear speed. MG1 speed is controlled so that ICE speed is in its optimum efficiency operation point (see Sec. 2.2.1) (MI; MASRUR, 2017).

6.1 PSD EFFICIENCY

The PSD efficiency is determined through the Davies method, seen in Chapter 3. Since the PSD is composed exclusively by cylindrical gears, the order of the screw system is $\lambda = 2$ and the hypotheses listed in Sec. 3.2.1 are adopted. To generate the motion and action matrices, the couplings in the schematic representation shown in Fig. 23(a) are labelled from a to f . Couplings a to d are revolute couplings and couplings e and f are gear couplings. Fig. 23(b) shows the PSD coupling graph. A reference frame O_{xyz} is attached to the PSD symmetrical axis. r_2 , r_3 and r_4 are the pitch radii of the sun, planet and ring gears, respectively.

Coupling a connects the ICE to the PSD, so the carrier speed corresponds to t_a

Figure 23 – PSD (a) schematic and (b) graph representations.



Source: From the author.

and is equivalent to the ICE speed. The ICE torque is transmitted to the PSD through coupling a and has magnitude T_a . Coupling b connects MG1 to the PSD, so the sun gear speed is indicated by t_b and is equal to MG1 speed. MG1 torque magnitude is represented by T_b . Coupling c connects the ring gear to the housing and is the output of the system. The ring gear speed, which is the PSD output speed, and MG2 speed are identical and have magnitude t_c . t_c is determined by the vehicle speed through Eqs. (9) and (10). The PSD output torque magnitude is T_c and defines MG2 torque magnitude.

Motion analysis

The six couplings of the PSD have 1 DOF: a rotation around z -axis. Their motion is represented by a twist whose ISA is a unit vector on z direction. The parameters for building the twists are listed in Tab. 8.

Table 8 – Twists' parameters of the PSD

Coupling	y coordinate	ISA direction	Twist's magnitude
a	0	$\{0; 0; 1\}^T$	t_a
b	0	$\{0; 0; 1\}^T$	t_b
c	0	$\{0; 0; 1\}^T$	t_c
d	$r_2 + r_3$	$\{0; 0; 1\}^T$	t_d
e	r_2	$\{0; 0; 1\}^T$	t_e
f	r_4	$\{0; 0; 1\}^T$	t_f

Source: From the author.

The fundamental circuit matrix $[B_M]_{2 \times 6}$, the unit motion matrix $[\hat{M}_D]_{2 \times 6}$ and the network unit motion matrix $[\hat{M}_N]_{4 \times 6}$ generated during the motion analysis are presented in Appendix C.

The motion analysis of the PSD is solved using $[\hat{M}_N]_{4 \times 6}$ as coefficient matrix of the homogeneous linear system of Eq. (22). $[\hat{M}_N]_{4 \times 6}$ has $F_N = 2$, meaning that two primary variables must be used to solve the linear system. As seen in Sec. 2.2.1, when calculating a HEV fuel economy, two speeds are known: the ICE speed and the transmission output speed. The transmission output speed comes from drive cycles and the ICE speed is imposed so that the ICE efficiency is maximum.

Thus, for solving the PSD motion analysis, t_a and t_c are chosen as primary variables, giving the motion matrix $[M]_{2 \times 6}$:

$$[M]_{2 \times 6} = \begin{matrix} \begin{matrix} \$a^m \\ \$b^m \\ \$c^m \\ \$d^m \\ \$e^m \\ \$f^m \end{matrix} \\ \begin{bmatrix} t_a & t_c \frac{\delta_2 r_3 r_4}{r_2} - t_a \left(\frac{\delta_2 r_3 r_4}{r_2} - 1 \right) & t_c & \delta_2 r_4 (t_a - t_c) & \frac{\delta_1}{r_2} & -\delta_2 (r_2 + r_3) (t_a - t_c) \\ 0 & 0 & 0 & \delta_1 & \delta_1 & -\delta_1 \end{bmatrix} \end{matrix} \begin{matrix} t \\ u \end{matrix} \quad (90)$$

where $\delta_1 = \frac{r_4 (r_2 + r_3) (t_a - t_c)}{r_2 + r_3 - r_4}$ and $\delta_2 = \frac{1}{r_2 + r_3 - r_4}$.

MG1 speed, i.e. t_b , is defined from Eq. (90) and is used in Algorithm 2 to obtain MG1 efficiency.

Action analysis

According to Sec. 3.2.1, the passive constraints are forces parallel to x-axis. The propulsion sources apply external torques on the PSD: the ICE torque T_a and the MG1 torque T_b . The external torque T_c is the PSD output torque. The power losses are modelled by six external torques. There are bearing losses on the revolute couplings a , b , c and d and gear meshing friction on couplings e and f . The frictional torque due to bearing losses is modelled through Eq. (35), while the friction torque generated by gear meshing friction is modelled through Eq. (32), resulting in the equations shown in Eq. (91):

$$\begin{aligned} T_{a_L} &= -\mu r_a \text{sign}(t_a) \text{sign}(U_a) U_a \\ T_{b_L} &= -\mu r_b \text{sign}(t_b) \text{sign}(U_b) U_b \\ T_{c_L} &= -\mu r_c \text{sign}(t_c) \text{sign}(U_c) U_c \\ T_{d_L} &= -\mu r_d \text{sign}(t_d) \text{sign}(U_d) U_d \\ T_{e_L} &= \zeta_e U_e \\ T_{f_L} &= \zeta_f U_f \end{aligned} \quad (91)$$

where r_a , r_b , r_c and r_d are the internal radii of the bearings at couplings a , b , c and d , respectively. It is assumed that the bearing friction coefficient is the same for all revolute couplings. ζ_e and ζ_f are the Coulomb friction factors of gear pairs e and f .

Tab. 9 contains the parameters needed to build the wrenches of the action analysis:

Table 9 – Wrenches' parameters of the PSD

Coupling	y coordinate	Action	ISA direction	Twist's magnitude
a	0	Torque	$\{0; 0; 1\}^T$	T_a
		Torque	$\{0; 0; 1\}^T$	T_{aL}
		Force	$\{1; 0; 0\}^T$	U_a
b	0	Torque	$\{0; 0; 1\}^T$	T_b
		Torque	$\{0; 0; 1\}^T$	T_{bL}
		Force	$\{1; 0; 0\}^T$	U_b
c	0	Torque	$\{0; 0; 1\}^T$	T_c
		Torque	$\{0; 0; 1\}^T$	T_{cL}
		Force	$\{1; 0; 0\}^T$	U_c
d	$r_2 + r_3$	Torque	$\{0; 0; 1\}^T$	T_{dL}
		Force	$\{1; 0; 0\}^T$	U_d
e	r_2	Torque	$\{0; 0; 1\}^T$	T_{eL}
		Force	$\{1; 0; 0\}^T$	U_e
f	r_4	Torque	$\{0; 0; 1\}^T$	T_{fL}
		Force	$\{1; 0; 0\}^T$	U_f

Source: From the author.

The fundamental cutset matrix $[Q_A]_{4 \times 9}$, the unit action matrix $[\hat{A}_D]_{2 \times 9}$ and the network unit action matrix $[\hat{A}_N]_{8 \times 9}$, ignoring power losses, are presented in Appendix C. When power losses are ignored, $[\hat{A}_N]_{8 \times 9}$ is the coefficient matrix of Eq. (25), resulting in a system with $C_N = 1$. So a single primary variable is needed to solve the linear system. As explained in Sec. 2.2.2, the primary variable is the ICE torque T_a , whose value is imposed so that the ICE presents the highest efficiency.

The condensed action matrix $[A]_{2 \times 6}$ of the PSD, ignoring power losses, is:

$$\begin{aligned}
 [A]_{2 \times 6} &= \\
 &= T_a \begin{bmatrix}
 \overset{\$T_a + U_a}{1} & \overset{\$T_b + U_b}{-\frac{r_2(r_2 + r_3 - r_4)}{(r_2 + r_3)(r_2 - r_4)}} & \overset{\$T_c + U_c}{\frac{r_3 r_4}{(r_2 + r_3)(r_2 - r_4)}} & \overset{\$U_d}{1} & \overset{\$U_e}{-\frac{r_2(r_2 + r_3 - r_4)}{(r_2 + r_3)(r_2 - r_4)}} & \overset{\$U_f}{-\frac{r_3 r_4}{(r_2 + r_3)(r_2 - r_4)}} \\
 -\frac{1}{r_2 + r_3} & \frac{r_2 + r_3 - r_4}{(r_2 + r_3)(r_2 - r_4)} & -\frac{r_3}{(r_2 + r_3)(r_2 - r_4)} & -\frac{1}{r_2 + r_3} & \frac{r_2 + r_3 - r_4}{(r_2 + r_3)(r_2 - r_4)} & \frac{r_3}{(r_2 + r_3)(r_2 - r_4)}
 \end{bmatrix} \begin{matrix} T \\ U \end{matrix}
 \end{aligned}$$

Equation (92) is used in the virtual power flow analysis carried out to define the Coulomb friction factors ζ_e and ζ_f . Eq. (92) is also useful for checking the signs of the forces for the bearing friction models shown in Eq. (91). The use of Eq. (92) for these two purposes is well exemplified in Chapter 5, Sec. 5.3.1, so it is not repeated here.

Equation (92) is also used to obtain an expression for T_b as function of T_a . This expression is manipulated so an equation of T_a as function of T_b is generated, shown in Eq. (93). Eq. (93) is used in the Algorithm 2 to compute the Prius fuel economy, detailed in Appendix D:

$$T_a = -T_b \frac{(r_2 + r_3)(r_2 - r_4)}{r_2(r_2 + r_3 - r_4)} \quad (93)$$

After determining ζ_e and ζ_f and the forces signs, the action analysis considering the friction torques T_{i_L} , for $i = a, b, c, d, e, f$, is carried out. The fundamental cutset matrix $[Q_A]_{4 \times 15}$, the unit action matrix $[\hat{A}_D]_{2 \times 15}$ and the augmented action matrix $[D_A]_{14 \times 15}$, including power losses, are presented in Appendix C.

The augmented action matrix $[D_A]_{14 \times 15}$ is the coefficient matrix of the homogeneous linear system shown in Eq. (25), resulting in $C_N = 1$. Again T_a is used as primary variable to solve the action analysis, generating the new action matrix:

$$[A]_{2 \times 15} = \begin{bmatrix} \$T_a^a & \$T_{aL}^a & \$U_a^a & \$T_b^a & \$T_{bL}^a & \$U_b^a & \$T_c^a & \$T_{cL}^a & \$U_c^a & \$T_{dL}^a & \dots & \dots & \$U_d^a & \$T_{eL}^a & \$U_e^a & \$T_{fL}^a & \$U_f^a \end{bmatrix} \quad (94)$$

Each column of Eq. (94) is a wrench representing the passive constraints and the external torques acting on the PSD. Due to their complexity, the elements of $[A]_{2 \times 15}$ are not presented in this Thesis, but they can be generated from the matrices presented in Appendix C.

MG1 torque T_b and the PSD output torque T_c are obtained from Eq. (94) and are used in the Algorithm 2 to compute the Prius fuel economy. Eq. (94) is also used to compute the PSD efficiency.

Efficiency

The propulsion sources power is calculated through Eq. (28). The power supplied by the ICE, $\mathcal{P}_a$, to the PSD is:

$$\mathcal{P}_a = \$T_a^m \cdot \$T_a^a \quad (95)$$

where $\$T_a^m$ and $\$T_a^a$ are the first columns of $[M]_{2 \times 6}$, shown in Eq. (90), and $[A]_{2 \times 15}$, shown in Eq. (94), respectively.

The power that MG1 provides or absorbs from the PSD is given by:

$$\mathcal{P}_b = \$_b^m \cdot \$_{T_b}^a \quad (96)$$

where $\$_b^m$ is the second column of $[M]_{2 \times 6}$, shown in Eq. (90), and $\$_{T_b}^a$ is the fourth column of $[A]_{2 \times 15}$, shown in Eq. (94).

The power that leaves the PSD through coupling c is:

$$\mathcal{P}_c = \$_c^m \cdot \$_{T_c}^a \quad (97)$$

where $\$_c^m$ is the third column of $[M]_{2 \times 6}$, shown in Eq. (90), and $\$_{T_c}^a$ is the seventh column of $[A]_{2 \times 15}$, shown in Eq. (94).

Since the PSD has 2 DOF, Eq. (29) must be adapted to comprise the sign variation of MG1, becoming Eq. (98):

$$\eta_{PSD} = -\frac{\sum \mathcal{P}_{(-)}}{\sum \mathcal{P}_{(+)}} \quad (98)$$

where $\sum \mathcal{P}_{(-)}$ is the sum of the power leaving the PSD and $\sum \mathcal{P}_{(+)}$ is the total power entering the PSD. Eq. (98) means that if the power is negative it should be computed in the numerator sum. Otherwise, it should be considered in the denominator.

Considering the sign convention stated in Sec. 3.3, since ICE always provides power to the PSD, $\mathcal{P}_a > 0$ during all the Prius operation, so $\sum \mathcal{P}_{(+)}$ contains at least $\mathcal{P}_a$. Similarly, $\sum \mathcal{P}_{(-)}$ should contain at least $\mathcal{P}_c$, as power always leaves the PSD through coupling c during the Prius operation. MG1 power, represented by $\mathcal{P}_b$, is added to $\sum \mathcal{P}_{(-)}$ or $\sum \mathcal{P}_{(+)}$, whether MG1 functions as motor or generator.

If $\mathcal{P}_b < 0$, Eq. (98) becomes:

$$\eta_{PSD} = -\frac{\mathcal{P}_c + \mathcal{P}_b}{\mathcal{P}_a} \quad (99)$$

Otherwise:

$$\eta_{PSD} = -\frac{\mathcal{P}_c}{\mathcal{P}_a + \mathcal{P}_b} \quad (100)$$

6.2 FUEL ECONOMY CALCULATION

Toyota Prius fuel economy is estimated under two conditions, considering the drive cycles introduced in Sec. 2.2:

1. 10-15 mode cycle, shown in Fig. 5. A fuel economy in km/L is obtained and compared with an official result for this drive cycle;
2. FTP75 drive cycle, shown in Fig. 1, associated with the HWFET cycle, Fig. 6. The composite fuel economy in L/100 km is obtained and compared with an official result.

The Prius' four operating modes are emulated in the fuel economy estimation (MI; MASRUR, 2017; EHSANI et al., 2018; HAYES; GOODARZI, 2018):

1. **Mode 0**: when starting off and travelling at low speeds, ICE and MG1 are shut off and MG2 provides the traction force. The ICE shaft is locked and is not spinning. ICE may start immediately if the battery pack SOC is below a threshold SOC_{ICE} . When the HEV speed is above a threshold v_{eb} , ICE starts. This mode can be called launch and backup mode.
2. **Mode 1**: when Prius is cruising, the ICE power is divided into two paths: a portion drives the wheels and another one drives MG1 to produce electricity. This mode is also called e-CVT mode 1.
3. **Mode 2**: when there is a sudden acceleration, ICE, MG1 and MG2 drive the wheels. This mode is also called e-CVT mode 2.
4. **Mode 3**: when Prius is decelerating or braking, MG2 acts as generator and produces electricity to charge the battery pack. At the same time, MG2 provides braking torque to the final drive. This mode can be called regenerative braking mode.

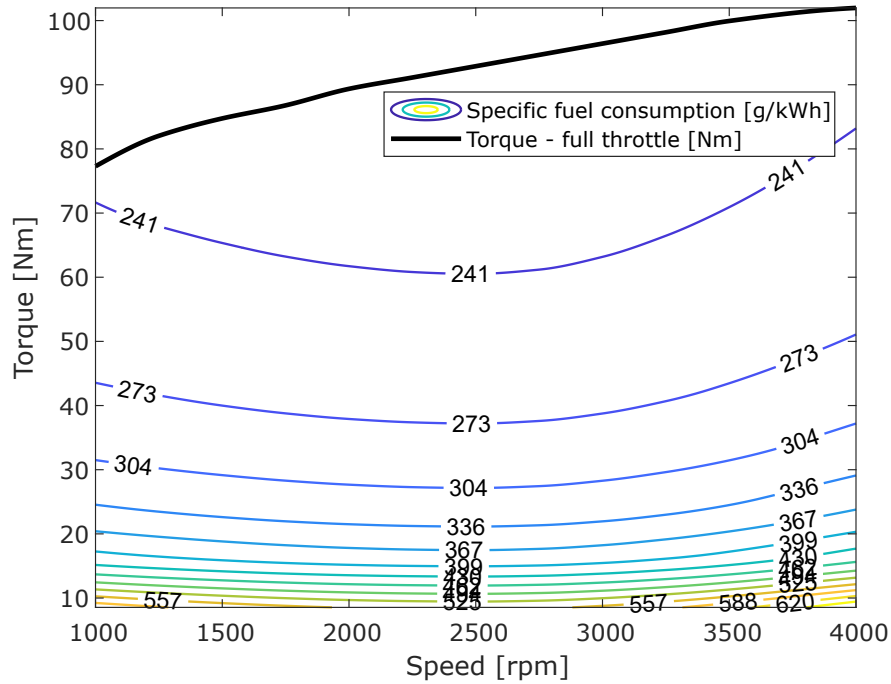
In e-CVT modes, MG1 power is adjusted so that the ICE turns at the desired speed. In the Davies method, described in Sec. 6.1, this phenomenon is represented by using t_a as primary variable in the motion analysis. MG1 power adjustment makes that the ICE operates at a relatively constant speed while the Prius speed changes during the drive cycles.

The EMS implemented for the Toyota Prius is as follows. For a given HEV speed and torque demand, determined through the drive cycle, the desired ICE operating point is defined based on the OOL. From the HEV speed and ICE speed, the desired MG1 speed is calculated through the Davies method. MG1 and PSD output torques are calculated from ICE torque through the Davies method. MG2 torque is determined by looking at the difference between the vehicle torque demand and the PSD output torque. The battery pack feeds or is recharged by MG1 and MG2 during the drive cycle.

Most of the vehicle data to compute Prius fuel economy in this Chapter comes from ADVISOR (MARKEL et al., 2002). Different sources are stated when necessary.

6.2.1 ICE maps and emission levels

The ICE maps are shown from Figs. 24 to 28. Fig. 24 contains the consumption map of the Prius ICE, whose code is 1NZ-FXE (TOYOTA-CLUB, 2021). The ICE is an Atkinson cycle engine, whose speed range goes from 1000 to 4000 rpm, and it presents a maximum power of 43 kW at 4000 rpm. The ICE maximum torque is 101.69 Nm at 4000 rpm.

Figure 24 – Fuel consumption map of Prius 1st generation ICE - model 1NZ-FXE

Source: Adapted from ADVISOR (MARKEL et al., 2002).

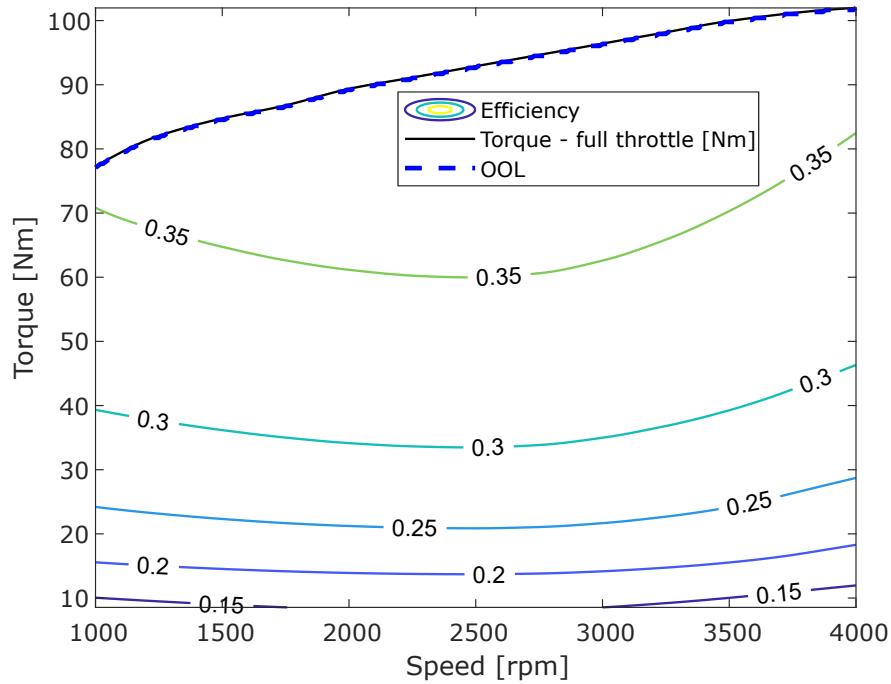
Applying Eq. (11) in the b_e values of Fig. 24, the map containing the η_{ICE} curves shown in Fig. 25 is generated. The ICE OOL is generated by taking the maximum η_{ICE} for each N_{ICE} value and is represented by the dashed blue line in Fig. 25. Note that the OOL coincides with the maximum torque curve, represented by the black full line.

The ICE optimum operation point, i.e. ICE torque and speed that gives the highest η_{ICE} , is found in the efficiency map of Fig. 25. The coordinates of the optimum operation point are $t_{a_{opt}} = 2420$ rpm and $T_{a_{opt}} = 92.3$ Nm, where $\eta_{ICE} = 39.25\%$. $t_{a_{opt}}$ and $T_{a_{opt}}$ are the primary variables used to compute η_{PSD} , whose process is described in Sec. 6.1.

The ICE emissions are mostly composed of unburned hydrocarbons (HC), carbon monoxide (CO) and nitrogen oxides (NO_x). The level of these emission elements are measured in standard drive cycles for validating a commercial vehicle. The exhaust gases are collected during tests and the final result is compared to emission standards, which stipulate some emission limits (EHSANI et al., 2018).

Figures 26, 27 and 28 show the HC, CO and NO_x emission maps of the 1st generation Prius 1st ICE. The emission levels, given in g/s, for each instant of the drive cycle are determined by using ICE speed and torque to navigate the emission maps and taking the corresponding emission level.

The Prius pollutant level in the whole cycle is obtained through the summation of

Figure 25 – Efficiency map of Prius 1st generation ICE - model 1NZ-FXE

Source: Adapted from ADVISOR (MARKEL et al., 2002).

$\dot{m}_{pol_i}$, the pollutant emission rate in g/s, for each time interval $\Delta\tau_i$, resulting in Eq. (101):

$$m_{pol} = \sum_{i=0}^n \dot{m}_{pol_i} \Delta\tau_i \quad (101)$$

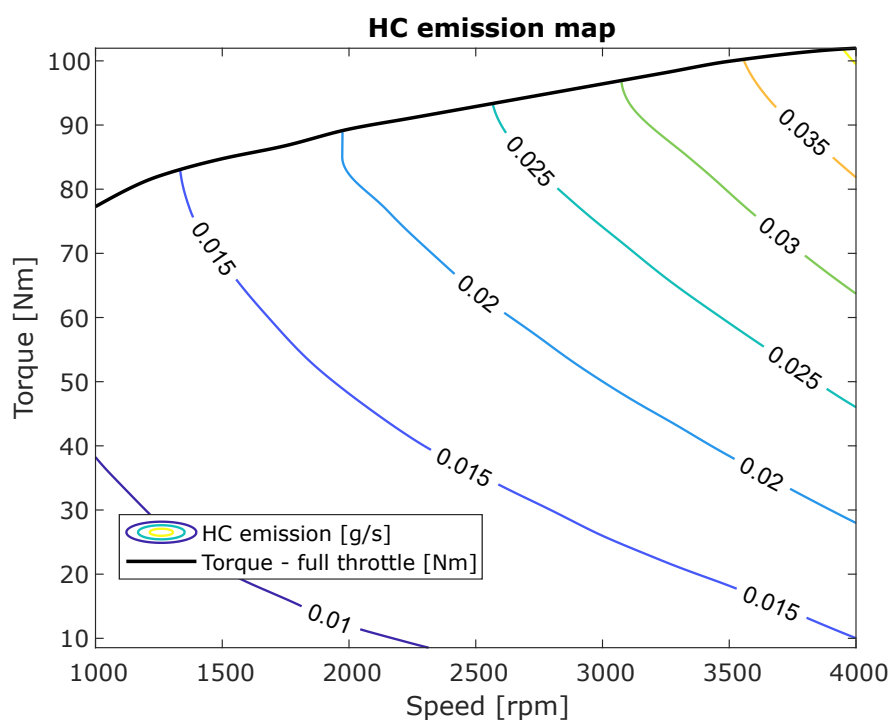
where m_{pol} is the accumulated pollutant mass, in grams, emitted by the HEV at the end of the drive cycle and n is the total number of points of the drive cycle.

Since it is the exhaust emissions that are measured in vehicle validation, the result of Eq. (101) must be multiplied by the conversion efficiency of the catalytic converter η_{cat} , which is around 95% (SANTOS; COSTA, 2008).

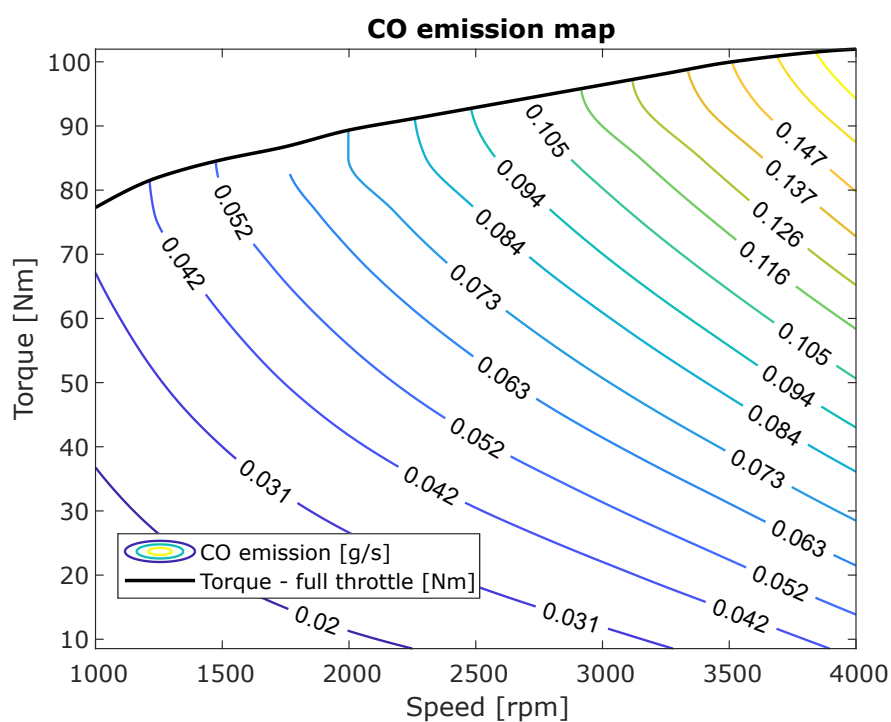
Usually the limits imposed by emission standards are given in g/km or g/mi. Thus, besides being multiplied by η_{cat} , m_{pol} must be divided by the drive cycle distance S_{cyc} to be compared with standard limits:

$$m_{pol}^{g/dist} = \frac{m_{pol}}{S_{cyc}} \eta_{cat} \quad (102)$$

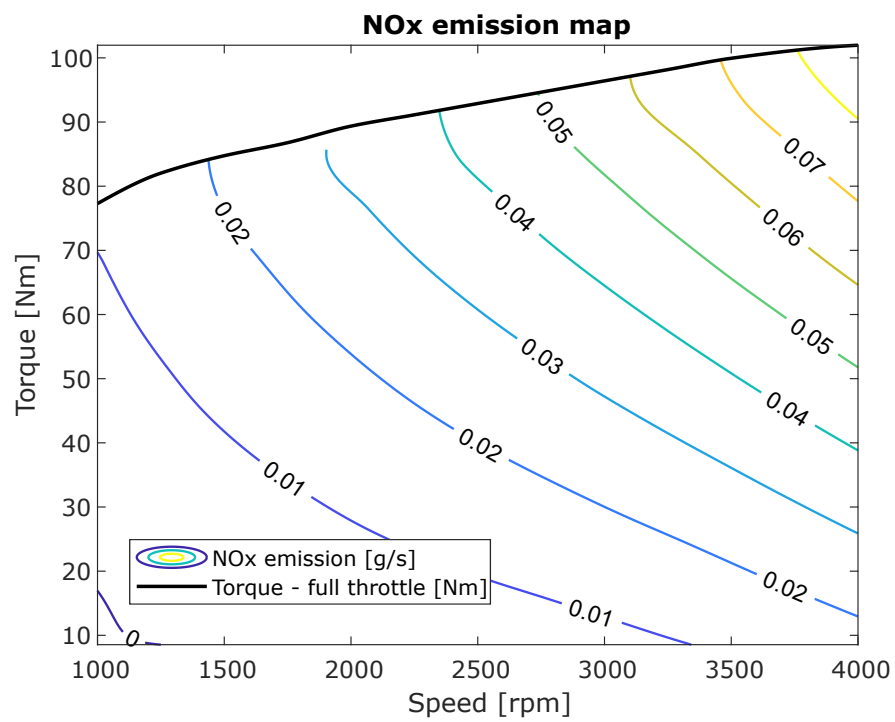
where $m_{pol}^{g/dist}$ is the pollutant mass per distance unit (km or mi).

Figure 26 – HC emission map of Prius 1st generation ICE - model 1NZ-FXE

Source: Adapted from ADVISOR (MARKEL et al., 2002).

Figure 27 – CO emission map of Prius 1st generation ICE - model 1NZ-FXE

Source: Adapted from ADVISOR (MARKEL et al., 2002).

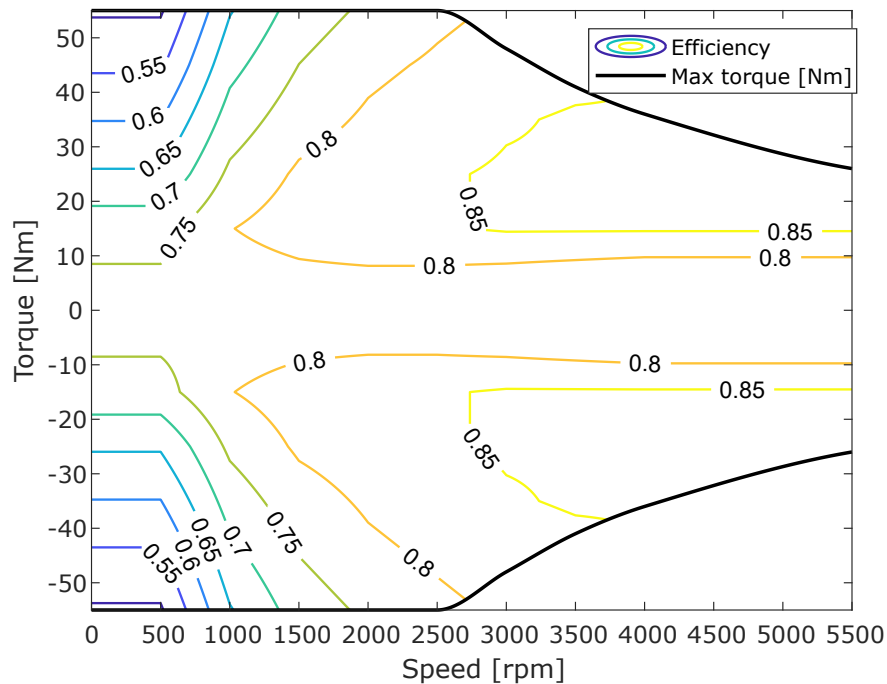
Figure 28 – NO_x emission map of Prius 1st generation ICE - model 1NZ-FXE

Source: Adapted from ADVISOR (MARKEL et al., 2002).

6.2.2 MG1 and MG2 maps

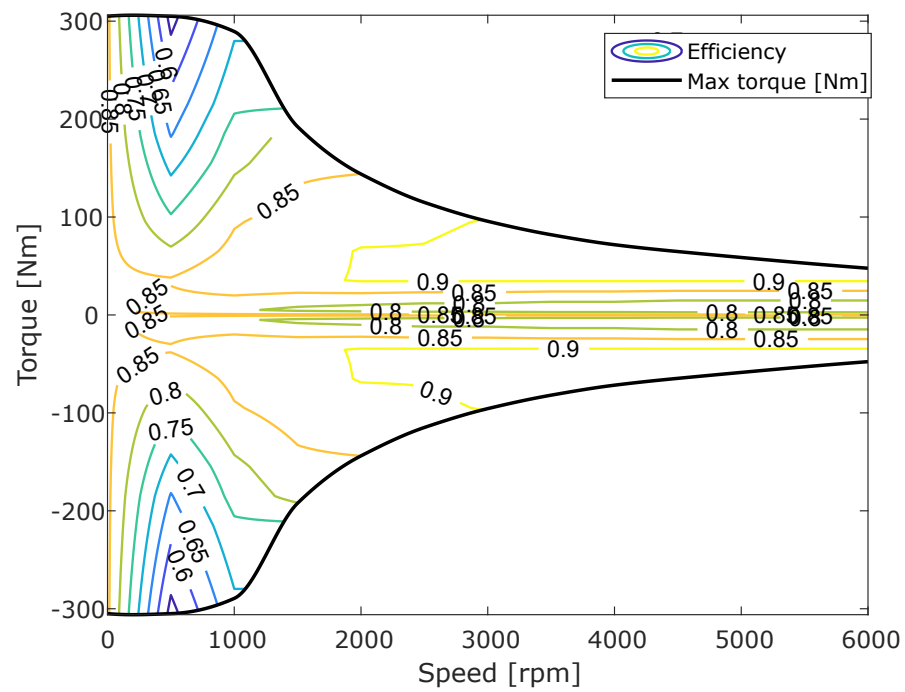
The MG1 and MG2 efficiency maps are shown in Figs. 29 and 30, respectively.

Figure 29 – MG1 efficiency map for Prius 1st generation



Source: Adapted from ADVISOR (MARKEL et al., 2002).

MG1 is a 15 kW permanent magnet motor, its speed range goes from -5500 rpm to +5500 rpm and its maximum torque is 55 Nm and its base speed, 2500 rpm. MG2 is a 30 kW permanent magnet motor whose speed goes up to 6000 rpm. MG2 maximum torque is 305 Nm and its base speed is 940 rpm.

Figure 30 – MG2 efficiency map for Prius 1st generation

Source: Adapted from ADVISOR (MARKEL et al., 2002).

6.2.3 Vehicle parameters

The parameters to be applied in the equations detailed in Chapter 2 are listed in this section. Tabs. 10 and 11 contain the environment and vehicle parameters needed to compute the vehicle's power demand $\mathcal{P}_{dem}$ through Eq. (8).

Table 10 – Environment parameters

Gravity acceleration	g	9.81 m/s ²
Road angle	α	0°
Air density	ρ_a	1.202 kg/m ³

Source: Adapted from ADVISOR (MARKEL et al., 2002).

Table 11 – Vehicle parameters

Effective radius of the wheel	r_d	0.287 m	(MI; MASRUR, 2017)
Gear ratio of the final drive	i_0	3.93	(MI; MASRUR, 2017)
Vehicle mass	M_{veh}	1398 kg	
Aerodynamic drag coefficient	C_D	0.3	
Frontal area	A_f	1.746 m ²	
Rotational inertia factor	δ	1.05	(EHSANI et al., 2018)
Power consumption for electrical appliances	$\mathcal{P}_{aux}$	700 W	
Prius speed threshold for starting ICE	v_{eb}	45 km/h	

Source: Adapted from ADVISOR (MARKEL et al., 2002).

The Prius battery pack parameters listed in Tab. 12 are used to determine the battery pack SOC through Eq. (17).

Table 12 – Battery pack parameters

Battery pack's charging/discharging effectiveness	η_{bat}	0.9	
Battery pack nominal capacity	C_{bat}	6.5 Ah	
Battery pack nominal voltage	V_{bat}	288 V	
SOC threshold for starting ICE	SOC_{ICE}	0.5	
Top of SOC operating range	SOC_t	0.7	(EHSANI et al., 2018)

Source: Adapted from ADVISOR (MARKEL et al., 2002).

The PSD parameters and dimensions are shown in Tab. 13. They are used in the efficiency calculation of the PSD, described in Sec. 6.1.

The bearings most commonly used in vehicle transmissions are rolling bearings and the preferred type is deep groove ball bearings (NAUNHEIMER et al., 2011). So the bearing friction coefficient shown in Tab. 13 corresponds to the friction coefficient of deep-groove ball bearings.

The gears module m and the bearings internal radii r_a , r_b , r_c and r_d were estimated from Fig. 22, based on the transmission length of 418 mm (WSU, 2017).

Table 13 – PSD parameters

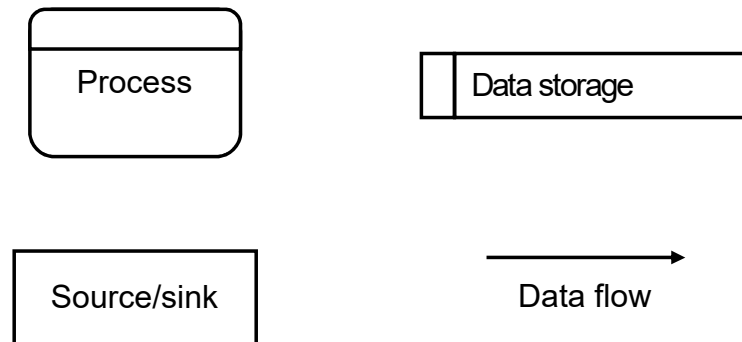
Number of teeth of the ring gear	z_4	78	(MI; MASRUR, 2017)
Number of teeth of the sun gear	z_2	30	(MI; MASRUR, 2017)
Number of teeth of the planet gears	z_3	23	(MI; MASRUR, 2017)
Gears module	m	1.5 mm	
Bearing friction coefficient	μ	0.0015	(NORTON, 2010)
Internal radius of the coupling a bearing	r_a	11.75 mm	
Internal radius of the coupling b bearing	r_b	17.10 mm	
Internal radius of the coupling c bearing	r_c	21.4 mm	
Internal radius of the coupling d bearing	r_d	5.5 mm	

Source: Mi and Masrur (2017), Norton (2010).

6.2.4 Computing Prius fuel economy

The pseudo-code of Algorithm 2, detailed in Appendix D, is developed to compute the fuel economy of the Prius 1st generation. The data flow diagrams (DFD) of this section visually represents how the the information flows through Algorithm 2. The symbols adopted in this Thesis for the DFDs are shown in Fig. 31.

Figure 31 – DFD symbols adopted in the Thesis



Source: From the author.

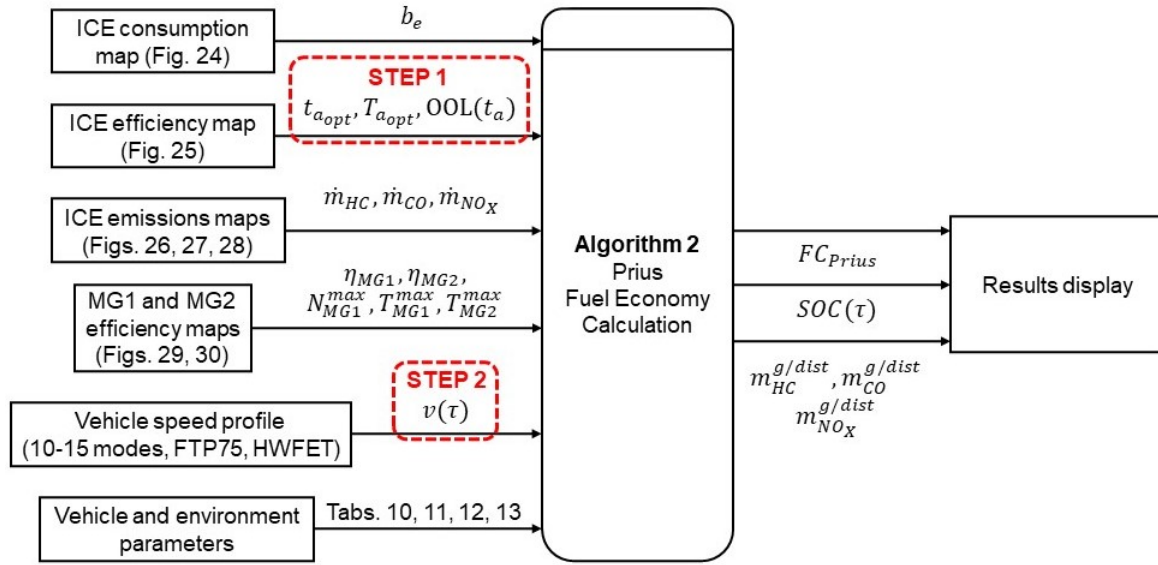
According to Fig. 31, in the DFDs of this section, a rectangle with round corners represents a process, while a rectangle with right corners stands for a source or a sink of data. Data flows are represented by arrows and data storage is represented by a rectangle crossed by a vertical line at the left.

The method developed in the Thesis, detailed in Chapter 4, is the basis of Algorithm 2. The steps listed in Chapter 4 are labelled and highlighted by slashed rectangles with round corners, when needed, in the DFDs of this Chapter.

Level 0 DFD

Fig. 32 contains the Level 0 DFD of Algorithm 2. The ICE consumption map shown in Fig. 24 is the source of the specific fuel consumption b_e for a given ICE speed

Figure 32 – Level 0 DFD of Algorithm 2



Source: From the author.

t_a and a given ICE torque T_a . The ICE efficiency map shown in Fig. 25 provides the ICE optimum operation point, represented by the ICE speed $t_{a_{opt}}$ and the ICE torque $T_{a_{opt}}$. Fig. 25 also provides the ICE optimum operation line (OOL). The definition of the ICE optimum operation point corresponds to the step 1 of method detailed in Chapter 4.

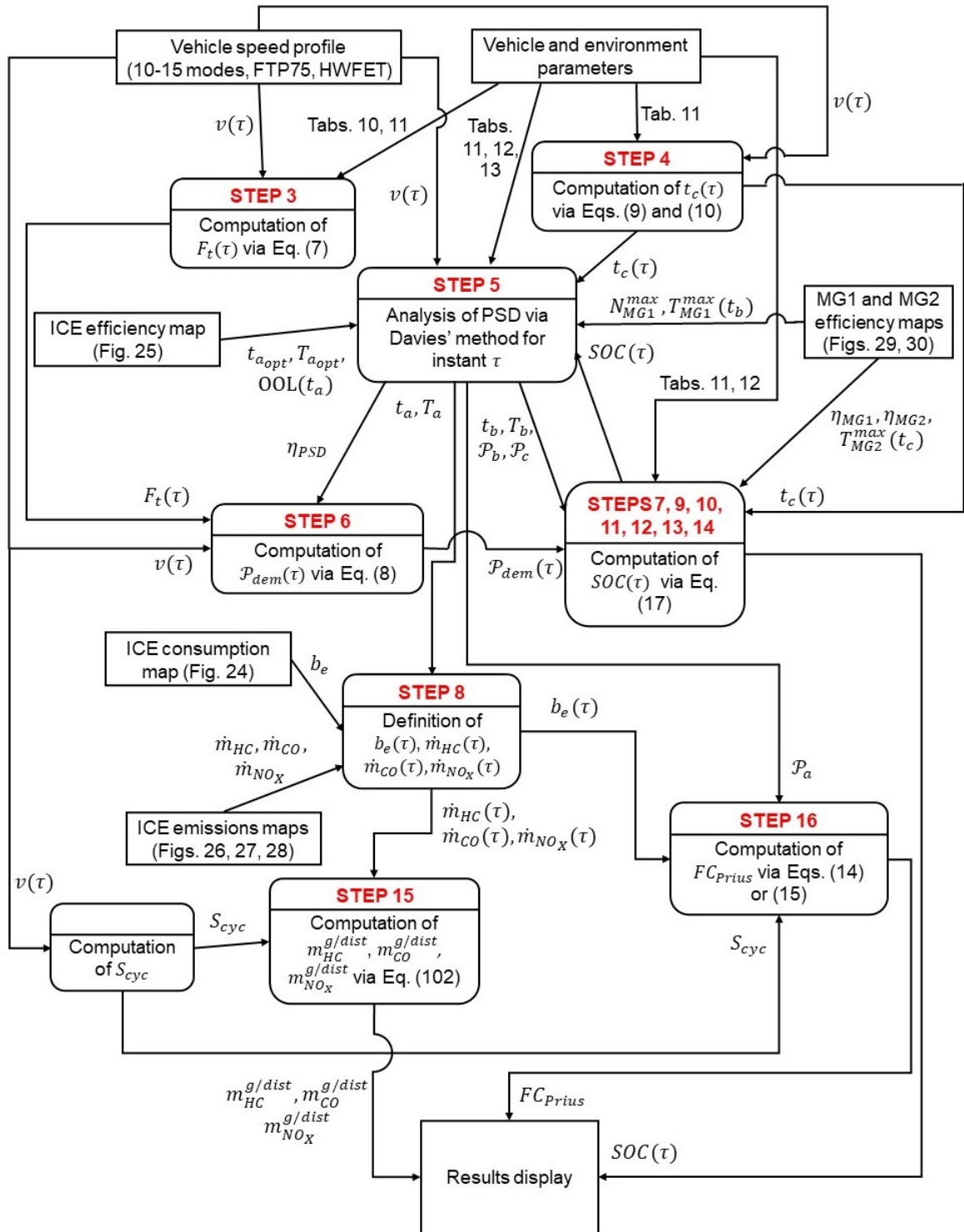
The emission maps shown in Figs. 26, 27 and 28 supply the HC ($\dot{m}_{HC}$), CO ($\dot{m}_{CO}$) and NO_x ($\dot{m}_{NO_x}$) emission rates, respectively. The sources of MG1 and MG2 efficiencies, η_{MG1} and η_{MG2} , are the efficiency maps contained in Figs. 29 and 30, respectively. Fig. 29 also provides MG1 maximum speed (N_{MG1}^{max}) and maximum torque (T_{MG1}^{max}), while Fig. 30 supplies T_{MG2}^{max} , the MG2 maximum torque.

The standard drive cycles listed in Sec. 2.2.3 are the source of vehicle speed profile as function of the time τ . The selection of the vehicle speed profile corresponds to the step 2 of method detailed in Chapter 4. Finally, the data listed in Tabs. 10, 11, 12 and 13 come from the vehicle and environment parameters.

The calculation of the Prius fuel economy performed by Algorithm 2 gives, for a given drive cycle, the fuel economy FC_{Prius} , the battery pack $SOC(\tau)$ as function of time and the HC, CO and NO_x mass per distance unit, represented by $m_{HC}^{g/dist}$, $m_{CO}^{g/dist}$ and $m_{NO_x}^{g/dist}$, respectively. The display of the results given by Algorithm 2 is the sink of the Level 0 DFD.

Level 1 DFD

Figure 33 – Level 1 DFD of Algorithm 2



Source: From the author.

The high level process "**Algorithm 2** Prius Fuel Economy Calculation", constituting the Level 0 DFD shown in Fig. 32, is decomposed into 9 main sub-processes, resulting in the Level 1 DFD, shown in Fig. 33.

The first sub-process of Fig. 33 corresponds to the step 3 of the method described in Chapter 4: computation of the Prius tractive force $F_t(\tau)$ through Eq. (7), for each instant τ of the vehicle speed profile. $F_t(\tau)$ depends on the environment and vehicle parameters listed in Tabs. 10 and 11, respectively, and the current Prius' speed $v(\tau)$.

The sub-process "Computation of $t_c(\tau)$ via Eqs. (9) and (10)" shown in Fig. 33 corresponds to the fourth step of the method of this Thesis. $t_c(\tau)$ is the PSD output speed for a given instant τ and is defined from the vehicle parameters r_d and i_0 shown in Tab. 11 and the current vehicle speed $v(\tau)$.

The sub-process "Analysis of PSD via Davies' method for instant τ ", composing the Level 1 DFD of Fig. 33, corresponds to the step 5 of the new method for calculating fuel economy. The application of the Davies method to the PSD is detailed in Sec. 6.1 and requires the current Prius speed $v(\tau)$, the vehicle and battery pack parameters listed in Tabs. 11 and 12, the PSD parameters listed in Tab. 13, the ICE optimum operation point and OOL, the PSD output speed $t_c(\tau)$, the $SOC(\tau)$, MG1 maximum speed (N_{MG1}^{max}) and MG1 maximum torque curve, shown in Fig. 29 and denoted in the DFDs as $T_{MG1}^{max}(t_b)$. The sub-process "Analysis of PSD via Davies' method for instant τ " is decomposed into three processes to generate the Level 2 DFD shown in Fig. 34, which will be detailed hereafter.

The step 6 of the new method detailed in Chapter 4 is represented by the sub-process "Computation of $\mathcal{P}_{dem}(\tau)$ via Eq. (8)". In this sub-process, the current Prius power demand $\mathcal{P}_{dem}(\tau)$ is calculated by applying $F_t(\tau)$ (coming from the sub-process "Computation of $F_t(\tau)$ via Eq. (7)"), the Prius current speed $v(\tau)$, and η_{PSD} (defined in "Analysis of PSD via Davies' method for instant τ ") in Eq. (8). The resulting $\mathcal{P}_{dem}(\tau)$ flows to the sub-process "Computation of $SOC(\tau)$ via Eq. (17)".

The sub-process "Computation of $SOC(\tau)$ via Eq. (17)" comprises step 7 and steps 9 to 14 of the method introduced in Chapter 4. Besides $\mathcal{P}_{dem}(\tau)$, this sub-process receives from the sources data regarding the vehicle and battery pack parameters, listed in Tabs. 11 and 12, respectively; and MG1 and MG2 efficiencies, represented by η_{MG1} and η_{MG2} , respectively. The MG2 maximum torque curve, denoted by $T_{MG2}^{max}(t_c)$, is also an input provided by the source MG2 efficiency map (Fig. 30). Other input data are the PSD output speed $t_c(\tau)$, MG1 speed (t_b), torque (T_b) and power ($\mathcal{P}_b$), and the PSD output power $\mathcal{P}_c$, the last four defined in the sub-process corresponding to the Davies method (step 5 of the Thesis method). From the input data, $SOC(\tau)$ is calculated and goes to the sub-process corresponding to the Davies' method and to the sink "Results display". The sub-process "Computation of $SOC(\tau)$ via Eq. (17)" is

decomposed into 7 processes to generate the Level 2 DFD shown in Fig. 35, which will be detailed hereafter.

Step 8 of the method described in Chapter 4 ("definition of ICE consumption and emissions") corresponds to the sub-process "Definition of $b_e(\tau)$, $\dot{m}_{HC}(\tau)$, $\dot{m}_{CO}(\tau)$, $\dot{m}_{NO_x}(\tau)$ ". This sub-process inputs are the b_e , coming from the source "ICE consumption map (Fig. 24)", and the pollutant rates $\dot{m}_{HC}$, $\dot{m}_{CO}$, $\dot{m}_{NO_x}$, provided by the source "ICE emission maps (Figs. 26, 27, 28)". ICE speed t_a and ICE torque T_a , defined in sub-process corresponding to the Davies method, are also inputs.

With T_a and t_a well defined for a given instant τ , the ICE specific fuel consumption for the instant τ , $b_e(\tau)$, and the emissions for the same instant, $\dot{m}_{HC}(\tau)$, $\dot{m}_{CO}(\tau)$ and $\dot{m}_{NO_x}(\tau)$ are obtained from the maps shown in Figs. 24 to 28. If the ICE shaft is locked ($t_a = 0$) or the ICE idles ($T_a = 0$), there is no fuel consumption ($b_e(\tau) = 0$) and no pollutants are emitted ($\dot{m}_{HC}(\tau) = \dot{m}_{CO}(\tau) = \dot{m}_{NO_x}(\tau) = 0$). If the ICE produces power ($t_a \neq 0$ and $T_a \neq 0$), $b_e(\tau)$, $\dot{m}_{HC}(\tau)$, $\dot{m}_{CO}(\tau)$ and $\dot{m}_{NO_x}(\tau)$ assume a non-null value for some given t_a and T_a .

The distance S_{cyc} travelled by the Prius over the whole drive cycle is defined in the sub-process "Computation of S_{cyc} ". The input of the sub-process "Computation of S_{cyc} " is the Prius speed profile. S_{cyc} is calculated through numerical integration of the Prius speed with respect to the time.

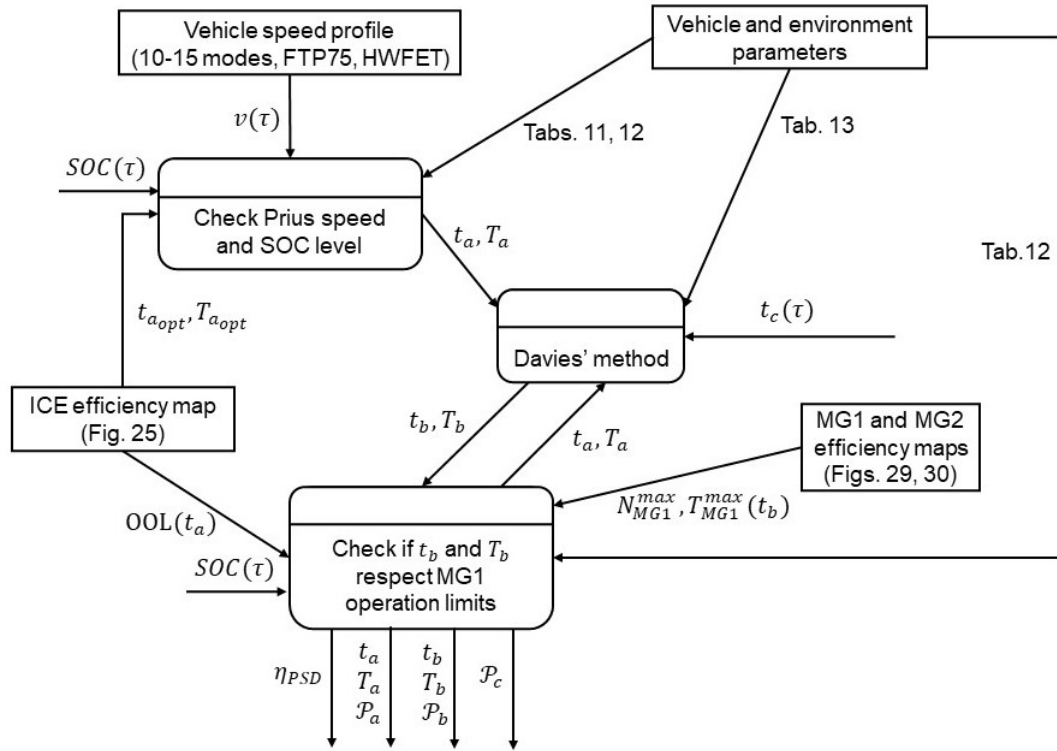
The resulting $\dot{m}_{HC}(\tau)$, $\dot{m}_{CO}(\tau)$ and $\dot{m}_{NO_x}(\tau)$ flow from the sub-process "Definition of $b_e(\tau)$, $\dot{m}_{HC}(\tau)$, $\dot{m}_{CO}(\tau)$, $\dot{m}_{NO_x}(\tau)$ " to the sub-process "Computation of $m_{HC}^{g/dist}$, $m_{CO}^{g/dist}$, $m_{NO_x}^{g/dist}$ via Eq. (102)", which corresponds to step 15 of the Thesis method ("Computation of the total mass of pollutants emitted"). Another input of this process is S_{cyc} . Using Eqs. (101) and (102), $m_{HC}^{g/dist}$, $m_{CO}^{g/dist}$ and $m_{NO_x}^{g/dist}$ are determined and flow to the sink "Results display".

The resulting $b_e(\tau)$ flows from the sub-process "Definition of $b_e(\tau)$, $\dot{m}_{HC}(\tau)$, $\dot{m}_{CO}(\tau)$, $\dot{m}_{NO_x}(\tau)$ " to the sub-process "Computation of FC_{Prius} via Eqs. (14) or (15)", which corresponds to step 16 of the Thesis method ("Computation of the fuel economy"). Other inputs of this sub-process are S_{cyc} and ICE power $\mathcal{P}_a$. Using Eq. (13) to determine the total fuel consumption and Eqs. (14) or (15) to compute the fuel economy, FC_{Prius} is determined and flows to the sink "Results display".

Level 2 DFDs

The sub-process "Analysis of PSD via Davies' method for instant τ " is decomposed into three processes to generate the Level 2 DFD shown in Fig. 34.

The current Prius speed $v(\tau)$, the Prius speed threshold for starting ICE v_{eb} (defined in Tab. 11), the current SOC(τ), the SOC threshold for starting ICE SOC_{ICE} and the top of SOC operating range SOC_t (both given in Tab. 12) are used to define if

Figure 34 – Level 2 DFD of the sub-process "Analysis of PSD via Davies' method for instant τ "

Source: From the author.

the ICE is turned on or turned off. The decision to start or not the ICE is represented by the process "Check Prius speed and SOC level", which also has as input the ICE optimum operation point.

It must be verified if $v(\tau) \leq v_{eb}$, i.e. it must be verified if the Prius is in mode 0, when is driven only by MG2. If $v(\tau) \leq v_{eb}$, it is verified if $SOC(\tau) > SOC_{ICE}$. If $SOC(\tau) > SOC_{ICE}$, the ICE is kept locked and does not supplies any power to the PSD, so t_a and T_a are null. If $SOC(\tau) < SOC_{ICE}$, the ICE starts to charge the batteries, operating at its optimum point, rotating at a speed $t_{a_{opt}}$ and providing a torque $T_{a_{opt}}$.

If $v(\tau) > v_{eb}$, the ICE turns on and operates at its optimum point if $SOC(\tau) > SOC_{ICE}$ to drive the Prius and charge the batteries. If $SOC(\tau) > SOC_t$, ICE idles and provides no torque, so only MG2 drives the Prius and the batteries are not charged. Therefore, $T_a = 0$ and $t_a = N_{ICE}^{idle} = 1200$ rpm, corresponding to the ICE idle speed, similarly to the ADVISOR simulations (MARKEL et al., 2002).

The data output of the process "Check Prius speed and SOC level" is the ICE speed t_a and ICE torque T_a . They are primary variables for the Davies' method, along with the PSD output speed $t_c(\tau)$, so they flow to the process "Davies' method", explained in detail in Sec. 6.1. The source "Vehicle and environment parameters" provides the

PSD parameters detailed in Tab. 13 to the process "Davies' method". The Davies method is used to calculate the PSD efficiency η_{PSD} , MG1 speed, torque and power, t_b , T_b and $\mathcal{P}_b$, respectively; and the PSD output power $\mathcal{P}_c$.

Before giving continuity to the fuel economy calculation, it must be verified if the values of t_b and T_b found via Davies' method respect the operation limits of MG1. The verification is represented by the process "Check if t_b and T_b respect MG1 operation limits" contained in the DFD Level 2 of Fig. 34. The input data of this process consist of t_b and T_b coming from the "Davies' method" process; the OOL provided by the source "ICE efficiency map (Fig. 25)"; Prius current state-of-charge $SOC(\tau)$; SOC_{ICE} (from Tab. 12); and the MG1 operation limits coming from the source "MG1 and MG2 efficiency maps (Figs. 29, 30)". The MG1 operation limits correspond to the maximum MG1 speed N_{MG1}^{max} , that is equal to 5500 rpm, and the MG1 maximum torque for a given MG1 speed, T_{MG1}^{max} .

If $|t_b| > N_{MG1}^{max}$, t_b must be recalculated using a different value of t_a so that $|t_b| < N_{MG1}^{max}$. If the Prius is operating in mode 0 or 3, the choice of the new t_a depends on $SOC(\tau)$. If $SOC(\tau) < SOC_{ICE}$, the ICE turns on and operates at its optimum point. If the $SOC(\tau) > SOC_{ICE}$, the ICE shaft is unlocked and the ICE idles ($t_a = N_{ICE}^{idle}$).

If $|t_b| > N_{MG1}^{max}$ but the Prius is operating in mode 1 or 2, the new t_a is calculated by multiplying N_{MG1}^{max} by i_{ab} , the ratio between the current t_a and t_b . With this new t_a , $|t_b|$ becomes equal to N_{MG1}^{max} . The new ICE torque T_a is obtained through the ICE OOL, using the new t_a as reference.

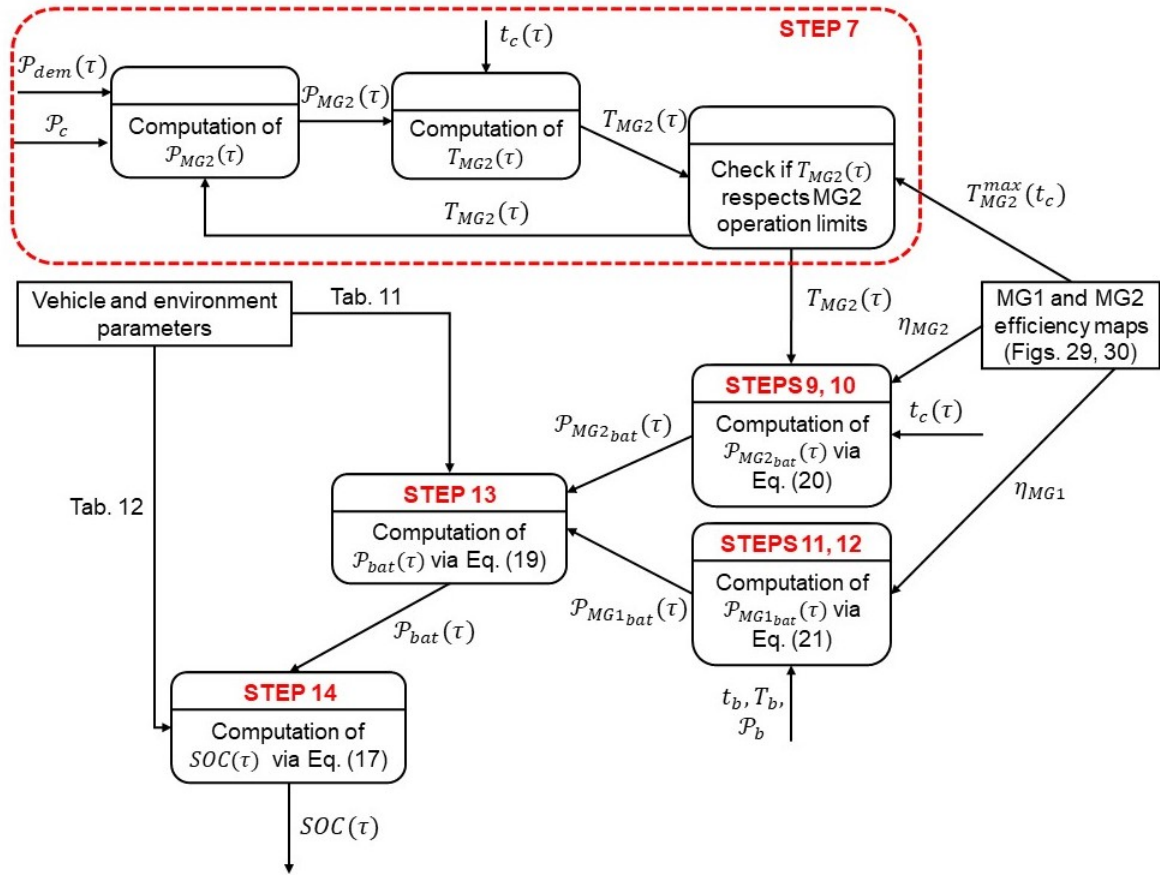
By changing t_a and T_a , the Davies' method must be applied again with the new primary variables (that is why there is data flow containing t_a and T_a from the process "Check if t_b and T_b respect MG1 operation limits" to the process "Davies' method").

It is also verified if $|T_b| > T_{MG1}^{max}$, where T_{MG1}^{max} is the MG1 maximum torque for a given speed t_b , defined from the maximum torque curve $T_{MG1}^{max}(t_b)$. If $|T_b| > T_{MG1}^{max}$, T_a is recalculated using Eq. (93), where T_b is replaced by T_{MG1}^{max} . The Davies' method is applied again with the new T_a , which assures that $T_b < T_{MG1}^{max}$.

Finally, with the MG1 operation limits respected, the data corresponding to η_{PSD} , t_a , T_a , $\mathcal{P}_a$, t_b , T_b , $\mathcal{P}_b$ and $\mathcal{P}_c$ flow to the other processes composing the DFD Level 1 shown in Fig. 33.

The sub-process "Computation of $SOC(\tau)$ via Eq. (17)" is decomposed into 7 processes to generate the Level 2 DFD shown in Fig. 35.

Step 7 of the Thesis method is "computation of the torque and power of the traction motor" (in Prius case, MG2) and is performed through the processes "Computation of $\mathcal{P}_{MG2}(\tau)$ ", "Computation of $T_{MG2}(\tau)$ " and "Check if $T_{MG2}(\tau)$ respects MG2 operation limits". The input of "Computation of $\mathcal{P}_{MG2}(\tau)$ " are $\mathcal{P}_{dem}(\tau)$ (defined in the sub-process "Computation of $\mathcal{P}_{dem}(\tau)$ via Eq. (8)" of the Level 1 DFD shown in Fig. 33) and the PSD output power $\mathcal{P}_c$ (defined via Davies' method). The current MG2

Figure 35 – Level 2 DFD of the sub-process "Computation of $SOC(\tau)$ via Eq. (17)"

Source: From the author.

power $P_{MG2}(\tau)$ is defined by the difference between $P_{dem}(\tau)$ and the absolute value of P_c .

The resulting $P_{MG2}(\tau)$ is the input of the process "Computation of $T_{MG2}(\tau)$ ", along with the PSD output speed $t_c(\tau)$. As seen in Sec. 6.1, MG2 speed is equal to $t_c(\tau)$. So, the current MG2 torque $T_{MG2}(\tau)$ is obtained by dividing $P_{MG2}(\tau)$ by $t_c(\tau)$.

The resulting $T_{MG2}(\tau)$ is the input of the process "Check if $T_{MG2}(\tau)$ respects MG2 operation limits". In this process, it is verified if $T_{MG2}(\tau)$ is higher than the MG2 maximum torque T_{MG2}^{max} for a given speed t_c . T_{MG2}^{max} is obtained from the MG2 maximum torque curve $T_{MG2}^{max}(t_c)$, provided by MG2 efficiency map (Fig. 30), which composes the source "MG1 and MG2 efficiency maps (Figs. 29, 30)".

The circumstance where $|T_{MG2}(\tau)| > T_{MG2}^{max}$ is observed only at the end of the HWFET cycle, in the last deceleration. So only when the Prius is in regenerative braking mode that $T_{MG2}(\tau)$ is recalculated. In this case, it is assumed that the regenerative braking is not enough to stop the vehicle, so the mechanical braking actuates simultaneously to the electric brakes (EHSANI et al., 2018). In this scenario, it is assumed that $|T_{MG2}(\tau)| = T_{MG2}^{max}$ and $P_{MG2}(\tau)$ is recalculated. If $|T_{MG2}(\tau)| > T_{MG2}^{max}$

when the Prius is accelerating, Algorithm 2 cannot calculate the Prius fuel economy. However, this phenomenon is not observed during the acceleration of the drive cycles considered in this Thesis.

Steps 9 ("Definition of M efficiency") and 10 ("Computation of the power transferred from the battery pack to M (or vice versa)") of the Thesis method correspond to the process "Computation of $\mathcal{P}_{MG2_{bat}}$ via Eq. (20)" of Level 2 DFD shown in Fig. 35. MG2 corresponds to the traction motor M. The process inputs are the MG2 speed $t_c(\tau)$, torque $T_{MG2}(\tau)$ (provided by step 7) and efficiency η_{MG2} (provided by the source "MG1 and MG2 efficiency maps (Figs. 29, 30)"). $t_c(\tau)$ and $T_{MG2}(\tau)$ define η_{MG2} for the instant τ . Then, the three values are used to compute the power $\mathcal{P}_{MG2_{bat}}$ transferred from the battery to MG2 (or vice versa) through Eq. (20).

Similarly, steps 11 ("Definition of G efficiency") and 12 ("Computation of the power transferred from G to the battery pack (or vice versa)") of the Thesis method correspond to the process "Computation of $\mathcal{P}_{MG1_{bat}}$ via Eq. (21)" of Level 2 DFD shown in Fig. 35. MG1 corresponds to the generator G. The process inputs are the MG1 speed t_b , torque T_b , power $\mathcal{P}_b$ and its efficiency η_{MG1} (coming from the source "MG1 and MG2 efficiency maps (Figs. 29, 30)"). t_b and T_b define η_{MG1} for the instant τ . Then, $\mathcal{P}_b$ and η_{MG1} in the instant τ are used to compute the power $\mathcal{P}_{MG1_{bat}}$ transferred from MG1 to the battery (or vice versa) through Eq. (21).

The resulting $\mathcal{P}_{MG1_{bat}}$ and $\mathcal{P}_{MG2_{bat}}$ are inputs of the process "Computation of $\mathcal{P}_{bat}(\tau)$ via Eq. (19)", which corresponds to step 13 of the method detailed in Chapter 4. Another input is the value of $\mathcal{P}_{aux}$, defined in Tab. 11 and belonging to the source "Vehicle and environment parameters". Applying these three inputs in Eq. (19) results in the real-time charging or discharging power of the battery pack $\mathcal{P}_{bat}(\tau)$, which is one of the two inputs of the remaining process of the Level 2 DFD shown in Fig. 35.

The last process of the DFD of Fig. 35 to be described is "Computation of $SOC(\tau)$ via Eq. (17)" and corresponds to the step 14 of the Thesis method ("Computation of the battery pack current state-of-charge (SOC)"). The second input of this process are the battery pack parameters listed in Tab. 12, belonging to the source "Vehicle and environment parameters". Applying $\mathcal{P}_{bat}(\tau)$ and the battery pack parameters in Eq. (17) results in $SOC(\tau)$, which is the process output.

6.3 PRIUS 1st GENERATION FUEL ECONOMY

Algorithm 2 is used to compute the Prius 1st generation fuel economy FC_{Prius} for the 10-15 mode, FTP75 and HWFET drive cycles. The resulting FC_{Prius} is compared to official values provided by Toyota. Graphics presenting the ICE, MG1, MG2, PSD and SOC behaviour during the drive cycles are presented in Appendix E.

6.3.1 10-15 mode drive cycle

Applying the Algorithm 2 to compute the Prius fuel economy for the 10-15 mode cycle results in $FC_{Prius}^{10-15} = 25.3$ km/L. The released official value for the Prius 1st generation in Japan is $FC_{Prius\ official}^{10-15} = 28$ km/L (TOYOTA, 2017), a difference of 9.7%. Tab. 14 summarizes the fuel consumption data for the 10-15 mode cycle.

Table 14 – Prius 1st Generation fuel economy for the 10-15 mode drive cycle

Calculated [km/L]	Official [km/L]	Difference [%]
25.3	28	9.7

Source: From the author.

No details on how Toyota measured $FC_{Prius\ official}^{10-15}$ were found, such as if some electrical appliances were on or off. It is known that there is a significant gap between on-road real fuel economy and certified fuel economy when the consumption is measured via the 10-15 mode test cycle. Among the reasons behind the gap there is the exclusion of the electric load used to drive some appliances such as oil pumps and compressors, discarded in the test (SANO; OHNO; KOTAKE, 2010). According to Japanese mobile phone site e-nenpi, which publishes fuel economy figures calculated on actual fuel and distance data provided by drivers, the difference between the certified fuel economy provided by Toyota and the actual average fuel economy achieved by the Prius 1st generation can be higher than 40% (E-NENPI, 2023). Therefore, the fuel economy obtained via Algorithm 2 is closer to the actual fuel economy than the official value.

The emission levels obtained for the 10-15 cycle are presented in Tab. 15. No official values of emissions were found to compare with the emission levels obtained using Algorithm 2.

Table 15 – Prius 1st Generation emission levels obtained through Algorithm 2 for the 10-15 mode drive cycle

HC [g/km]	CO [g/km]	NO _x [g/km]
0.0249	0.0958	0.0438

Source: From the author.

6.3.2 FTP75 and HWFET drive cycles

Prius fuel economy is calculated for the FTP75 and HWFET drive cycles using Algorithm 2. The fuel economy obtained for the FTP75 drive cycle is $FC_{Prius}^{FTP} = 4.13$ L/100 km, while the resulting fuel economy for the HWFET cycle is $FC_{Prius}^{HW} = 4.19$ L/100 km. FC_{Prius}^{FTP} and FC_{Prius}^{HW} are applied in Eq. (16) to compute the composite fuel economy $FC_{C_{Prius}}$, which is equal to 4.15 L/100 km. The official value released by

Toyota for the composite fuel economy is $FC_{C_{Prius}}^{official} = 4.08$ L/100 km (CLIFFORD, 2015), a difference of 1.7%. Tab. 16 summarizes the fuel economy data for the FTP75 and HWFET drive cycles.

Table 16 – Prius 1st Generation fuel economy for the FTP75 and HWFET drive cycles

FC_{Prius}^{FTP} [L/100 km]	FC_{Prius}^{HW} [L/100 km]	$FC_{C_{Prius}}$ [L/100 km]	$FC_{C_{Prius}}^{official}$ [L/100 km]	Difference FC_C
4.13	4.19	4.15	4.08	1.7%

Source: From the author.

The emission levels obtained for the FTP75 and HWFET cycles are presented in Tab. 17. Their unit is in g/mi so the FTP75 results can be compared to the emission limits of the United States exhaust emission standard Tier 1, in effect from 1994 to 1999. The Tier 1 limits are defined for the FTP75 drive cycle (EPA, 2016).

Table 17 – Prius 1st Generation emission levels obtained through Algorithm 2 for the FTP75 and HWFET drive cycles, along with Tier 1 limits

	FTP75	Tier 1 limits	HWFET
HC [g/mi]	0.0418	0.41	0.0423
CO [g/mi]	0.1609	3.4	0.1631
NO _x [g/mi]	0.0735	0.4	0.0746

Source: From the author.

Note that the emission results are far bellow the Tier 1 limits, demonstrating the environmental advantage of HEVs.

The difference of fuel economy between the results obtained for the 10-15 mode and FTP75/HWFET drive cycles and the official values indicate that Algorithm 2 has potential to be used in the design process of new HEVs, to predict fuel consumption in a reliable way.

7 OPTIMIZATION OF A HEV POWERTRAIN

In this Chapter, the mechanical efficiency of a recently patented hybrid transmission, composed of bevel gears, is calculated via Davies' method. Then, the fuel economy of a HEV with the same characteristics of the Prius 1st generation, described in Chapter 6, is calculated using Algorithm 2. The difference between the HEV of this Chapter and the Prius is that the HEV transmission is the hybrid transmission composed of bevel gears, instead of the PSD. The resulting fuel economy is compared to the results obtained for the Prius, shown in Sec. 6.3. Next, a dimensional synthesis of the hybrid transmission is done through the fuel economy optimization.

7.1 DIFFERENTIAL-BASED HYBRID SYSTEM

Xiaohua et al. (n.d.) developed an innovative power-split device for HEVs aiming to not infringe the Toyota patent of the PSD. The new power-split device is based on the structure and working principle of an automotive bevel gear differential, usually found in rear wheel drive vehicles. It is not clear if there is a commercial version of this HEV transmission currently in the market.

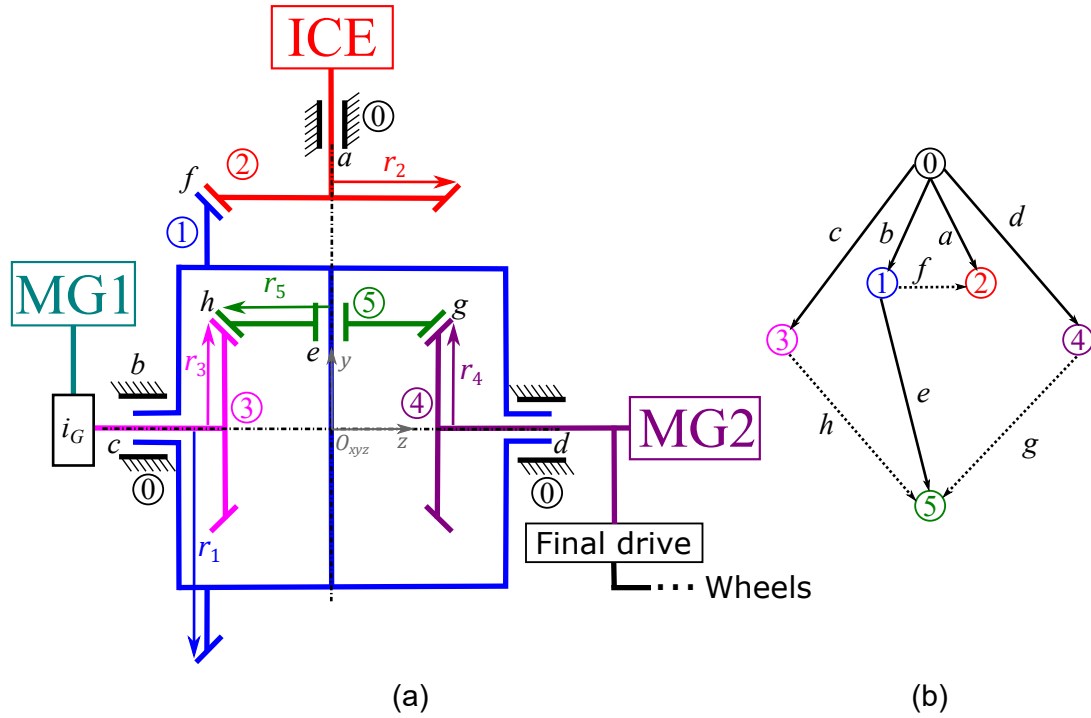
The new system is called Differential-based Hybrid System (DHS) and its schematic representation is shown in Fig. 36(a), where the DHS links are numbered from 0 to 5.

Similar to the Prius, MG1 acts mainly as a generator and MG2, as traction motor. The ICE is connected to the input pinion (gear 2), that drives the DHS cage (link 1). The generator MG1 is connected to the left-side gear (link 3) through a single speed ratio gear reducer with ratio i_G , so MG1 can be equal to Prius' MG1. The traction motor MG2 is connected to the right-side gear (link 4). Link 5 is the planet gear and link 0 is the DHS housing (fixed link) (ZENG; WANG, J., 2018). r_1 , r_2 , r_3 , r_4 and r_5 are the pitch radii of gears 1, 2, 3, 4 and 5, respectively.

The DHS operation is similar to the PSD: the DHS is always running when the HEV is moving, since there is no clutch. The motor (MG2) speed is directly proportional to the vehicle speed and determines gear 4 speed. The generator (MG1) speed is controlled so that ICE speed is in its optimum efficiency operation point.

The DHS inventors published some papers on the DHS design, but none of them focused on the DHS efficiency. A preliminary design of the DHS by minimizing the volume, through a genetic algorithm, is made by Yu et al. (2013). The temperature field of the DHS under typical operating conditions using the thermal network method is predicted by Jixin Wang et al. (2014). Some reference data for the design of the DHS lubrication and cooling system is generated by Yunhui Zhang et al. (2013), by applying thermal elastohydrodynamic lubrication at the contact zone of planet gears and half-shaft gears. Zeng et al. (2014) conduct a simulation on ADVISOR to verify

Figure 36 – DHS (a) schematic and (b) graph representations.



Source: From the author.

gears 3 and 4 speed, a finite element analysis and a bench test to check if the DHS can be applied as power-split device. Other bench tests were previously carried out by Zeng, Nie, and Wanga (2011).

Zeng and Jixin Wang (2018) carry out a fuel economy simulation in ADVISOR, replacing the PSD of the Prius 1st generation, described in Chapter 6, by the DHS. The drive cycle used was the NEDC, resulting in a fuel economy of 4.8 L/100 km. In this simulation, a constant efficiency of 85% was assumed for the whole powertrain.

Therefore, to the best of the author's knowledge, this Thesis contains the first study focusing on the DHS mechanical efficiency and its impact on a HEV fuel economy.

7.2 DHS EFFICIENCY

The DHS efficiency is determined through the Davies method, seen in Chapter 3. Since the DHS is composed exclusively of bevel gears, the order of the screw system is $\lambda = 3$ and the hypotheses listed in Sec. 3.2.2 are adopted.

To generate the motion and action matrices, the couplings in the schematic representation of Fig. 36 are labelled from *a* to *h*. Couplings *a* to *e* are revolute pairs, while couplings *f*, *g* and *h* are gear pairs. The DHS topology is represented by the coupling graph of Fig. 36(b). A reference frame O_{xyz} is attached to the point where the axes of gears 2, 5, 3 and 4 intersect, with the *y*-axis parallel to the revolute couplings *a*

and e and the z -axis parallel to couplings b , c and d .

Coupling a connects the ICE to the DHS, so the input pinion speed corresponds to s_a and is equivalent to ICE speed. The ICE torque is transmitted to the DHS through coupling a and has magnitude S_a . Coupling c connects the MG1 reducer to the DHS, so the left-side gear speed (gear 3) is indicated by t_c and is equal to MG1 speed divided by i_G ($i_G > 1$). The torque magnitude T_c is the MG1 torque multiplied by i_G and is transmitted to the DHS through coupling c . Coupling d connects the right-side gear (gear 4) to the housing and is the DHS output. Gear 4 speed, which is the DHS output speed, and MG2 speed are identical and have magnitude t_d . t_d is determined by the HEV speed through Eqs. (9) and (10). The DHS output torque is T_d and defines the magnitude of the MG2 torque.

Motion analysis

All couplings of the DHS has 1 DOF. The single motion allowed by the couplings is a rotation whose ISAs passes through the origin of O_{xyz} . The rotation axes of couplings a and e are parallel to the y -axis and their twists have magnitude s_a and s_e , respectively. The twists of couplings b , c and d are parallel to the z -axis and have magnitude t_b , t_c and t_d , respectively. The ISAs of the gear pairs' twists are straight lines connecting the gear pairs' positions to the origin of O_{xyz} . The gear pairs' twists have magnitude p_i , with $i = f, g, h$. These data are summarized in Tab. 18.

Table 18 – Twists' parameters of the DHS

Coupling	Planar location		ISA direction	Twist's magnitude
	y	z		
a	0	0	$\{0; 1; 0\}^T$	s_a
b	0	0	$\{0; 0; 1\}^T$	t_b
c	0	0	$\{0; 0; 1\}^T$	t_c
d	0	0	$\{0; 0; 1\}^T$	t_d
e	0	0	$\{0; 1; 0\}^T$	s_e
f	r_1	$-r_2$	$\left\{0; \frac{r_1}{\sqrt{r_1^2+r_2^2}}; -\frac{r_2}{\sqrt{r_1^2+r_2^2}}\right\}^T$	p_f
g	r_4	r_5	$\left\{0; \frac{r_4}{\sqrt{r_4^2+r_5^2}}; \frac{r_5}{\sqrt{r_4^2+r_5^2}}\right\}^T$	p_g
h	r_3	$-r_5$	$\left\{0; \frac{r_3}{\sqrt{r_3^2+r_5^2}}; -\frac{r_5}{\sqrt{r_3^2+r_5^2}}\right\}^T$	p_h

Source: From the author.

The fundamental circuit matrix $[B_M]_{3 \times 8}$, the unit motion matrix $[\hat{M}_D]_{3 \times 8}$ and the network unit motion matrix $[\hat{M}_N]_{9 \times 8}$ generated during the motion analysis are presented in Appendix F.

The motion analysis of the DHS is solved using $[\hat{M}_N]_{9 \times 8}$ as coefficient matrix of the homogeneous linear system of Eq. (22). $[\hat{M}_N]_{9 \times 8}$ has $F_N = 2$, meaning that two

primary variables are needed to solve the linear system. As seen in Sec. 2.2.1 and similarly to what was done for the Prius, when calculating a HEV fuel economy, two speeds are known: the ICE speed and the transmission output speed. The transmission output speed comes from the drive cycles and is calculated using Eqs. (9) and (10). The ICE speed is imposed so that the ICE efficiency is maximum.

Thus, s_a and t_d are the primary variables used for solving the DHS motion analysis, giving the motion matrix $[M]_{2 \times 8}$:

$$[M]_{3 \times 8} = \begin{bmatrix} \overset{\$a^m}{s_a} & \overset{\$b^m}{0} & \overset{\$c^m}{0} & \overset{\$d^m}{0} & \overset{\$e^m}{r_3 \frac{r_2 s_a - r_1 t_d}{r_1 r_5}} & \overset{\$f^m}{s_a} & \overset{\$g^m}{r_3 \frac{r_2 s_a - r_1 t_d}{r_1 r_5}} & \overset{\$h^m}{r_3 \frac{r_2 s_a - r_1 t_d}{r_1 r_5}} \\ 0 & s_a \frac{r_2}{r_1} & 2s_a \frac{r_2}{r_1} - t_d & t_d & 0 & -s_a \frac{r_2}{r_1} & \frac{r_2 s_a - r_1 t_d}{r_1} & -\frac{(r_2 s_a - r_1 t_d)}{r_1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} s \\ t \\ u \end{matrix} \quad (103)$$

where the equality $r_3 = r_4$ was implemented.

The speed of the reducer connecting the DHS to MG1, i.e. t_c , is defined from Eq. (103) and is used in Algorithm 2 to obtain the speed and the efficiency of MG1.

Action analysis

According to Sec. 3.2.2, the passive constraints are forces parallel to the x-axis and torques parallel to the plane $x = 0$. The propulsion sources apply external torques on the DHS: the ICE torque S_a and MG1 reducer torque T_c . The external torque T_d is the DHS output torque. U_i , $i = a, b, c, d, e$ is the magnitude of the reaction forces of the revolutes couplings and U_j , $j = f, g, h$ is the force transmitted by the gear couplings. T_a and T_e are reaction torques on z-direction on couplings a and e , respectively, and S_b , S_c and S_d are reaction torques on y-direction present in couplings b , c and d .

The power losses are modelled by eight external torques. There are bearing losses on the revolute couplings a , b , c , d and e and gear meshing friction on couplings f , g and h . The frictional torque due to bearing losses is modelled through Eq. (35), while the friction torque generated by gear meshing friction is modelled through Eq. (32), resulting in the equations shown in Eq. (104)

$$\begin{aligned}
S_{a_L} &= -\mu r_a \text{sign}(s_a) \text{sign}(U_a) U_a \\
T_{b_L} &= -\mu r_b \text{sign}(t_b) \text{sign}(U_b) U_b \\
T_{c_L} &= -\mu r_c \text{sign}(t_c) \text{sign}(U_c) U_c \\
T_{d_L} &= -\mu r_d \text{sign}(t_d) \text{sign}(U_d) U_d \\
S_{e_L} &= -\mu_e r_e \text{sign}(s_e) \text{sign}(U_e) U_e \\
P_{f_L} &= \zeta_f U_f \\
P_{g_L} &= \zeta_g U_g \\
P_{h_L} &= \zeta_h U_h
\end{aligned} \tag{104}$$

where P_f , P_g and P_h are the friction torques on the gear pairs. r_a , r_b , r_c , r_d and r_e are the internal radii of the bearings at couplings a , b , c , d and e , respectively. It is assumed that the bearings of couplings a , b , c and d are deep-groove ball bearings, the most used type of bearing in automotive transmissions (NAUNHEIMER et al., 2011). The bearing of coupling e is a needle roller bearing, as explicitly said by the DHS inventors (YU et al., 2013). So, the bearing friction coefficient μ is the same for all revolute couplings except coupling e , whose bearing friction coefficient is μ_e . ζ_f , ζ_g and ζ_h are the Coulomb friction factors of gear pairs f , g and h .

Tab. 19 contains the parameters needed for building the wrenches of the action analysis:

The fundamental cutset matrix $[Q_A]_{5 \times 16}$, the unit action matrix $[\hat{A}_D]_{2 \times 16}$ and the network unit action matrix $[\hat{A}_N]_{15 \times 16}$, ignoring power losses, are presented in Appendix F. When power losses are ignored, $[\hat{A}_N]_{15 \times 16}$ is the coefficient matrix of Eq. (25), resulting in a system with $C_N = 1$. So a single primary variable is needed to solve the linear system. As explained in Sec. 2.2.2, the primary variable is the ICE torque S_a , whose value is imposed so that the ICE presents the highest efficiency.

The DHS condensed action matrix $[A]_{3 \times 8}$, ignoring power losses, is shown in Eq. (105).

$$\begin{aligned}
[A]_{3 \times 8} &= \\
&= S_a \begin{bmatrix}
\overset{\$a_{S_a+T_a+U_a}}{1} & \overset{\$a_{S_b+U_b}}{-1} & \overset{\$a_{S_c+T_c+U_c}}{-\frac{r_1 r_5}{2 r_2 r_3}} & \overset{\$a_{S_d+T_d+U_d}}{\frac{r_1 r_5}{2 r_2 r_3}} & \overset{\$a_{T_e+U_e}}{0} & \overset{\$a_{U_f}}{-1} & \overset{\$a_{U_g}}{\frac{r_1 r_5}{2 r_2 r_3}} & \overset{\$a_{U_h}}{-\frac{r_1 r_5}{2 r_2 r_3}} \\
\frac{r_1}{r_2} & 0 & -\frac{r_1}{2 r_2} & -\frac{r_1}{2 r_2} & \frac{r_1}{r_2} & -\frac{r_1}{r_2} & -\frac{r_1}{2 r_2} & -\frac{r_1}{2 r_2} \\
-\frac{1}{r_2} & -\frac{(r_1-r_3)}{r_2 r_3} & \frac{r_1}{2 r_2 r_3} & \frac{r_1}{2 r_2 r_3} & -\frac{r_1}{r_2 r_3} & \frac{1}{r_2} & \frac{r_1}{2 r_2 r_3} & \frac{r_1}{2 r_2 r_3}
\end{bmatrix} \begin{matrix} S \\ T \\ U \end{matrix}
\end{aligned} \tag{105}$$

where the identity $r_3 = r_4$ is applied to shorten the matrix. The wrenches composing Eq. (105) are used to determine the power flow direction of the ideal DHS. This way, the Coulomb friction factors ζ_f , ζ_g and ζ_h are obtained. Moreover, Eq. (105) is used to

Table 19 – Wrenches' parameters of the DHS

Coupling	Planar location		Action	ISA direction	Magnitude
	y	z			
a	0	0	Torque	$\{0; 1; 0\}^T$	S_a
			Torque	$\{0; 1; 0\}^T$	S_{a_L}
			Torque	$\{0; 0; 1\}^T$	T_a
			Force	$\{1; 0; 0\}^T$	U_a
b	0	0	Torque	$\{0; 1; 0\}^T$	S_b
			Torque	$\{0; 0; 1\}^T$	T_{b_L}
			Force	$\{1; 0; 0\}^T$	U_b
			Torque	$\{0; 1; 0\}^T$	S_c
c	0	0	Torque	$\{0; 0; 1\}^T$	T_c
			Torque	$\{0; 0; 1\}^T$	T_{c_L}
			Force	$\{1; 0; 0\}^T$	U_c
			Torque	$\{0; 1; 0\}^T$	S_d
d	0	0	Torque	$\{0; 0; 1\}^T$	T_d
			Torque	$\{0; 0; 1\}^T$	T_{d_L}
			Force	$\{1; 0; 0\}^T$	U_d
			Torque	$\{0; 1; 0\}^T$	S_{e_L}
e	0	0	Torque	$\{0; 0; 1\}^T$	T_e
			Force	$\{1; 0; 0\}^T$	U_e
			Torque	$\left\{0; \frac{r_1}{\sqrt{r_1^2+r_2^2}}; -\frac{r_2}{\sqrt{r_1^2+r_2^2}}\right\}^T$	P_{f_L}
f	r_1	$-r_2$	Force	$\{1; 0; 0\}^T$	U_f
			Torque	$\left\{0; \frac{r_4}{\sqrt{r_4^2+r_5^2}}; \frac{r_5}{\sqrt{r_4^2+r_5^2}}\right\}^T$	P_{g_L}
g	r_4	r_5	Force	$\{1; 0; 0\}^T$	U_g
			Torque	$\left\{0; \frac{r_3}{\sqrt{r_3^2+r_5^2}}; -\frac{r_5}{\sqrt{r_3^2+r_5^2}}\right\}^T$	P_{h_L}
h	r_3	$-r_5$	Force	$\{1; 0; 0\}^T$	U_h

Source: From the author.

define the force sign functions of the bearing friction torque models shown in Eq. (104). These processes are explained in detail in Chapter 5 and are not repeated here.

After determining ζ_f , ζ_g and ζ_h and the forces signs, the action analysis considering the friction torques listed in Eq. (104) is carried out. The fundamental cutset matrix $[Q_A]_{5 \times 24}$, the unit action matrix $[\hat{A}_D]_{3 \times 24}$ and the augmented action matrix $[D_A]_{23 \times 24}$, including power losses, are presented in Appendix F.

The augmented action matrix $[D_A]_{23 \times 24}$ is the coefficient matrix of the homogeneous linear system shown in Eq. (25), resulting in $C_N = 1$. Again S_a is used as primary variable to solve the action analysis, generating the new action matrix:

$$[A]_{3 \times 24} = \begin{bmatrix} \$S_a^a & \$S_{a_L}^a & \$T_a^a & \$U_a^a & \$S_b^a & \$T_{b_L}^a & \$U_b^a & \dots & \dots & \$S_c^a & \$T_c^a & \$T_{c_L}^a & \$U_c^a & \$S_d^a & \$T_d^a & \$T_{d_L}^a & \$U_d^a & \dots & \dots & \$S_{e_L}^a & \$T_e^a & \$U_e^a & \$P_{f_L}^a & \$U_f^a & \$P_{g_L}^a & \$U_g^a & \$P_{h_L}^a & \$U_h^a \end{bmatrix} \quad (106)$$

Each column of Eq. (106) is a wrench representing the passive constraints and the external torques acting on the DHS. Due to their complexity, the elements of $[A]_{3 \times 24}$ are not presented in this Thesis, but they can be obtained through the manipulation of the matrices presented in Appendix F.

The torque of the MG1 reducer T_c is obtained from Eq. (106) and then divided by i_G to calculate MG1 torque. The DHS output torque T_d also comes from Eq. (106). Both torques are used in Algorithm 2 to compute the HEV fuel economy. Moreover, Eq. (106) is necessary to calculate the DHS efficiency.

Efficiency

The propulsion sources power is calculated through Eq. (28). The power supplied by the ICE, $\mathcal{P}_a$, to the DHS is:

$$\mathcal{P}_a = \$_a^m \cdot \$_{S_a}^a \quad (107)$$

where $\$_a^m$ and $\$_{S_a}^a$ are the first columns of $[M]_{3 \times 8}$, shown in Eq. (103), and $[A]_{3 \times 24}$, shown in Eq. (106), respectively.

The power that the MG1 reducer provides or absorbs from the DHS is given by:

$$\mathcal{P}_c = \$_c^m \cdot \$_{T_c}^a \quad (108)$$

where $\$_c^m$ is the third column of $[M]_{3 \times 8}$, shown in Eq. (103), and $\$_{T_c}^a$ is the ninth column of $[A]_{3 \times 24}$, shown in Eq. (106).

The power that leaves the DHS through coupling d is:

$$\mathcal{P}_d = \$_d^m \cdot \$_{T_d}^a \quad (109)$$

where $\$_d^m$ is the fourth column of $[M]_{3 \times 8}$, shown in Eq. (103), and $\$_{T_d}^a$ is the 13th column of $[A]_{3 \times 24}$, shown in Eq. (106).

Similarly to the PSD, the DHS has 2 DOF and the sign of $\mathcal{P}_c$ can be positive or negative, so Eq. (98) must be used to compute the DHS efficiency.

Considering the sign convention stated in Sec. 3.3, since ICE always provides power to the DHS, $\mathcal{P}_a > 0$ during all the HEV operation, so $\sum \mathcal{P}_{(+)}$ contains at least $\mathcal{P}_a$. Similarly, $\sum \mathcal{P}_{(-)}$ should contain at least $\mathcal{P}_d$, as power always leaves the DHS through coupling d during the HEV operation. The power of the MG1 reducer, represented by $\mathcal{P}_c$, is added to $\sum \mathcal{P}_{(-)}$ or $\sum \mathcal{P}_{(+)}$, whether MG1 functions as motor or generator.

If $\mathcal{P}_c < 0$, Eq. (98) becomes:

$$\eta_{DHS} = -\frac{\mathcal{P}_c + \mathcal{P}_d}{\mathcal{P}_a} \quad (110)$$

Otherwise:

$$\eta_{DHS} = -\frac{\mathcal{P}_d}{\mathcal{P}_a + \mathcal{P}_c} \quad (111)$$

The DHS efficiency is used in the fuel economy calculation, being applied in the vehicle power demand equation, defined by Eq. (8).

7.3 FUEL ECONOMY CALCULATION

The fuel economy of the HEV equipped with the DHS is estimated under the FTP75 and HWFET cycles, introduced in Sec. 2.2. The FTP75 cycle is shown in Fig. 1 and the HWFET drive cycle is shown in Fig. 6. As done for the Prius in Chapter 6, the composite fuel economy, in L/100 km, associating the FTP75 and HWFET cycles, is calculated and compared with the results presented in Sec 6.3.2 for the Prius 1st generation.

It is assumed that the HEV equipped with the DHS is the Prius 1st generation with the PSD, described in Chapter 6, replaced by the DHS. The HEV has the same four operating modes listed in Sec. 6.2 and follows the same EMS described in the same section. Thus, the fuel economy of the HEV equipped with the DHS is calculated using Algorithm 2.

The HEV has the same ICE model of the Prius, so the fuel consumption, efficiency and emission maps shown in Figs. 24 to 28, presented in Sec. 6.2.1, are adopted in this Chapter.

The HEV generator is equal to Prius' MG1, while the traction motor is equal to MG2. So MG1 and MG2 have the efficiency maps shown in Figs. 29 and 30, respectively, presented in Sec. 6.2.2.

Almost all vehicle parameters listed in Sec. 6.2.3 are adopted in this section. Tabs. 10, 11 and 12 are used in this Chapter to compute fuel economy. In addition to these three tables, Tab. 20 contains the DHS parameters used to compute η_{DHS} . The gears' module m , number of teeth z_3 , z_4 and z_5 , and the internal radius of the coupling e bearing r_e are provided by Zeng et al. (2014). The MG1 reducer gear ratio i_G is informed by Zeng and Jixin Wang (2018).

The number of teeth of the DHS cage (link 1) and of the input pinion (link 2), indicated by z_1 and z_2 , respectively, are estimated from technical drawings provided by Zeng et al. (2014), using $m = 5.4$ mm and the inner diameter of the needle roller bearing as reference. The same process is carried out to estimate the bearings internal radii r_a , r_b , r_c and r_d .

Algorithm 2, explained in Appendix D, is used to calculate the composite fuel economy of the HEV equipped with the DHS, with some adaptation on lines 59 and 60. In these lines, ICE torque is recalculated using Eq. (93) so that MG1 torque does not surpass its upper bound. For the HEV equipped with the DHS, this process does not

Table 20 – DHS parameters

Number of teeth of the DHS cage	z_1	34	
Number of teeth of the input pinion	z_2	34	
Number of teeth of the left-side gear	z_3	14	
Number of teeth of the right-side gear	z_4	14	
Number of teeth of the planet gear	z_5	10	
Gears module	m	5.4 mm	
Deep groove ball bearing friction coefficient	μ	0.0015	(NORTON, 2010)
Needle roller bearing friction coefficient	μ_e	0.0045	(NORTON, 2010)
Internal radius of the coupling a bearing	r_a	11.75 mm	
Internal radius of the coupling b bearing	r_b	22.15 mm	
Internal radius of the coupling c bearing	r_c	20.1 mm	
Internal radius of the coupling d bearing	r_d	20.1 mm	
Internal radius of the coupling e bearing	r_e	8 mm	
Ratio of the MG1 reducer	i_G	1.8	(ZENG; WANG, J., 2018)

Source: Zeng et al. (2014).

happen: if MG1 torque is higher than the maximum torque for a given speed, the **for** loop is broken. The MG1 torque does not surpass its maximum value in FTP75 and HWFET cycles when the parameters of Tab. 20 are considered. The **for** loop is broken during the evaluation of some individuals of a given population in the optimization described in Sec. 7.4.

7.3.1 Fuel economy results

Applying Algorithm 2 to compute the fuel economy of the HEV equipped with the DHS for the FTP75 drive cycle results in $FC_{DHS}^{FTP} = 4.25$ L/100 km. For the HWFET drive cycle, it gives $FC_{DHS}^{HW} = 4.36$ L/100 km. FC_{DHS}^{FTP} and FC_{DHS}^{HW} are applied in Eq. (16) to compute the composite fuel economy $FC_{C_{DHS}}$, which is equal to 4.3 L/100 km. Graphics presenting the ICE, MG2, MG1, DHS and SOC behaviour during the cycles are presented in Appendix G.

Tab. 21 summarizes the fuel economy results and place them side by side with the results obtained for the the Prius 1st Generation, shown in Chapter 6, Sec. 6.3.2.

Table 21 – Fuel economy of the HEV equipped with DHS for the FTP75 and HWFET drive cycles, compared with Prius 1st Generation

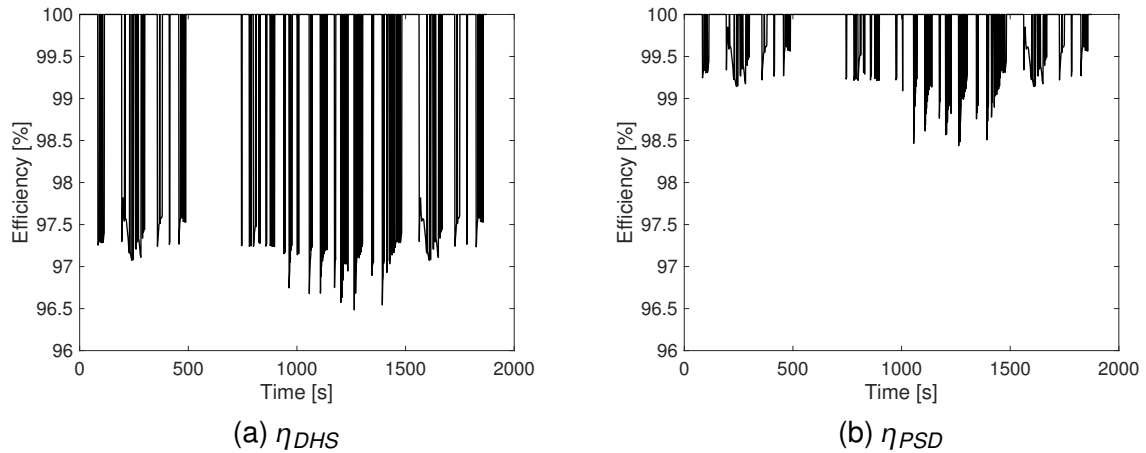
	HEV with DHS [L/100 km]	Prius [L/100 km]	Difference [%]
FTP75	4.25	4.13	2.8
HWFET	4.36	4.19	3.9
Composite	4.3	4.15	3.5

Source: From the author.

Note that, on average, the HEV equipped with DHS consumes 3.5% more fuel than the Prius, according to Tab. 21. The reason behind this difference might be the mechanical efficiency of the transmission. Fig. 37 illustrates this reasoning, showing

the transmission efficiency variation during the FTP75 drive cycle for the HEV equipped with the DHS, represented by η_{DHS} , and for the Toyota Prius, given by η_{PSD} .

Figure 37 – Transmission efficiency during the FTP75 drive cycle for (a) the HEV equipped with the DHS and (b) Toyota Prius



Source: From the author.

Comparing Fig. 37(a) with Fig. 37(b), it can be seen that η_{PSD} is slightly higher than η_{DHS} during the cycle, with a minimum value of 98.5% in the middle of the cycle. The lowest value of η_{DHS} is 96.5%. Moreover, η_{DHS} remains below 97.5% almost the whole FTP75 cycle, while η_{PSD} remains above 99% most of the time. A similar behaviour for η_{PSD} and η_{DHS} in the HWFET can be observed in their respective graphics in Appendices E and G.

The emission levels of the HEV equipped with the DHS for the FTP75 cycle are presented in Tab. 22, in comparison with emission levels obtained for the Prius and the Tier 1 limits.

Table 22 – Emission levels of the HEV equipped with the DHS and the Prius 1st Generation, and Tier 1 limits for the FTP75 drive cycle

	HEV with DHS	Prius	Tier 1 limits
HC [g/mi]	0.0435	0.0418	0.41
CO [g/mi]	0.1657	0.1609	3.4
NO _x [g/mi]	0.0752	0.0735	0.4

Source: From the author.

The emission levels of the HEV equipped with the DHS are slightly higher than the levels obtained for the Prius. However, they are still far below the Tier 1 limits, so pollutants level is not an issue for DHS. Tab. 23 shows the emission levels for the HWFET drive cycle, comparing the HEV equipped with DHS and the Prius.

Table 23 – Emission levels of the HEV equipped with the DHS and the Prius 1st Generation for the HWFET drive cycle

	HEV with DHS	Prius
HC [g/mi]	0.0442	0.0423
CO [g/mi]	0.1701	0.1631
NO _x [g/mi]	0.0778	0.0746

Source: From the author.

The difference of emission levels between the HEV equipped with the DHS and the original Prius may originate from a slight modification of the ICE operation point in some instants of the drive cycle due to the switch of transmission system.

7.4 FUEL ECONOMY OPTIMIZATION

The graphics shown in Fig. 37 demonstrate that there is room for improving the fuel economy of the HEV equipped with the DHS by maximizing η_{DHS} . Therefore, an optimization problem is defined aiming to minimize composite fuel economy through the modification of the DHS dimensions, which directly impacts η_{DHS} .

7.4.1 Optimization problem

The objective function of the optimization problem is the composite fuel economy $FC_{C_{DHS}}$, given by Eq. (16). The fuel economy for the FTP75 and HWFET drive cycles is calculated using Algorithm 2. The design variables are the MG1 reducer ratio i_G , the gears module m , and the gears' number of teeth z_1 , z_2 , z_3 and z_5 . z_4 is not a design variable because during the optimization is already imposed that $z_3 = z_4$. It should be noted that the gears in this Chapter are not profile shifted gears, so the pitch radii is simply defined as $r_i = mz_i/2$, for $i = 1,2,3,4,5$.

The cost function $FC_{C_{DHS}}$ is not an explicit function of the design variables, but $FC_{C_{DHS}}$ depends implicitly on m and z_i , for $i = 1,2,3,4,5$. Following the Thesis method, detailed in Chapter 4, the DHS efficiency η_{DHS} is obtained via Davies' method (step 5 of the Thesis method, explained in Sec. 7.2 for the DHS) and the resulting expression for η_{DHS} is as function of the pitch radii of the gears, i.e. $\eta_{DHS} = f(r_i)$, for $i = 1,2,3,4,5$. Since $r_i = mz_i/2$, $\eta_{DHS} = f(m, z_i)$. In step 6 of the Thesis method ("*Computation of the vehicle power demand*"), η_{DHS} is used to compute the vehicle power demand $\mathcal{P}_{dem}$ through Eq. (8), so $\mathcal{P}_{dem} = f(m, z_i)$.

As seen in step 7 ("*Computation of the torque and power of the traction motor*"), the traction motor MG2 power and its torque are defined from $\mathcal{P}_{dem}$. Since MG2 efficiency is defined from MG2 torque (step 9: "*Definition of M efficiency*") and the battery pack SOC is calculated using MG2 efficiency (see Sec. 2.3 to review steps 10

to 14 of the Thesis method), the design variables m and z_i influence the SOC. The SOC, in turn, determine whether the ICE turns on or off, impacting the fuel economy. Since $FC_{C_{DHS}}$ is a weighted composite value of fuel economies, $FC_{C_{DHS}}$ depends indirectly on m and z_i .

The module m is a discrete design variable that assumes values between 1 mm and 10 mm, following the Metric Gearing Standards (ISO, 1996). Gears module for automotive gearboxes usually varies from 2.25 to 3 mm (FANBIAO; LIANGHUA; BAOSHAN, 2015), so m is set to assume values of the same order of magnitude. z_1 , z_2 , z_3 and z_5 are integer design variables and can vary from 12 to 50 teeth. The lower bound comes from the suggestion that the minimum number of teeth for a bevel pinion gear is 12 (OSAKUE; LUCKY, 2018) and the upper bound of 50 teeth is imposed so that the risk of violating volume constraints decreases.

The gear ratio i_G varies from 1 to 10. i_G must be greater than 1 so a generator equal to Prius' MG1 can be used in the HEV equipped with the DHS. The upper bound of 10 corresponds to the recommended maximum gear ratio for a simple spur gear pair (NORTON, 2010).

The constraint functions are related to the DHS size and the maximum stresses the gear teeth withstand.

The gear ratio between gears 1 (DHS cage) and 2 (input pinion) must be less than 10, corresponding to the recommended maximum gear ratio for a simple bevel gear pair (NORTON, 2010). So, $z_1/z_2 \leq 10$.

The DHS volume V_{DHS} must be smaller than or equal to the Prius PSD volume V_{PSD} , i.e. $V_{DHS} \leq V_{PSD}$. The PSD is assumed as a cylinder with a diameter of $D = 115$ mm and a height of $H = 40$ mm and its volume is calculated as:

$$V_{PSD} = H\pi \left(\frac{D}{2}\right)^2 \quad (112)$$

The dimensions D and H are estimated from Fig. 22, based on the transmission length of 418 mm (WSU, 2017).

The DHS volume V_{DHS} is calculated according to Eq. (113) (ZENG; WANG, J., 2018):

$$V_{DHS} = \frac{n_p}{12}\pi b \cos(\Gamma_5) \left(3m^2 z_5^2 - 6mz_5 b \sin(\Gamma_5) + 4b^2 \sin^2(\Gamma_5)\right) + \frac{n_h}{12}\pi b \cos(\Gamma_3) \left(3m^2 z_3^2 - 6mz_3 b \sin(\Gamma_3) + 4b^2 \sin^2(\Gamma_3)\right) \quad (113)$$

where n_p is the number of planet gears (gear 5) and n_h is the number of half-axle gears (gears 3 and 4). The DHS has $n_p = 2$ planet gears (Fig. 36 contains only the planet

gear 5) and $n_h = 2$ half-axle gears. The coefficients Γ_3 and Γ_5 are the pitch cone angles of gears 3 and 5, respectively, and can be calculated as:

$$\Gamma_5 = \tan^{-1} \left(\frac{z_5}{z_3} \right) \quad (114)$$

$$\Gamma_3 = \frac{\pi}{2} - \Gamma_5 \quad (115)$$

The face-width b is computed as $b = \gamma \varphi_R$, where γ is the cone distance of the planet gear and is defined as $\gamma = m \frac{z_3^2 + z_5^2}{2}$. $\varphi_R = 1/3$ and corresponds to the coefficient of face width.

Note that gears 1 and 2 are excluded from the volume constraint.

Other constraint imposed is that $z_1 \geq z_3$, as the DHS cage envelops gear 3 and both have the same module m . Besides, it is imposed that $z_1 \geq z_2$, aiming to reduce gear 2 size.

The stresses to which the gears teeth are submitted are divided into two kinds: bending stress at the root of the tooth (σ_b) and contact stress at the tooth surface or pitting resistance (σ_c). The AGMA bending stress equation for straight bevel gears is given by (BUDYNAS; NISBETT, 2020):

$$\sigma_b = \frac{W_t}{b} \frac{K_A K_V}{m} \frac{Y_x K_{H\beta}}{Y_\beta Y_J} \quad (116)$$

where W_t is the tangential force acting on the tooth flank, K_A is the overload factor, K_V is the dynamic factor, Y_x is the size factor for bending strength, $K_{H\beta}$ is the load distribution factor, Y_β is the lengthwise curvature factor for bending strength and Y_J is the bending strength geometry factor.

The peak of the bending stress σ_b must be lesser than the permissible bending stress S_{fb} , calculated as (BUDYNAS; NISBETT, 2020):

$$S_{fb} = \frac{Y_{NT}}{K_T Y_Z} S'_{fb} \quad (117)$$

where Y_{NT} is the stress-cycle factor for bending strength, K_T is the temperature factor, Y_Z is the reliability factor for bending strength and S'_{fb} is the allowable bending stress number.

The safety factor SF_b against bending failure is given by the ratio (BUDYNAS; NISBETT, 2020):

$$SF_b = \frac{S_{fb}}{\sigma_b} \quad (118)$$

The AGMA pitting resistance σ_c formula for straight bevel gears is given by (BUDYNAS; NISBETT, 2020):

$$\sigma_c = C_p \sqrt{\frac{W_t}{bd_p Z_I} K_A K_V K_{H\beta} Z_x Z_{xc}} \quad (119)$$

where C_p is the elastic coefficient for pitting resistance, d_p is the pitch diameter of the smallest gear of the pair, Z_I is the pitting resistance geometry factor, Z_x is the size factor for pitting resistance and Z_{xc} is the crowning factor for pitting resistance.

The pitting resistance σ_c must be lesser than the permissible contact stress number S_{fc} , defined as (BUDYNAS; NISBETT, 2020)

$$S_{fc} = \frac{Z_{NT} Z_W}{K_T Z_z} S'_{fc} \quad (120)$$

where Z_{NT} is the stress cycle factor for pitting resistance, Z_W is hardness ratio factor for pitting resistance, Z_z is the reliability factor for pitting resistance and S'_{fc} is the allowable contact stress number.

The safety factor SF_c against surface failure is given by the ratio:

$$SF_c = \left(\frac{S_{fc}}{\sigma_c} \right)^2 \quad (121)$$

The stress constraints in the optimization problem are defined by the safety factors SF_b and SF_c , which must be greater than or equal to 1.6 (NAUNHEIMER et al., 2011).

The DHS gears are made of carburized and case hardened steel, grade 3, so $SF'_b = 275$ MPa and $SF'_c = 1720$ MPa (BUDYNAS; NISBETT, 2020). Appendix H contains a detailed explanation on the gear design factors values chosen for calculating the DHS stresses. Tab. 24 summarizes the factors values adopted that do not change with the gears' size:

Table 24 – Constant gear design factors for DHS stress calculation

K_A^f	K_A^g	K_A^h	Y_β	Y_{NT}	K_T	Y_z	C_p [N/mm ²]	Z_{xc}	Z_{NT}	Z_W	Z_z
1.35	1.25	1.25	1	0.9	1	1.5	190	1.5	0.87	1	1.22

Source: From the author.

The superscripts f , g and h in the K_A factor indicates that the K_A value is defined for coupling f , g and h , respectively.

The dynamic factor K_V depends on the linear velocity at the pitch line at the outside pitch diameter of the smallest gear of the pair. So K_V depends on m and the number of teeth. Y_x is defined from m , $K_{H\beta}$ and Z_x are defined according to the gears face width, and Y_J and Z_I are defined from the number of teeth of both gears composing the pair. Since the optimization problem is solved through GA and these factors are determined according to the design variables m and z_1 , z_2 , z_3 and z_5 , they

are recalculated for each new individual solution composing the current population. Thus, these factors are not included in Tab. 24.

Eq. (122) summarizes the optimization problem of this Chapter:

$$\text{Min } FC_{C_{DHS}}(m, z_1, z_2, z_3, z_5, i_G)$$

Subject to

$$\left\{ \begin{array}{l} m \in \text{standard ISO module set} \\ 12 \leq z_1, z_2, z_3, z_5 \leq 50 \\ 1 \leq i_G \leq 10 \\ \frac{z_1}{z_2} \leq 10 \\ V_{DHS} \leq V_{PSD} \\ z_3 \leq z_1 \\ z_2 \leq z_1 \\ SF_b \geq 1.6 \\ SF_c \geq 1.6 \end{array} \right.$$

(122)

The optimization problem is solved through a genetic algorithm (GA).

7.4.2 Results and discussion

After 118 generations and the cost function being evaluated 7141 times, the GA has found the design variables that enhance composite fuel economy $FC_{C_{DHS}}$ while the constraint functions are respected. The optimization was run in a server with a processor Intel(R) Xeon(R) Silver 4208 CPU @ 2.10GHz and demanded 51 h 50 min 42 s to converge to a local minimum. The new module, number of teeth and i_G are presented in Tab. 25, side-by-side with the DHS original dimensions.

Table 25 – Original and optimized DHS design variables

	Original DHS	Optimized DHS
m [mm]	5.4	3
z_1	34	48
z_2	34	48
z_3	14	40
z_5	10	20
i_G	1.8	2.88

Source: From the author.

Note in Tab. 25 that z_1 and z_2 remain equivalent after the optimization, although there is no constraint imposing that $z_1 = z_2$. Moreover, the optimization increases the number of teeth of all gears, but compensates the increase by diminishing the module m , attempting to respect the volume constraints.

The design variables found through the optimization lead to a composite fuel economy of $FC_{C_{DHS}}^{opt} = 4.2$ L/100 km, a reduction of 2.2% in relation to the result obtained in Sec. 7.3: $FC_{C_{DHS}} = 4.3$ L/100 km.

The fuel economy results obtained by running Algorithm 2 for the FTP75 and HWFET cycles, using the original and optimized design variables shown in Tab. 25, are presented in Tab. 26, along with the composite fuel economy $FC_{C_{DHS}}$.

Table 26 – Original and optimized fuel economy

	Original DHS	Optimized DHS	Reduction [%]
FC_{DHS}^{FTP} [L/100 km]	4.25	4.10	3.33
FC_{DHS}^{HW} [L/100 km]	4.36	4.33	0.81
$FC_{C_{DHS}}$ [L/100 km]	4.3	4.2	2.2

Source: From the author.

According to Tab. 26, the drive cycle which presented the highest fuel economy reduction is the FTP75 cycle. Since the weight of the FTP75 cycle is higher than the weight of the HWFET on the composite fuel economy $FC_{C_{DHS}}$ calculation, the fuel economy enhancement obtained for the FTP75 cycle has a major contribution on the $FC_{C_{DHS}}$ minimization.

Many factors may have contributed to the $FC_{C_{DHS}}$ reduction, such as a slight increase on the DHS efficiency on some points of the drive cycles; or the changing of the propulsion sources operation point in some instants of the cycle. A brief analysis on the causes behind the improvement of $FC_{C_{DHS}}$ due to the optimization is presented in Appendix I.

8 CONCLUSIONS

Due to ever-tightening government regulations, the development of innovative EV and HEV powertrain technology is a fertile field of research. Methods for decreasing harmful emissions and increasing fuel economy are sought, but most of them do not focus on the transmission mechanical efficiency. Hybrid transmissions are composed of PGTs and their mechanical efficiency has direct impact on the powertrain performance. Nonetheless, the methods found in literature to calculate fuel economy do not consider the variation of PGT efficiency during the HEV operation.

This Thesis presents a novel and complete method to determine consumption and emissions of EVs and HEVs, which integrates the propulsion sources efficiency with the variability of the mechanical efficiency of transmissions composed of PGTs. The PGTs efficiency is computed through the Davies method, what allows to emulate the transmission efficiency variation due to vehicle's power demand fluctuation on a drive cycle. The method developed in the Thesis is detailed in Chapter 4.

The Thesis method is applied to three case studies: an inspection robot for overhead power lines, representing an EV, on Chapter 5; the Toyota Prius 1st Generation on Chapter 6; and a HEV similar to Prius, but equipped with a power-split device composed of bevel gears called DHS, on Chapter 7. Tab. 27 summarizes the case studies of the Thesis, listing their main characteristics and results. "p.p." stands for "percentage points", "max" for "maximizing" and "min" for "minimizing" in Tab. 27.

Table 27 – Summary of the Thesis' case studies

	#1	#2	#3
	Inspection robot	Toyota Prius	HEV with DHS
Vehicle type	Electric	Hybrid	Hybrid
Propulsion source efficiency	Constant	Variable	Variable
Transmission efficiency	Constant	Variable	Variable
Thesis method + Optimization	Yes	No	Yes
Parameter optimized	η_{PGT} (max)	-	FC_{DHS} (min)
Optimization gain	8.94 p.p.	-	2.2 p.p.

Source: From the author.

In the first case study, the method developed in the Thesis is applied to an inspection robot which moves along the conductor cables of distribution power lines. The propulsion source of the inspection robot is a DC motor, so the robot is analogous to an EV. That is why its vehicle type is classified as "electric" in Tab. 27. The inspection robot power consumption is quantified through the batteries SOC, which is computed for a speed profile emulating the robot travelling on a conductor connecting two consecutive poles. The impact of the efficiency of the gear reducer of the robot's traction motor on the SOC is analyzed. The gear reducer efficiency is constant, so as the efficiency of the DC motor, as stated in Tab. 27. The reducer is a PGT with original efficiency of

$\eta_{PGT} = 78.81\%$, which leads to a final SOC, at end of the robot's route, of 99.95%. An optimization problem is defined to maximize η_{PGT} , as indicated in Tab. 27, by altering the gears module and number of teeth. η_{PGT} increases by almost 9 percentage points, as stated in the last line of Tab. 27, but the PGT efficiency enhancement does not reflect on the batteries SOC: the final SOC slightly increases from 99.95% to 99.97%. Therefore, the impact of the PGT efficiency on the robot's driving range is minimal under the conditions adopted to emulate the robot operation in Chapter 5. More significant results could be achieved by making some assumptions closer to the real conditions under which the robot operates.

The second case study is a more sophisticated application of the Thesis method. Instead of calculating the consumption of a vehicle with a single propulsion source, the method is applied to determine the consumption of a vehicle with three propulsion sources: the Toyota Prius 1st Generation, which is propelled by an ICE and two motor-generators. Thus, the vehicle of the second case study is classified as "hybrid" in Tab. 27. The estimation of the Toyota Prius fuel economy integrated ICE, MG1, MG2 and PSD efficiencies using the propulsion sources maps and the Davies method to calculate the PSD efficiency. In this case study, the efficiency of the propulsion sources and the efficiency of the transmission vary, as pointed out in Tab. 27, adding a level of complexity in the study of the consumption of the Toyota Prius 1st Generation.

It is possible to integrate the efficiency maps of the propulsion sources with the variable efficiency of the transmission in the fuel economy calculation due to the Davies method, a crucial step in the method described in Chapter 4. Thus, the study of the Toyota Prius gives prominence to the advantages of using the Thesis method.

The goal of the second case study is validating the method developed in the Thesis, by comparing the resulting fuel economy with official values. Thus, no optimization is carried out for the Toyota Prius, what is indicated in the last three lines of Tab. 27. The fuel economy FC_{Prius} is estimated for the Japanese 10-15 mode drive cycle and the composite fuel economy $FC_{C_{Prius}}$ is estimated for the FTP75 and HWFET drive cycles. The fuel economy results are compared to official values released by Toyota: the fuel economy calculated through Algorithm 2 for the 10-15 mode cycle presented a difference of 9.7% from the official value, while the composite fuel economy calculated through the method of this Thesis differs only 1.7% from the official value. However, due to the gap between on-road real fuel economy and certified fuel economy, the fuel economy values calculated using the Thesis method are closer to the actual consumption than the official values.

The differences between the fuel economy values obtained through the method developed on this Thesis and official values may result from the hypotheses adopted in Algorithm 2 and some assumptions made due to lack of information. Despite of that, the variation from official fuel economy values is minimal, suggesting that this Thesis

method is robust and has potential to predict consumption and emissions in a reliable way on the design phase of a new HEV. So, the main objective of the Thesis, which is developing a method to simulate fuel consumption and emissions of EVs and HEVs integrating power sources efficiency with the variability of the transmission efficiency, is achieved.

Finally, the last case study comprises the fuel economy estimation of a HEV equipped with the DHS. The HEV is similar to the Toyota Prius presented in Chapter 6, but instead of using a PGT as power-split device, the ICE, MG1 and MG2 power is managed by the DHS. Thus, the vehicle of the third case study is classified as "hybrid" and its propulsion sources and transmission have variable efficiencies, as stated in Tab. 27. The complex geometry of the DHS is the complicating factor of this case study in relation to the others, as the transmission is composed of bevel gears. The DHS case study emphasizes the differential of using the Davies method to estimate the efficiency of PGTs, as the efficiency of the DHS, a bevel PGT, is easily defined using the electromechanical analogy. Consequently, the contribution of the method to compute consumption developed in the Thesis is highlighted in this case study as well, since the Thesis method has as one of its most important steps the application of the Davies method to determine the transmission efficiency.

The HEV composite fuel economy is calculated using Algorithm 2 and the FTP75 and HWFET drive cycles, resulting in a value slightly higher than the one obtained for the Toyota Prius. The HEV equipped with the DHS consumes more fuel than the Toyota Prius because the DHS efficiency is lower than the PSD efficiency during the drive cycles.

An optimization problem is defined aiming to minimize the composite fuel economy $FC_{C_{DHS}}$ of the HEV equipped with the DHS, as indicated in Tab. 27. The goal is to diminish $FC_{C_{DHS}}$ by increasing the DHS efficiency η_{DHS} , which depends on the gears dimensions. So the design variables of the optimization problem are the module and the number of teeth, in addition to the gear ratio of the reducer connecting the DHS to the generator. The $FC_{C_{DHS}}$ obtained through the optimization is 2.2% lower than the original $FC_{C_{DHS}}$, as summarized in Tab. 27. The $FC_{C_{DHS}}$ reduction may result from a slight increase of the DHS efficiency in some points of the drive cycles; or from the changing of the propulsion sources operations points due to the new gears' dimensions and the new gear ratio of the generator reducer.

Enhanced transmission mechanical efficiency, however small, associated with the development of more efficient engines and motor-generators, can make the difference when it comes to validating a new HEV to be launched on the market. Therefore, more studies are necessary to understand the interaction between the efficiency of the propulsion sources and the transmission efficiency and how the fuel economy is affected by them.

8.1 CONTRIBUTIONS OF THIS THESIS

The original contributions of this work can be summarized as follows:

- Development of a method to predict fuel economy and emissions of HEVs which integrates the efficiencies of the whole powertrain (ICE, M, G and transmission efficiencies). The variation of the powertrain efficiency, including the transmission efficiency, during the HEV operation is comprised by the method developed in the Thesis. The method proposed in this Thesis gives fuel economy values with satisfactory precision for the Toyota Prius 1st Generation, demonstrating the method robustness;
- An equation to compute the mechanical efficiency of PGTs with more than 1 DOF through the Davies method, given by Eq. (98), is introduced for the first time;
- A study on the DHS mechanical efficiency, an automotive transmission composed of bevel gears, is carried out for the first time;
- Generation of an analytical expression to compute the gear meshing friction torque, as function of the ordinary efficiency, for bevel gear pairs with 90° shafts, demonstrated in Appendix A.

Another contribution of the Thesis is the generality of the method developed. The Thesis focuses on EVs and HEVs, however, nothing prevents the use of the Thesis method, with some adaptations, to compute the energy consumption of other machines that possess a PGT to transfer power. The most evident application that is not dealt in this document is the use of the method of Chapter 4 to calculate the fuel economy of a conventional vehicle with automatic transmission, propelled exclusively by an ICE. Calculating the fuel economy of a conventional vehicle is even simpler than calculating the consumption of a HEV, as it is unnecessary to execute the steps related to the electric propulsion sources. Other machines that can have their consumption estimated through the Thesis method are industrial robots with spherical wrists composed of bevel gears, actuators used in the aerospace field, machine tools, wind turbines, among others.

8.2 PUBLICATIONS

The following papers related to the Thesis were published:

- SOUZA, M. B. de; VIEIRA, R. de S. ; MARTINS, D. Synthesis of epicyclic gear trains with one and two degrees of freedom from kinematic chains belonging to minimal sets. INTERNATIONAL JOURNAL OF MECHANISMS AND ROBOTIC SYSTEMS, v. 4, p. 383-400, 2018.

- SOUZA, M. B. de; VIEIRA, R. de S.; MARTINS, D.; LAUS, L. P. Dimensional Synthesis of a Dedicated Hybrid Transmission Through Efficiency Optimization of Gear Trains. In: Uhl T.. (Org.). Mechanisms and Machine Science. 1ed.: Springer International Publishing, 2019, v. 73, p. 1101-1110.
- SOUZA, M. B. de; LIMA, V. S.; VIEIRA, R. de S.; MARTINS, D. DIMENSIONAL SYNTHESIS OF COMPLEX GEAR TRAINS FOR RAPID PROTOTYPING WITH 3D PRINTING. In: 25th International Congress of Mechanical Engineering, 2019. Proceedings of the 25th International Congress of Mechanical Engineering.
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- SOUZA, M. B. de; FERNANDES, G. Q.; VIEIRA, R. de S.; NICOLAZZI, L.; KINCELER, R. A Methodology to Field Testing Friction Between Aluminum Wires and Wheels of Cable Riding Robots. In: 26th International Congress of Mechanical Engineering, 2021, Florianópolis. Proceedings of the 26th International Congress of Mechanical Engineering, 2021.
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- MORLIN, F. V.; CARBONI, A. P.; SOUZA, M. B. de; MARTINS, D. Degeneracy Detection in Epicyclic Gear Trains Using a Matroid-Based Algorithm. In: Pucheta M., Cardona A., Preidikman S., Hecker R.. (Org.). Mechanisms and Machine Science. 1ed.Cham: Springer International Publishing, 2021, v. 94, p. 11-18.
- BARRETO, R. L. P.; MORLIN, F. V. ; SOUZA, M. B. de ; CARBONI, A. P. ; MARTINS, D. Multiloop origami inspired spherical mechanisms. MECHANISM AND MACHINE THEORY, v. 155, p. 104063, 2021.

8.3 FUTURE WORKS

Future works on this Thesis includes:

- Applying the Thesis method in other commercial HEVs and compare the result with official values of fuel economy, to emphasize the robustness of the method;
- Formulating a friction model to quantify losses due to hysteresis caused by gear backlash;
- Gear trains are lubricated with oil, whose viscosity varies when there is a significant temperature change. A viscosity variation may increase the viscous friction intensity. So, future works include formulating a friction model to quantify the losses coming from the changing of oil viscosity provoked by temperature oscillations;
- Considering in the method the fuel consumed to keep the transmission's oil pump functioning when the ICE is operating at idling speed, specially under urban traffic conditions. The oil pump effect could be included as a power sink connected to the PGT, so the effect of the oil pumping on the transmission efficiency could be quantified;
- Evaluating the impact of the PGT optimization on the inspection robot consumption by emulating its operation under more realistic conditions;
- Developing an algorithm similar to Algorithm 2 to compute EV driving range. The traction motor efficiency map would be used instead of considering the motor efficiency constant as in Chapter 5;
- Considering the inverter and the impact of its efficiency on the batteries' SOC;
- Applying the method developed in the Thesis in a HEV with a multi-speed transmission, which contains clutches and brakes;
- Developing an EMS for HEVs with multi-speed transmissions where the transmission efficiency is the main selection criterion for the driving mode choice;
- Decreasing the computational cost of the Algorithm 2 to embed it in HEVs control system, for real-time application;
- Including the profile-shift coefficients as design variables of an optimization problem aiming to maximize the gear train efficiency. The coefficients would be considered by using the working pitch diameter instead of the pitch diameter to define the position of the gear pairs;
- Evaluating the gear inertia in the new method. The inertia evaluation is possible after including the profile-shift coefficients in the Davies method;
- Adapting the Thesis method to become a tool to evaluate the viability of new hybrid transmissions.

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GLOSSARY

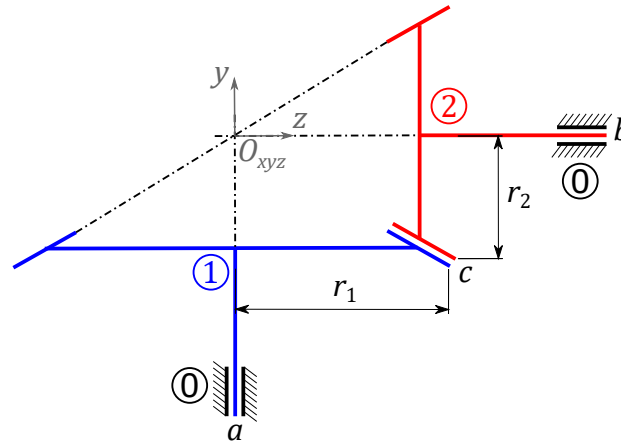
ADVISOR ADVISOR (from "Advanced Vehicle Simulator") is a drivetrain analysis tool developed in MATLAB/Simulink® to compare fuel economy and emission performance between different drivetrain configurations. Developed by the National Renewable Energy Laboratory (NREL) to assist the U.S. Department of Energy in developing and understanding hybrid electric vehicles through the Hybrid Vehicle Propulsion System contracts with some American car manufacturers (HAYES; GOODARZI, 2018).

APPENDIX A – EFFICIENCY OF A BEVEL GEAR PAIR

This appendix shows the steps of the Davies method applied to a simple bevel gear train to generate Eq. (34), that calculates the Coulomb friction factor for straight bevel gears with perpendicular axes. The simplifying hypotheses listed in Sec. 3.2.2 are assumed herein.

The simple bevel gear train is schematically represented in Fig. 38 and consists of three links: a fixed link 0 and gears 1 and 2. The axes of gears 1 and 2 are perpendicular, since the most common angle between the axes of two meshing bevel gears is 90° (NORTON, 2010). A reference frame O_{xyz} is attached to the intersection point of the gears' axes. r_1 and r_2 stand for the pitch radii of gears 1 and 2, respectively.

Figure 38 – Simple bevel gear train with perpendicular axes.



Source: From the author.

The gear train couplings are labeled a , b and c . Coupling c is a gear pair and couplings a and b are revolute pairs. In gear trains' applications, a torque source, such as a motor, is connected to one link, while a load, working as a torque sink, is connected to another. The torque source and sink produce external torques that are internalised into the network by considering a and b active couplings.

A.1 MOTION ANALYSIS

The topology of the bevel gear train of Fig. 38 is represented by the coupling graph of Fig. 39(a), where the full edges stand for the revolute pairs and the dashed edge, for the gear pair. The three couplings of the simple bevel gear train allow one single motion, so the motion graph is equivalent to the coupling graph of Fig. 39(a).

The fundamental circuit matrix created from the motion graph, using edges a and b as branches of the spanning tree, is shown in Eq. (123):

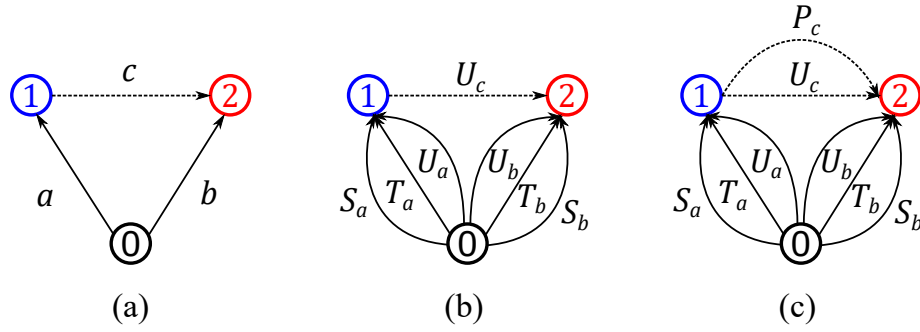


Figure 39 – Simple bevel gear train graphs: (a) original coupling graph/motion graph, (b) action graph with torque source and sink internalised and (c) action graph with frictional torque internalised.

$$[B_M]_{1 \times 3} = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \quad (123)$$

The single motion allowed by the couplings is a rotation. Coupling a allows a rotation around y -axis, while coupling b allows a rotation around z -axis. By adopting the Davies' notation, the speed magnitude of the revolute couplings' twists are indicated by s_a for coupling a and t_b for coupling b . The ISA of coupling c 's twist $\$c^m$ passes through a line connecting the origin O_{xyz} and the coupling c position in the yz -plane. So $\$c^m$ has two components: one rotation around y -axis and another one around z -axis and its magnitude is expressed by p_c . Table 28 summarizes these data. The twists' ISA direction is given by a unit vector in the third column of Tab. 28.

Table 28 – Twists' parameters of the simple bevel gear train

Coupling	Planar location		ISA direction	Twist's magnitude
	y	z		
a	0	0	$\{0; 1; 0\}^T$	s_a
b	0	0	$\{0; 0; 1\}^T$	t_b
c	$-r_2$	r_1	$\{0; -r_2/\sqrt{r_1^2 + r_2^2}; r_1/\sqrt{r_1^2 + r_2^2}\}^T$	p_c

The twists representing the motions of the simple bevel gear train are built from the parameters of Tab. 28. The twists and the fundamental circuit matrix of Eq. (123) generate the coefficient matrix $[\hat{M}_N]_{3 \times 3}$ of Eq. (22). The simple bevel gear train has $F_N = 1$, so one primary variable is enough to solve the homogeneous linear system. Choosing s_a , the angular speed of gear 1 in relation to the fixed link 0, as the primary variable leads to the motion matrix of Eq. (124), where each column corresponds to the couplings' twists.

$$[M]_{3 \times 3} = s_a \begin{bmatrix} 1 & 0 & -1 \\ 0 & \frac{r_1}{r_2} & \frac{r_1}{r_2} \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} s \\ t \\ u \end{matrix} \quad (124)$$

The row labels s , t and u on the right side of $[M]_{3 \times 3}$ indicate the twist coordinate, while column labels $\$^m_a$, $\$^m_b$ and $\$^m_c$ on the top of the matrix indicate the twist.

A.2 ACTION ANALYSIS

The passive constraints of couplings a , b and c are forces parallel to x -axis and torques parallel to the plane $x = 0$, whose wrenches are reciprocal to the couplings' twists. The forces' magnitudes are U_a , U_b and U_c for couplings a , b and c , respectively. Coupling a allows a rotation around y -axis, so the passive torque constraint is around z -axis and its magnitude is indicated by T_a . On the other hand, coupling b is a revolute joint on z -axis, so its passive torque constraint is around y -axis, whose magnitude is given by S_b .

The external torque source and sink have the same direction of the revolute pairs' twists. The external torque acting on coupling a is on y -axis direction and has magnitude S_a . The external torque acting on coupling b is on z -axis direction and has magnitude T_b . The passive constraints and the torque source and sink are represented in the action graph of Fig. 39(b).

Power losses due to meshing friction are introduced by adding an edge representing a frictional torque in Fig. 39(c), with magnitude P_c . The wrench quantifying the frictional torque in gear pair c has the same direction of the twist $\$^m_c$. The information about the actions given above are summarized in Tab. 29.

Table 29 – Wrenches' parameters of the simple bevel gear train

Coupling	Planar location		Action	ISA direction	Magnitude
	y	z			
a	0	0	Torque	$\{0; 1; 0\}^T$	S_a
			Torque	$\{0; 0; 1\}^T$	T_a
			Force	$\{1; 0; 0\}^T$	U_a
b	0	0	Torque	$\{0; 1; 0\}^T$	S_b
			Torque	$\{0; 0; 1\}^T$	T_b
			Force	$\{1; 0; 0\}^T$	U_b
c	$-r_2$	r_1	Torque	$\{0; -r_2/\sqrt{r_1^2 + r_2^2}; r_1/\sqrt{r_1^2 + r_2^2}\}^T$	P_c
			Force	$\{1; 0; 0\}^T$	U_c

The wrenches representing the actions transmitted by the couplings are

generated from the parameters of Tab. 29 and are assembled to create the unit action matrix of Eq. (125):

$$[\hat{A}_D]_{3 \times 8} = \begin{array}{cccccc} \begin{array}{c} S_a \\ T_a \\ U_a \end{array} & \begin{array}{c} S_b \\ T_b \\ U_b \end{array} & \begin{array}{c} P_c \\ U_c \end{array} & \begin{array}{c} S \\ T \\ U \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & -\frac{r_2}{\sqrt{r_1^2+r_2^2}} & r_1 \\ 0 & 1 & 0 & 0 & 1 & 0 & \frac{r_1}{\sqrt{r_1^2+r_2^2}} & r_2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array} \quad (125)$$

The row labels S , T and U on the right of $[\hat{A}_D]_{3 \times 8}$ indicate the wrench coordinate, while the column labels on the top of $[\hat{A}_D]_{3 \times 8}$ indicate the wrench representing each action.

By considering again edges a and b as branches of a spanning tree of the action graph shown in Fig. 39(c), the fundamental cutset matrix is generated:

$$[Q_A]_{2 \times 8} = \begin{array}{cccccc} a & a & a & b & b & b & c & c \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \quad (126)$$

By associating Eqs. (125) and (126), the network unit action matrix $[\hat{A}_N]_{6 \times 8}$ is generated according to Eq. (26), resulting in:

$$[\hat{A}_N]_{6 \times 8} = \begin{array}{cccccc} \begin{array}{c} S_a \\ T_a \\ U_a \end{array} & \begin{array}{c} S_b \\ T_b \\ U_b \end{array} & \begin{array}{c} P_c \\ U_c \end{array} & \begin{array}{c} S \\ T \\ U \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \frac{r_2}{\sqrt{r_1^2+r_2^2}} & -r_1 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\frac{r_1}{\sqrt{r_1^2+r_2^2}} & -r_2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{r_2}{\sqrt{r_1^2+r_2^2}} & r_1 \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{r_1}{\sqrt{r_1^2+r_2^2}} & r_2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array} \quad (127)$$

A.2.1 Constitutive equations

The friction models presented in Sec.3.5 are appended at the network unit action matrix as constitutive equations to compute the magnitude of the wrenches representing friction torques. Power losses are caused only by meshing friction in this example, so only one constitutive equation is added to the matrix $[\hat{A}_N]_{6 \times 8}$ shown in Eq. (127). The Coulomb friction model, presented in Tab. 3, is used to quantify meshing friction losses, leading to the following constitutive equation:

$$P_c = -\zeta_c \text{sign}(U_c) \text{sign}(p_c) U_c \quad (128)$$

where ζ_c is the Coulomb friction factor and has unit of length, converting the force of magnitude U_c , transmitted by the gear pair c , into a friction torque of magnitude P_c .

Eq. (128) is appended to $[\hat{A}_N]_{6 \times 8}$ in matrix form, generating the augmented action matrix $[D_A]_{7 \times 8}$:

$$[D_A]_{7 \times 8} = \begin{array}{cccccc} S_a & T_a & U_a & S_b & T_b & U_b & P_c & U_c \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \frac{r_2}{\sqrt{r_1^2+r_2^2}} & -r_1 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\frac{r_1}{\sqrt{r_1^2+r_2^2}} & -r_2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{r_2}{\sqrt{r_1^2+r_2^2}} & r_1 \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{r_1}{\sqrt{r_1^2+r_2^2}} & r_2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & c\zeta_c \end{bmatrix} & \begin{matrix} S \\ T \\ U \\ S \\ T \\ U \\ c.e. \end{matrix} \end{array} \quad (129)$$

where $c = \text{sign}(p_c)\text{sign}(U_c)$ and the row label *c.e.* indicates that the last row of $[D_A]_{7 \times 8}$ comes from the constitutive equation of Eq. (128).

The matrix $[D_A]_{7 \times 8}$ is used as coefficient matrix in Eq. (25) instead of $[\hat{A}_N]_{\lambda \times C}$. The net degree of constraint of $[D_A]_{7 \times 8}$ is $C_N = 1$, meaning that one primary variable is necessary to solve the homogeneous linear system. Choosing S_a , the external torque acting on coupling a , as primary variable, the solution of the linear system is given by the vector $\{\vec{\psi}\}_{8 \times 1}$, which contains the wrenches magnitudes:

$$\{\vec{\psi}\}_{8 \times 1} = S_a \begin{pmatrix} 1 \\ \frac{r_2 \sqrt{r_1^2+r_2^2} - r_1 \zeta_c c}{r_1 \sqrt{r_1^2+r_2^2} + r_2 \zeta_c c} \\ \frac{\sqrt{r_1^2+r_2^2}}{r_1 \sqrt{r_1^2+r_2^2} + r_2 \zeta_c c} \\ -1 \\ -\frac{r_2 \sqrt{r_1^2+r_2^2} - r_1 \zeta_c c}{r_1 \sqrt{r_1^2+r_2^2} + r_2 \zeta_c c} \\ -\frac{\sqrt{r_1^2+r_2^2}}{r_1 \sqrt{r_1^2+r_2^2} + r_2 \zeta_c c} \\ -\frac{\zeta_c c \sqrt{r_1^2+r_2^2}}{r_1 \sqrt{r_1^2+r_2^2} + r_2 \zeta_c c} \\ \frac{\sqrt{r_1^2+r_2^2}}{r_1 \sqrt{r_1^2+r_2^2} + r_2 \zeta_c c} \end{pmatrix} \begin{matrix} S_a \\ T_a \\ U_a \\ S_b \\ T_b \\ U_b \\ P_c \\ U_c \end{matrix} \quad (130)$$

where the row labels on the right of $\{\vec{\psi}\}_{8 \times 1}$ indicate the magnitude to which the vector element corresponds. According to Eq. (130), the magnitude of the force transmitted by gear pair c is given by:

$$U_c = \frac{S_a \sqrt{r_1^2 + r_2^2}}{r_1 \sqrt{r_1^2 + r_2^2 + r_2 \zeta_c}} \quad (131)$$

Since ζ_c is positive and $\zeta_c < r_1, r_2$, in practice, the denominator of Eq. (131) will always be positive. So, the sign of U_c is the same of S_a , i.e. $\text{sign}(U_c) = \text{sign}(S_a)$. Similarly, from Eq. (124), we have $\text{sign}(p_c) = \text{sign}(s_a)$.

The action matrix $[A]_{\lambda \times C}$ containing the wrenches transmitted through the network is obtained by multiplying each column of the unit action matrix $[\hat{A}_D]_{\lambda \times C}$ by the respective magnitudes. Applying the unit action matrix $[\hat{A}_D]_{3 \times 8}$ shown in Eq. (125) and the vector solution of Eq. (130) in Eq. (27) results in the action matrix of the simple bevel gear train analyzed. Eq. (132) shows the condensed version of the matrix obtained through Eq. (27), generated by adding together the columns related to same coupling, e.g. $\$a = \$S_a + \$T_a + \$U_a$.

$$[A]_{3 \times 3} = S_a \begin{bmatrix} \overset{\$a}{1} & \overset{\$b}{-1} & \overset{\$c}{1} \\ \frac{r_2 \sqrt{r_1^2 + r_2^2 - a r_1 \zeta_c}}{r_1 \sqrt{r_1^2 + r_2^2 + a r_2 \zeta_c}} & -\frac{(r_2 \sqrt{r_1^2 + r_2^2 - a r_1 \zeta_c})}{r_1 \sqrt{r_1^2 + r_2^2 + a r_2 \zeta_c}} & \frac{r_2 \sqrt{r_1^2 + r_2^2 - a r_1 \zeta_c}}{r_1 \sqrt{r_1^2 + r_2^2 + a r_2 \zeta_c}} \\ \frac{\sqrt{r_1^2 + r_2^2}}{r_1 \sqrt{r_1^2 + r_2^2 + a r_2 \zeta_c}} & -\frac{\sqrt{r_1^2 + r_2^2}}{r_1 \sqrt{r_1^2 + r_2^2 + a r_2 \zeta_c}} & \frac{\sqrt{r_1^2 + r_2^2}}{r_1 \sqrt{r_1^2 + r_2^2 + a r_2 \zeta_c}} \end{bmatrix} \begin{matrix} S \\ T \\ U \end{matrix} \quad (132)$$

where $a = \text{sign}(s_a) \text{sign}(S_a)$.

A.3 POWER, POWER FLOW AND EFFICIENCY

The motion matrix of Eq. (124) and the action matrix of Eq. (132) are used to calculate the power that enters or leaves the network in each coupling, the power flow transmitted into the network through each coupling and the efficiency.

The power flow that enters or leaves through coupling a is given by:

$$\mathcal{P}_a = \$a^m \cdot \$a^a = S_a s_a \quad (133)$$

The twist $\$a^m$ and the wrench $\$a^a$ are the first columns of $[M]_{3 \times 3}$, shown in Eq. (124), and $[A]_{3 \times 3}$, shown in Eq. (132), respectively. If S_a and s_a have the same sign, $\mathcal{P}_a$ is positive and power enters the network through coupling a . Otherwise, if the signs of S_a and s_a are opposite, $\mathcal{P}_a$ is negative and power leaves the network through coupling a .

Similarly, for couplings b and c :

$$\mathcal{P}_b = \$^m_b \cdot \$^a_b = -S_a s_a \frac{r_1(r_2 \sqrt{r_1^2 + r_2^2} - a r_1 \zeta_c)}{r_2(r_1 \sqrt{r_1^2 + r_2^2} + a r_2 \zeta_c)} \quad (134)$$

$$\mathcal{P}_c = \$^m_c \cdot \$^a_c = -|S_a| |s_a| \frac{\zeta_c(r_1^2 + r_2^2)}{r_2(r_1 \sqrt{r_1^2 + r_2^2} + a r_2 \zeta_c)} \quad (135)$$

The twist $\$^m_b$ and the wrench $\$^a_b$ of Eq. (134) are the second columns of $[M]_{3 \times 3}$ and $[M]_{3 \times 3}$, respectively. The screws $\$^m_c$ and $\$^a_c$ of Eq. (135) are the third columns of $[M]_{3 \times 3}$ and $[M]_{3 \times 3}$, respectively.

Since $\zeta_c > 0$ and $\zeta_c < r_1, r_2$, $\mathcal{P}_c < 0$ independently of the sign of S_a and s_a , meaning that power always leaves through coupling c due to meshing friction.

The power transferred from gear 1 to gear 2, considering the power lost in the process, is calculated as:

$$\mathcal{P}_{12} = \$^m_a \cdot \$^a_c = S_a s_a \quad (136)$$

where $\$^m_a$ is the twist representing the motion of gear 1 in relation to the fixed link 0 and $\$^a_c$ is the wrench representing the actions between gears 1 and 2.

The power transferred from gear 2 to gear 1, taking into account power losses, is given by:

$$\mathcal{P}_{21} = \$^m_b \cdot \$^a_c = S_a s_a \frac{r_1(r_2 \sqrt{r_1^2 + r_2^2} - a r_1 \zeta_c)}{r_2(r_1 \sqrt{r_1^2 + r_2^2} + a r_2 \zeta_c)} \quad (137)$$

Similarly to the equations of $\mathcal{P}_a$, $\mathcal{P}_b$ and $\mathcal{P}_c$, the screws of Eqs. (136) and (137) are taken from the matrices of Eqs. (124) and (132).

The ordinary efficiency of a gear pair depends on the power flow direction and is determined by the ratio between the power transferred by the driver gear and the power received by the follower gear, considering power losses.

There are two scenarios: in the first one, gear 1 drives gear 2, meaning that S_a and s_a have the same sign so $a = \text{sign}(S_a)\text{sign}(s_a) = 1$ and $S_a s_a > 0$. Thus coupling a is the input and coupling b , the output. If b is the output, $\mathcal{P}_b < 0$. According to Eq. (134), it is necessary that $\zeta_c < (r_2/r_1)\sqrt{r_1^2 + r_2^2}$ so $\mathcal{P}_b$ is negative, which is true in practice.

For the first scenario, the ordinary efficiency η_{12} is defined as:

$$\eta_{12} = \frac{\mathcal{P}_{\text{follower}}}{\mathcal{P}_{\text{driver}}} = \frac{\mathcal{P}_{21}}{\mathcal{P}_{12}} = \frac{r_1(r_2 \sqrt{r_1^2 + r_2^2} - r_1 \zeta_c)}{r_2(r_1 \sqrt{r_1^2 + r_2^2} + r_2 \zeta_c)} \quad (138)$$

where the subscript of η_{12} indicates that gear 1 drives gear 2.

Eq. (138) allows to estimate ζ_c from the ordinary efficiency of the gear pair, as η_{12} can be calculated or obtained experimentally. For instance, η_{12} can be assumed as 0.98 for straight bevel gears (PATHAK; JAIN; PALANI, 2016). Isolating ζ_c in Eq. (138) leads to:

$$\zeta_c = \zeta_{12} = -\frac{r_1 r_2 \sqrt{r_1^2 + r_2^2} (\eta_{12} - 1)}{r_1^2 + \eta_{12} r_2^2} \quad (139)$$

Eq. (139) should be use to calculate ζ_c when coupling a is the input.

In the second scenario, gear 2 drives gear 1 and the signs of S_a and s_a are opposite, i.e. $a = \text{sign}(S_a)\text{sign}(s_a) = -1$. So coupling b is the input and coupling a , the output. The ordinary efficiency for the second scenario is:

$$\eta_{21} = \frac{P_{\text{follower}}}{P_{\text{driver}}} = \frac{P_{12}}{P_{21}} = \frac{r_2(r_1 \sqrt{r_1^2 + r_2^2} - r_2 \zeta_c)}{r_1(r_2 \sqrt{r_1^2 + r_2^2} + r_2 \zeta_c)} \quad (140)$$

where the subscript of η_{21} indicates that gear 2 drives gear 1.

Similarly to the first scenario, ζ_c can estimated from η_{21} , resulting in:

$$\zeta_c = \zeta_{21} = -\frac{r_1 r_2 \sqrt{r_1^2 + r_2^2} (\eta_{21} - 1)}{r_2^2 + \eta_{21} r_1^2} \quad (141)$$

Eq. (141) is generated by isolating ζ_c in Eq. (140) or by replacing η_{12} by η_{21}^{-1} in Eq. (139) and reverting the sign of the result to compensate for the different signs of S_a and s_a . Eq. (141) is used to compute ζ_c when coupling b is the input.

The ordinary efficiencies η_{12} and η_{21} have the same numerical value, although Eqs. (138) and (140) are generated under different circumstances, depending on the power flow direction.

The sign function can be avoided when ζ_c is allowed to be negative when necessary, assuming that power losses are not high enough to change power flow direction. Thus, the Coulomb friction model can be simplified to:

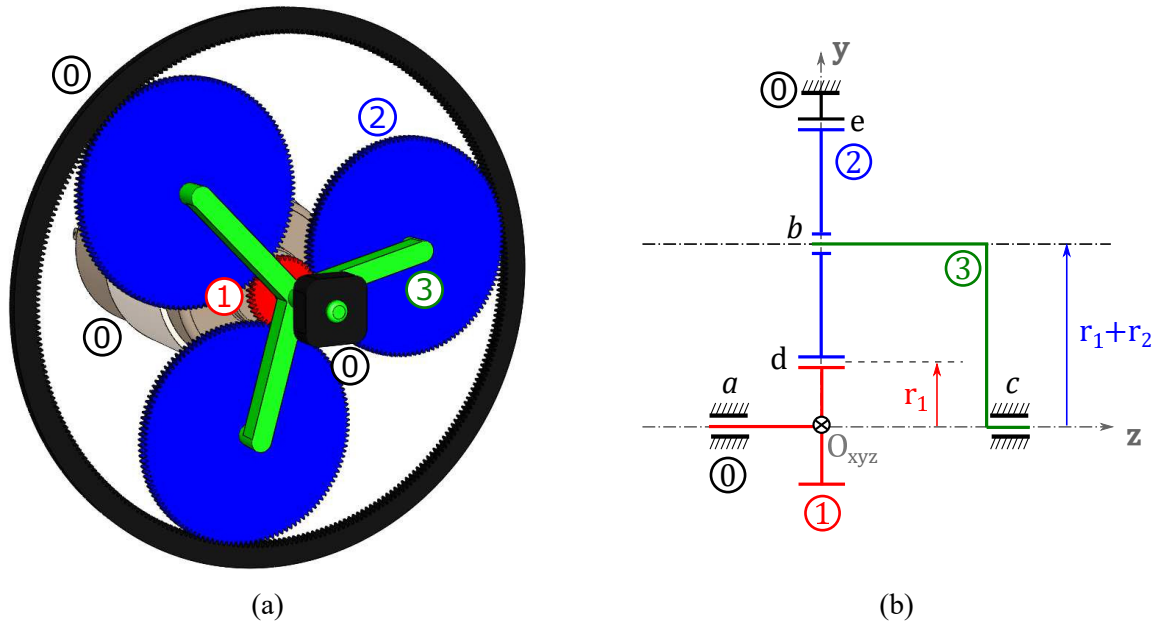
$$P_{Li} = \zeta_i U_i \quad (142)$$

This simplification helps to use Eqs. (139) and (141) in the efficiency calculation of complex PGTs composed of multiple bevel gear pairs, such as an automotive differential gearbox.

APPENDIX B – BENDING AND CONTACT STRESSES ON SPUR GEARS

This appendix details the parameters used to calculate bending and contact stresses on spur cylindrical gears. This appendix focuses on justifying the factors adopted in Chapter 5, where bending and contact stresses are constraint functions of the optimization problem regarding the DC motor's reducer of the inspection robot for overhead powerlines. Fig. 40 shows the reducer, which is a PGT. Fig. 40(a) is the PGT CAD model and Fig. 40(b) is the schematic representation, where the gears' pitch radii and the couplings' labels are indicated.

Figure 40 – DC motor's reducer (a) CAD model and (b) schematic representation.



Source: From the author.

B.1 BENDING STRESSES

For the PGT shown in Fig. 40, the bending stress σ_b must be calculated for all gears: sun gear (link 1), planet gear (link 2) and ring gear (link 3). The AGMA bending stress equation is given by (NORTON, 2010; AGMA, 2004):

$$\sigma_b = \frac{W_t}{bmJ} \frac{K_a K_m}{K_v} K_S K_B K_I \quad (143)$$

where b is the tooth's face width, m is the gear module, W_t is the tangential force acting on the tooth flank, J is the bending strength geometry factor, K_a is the application factor, K_v is the dynamic factor, K_m is the load distribution factor, K_S is the size factor, K_B is the rim thickness factor and K_I is the idler factor.

Equation (143) is valid only for external gears and AGMA does not have an official standard to calculate bending stresses in internal gears. An adaptation of the AGMA method for internal gears is adopted in this work (LISLE; SHAW; FRAZER, 2019):

$$\sigma_b^{int} = \frac{W_t K_f}{b Y} \quad (144)$$

where Y is the form factor and K_f is the stress concentration factor.

The bending stress on sun and planet gears (links 1 and 2) are calculated using Eq. (143), while the bending stress on the ring gear is calculated through Eq. (144).

The tangential transmitted force W_t has the same magnitude for the three gears and is calculated as:

$$W_t = \frac{T_a}{r_1} \quad (145)$$

where T_a is the motor torque, transmitted to the PGT through coupling a , and r_1 is the sun gear pitch radius.

The nominal value for the face width is $b = 12m$ (NORTON, 2010).

Bending strength geometry factor J

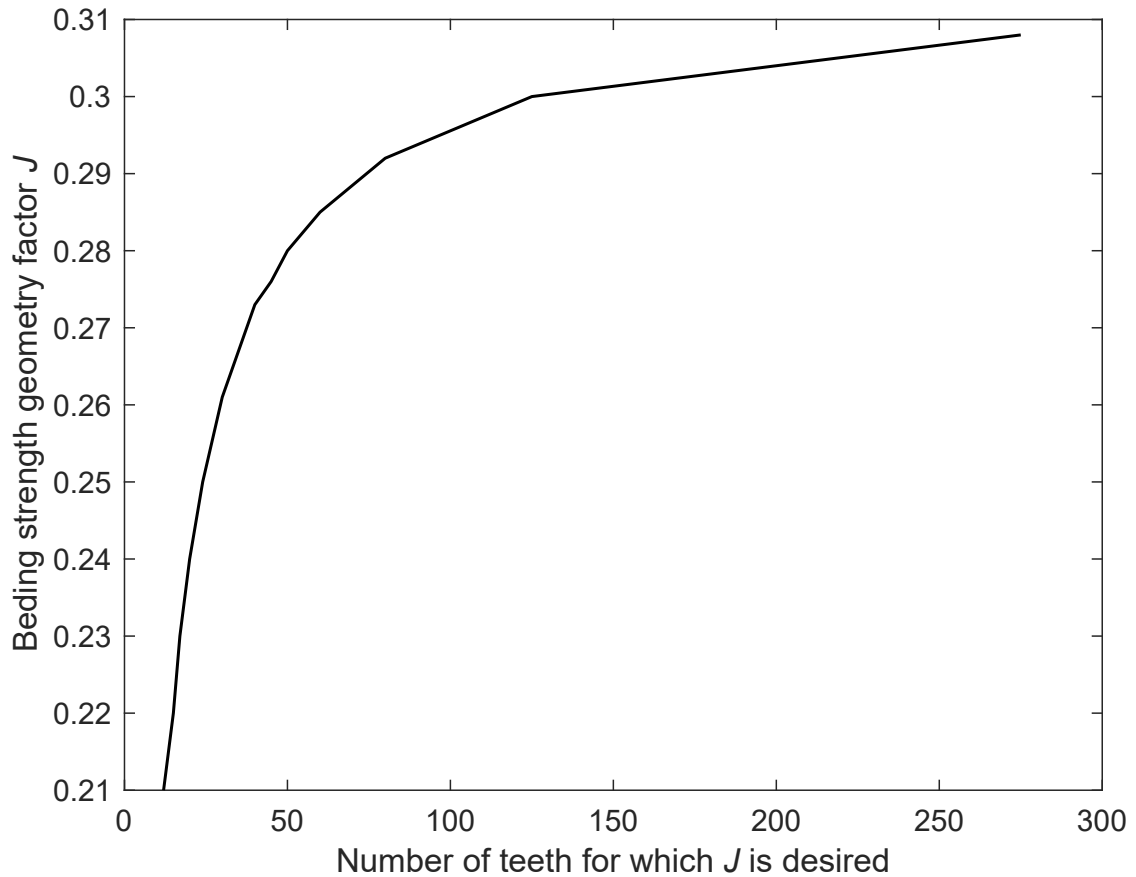
The bending strength geometry factor J serves to take the tooth geometry into account to determine its effective strength at the root fillet. J varies with the number of teeth on the pinion and mating gear and the pressure angle. In this thesis, it is assumed the worst case: only one pair of teeth take all the load at the tip (NORTON, 2010), making that J depends exclusively on the number of teeth for which J is desired (BUDYNAS; NISBETT, 2020). Fig. 41 shows a curve of J as function of the number of teeth, for a pressure angle of 20° . During the optimization, J is obtained from this curve through linear interpolation.

Application factor K_a

The application factor emulates the dynamic loads to which the gear teeth are submitted, based on the "shockiness" of the power source connected to the gear train. Tab. 30 contains some AGMA-suggested values for K_a .

The driving machine of the PGT is the DC motor, which falls into the category "uniform" in the first column of Tab. 30. The driven machine is the inspection robot and it is assumed that it moves smoothly, so the inspection robot also falls into the category "uniform". Thus, $K_a = 1$ in Chapter 5.

Load distribution factor K_m

Figure 41 – Geometry factor J for load applied at the tip of tooth and pressure angle 20° .

Source: Adapted from Budynas and Nisbett (2020).

Table 30 – Application factors

Driving machine	Driven machine		
	Uniform	Moderate shock	Heavy shock
Uniform	1	1.25	1.75 or higher
Light shock	1.25	1.50	2 or higher
Medium shock	1.5	1.75	2.25 or higher

Source: Adapted from Norton (2010).

Any misalignment in the tooth form makes that W_t to be unevenly distributed over the tooth face width. The larger the face width, more pronounced becomes this problem. The load distribution factor K_m makes that phenomenon is accounted in the bending stress calculation (NORTON, 2010). Tab. 31 contains some suggested values for K_m as function of the face width b .

Table 31 – Load distribution factors

Face width b [mm]	K_m
< 50	1.6
150	1.7
250	1.8
≥ 500	2

Source: Adapted from Norton (2010).

In Chapter 5, $0.1 \text{ mm} \leq m \leq 2 \text{ mm}$, so $b < 50 \text{ mm}$. Thus, for the reducer optimization, $K_m = 1.6$.

Dynamic factor K_v

The dynamic factor K_v serves to consider the internally generated vibration loads from tooth-tooth impacts induced by nonconjugate meshing of the gear teeth (NORTON, 2010). K_v is calculated through the empirical equation Eq. (146):

$$K_v = \left(\frac{A}{A + \sqrt{200v_t}} \right)^B \quad (146)$$

where v_t is the linear velocity of the gear teeth at the pitch point, called pitch-line velocity, in m/s, and the factors A and B are defined as:

$$A = 50 + 56(1 - B) \quad (147)$$

$$B = \frac{(12 - Q_v)^{2/3}}{4}, \text{ for } 6 \leq Q_v \leq 11 \quad (148)$$

The quality index Q_v quantifies the gear accuracy and is based on the pitch-line velocity. The pitch-line velocity is the same for the sun, planet and ring gears and is given by:

$$v_t = t_a r_1 \quad (149)$$

where t_a is the DC motor speed.

For pitch-line velocities lower than 4 m/s, $6 \leq Q_v \leq 8$ (NORTON, 2010). In the reducer optimization of Chapter 5, $v_t < 4 \text{ m/s}$ for most of the design variables sets, so $Q_v = 7$ is adopted.

Size factor K_s

The size factor K_S reflects nonuniformity of material properties due to size. AGMA has not yet been established standard values for K_S when there is a detrimental size effect. so it recommends to adopt $K_S = 1$ (BUDYNAS; NISBETT, 2020).

Rim thickness factor K_B

This factor takes into account situations where the gear is made with a rim and spokes rather than a solid disk. This design can provoke a radial fracture across the rim rather than through the tooth root. The sun and ring gears of the reducer are solid-disk gears, so $K_B = 1$ (NORTON, 2010).

Idler factor K_I

Idler gears are subjected to more cycles of stress per unit time and to larger-magnitude alternating loads. The idler factor K_I serves to take into account this situation, so for the planet gear, which is an idler gear, $K_I = 1.42$. For the sun gear, $K_I = 1$, as it is a nonidler gear (NORTON, 2010).

Form factor Y

The form factor Y is used to compute the bending stress of the ring gear and indicates that compression loads due to the radial component of the tangential force are neglected (BUDYNAS; NISBETT, 2020). Tab. 32 contains the values adopted for Y as function of the ring gear number of teeth z_0 . Y is obtained through linear interpolation when z_0 does not appear in the first columns of Tab. 32.

Stress concentration factor K_f

The stress concentration factor K_f is used in the bending stress calculation of the ring gear and is calculated according to Eq. (150) (BUDYNAS; NISBETT, 2020):

$$K_f = H + \left(\frac{t}{r}\right)^L \left(\frac{t}{l}\right)^M \quad (150)$$

Table 32 – Values for form factor Y for a pressure angle of 20°

Number of teeth	Y
12	0.245
13	0.261
14	0.277
15	0.290
16	0.296
17	0.303
18	0.309
19	0.314
20	0.322
21	0.328
22	0.331
24	0.337
26	0.346
28	0.353
30	0.359
34	0.371
38	0.384
43	0.397
50	0.409
60	0.422
75	0.435
100	0.447
150	0.460
300	0.472
400	0.480

Source: Adapted from Budynas and Nisbett (2020).

$$\text{where } H = 0.34 - 0.458366 \, 2\varphi$$

$$L = 0.316 - 0.458366 \, 2\varphi$$

$$M = 0.290 - 0.458366 \, 2\varphi$$

$$r = \frac{(h_f - r_f)^2}{r_0 + h_f - r_f}$$

where φ is the pressure angle in rad, h_f is the dedendum, standardized as $h_f = 1.25m$ for internal gears, and r_f is the fillet radius, assumed as $r_f = 0.15m$ (LISLE; SHAW; FRAZER, 2019).

The parameters t and l come from the simplification adopted to derive the bending stress equation, by considering the gear tooth analogue to a rectangular cantilever beam of cross-sectional dimensions b and t , with length l and submitted to a load W_t at its free extremity (BUDYNAS; NISBETT, 2020).

The length l is analogue to the tooth depth and is given by:

$$l = h_a + h_f \quad (151)$$

where h_a is the addendum and is standardized as $h_a = m$.

The dimension t is calculated as:

$$t = \sqrt{4I \frac{3Ym}{2}} \quad (152)$$

Corrected allowable bending stress

The peak of the bending stress σ_b must be lesser than the corrected allowable bending stress S_{fb} , calculated as (NORTON, 2010):

$$S_{fb} = \frac{K_L}{K_T K_R} S'_{fb} \quad (153)$$

where K_L is the life factor, K_T is the temperature factor, K_R is the reliability factor and S'_{fb} is the published AGMA bending-fatigue strength.

The published AGMA bending-fatigue strength is based on 10^7 load cycles. The life factor K_L modifies the gear strength for lives other than 10^7 cycles. In this thesis, $K_L = 1$ as it is assumed that the gears' life is of 10^7 cycles (BUDYNAS; NISBETT, 2020).

The temperature factor K_T can be set to 1 for temperatures below 121°C , so $K_T = 1$ in Chapter 5 (NORTON, 2010).

Finally, the reliability factor K_R is set to 1, supposing a 99% reliability (NORTON, 2010).

B.2 CONTACT STRESSES OR PITTING RESISTANCE

The contact stress σ_c must be calculated for each gear pair of the PGT shown in Fig. 40: coupling d , connecting the sun gear to the planet gear, and coupling e , connecting the planet gear to the ring gear. The AGMA pitting resistance formula is given by (AGMA, 2004; NORTON, 2010):

$$\sigma_c = C_p \sqrt{\frac{W_t}{bld} \frac{K_a K_m}{K_v} K_S C_f} \quad (154)$$

where d is the pitch diameter of the smaller of the two gears in mesh, l is a dimensionless surface geometry factor, C_p is an elastic coefficient and C_f is the surface finish factor.

Elastic coefficient C_p

The elastic coefficient accounts for differences in tooth materials. When both pinion and mating gear are made of steel, $C_p = 191 \text{ MPa}^{0.5}$ (NORTON, 2010). It is

supposed that sun, planet and ring gears are made of steel, so this C_p value is adopted in Chapter 5.

Surface geometry factor I

The surface geometry factor I considers the effect of the radii of curvature of the gear teeth and the pressure angle φ on the contact stress. I is computed as:

$$I = \frac{\cos(\varphi)}{d_p \left(\frac{1}{\rho_p} \pm \frac{1}{\rho_g} \right)} \quad (155)$$

where ρ_p and ρ_g are the curvature radii of the pinion and the gear teeth, respectively, and d_p is the pinion pitch diameter (NORTON, 2010). The $\pm$ sign accounts for external and internal gearsets. For external gearing, the plus sign is used. That is the case of coupling d (sun gear and planet gear). For internal gearing, the minus sign is used, e.g. in coupling e , connecting the planet gear to the sun gear.

The radii of curvature ρ_p and ρ_g are given by:

$$\rho_p = \sqrt{\left(r_p + \frac{1 + x_p}{p_d} \right)^2 - (r_p \cos(\varphi))^2} - \frac{\pi}{p_d} \cos \varphi \quad (156)$$

$$\rho_g = C \sin(\varphi) \mp \rho_p \quad (157)$$

where r_p is the pitch radius of the pinion, p_d is the diametral pitch (defined as $p_d = 1/m$), C is the center distance between pinion and gear and x_p is the pinion addendum coefficient, assumed as $x_p = 0$ in Chapter 5, as the PGT is composed of gear with standard, full-depth teeth (NORTON, 2010).

The equation to obtain ρ_g has a $\mp$. The minus sign is used for external gearing, while the plus sign is for internal gearing.

Surface finish factor C_f

The surface finish factor C_f accounts for unusually rough surface finishes on gear teeth. AGMA has not yet established standards values for C_f and recommends to adopt $C_f = 1$.

Corrected allowable contact stress

The pitting resistance σ_c must be lesser than the corrected allowable contact stress S_{fc} :

$$S_{fc} = \frac{C_L C_H}{K_T K_R} S'_{fc} \quad (158)$$

where C_L is the surface-life factor, C_H is the hardness ratio factor for pitting resistance and S'_{fc} is the published AGMA surface-fatigue strength.

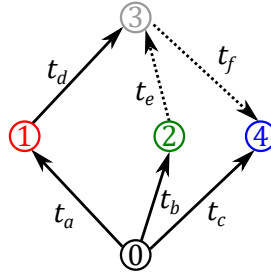
The published AGMA surface-fatigue strength is based on 10^7 load cycles. Similarly to K_L , the life factor C_L modifies the gear strength for lives other than 10^7 cycles. In this thesis, $C_L = 1$ as it is assumed that the gears' life is of 10^7 cycles (BUDYNAS; NISBETT, 2020).

The hardness ratio factor for pitting resistance C_H accounts for the relative hardness of pinion and gear. It is assumed that the gears composing the PGT have approximately equal hardness, so $C_H = 1$.

APPENDIX C – DAVIES' METHOD MATRICES FOR DETERMINING PSD EFFICIENCY

C.1 MOTION ANALYSIS

Figure 42 – PSD motion graph.



Source: From the author.

Fundamental circuit matrix $[B_M]_{2 \times 6}$

Considering edges e and f as chords:

$$[B_M]_{2 \times 6} = \begin{matrix} & \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{bmatrix} -1 & 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 1 & 0 & 1 \end{bmatrix} & \begin{matrix} e \\ f \end{matrix} \end{matrix} \quad (159)$$

Unit motion matrix $[\hat{M}_D]_{2 \times 6}$

$$[\hat{M}_D]_{2 \times 6} = \begin{matrix} & \begin{matrix} t_a & t_b & t_c & t_d & t_e & t_f \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & r_2 + r_3 & r_2 & r_4 \end{bmatrix} & \begin{matrix} t \\ u \end{matrix} \end{matrix} \quad (160)$$

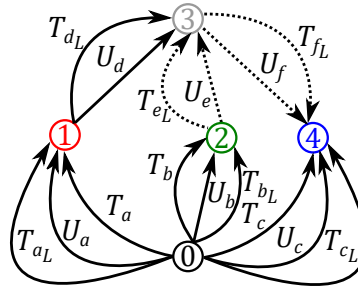
Network unit motion matrix $[\hat{M}_N]_{4 \times 6}$

$$[\hat{M}_N]_{4 \times 6} = \begin{matrix} & \begin{matrix} t_a & t_b & t_c & t_d & t_e & t_f \end{matrix} \\ \begin{bmatrix} -1 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -r_2 - r_3 & r_2 & 0 \\ 1 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & r_2 + r_3 & 0 & r_4 \end{bmatrix} & \begin{matrix} t \\ u \\ t \\ u \end{matrix} \end{matrix} \quad (161)$$

C.2 ACTION ANALYSIS

Fundamental cutset matrix ignoring power losses $[Q_A]_{4 \times 9}$

Figure 43 – PSD action graph.



Source: From the author.

Considering edges T_a , T_b , T_c and U_d as branches:

$$[Q_A]_{4 \times 9} = \begin{matrix} & \begin{matrix} a & a & b & b & c & c & d & e & f \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \end{bmatrix} & \begin{matrix} T_a \\ T_b \\ T_c \\ U_d \end{matrix} \end{matrix} \quad (162)$$

Unit action matrix ignoring power losses $[\hat{A}_D]_{2 \times 9}$

$$[\hat{A}_D]_{2 \times 9} = \begin{matrix} & \begin{matrix} T_a & U_a & T_b & U_b & T_c & U_c & U_d & U_e & U_f \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & -r_2 - r_3 & -r_2 & -r_4 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} & \begin{matrix} t \\ u \end{matrix} \end{matrix} \quad (163)$$

Network unit action matrix ignoring power losses $[\hat{A}_N]_{8 \times 9}$

$$[\hat{A}_N]_{2 \times 9} = \begin{matrix} & \begin{matrix} T_a & U_a & T_b & U_b & T_c & U_c & U_d & U_e & U_f \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -r_2 & r_4 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & r_2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -r_4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -r_2 - r_3 & -r_2 & r_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \end{bmatrix} & \begin{matrix} T \\ U \\ T \\ U \\ T \\ U \\ T \\ U \end{matrix} \end{matrix} \quad (164)$$

Fundamental cutset matrix considering power losses $[Q_A]_{4 \times 15}$ Considering edges T_a , T_b , T_c and U_d as branches:

$$[Q_A]_{4 \times 15} = \begin{matrix} & \begin{matrix} a & a & a & b & b & b & c & c & c & d & d & e & e & f & f \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix} & \begin{matrix} T_a \\ T_b \\ T_c \\ U_d \end{matrix} \end{matrix} \quad (165)$$

Unit action matrix considering power losses $[\hat{A}_D]_{2 \times 15}$

$$[\hat{A}_D]_{2 \times 15} = \begin{matrix} & \begin{matrix} T_a & T_{a_L} & U_a & T_b & T_{b_L} & U_b & T_c & T_{c_L} & U_c & T_{d_L} & U_d & T_{e_L} & U_e & T_{f_L} & U_f \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & -r_2 - r_3 & 1 & -r_2 & 1 & -r_4 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} & \begin{matrix} T \\ U \end{matrix} \end{matrix} \quad (166)$$

Augmented action matrix $[D_A]_{14 \times 15}$

$$[D_A]_{14 \times 15} = \begin{array}{c} \begin{array}{cccccccccccccc} T_a & T_{a_L} & U_a & T_b & T_{b_L} & U_b & T_c & T_{c_L} & U_c & T_{d_L} & U_d & T_{e_L} & U_e & T_{f_L} & U_f \end{array} \\ \left[\begin{array}{cccccccccccccc|c} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -r_2 & -1 & r_4 & T \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & U \\ \hline 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & r_2 & 0 & 0 & T \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & U \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -r_4 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & U \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -r_2 - r_3 & 1 & -r_2 & -1 & r_4 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & U \\ \hline 0 & 1 & a\mu r_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c.e. \\ 0 & 0 & 0 & 0 & 1 & b\mu r_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c.e. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & c\mu r_c & 0 & 0 & 0 & 0 & 0 & 0 & c.e. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & d\mu r_d & 0 & 0 & 0 & 0 & c.e. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\zeta_e & 0 & 0 & c.e. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\zeta_f & c.e. \end{array} \right] \end{array} \quad (167)$$

where $a = \text{sign}(U_a) \text{sign}(t_a)$, $b = \text{sign}(U_b) \text{sign}(t_b)$, $c = \text{sign}(U_c) \text{sign}(t_c)$ and $d = \text{sign}(U_d) \text{sign}(t_d)$

APPENDIX D – ALGORITHM FOR COMPUTING PRIUS FUEL ECONOMY

The pseudo-code of Algorithm 2 is developed to compute Prius fuel economy in Chapter 6 and is applied to the HEV equipped with a DHS in Chapter 7. It is assumed that the initial SOC, SOC_0 , is 0.7 in the first iteration of the **for** loop and that the ICE idle speed N_{ICE}^{idle} is 1200 rpm, similarly to the ADVISOR simulations (MARKEL et al., 2002). A time step $\Delta\tau = 1$ s is considered.

In Algorithm 2, N_{MG1}^{max} is the maximum MG1 speed and is equal to 5500 rpm. i_{ab} is the ratio between ICE speed and MG1 speed. T_{MG1}^{max} is the MG1 maximum torque for a given speed. MG2 power and torque are represented by $\mathcal{P}_{MG2}$ and T_{MG2} , respectively. T_{MG2}^{max} is the MG2 maximum torque for a given speed. η_{MG1} and η_{MG2} stand for MG1 and MG2 efficiencies. The Prius fuel economy is represented by FC_{Prius} .

Algorithm 2 Pseudo-code to compute Prius fuel economy

```

1: for each instant  $\tau$  of the drive cycle do
2:    $SOC(\tau) \leftarrow SOC_0$ 
3:   if  $\tau = 1$  then
4:      $dv \leftarrow 0$ 
5:   else
6:      $dv \leftarrow v(\tau) - v(\tau - 1)$ 
7:   end if
8:    $dt \leftarrow 1$  ▷  $\Delta\tau = 1$  s
9:   Compute rolling resistance coefficient  $f_r$  via Eq. (2)
10:  Compute tractive force  $F_t$  via Eq. (7)
11:  Compute drive wheel speed  $N_w$  via Eq. (9)
12:  Compute PSD output speed  $t_c$  via Eq. (10)
13:  if  $dv \geq 0$  then
14:    if  $v(\tau) = 0$  or  $v(\tau) \leq v_{eb}$  then ▷ Mode 0
15:      if  $SOC(\tau) > SOC_{ICE}$  then ▷ ICE turned off
16:         $t_a \leftarrow 0$ 
17:         $T_a \leftarrow 0$ 
18:      else ▷ ICE turns on
19:         $t_a \leftarrow t_{a_{opt}}$ 
20:         $T_a \leftarrow T_{a_{opt}}$ 
21:      end if
22:    else ▷  $v(\tau) > v_{eb}$ 
23:      if  $SOC(\tau) \leq SOC_t$  then
24:         $t_a \leftarrow t_{a_{opt}}$ 
25:         $T_a \leftarrow T_{a_{opt}}$ 
26:      else

```

```

27:          $t_a \leftarrow N_{ICE}^{idle}$ 
28:          $T_a \leftarrow 0$ 
29:     end if
30: end if
31: else ▷  $dv < 0 \rightarrow$  deceleration: regenerative breaking
32:     if  $SOC(\tau) > SOC_{ICE}$  then
33:          $t_a \leftarrow 0$ 
34:          $T_a \leftarrow 0$ 
35:     else
36:          $t_a \leftarrow t_{a_{opt}}$ 
37:          $T_a \leftarrow T_{a_{opt}}$ 
38:     end if
39: end if
40:  $[\eta_{PSD}, t_b, T_b, T_c, \mathcal{P}_a, \mathcal{P}_b, \mathcal{P}_c] = \text{PSD-efficiency}(t_a, t_c, T_a)$  ▷ Davies' method -
    Sec. 6.1
41: if  $|t_b| > N_{MG1}^{max}$  then ▷ Check if  $t_b$  is higher than MG1 max speed (5500 rpm)
42:     if  $T_a = 0$  and  $t_a = 0$  then ▷ Modes 0 or 3
43:         if  $SOC(\tau) \leq SOC_{ICE}$  then
44:              $t_a \leftarrow t_{a_{opt}}$ 
45:              $T_a \leftarrow T_{a_{opt}}$ 
46:         else
47:              $t_a \leftarrow N_{ICE}^{idle}$ 
48:              $T_a \leftarrow 0$ 
49:         end if
50:     else ▷ Mode 1 or 2
51:          $i_{ab} = t_a/t_b$ 
52:          $t_a \leftarrow i_{ab} N_{MG1}^{max}$ 
53:          $T_a \leftarrow \text{OOL}(t_a)$  ▷  $T_a$  is defined from OOL for the new  $t_a$ 
54:     end if
55:      $[\eta_{PSD}, t_b, T_b, T_c, \mathcal{P}_a, \mathcal{P}_b, \mathcal{P}_c] = \text{PSD-efficiency}(t_a, t_c, T_a)$ 
56: end if
57:  $T_{MG1}^{max} \leftarrow \text{MG1max-torque}(t_b)$ 
58: if  $|T_b| > T_{MG1}^{max}$  then ▷ Check if  $T_b$  is higher than MG1 max torque for given
    speed  $t_b$ 
59:     Compute  $T_a$  via Eq. (93), using  $T_b = T_{MG1}^{max}$ 
60:      $[\eta_{PSD}, t_b, T_b, T_c, \mathcal{P}_a, \mathcal{P}_b, \mathcal{P}_c] = \text{PSD-efficiency}(t_a, t_c, T_a)$ 
61: end if
62: Compute power demand  $\mathcal{P}_{dem}$  via Eq. (8)
63:  $\mathcal{P}_{MG2} \leftarrow \mathcal{P}_{dem} - |\mathcal{P}_c|$ 

```

```

64:   if  $t_c = 0$  then
65:        $T_{MG2} \leftarrow 0$ 
66:   else
67:        $T_{MG2} \leftarrow \mathcal{P}_{MG2}/|t_c|$ 
68:   end if
69:    $T_{MG2}^{max} \leftarrow \text{MG2max-torque}(t_c)$ 
70:   if  $|T_{MG2}| > T_{MG2}^{max}$  then  $\triangleright$  Check if  $T_{MG2}$  is higher than MG2 max torque for
    given speed  $t_c$ 
71:       if  $dv < 0$  and  $T_a = 0$  then  $\triangleright$  Deceleration - mode 3
72:            $T_{MG2} \leftarrow \text{sgn}(T_{MG2}) T_{MG2}^{max}$ 
73:            $\mathcal{P}_{MG2} \leftarrow T_{MG2} t_c$   $\triangleright$  Mechanical braking
74:       else
75:            $FC_{Prius} \leftarrow 0$ 
76:           break
77:       end if
78:   end if
79:   if  $t_a = 0$  or  $t_a = N_{ICE}^{idle}$  then
80:        $b_e(\tau) \leftarrow 0$ 
81:        $\dot{m}_{fuel}(\tau) \leftarrow 0$ 
82:        $\dot{m}_{HC}(\tau) \leftarrow 0$ 
83:        $\dot{m}_{CO}(\tau) \leftarrow 0$ 
84:        $\dot{m}_{NO_x}(\tau) \leftarrow 0$ 
85:   else
86:        $b_e(\tau) \leftarrow \text{ICE-maps}(t_a, T_a)$   $\triangleright$  ICE fuel consumption map
87:       Compute  $\dot{m}_{fuel}(\tau)$  via Eq. (12)
88:        $\dot{m}_{HC}(\tau) \leftarrow \text{ICE-maps}(t_a, T_a)$   $\triangleright$  ICE HC map
89:        $\dot{m}_{CO}(\tau) \leftarrow \text{ICE-maps}(t_a, T_a)$   $\triangleright$  ICE CO map
90:        $\dot{m}_{NO_x}(\tau) \leftarrow \text{ICE-maps}(t_a, T_a)$   $\triangleright$  ICE  $NO_x$  map
91:   end if
92:    $\eta_{MG1}(\tau) \leftarrow \text{MG1-map}(t_b, T_b)$ 
93:   Compute  $\mathcal{P}_{MG1_{bat}}(\tau)$  via Eq. (21)
94:    $\eta_{MG2}(\tau) \leftarrow \text{MG2-map}(t_c, T_{MG2})$ 
95:   Compute  $\mathcal{P}_{MG2_{bat}}(\tau)$  via Eq. (20)
96:   Compute  $\mathcal{P}_{bat}(\tau)$  via Eq. (19)
97:   Compute  $SOC(\tau)$  via Eq. (17)
98:    $SOC_0 \leftarrow SOC(\tau)$ 
99: end for
100:  $S_{cyc} \leftarrow \text{trapz}(v(\tau))$ , for  $\tau = 1, \dots, n$ 
101: Compute  $m_{HC}$ ,  $m_{CO}$  and  $m_{NO_x}$  via Eq. (101)

```

-
- 102: Compute $m_{HC}^{g/dist}$, $m_{CO}^{g/dist}$ and $m_{NO_x}^{g/dist}$ via Eq. (102)
 103: Compute m_{fuel} via Eq. (13)
 104: Compute FC_{Prius} via Eq. (14) or Eq. (15)
-

Algorithm 2 is interpreted as follows. The Prius initial SOC is 0.7, so in instant $\tau = 1$ s, $SOC(1) = 0.7$. Then, for each instant of the drive cycle, the variables dv and dt are determined (lines 3 to 8) to be applied in Eq. (6) so that the tractive force F_t is calculated (line 10). The rolling resistance coefficient f_r is also defined (line 9) to be applied in Eq. (7) to obtain F_t .

The explanation given in Sec. 2.2.1 for obtaining the PSD output speed (t_c) is applied in lines 11 and 12. Thus, one of the primary variables for the PSD motion analysis, detailed in Sec. 6.1, is defined. The second primary variable is the ICE speed t_a , which depends on the current Prius operating mode. The primary variable for the action analysis, detailed in Sec. 6.1, is the ICE torque T_a , whose definition also depends on the Prius operating mode (lines 13 to 39).

In line 13, it is verified if the Prius is accelerating ($dv > 0$). If the HEV is accelerating, it must be verified if its speed is below $v_{eb} = 45$ km/h, i.e. it must be verified if the Prius is in mode 0, when is driven only by MG2 (line 14). If the Prius speed is below v_{eb} , it is verified if the current battery pack SOC is above SOC_{ICE} (line 15). If $SOC(\tau) > SOC_{ICE}$, the ICE is kept locked and does not supplies any power to the PSD, so t_a and T_a are null (lines 16 and 17). If $SOC(\tau) < SOC_{ICE}$ (line 18), the ICE starts and operates at its optimum point, rotating at a speed $t_{a_{opt}}$ and providing a torque $T_{a_{opt}}$ (lines 19 and 20). This way, the battery pack is recharged during mode 0.

If the Prius is accelerating but its speed is above v_{eb} (line 22), the ICE turns on and operates at its optimum point if the current SOC is below SOC_t (lines 23, 24 and 25). If the current SOC is above SOC_t , ICE idles and provides no torque (lines 26, 27 and 28), so only MG2 drives the Prius.

If the Prius is decelerating (line 31), it operates under mode 3 (regenerative braking). If the current SOC is above SOC_{ICE} , the ICE turns off, so $t_a = 0$ and $T_a = 0$ (lines 32, 33 and 34). Otherwise, the ICE turns on and operates at its optimum point (lines 35, 36 and 37)

At line 40, the primary variables of the Davies method are defined and the PSD efficiency η_{PSD} can be calculated. η_{PSD} is obtained through a function called **PSD-efficiency**, whose parameters are the primary variables t_c , t_a and T_a . Besides η_{PSD} , the function returns MG1 speed, torque and power, t_b , T_b and $\mathcal{P}_b$, respectively; the PSD output torque T_c and output power $\mathcal{P}_c$; and the ICE power $\mathcal{P}_a$.

Line 41 verifies if the MG1 speed t_b found in line 40 is higher than the MG1 maximum speed ($N_{MG1}^{max} = 5500$ rpm). If that is the case, t_b must be recalculated using a different value of t_a so that $t_b < N_{MG1}^{max}$. By changing t_a , T_a must also be changed, so the function **PSD-efficiency** must be applied again with the new primary variables. If

the Prius is operating in mode 0 or 3 (line 42), the choice of the new t_a depends on the current SOC. If the SOC is below SOC_{ICE} , the ICE turns on and operates at its optimum point (lines 43, 44 and 45). If the SOC is above SOC_{ICE} , the ICE shaft is unlocked and the ICE idles (lines 46, 47 and 48).

If $|t_b| > N_{MG1}^{max}$ but the Prius is operating in mode 1 or 2 (line 50), the new t_a is calculated by multiplying N_{MG1}^{max} by the ratio between the current t_a and t_b (lines 51 and 52). The new ICE torque T_a is obtained through the function **OOL**, whose parameter t_a is the new ICE speed (line 53). The function **OOL** returns the ICE torque belonging to the OOL, shown in Fig. 25, for a given ICE speed.

In line 55, the function **PSD-efficiency** is applied again with the new primary variables t_a and T_a , which assure that the new resulting t_b is below N_{MG1}^{max} .

The function **MG1max-torque** in line 57 returns the MG1 torque T_{MG1}^{max} on the maximum torque curve shown in Fig. 29, for a given MG1 speed t_b . Then, in line 58, it is verified if MG1 torque T_b is higher than T_{MG1}^{max} . If $|T_b| > T_{MG1}^{max}$, T_a is recalculated using Eq. (93), where T_b is replaced by T_{MG1}^{max} (line 59). In line 60, the function **PSD-efficiency** is applied again with the new T_a , which assures that the resulting T_b is less than T_{MG1}^{max} .

In line 62, the current Prius power demand $\mathcal{P}_{dem}$ is calculated by applying $F_t(\tau)$, calculated in line 10, Prius speed $v(\tau)$ and η_{PSD} in Eq. (8). The current MG2 power $\mathcal{P}_{MG2}$ is defined by the difference between $\mathcal{P}_{dem}$ and the absolute value of the PSD output power $\mathcal{P}_c$ (line 63). The current MG2 torque T_{MG2} is obtained by dividing $\mathcal{P}_{MG2}$ by the MG2 speed t_c (lines 64 to 68).

Similarly to MG1 torque, it is verified if T_{MG2} is higher than the MG2 maximum torque T_{MG2}^{max} for a given speed t_c (line 70). T_{MG2}^{max} is obtained in line 69, where the function **MG2max-torque** returns the MG2 torque on the maximum torque line shown in Fig. 30 for a given t_c .

The circumstance where $|T_{MG2}| > T_{MG2}^{max}$ is observed only at the end of the HWFET cycle, in the last deceleration. So only when the Prius is in regenerative braking mode that T_{MG2} is recalculated (line 71). In this case, it is assumed that the regenerative braking is not enough to stop the vehicle, so the mechanical braking actuates simultaneously to the electric brakes (EHSANI et al., 2018). In this scenario, it is assumed that MG2 provides its maximum torque for the speed t_c (line 72) and $\mathcal{P}_{MG2}$ is recalculated (line 73).

If $|T_{MG2}| > T_{MG2}^{max}$ when the Prius is accelerating (line 74), the loop for is broken (line 76) and the Prius fuel economy FC_{Prius} is not calculated (assumed as 0 in line 75). However, this phenomenon is not observed during the acceleration of the drive cycles considered in this Thesis.

With T_a and t_a well defined, the ICE specific fuel consumption and emissions can be obtained from the maps shown in Figs. 24 to 28 (lines 79 to 91). If the ICE shaft is locked or the ICE idles (line 79), there is no fuel consumption (lines 80 and 81) and

no pollutants are emitted (lines 82, 83, and 84). If the ICE produces power (line 85), the function **ICE-maps** returns the specific fuel consumption $b_e(\tau)$ (line 86) and the pollutants emitted in g/s (lines 88, 89 and 90) for some given ICE speed and torque. **ICE-maps** uses t_a and T_a to navigate in the ICE maps: $b_e(\tau)$ comes from Fig. 24, $\dot{m}_{HC}(\tau)$, $\dot{m}_{CO}(\tau)$ and $\dot{m}_{NO_x}(\tau)$ come from Figs. 26, 27 and 28, respectively.

MG1 and MG2 efficiencies, given by η_{MG1} and η_{MG2} , are returned by the functions **MG1-map** and **MG2-map**, respectively. In line 92, **MG1-map** uses MG1 speed and torque to navigate in the efficiency map shown in Fig. 29. Next, η_{MG1} is used to compute the power transferred from the battery to MG1 (or vice versa) through Eq. (21) (line 93).

Likewise, in line 94, **MG2-map** does the same for MG2: it searches on the efficiency map shown in Fig. 30 the corresponding η_{MG2} for some given MG2 speed and torque. Next, η_{MG2} is used to compute the power transferred from the battery to MG2 (or vice versa) through Eq. (20) (line 95).

The current battery pack power $\mathcal{P}_{bat}$ is calculate in line 96 and then the current SOC $SOC(\tau)$ is calculated in line 97. In line 98, the value of SOC_0 is updated to the current SOC, for the next iteration.

After the loop **for** is finished, the distance travelled by the Prius is calculated through the function **trapz**, which numerically integrates the Prius speed to return S_{cyc} (line 100). The emission levels for the whole drive cycle, in mass per distance unit, are calculated in lines 101 and 102. Finally, the Prius fuel economy FC_{Prius} is calculated in lines 103 and 104.

APPENDIX E – GRAPHICS GENERATED DURING PRIUS FUEL ECONOMY CALCULATION

E.1 10-15 MODE DRIVE CYCLE

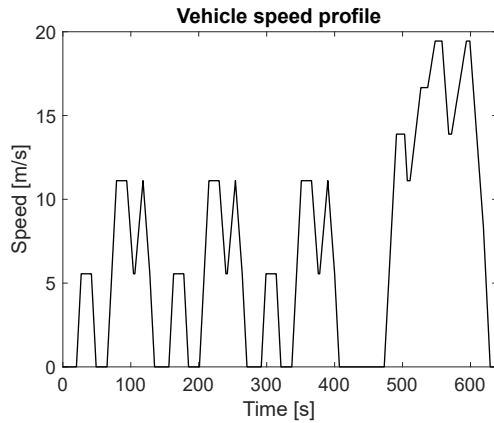


Figure 44 – 10-15 mode cycle.

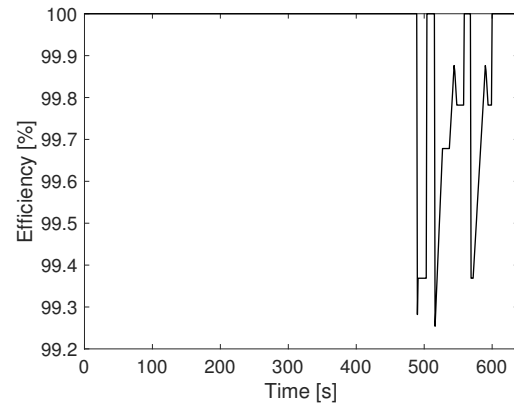


Figure 45 – η_{PSD} on 10-15 mode cycle.

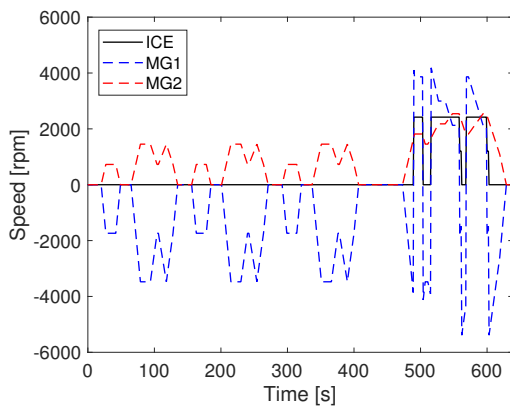


Figure 46 – ICE, MG1, MG2 speed on 10-15 mode cycle.

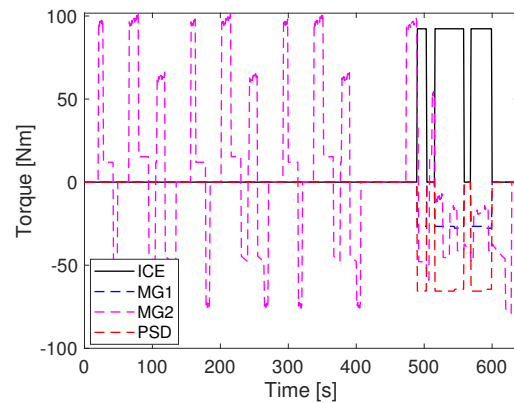


Figure 47 – ICE, MG1, MG2, PSD torque on 10-15 mode cycle.

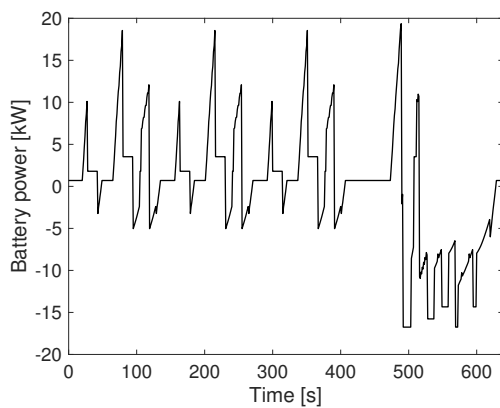


Figure 48 – Battery power on 10-15 mode cycle.

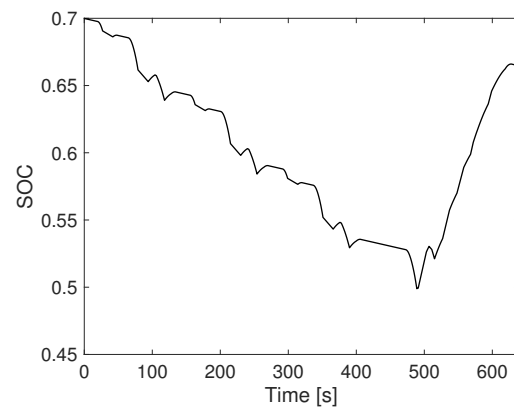


Figure 49 – Battery SOC on 10-15 mode cycle.

E.2 FTP75 DRIVE CYCLE

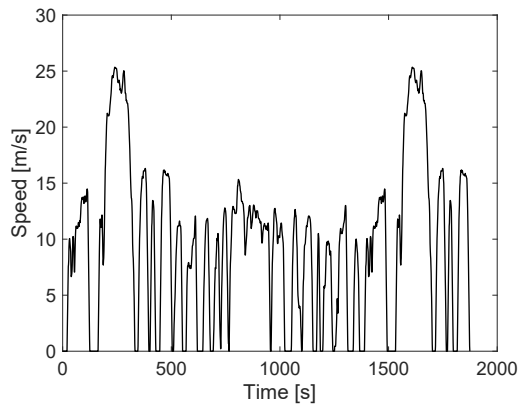


Figure 50 – FTP75 cycle.

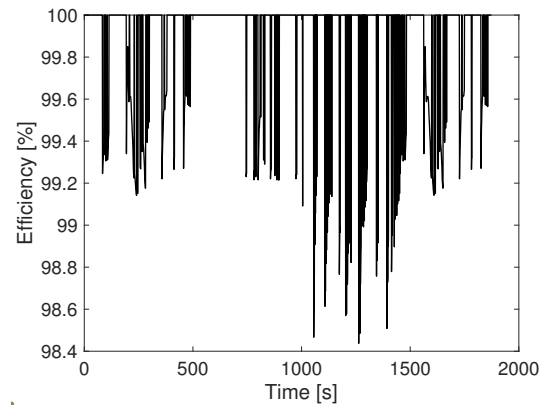
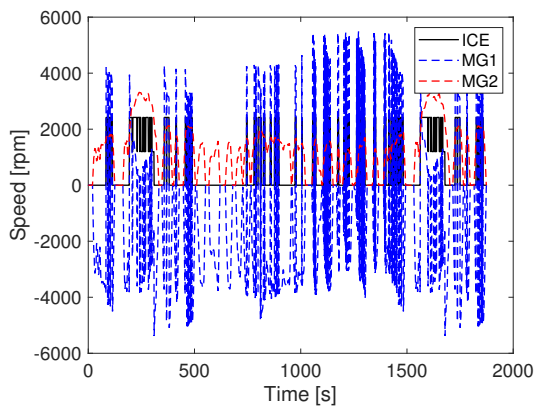
Figure 51 – η_{PSD} on FTP75.

Figure 52 – ICE, MG1, MG2 speed on FTP75.

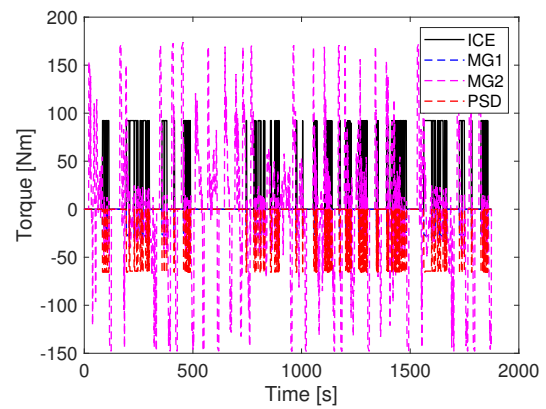


Figure 53 – ICE, MG1, MG2, PSD torque on FTP75.

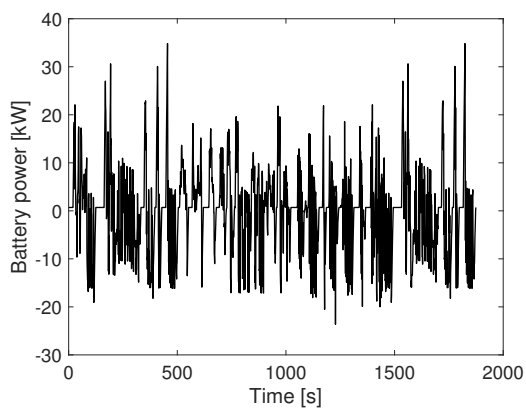


Figure 54 – Battery power on FTP75.

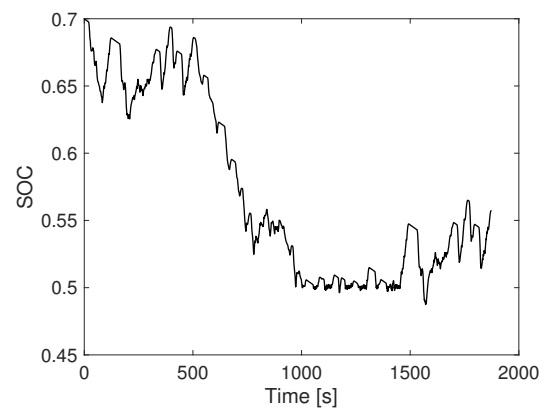


Figure 55 – Battery SOC on FTP75.

E.3 HWFET DRIVE CYCLE

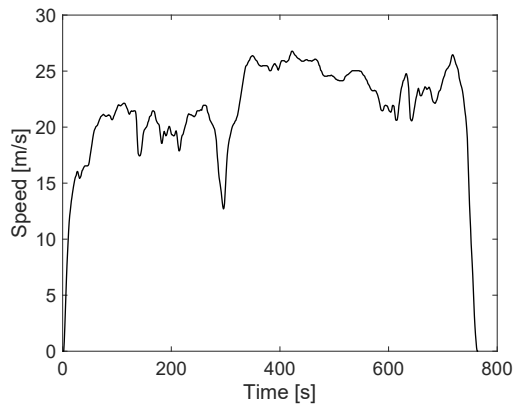


Figure 56 – HWFET cycle.

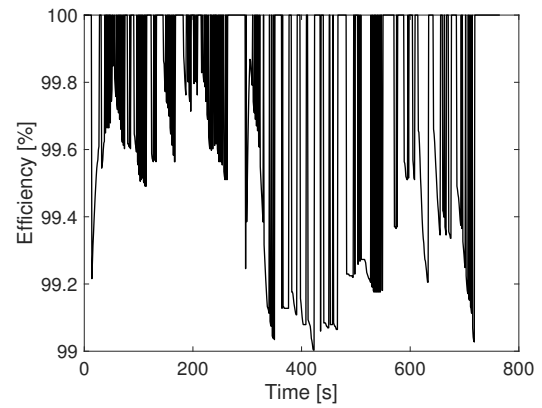
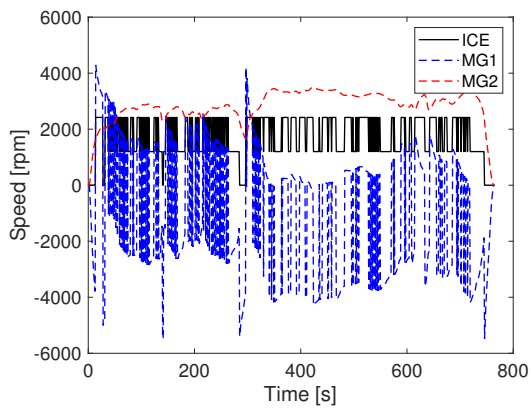
Figure 57 – η_{PSD} on HWFET.

Figure 58 – ICE, MG1, MG2 speed on HWFET.

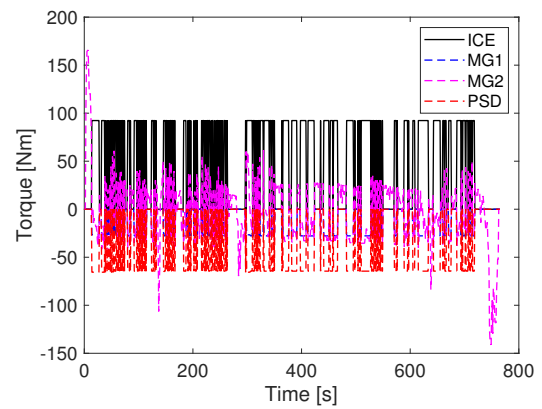


Figure 59 – ICE, MG1, MG2, PSD torque on HWFET.

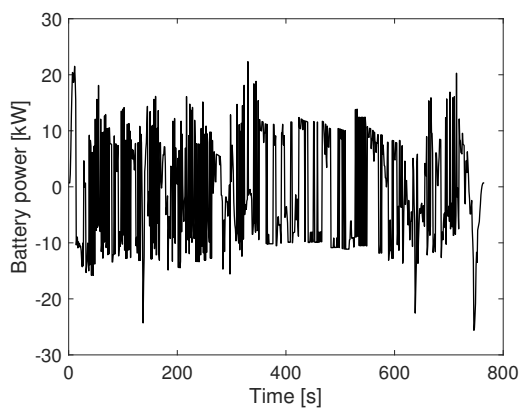


Figure 60 – Battery power on HWFET.

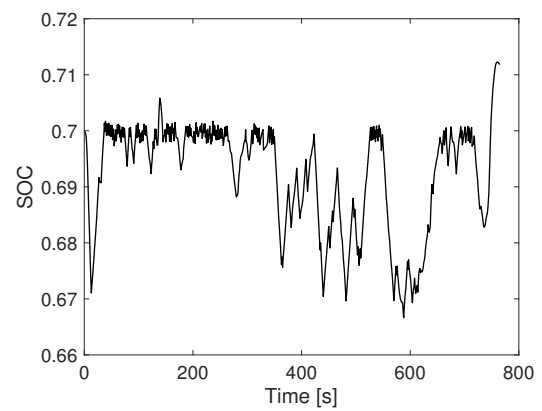
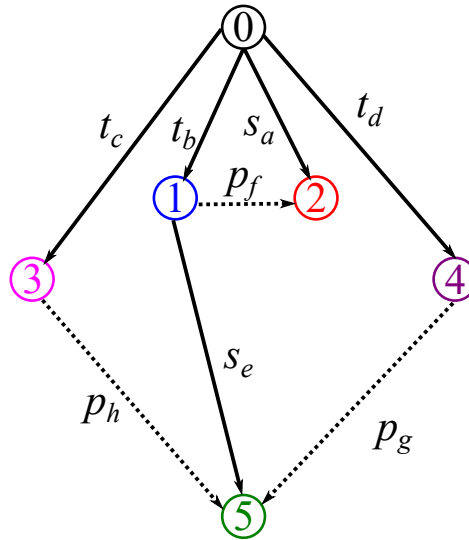


Figure 61 – Battery SOC on HWFET.

APPENDIX F – DAVIES' METHOD MATRICES FOR DETERMINING DHS EFFICIENCY

F.1 MOTION ANALYSIS

Figure 62 – DHS motion graph.



Source: From the author.

Fundamental circuit matrix $[B_M]_{3 \times 8}$

Considering edges f , g and h as chords:

$$[B_M]_{3 \times 8} = \begin{bmatrix} a & b & c & d & e & f & g & h \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} f \\ g \\ h \end{matrix} \quad (168)$$

Unit motion matrix $[\hat{M}_D]_{3 \times 8}$

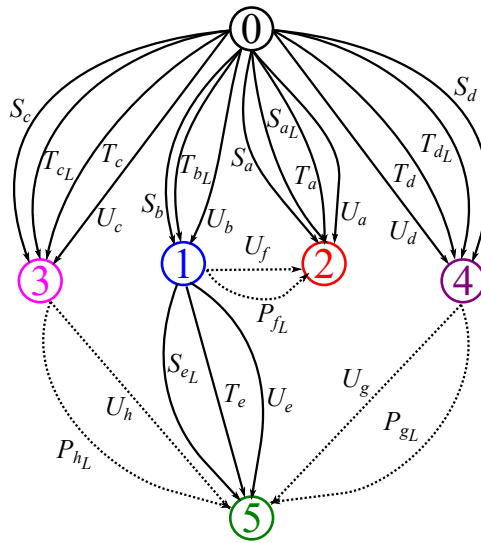
$$[\hat{M}_D]_{3 \times 8} = \begin{bmatrix} s_a & t_b & t_c & t_d & s_e & p_f & p_g & p_h \\ 1 & 0 & 0 & 0 & 1 & \frac{r_1}{\sqrt{r_1^2 + r_2^2}} & \frac{r_4}{\sqrt{r_4^2 + r_5^2}} & \frac{r_3}{\sqrt{r_3^2 + r_5^2}} \\ 0 & 1 & 1 & 1 & 0 & -\frac{r_2}{\sqrt{r_1^2 + r_2^2}} & \frac{r_5}{\sqrt{r_4^2 + r_5^2}} & -\frac{r_5}{\sqrt{r_3^2 + r_5^2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} s \\ t \\ u \end{matrix} \quad (169)$$

Network unit motion matrix $[\hat{M}_N]_{9 \times 8}$

$$[\hat{M}_N]_{9 \times 8} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & \frac{r_1}{\sqrt{r_1^2 + r_2^2}} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{r_2}{\sqrt{r_1^2 + r_2^2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \frac{r_4}{\sqrt{r_4^2 + r_5^2}} & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & \frac{r_5}{\sqrt{r_4^2 + r_5^2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & \frac{r_3}{\sqrt{r_3^2 + r_5^2}} \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & -\frac{r_5}{\sqrt{r_3^2 + r_5^2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} s \\ t \\ u \\ s \\ t \\ u \\ s \\ t \\ u \end{matrix} \quad (170)$$

F.2 ACTION ANALYSIS

Figure 63 – DHS action graph.



Source: From the author.

Fundamental cutset matrix ignoring power losses $[Q_A]_{5 \times 16}$

Considering edges S_a , S_b , S_c , S_d and T_e as branches:

$$[Q_A]_{5 \times 16} = \begin{matrix} & \begin{matrix} a & a & a & b & b & c & c & c & d & d & d & e & e & f & g & h \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} & \begin{matrix} S_a \\ S_b \\ S_c \\ S_d \\ T_e \end{matrix} \end{matrix} \quad (171)$$

Unit action matrix ignoring power losses $[\hat{A}_D]_{3 \times 16}$

$$[\hat{A}_D]_{3 \times 16} = \begin{matrix} \begin{matrix} S_a & T_a & U_a & S_b & U_b & S_c & T_c & U_c & S_d & T_d & U_d & T_e & U_e & U_f & U_g & U_h \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -r_2 & r_5 & -r_5 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & -r_1 & -r_4 & -r_3 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} & \begin{matrix} S \\ T \\ U \end{matrix} \end{matrix} \quad (172)$$

Network unit action matrix ignoring power losses $[\hat{A}_N]_{15 \times 16}$

$$[\hat{A}_N]_{15 \times 16} = \begin{array}{c} \begin{array}{cccccccccccccccc} S_a & T_a & U_a & S_b & U_b & S_c & T_c & U_c & S_d & T_d & U_d & T_e & U_e & U_f & U_g & U_h \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_2 & r_5 & -r_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_1 & -r_3 & -r_3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -r_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_5 & -r_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -r_3 & -r_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{array}{l} S \\ T \\ U \\ \hline S \\ T \\ U \\ \hline S \\ T \\ U \\ \hline S \\ T \\ U \\ \hline S \\ T \\ U \end{array} \end{array} \quad (173)$$

Fundamental cutset matrix considering power losses $[Q_A]_{5 \times 24}$

Considering edges S_a , S_b , S_c , S_d and T_e as branches:

$$[Q_A]_{5 \times 24} = \begin{bmatrix} \textcolor{red}{a} & \textcolor{red}{a} & \textcolor{red}{a} & \textcolor{red}{a} & \textcolor{red}{b} & \textcolor{red}{b} & \textcolor{red}{b} & \textcolor{red}{c} & \textcolor{red}{c} & \textcolor{red}{c} & \textcolor{red}{c} & \textcolor{red}{d} & \textcolor{red}{d} & \textcolor{red}{d} & \textcolor{red}{d} & \textcolor{red}{e} & \textcolor{red}{e} & \textcolor{red}{e} & \textcolor{red}{f} & \textcolor{red}{f} & \textcolor{red}{g} & \textcolor{red}{g} & \textcolor{red}{h} & \textcolor{red}{h} \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} S_a \\ S_b \\ S_c \\ S_d \\ T_e \end{matrix} \quad (174)$$

Unit action matrix considering power losses $[\hat{A}_D]_{3 \times 24}$

$$[\hat{A}_D]_{3 \times 16}^T = \begin{array}{ccc|c} & S & T & U \\ \hline & 1 & 0 & 0 & S_a \\ & 1 & 0 & 0 & S_{aL} \\ & 0 & 1 & 0 & T_a \\ & 0 & 0 & 1 & U_a \\ & 1 & 0 & 0 & S_b \\ & 0 & 1 & 0 & T_{bL} \\ & 0 & 0 & 1 & U_b \\ & 1 & 0 & 0 & S_c \\ & 0 & 1 & 0 & T_c \\ & 0 & 1 & 0 & T_{cL} \\ & 0 & 0 & 1 & U_c \\ & 1 & 0 & 0 & S_d \\ & 0 & 1 & 0 & T_d \\ & 0 & 1 & 0 & T_{dL} \\ & 0 & 0 & 1 & U_d \\ & 1 & 0 & 0 & S_{eL} \\ & 0 & 1 & 0 & T_e \\ & 0 & 0 & 1 & U_e \\ & r_1\beta_2 & -r_2\beta_2 & 0 & P_{fL} \\ & -r_2 & -r_1 & 1 & U_f \\ & r_3\beta_1 & r_5\beta_1 & 0 & P_{gL} \\ & r_5 & -r_3 & 1 & U_g \\ & r_3\beta_1 & -r_5\beta_1 & 0 & P_{hL} \\ & -r_5 & -r_3 & 1 & U_h \end{array} \quad (175)$$

where $\beta_1 = \frac{1}{\sqrt{r_3^2 + r_5^2}}$ and $\beta_2 = \frac{1}{\sqrt{r_1^2 + r_2^2}}$.

Augmented action matrix $[D_A]_{23 \times 24}$

$[D_A]_{23 \times 24} =$

$$= \begin{bmatrix} S_a & S_{a_L} & T_a & U_a & S_b & T_{b_L} & U_b & S_c & T_c & T_{c_L} & U_c & S_d & T_d & T_{d_L} & U_d & S_{e_L} & T_e & U_e & P_{f_L} & U_f & P_{g_L} & U_g & P_{h_L} & U_h \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 r_1 & -r_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_2 r_2 & -r_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_2 r_1 & r_2 & \beta_1 r_3 & r_5 & \beta_1 r_3 & -r_5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 r_2 & r_1 & \beta_1 r_5 & -r_3 & -\beta_1 r_5 & -r_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_1 r_3 & r_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_1 r_5 & r_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_1 r_3 & -r_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_1 r_5 & r_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \beta_1 r_3 & r_5 & \beta_1 r_3 & -r_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \beta_1 r_5 & -r_3 & -\beta_1 r_5 & -r_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & a\mu r_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & b\mu r_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & c\mu r_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & d\mu r_d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & e\mu r_e & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\zeta_f & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\zeta_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\zeta_h & 0 \end{bmatrix}$$

(176)

where $a = \text{sign}(U_a) \text{sign}(s_a)$, $b = \text{sign}(U_b) \text{sign}(t_b)$, $c = \text{sign}(U_c) \text{sign}(t_c)$, $d = \text{sign}(U_d) \text{sign}(t_d)$ and $e = \text{sign}(U_e) \text{sign}(s_e)$.

APPENDIX G – GRAPHICS GENERATED DURING THE CALCULATION OF THE FUEL ECONOMY OF A HEV EQUIPPED WITH DHS

G.1 FTP75 DRIVE CYCLE

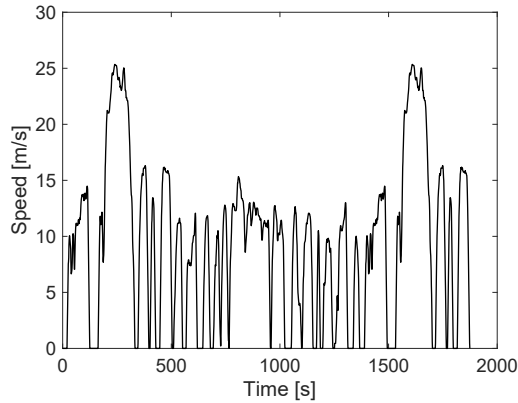


Figure 64 – FTP75 cycle.

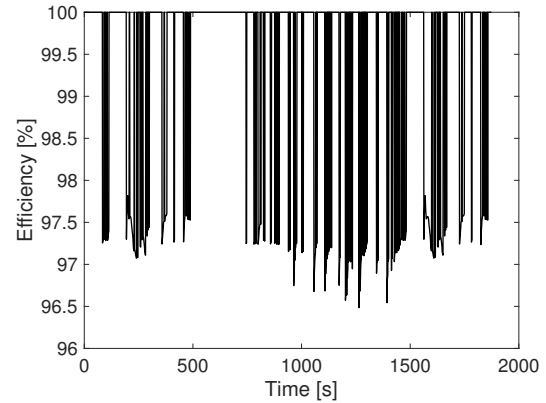


Figure 65 – η_{DHS} on FTP75.

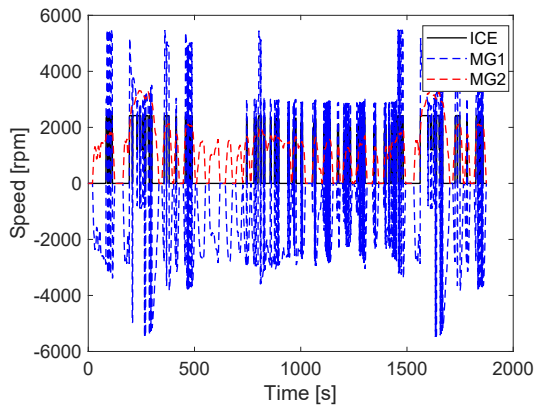


Figure 66 – ICE, MG1, MG2 speed on FTP75.

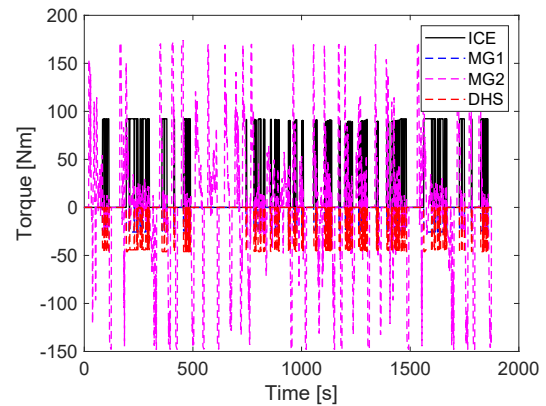


Figure 67 – ICE, MG1, MG2, PSD torque on FTP75.

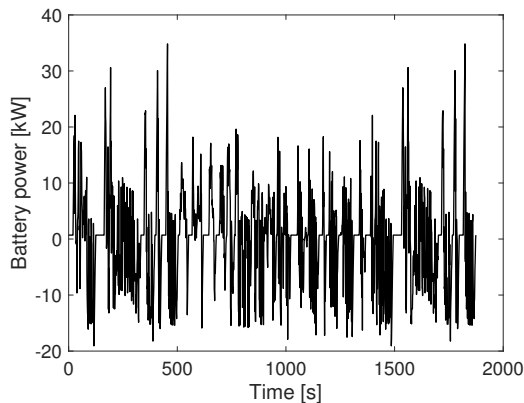


Figure 68 – Battery power on FTP75.

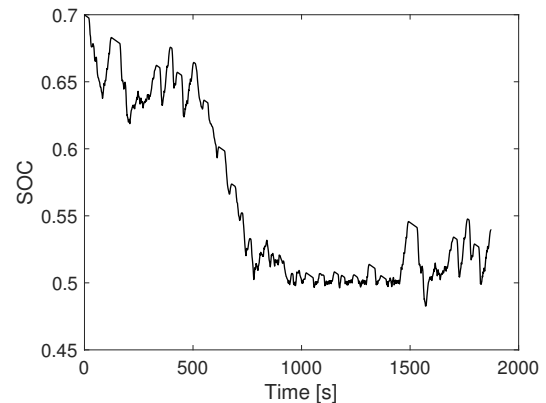


Figure 69 – Battery SOC on FTP75.

G.2 HWFET DRIVE CYCLE

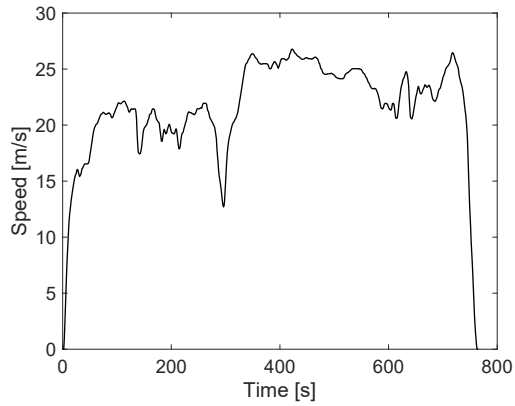


Figure 70 – HWFET cycle.

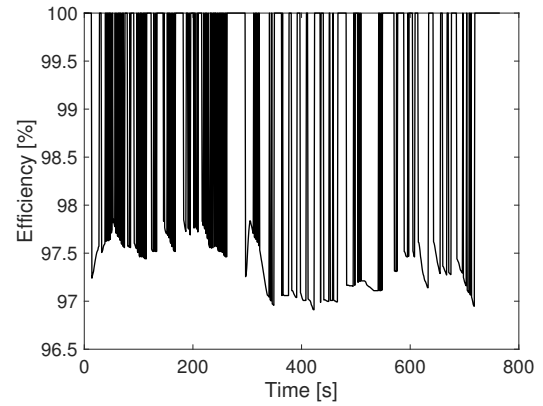


Figure 71 – η_{DHS} on HWFET.

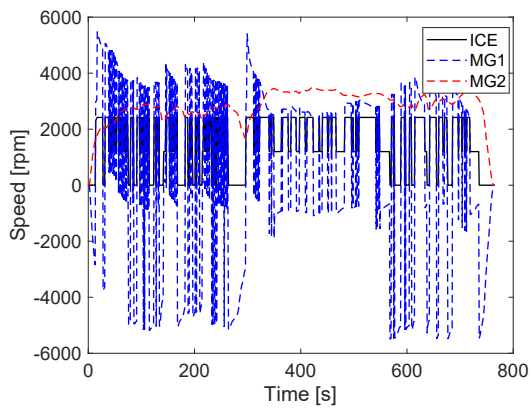


Figure 72 – ICE, MG1, MG2 speed on HWFET.

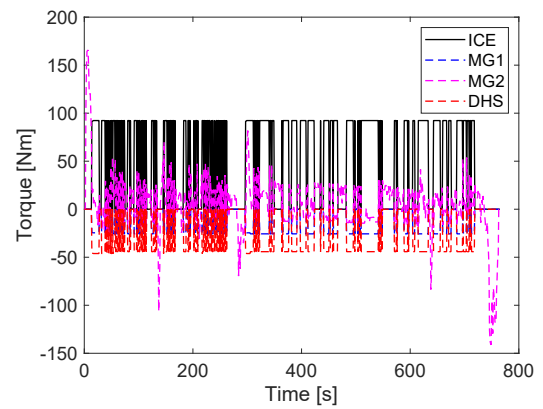


Figure 73 – ICE, MG1, MG2, PSD torque on HWFET.

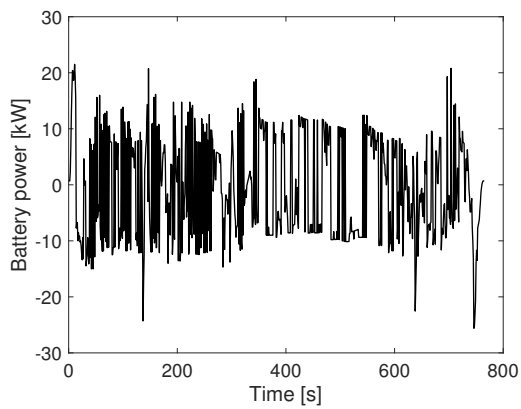


Figure 74 – Battery power on HWFET.

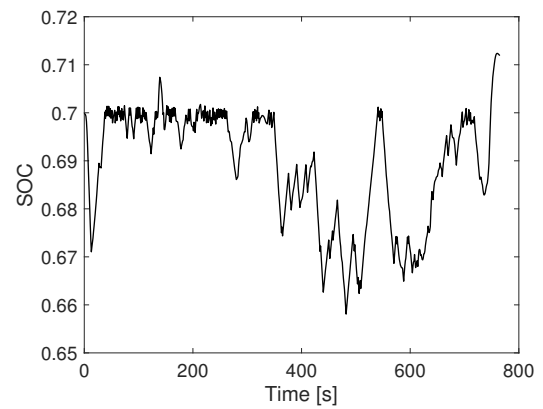


Figure 75 – Battery SOC on HWFET.

distribution factor, Y_β is the lengthwise curvature factor for bending strength and Y_J is the bending strength geometry factor.

The tangential force acting on the tooth flank W_t is defined by the input torque of the gear pair T_{in} :

$$W_t = \frac{2T_{in}}{d_p} \quad (178)$$

where d_p is the pitch diameter of the smallest gear of the pair. For gear pair 1-2 (DHS cage and input pinion), $T_{in} = S_a$, that is the torque on the ICE OOL (Fig. 25) for a given ICE speed $s_{a_{opt}}$. T_{in} for gear pairs 3-5 (left-side gear and planet gear) and 4-5 (right-side gear and planet gear) corresponds to T_c and T_d , respectively. For the teeth stress calculation, T_c and T_d are determined via Davies' method, from the action analysis ignoring power losses, see Sec. 7.2:

$$T_c^{W_t} = \left| -S_a \frac{r_1 r_3}{r_2(r_3 + r_4)} \right| \quad (179)$$

$$T_d^{W_t} = \left| -S_a \frac{r_1 r_4}{r_2(r_3 + r_4)} \right| \quad (180)$$

The face width b must be taken from the smallest gear of the pair and is defined as:

$$b = \frac{\gamma_p}{3} \quad (181)$$

where γ_p is the cone distance of the smallest gear of the pair and is calculated as:

$$\gamma_p = \frac{d_p}{2 \sin(\Gamma_p)} \quad (182)$$

The pitch cone angle Γ_p is the angle of the smallest gear of the pair. For a gear pair composed of gears α and β , the pitch cone angles are calculated as:

$$\Gamma_\alpha = \tan^{-1} \left(\frac{z_\alpha}{z_\beta} \right) \quad (183)$$

$$\Gamma_\beta = \frac{\pi}{2} - \Gamma_\alpha \quad (184)$$

Overload factor K_A

The overload factor emulates the dynamic loads to which the gear teeth are submitted, based on the "shockiness" of the power source connected to the gear train. Tab. 33 contains some AGMA-suggested values for K_A for speed-decreasing drives.

Table 33 – Overload factors

Character of the prime mover	Character of the load on driven machine			
	Uniform	Light shock	Medium shock	Heavy shock
Uniform	1	1.25	1.5	1.75 or higher
Light shock	1.1	1.35	1.60	1.85 or higher
Medium shock	1.25	1.5	1.75	2 or higher
Heavy shock	1.5	1.75	2	2.25 or higher

Source: Adapted from Budynas and Nisbett (2020).

The driven machine is the DHS and it is assumed that the DHS has the character of light shock. The prime mover of the pair 1-2 is a multi-cylinder engine, a light shock prime mover (NORTON, 2010). So for pair 1-2 (coupling f), the overload factor is $K_A^f = 1.35$.

The prime movers of pair 3-5 and 4-5 are electric machines, characterized by being uniform prime movers (NORTON, 2010). So for pairs 3-5 (coupling h) and 4-5 (coupling g), the overload factors are $K_A^g = K_A^h = 1.25$.

Dynamic factor K_V

The dynamic factor takes into account the effect of gear-tooth quality related to speed and load, and the increase in stress due to vibration loads. AGMA uses a transmission accuracy number Q_V to quantify the precision with which tooth profiles are spaced along the pitch circle. AGMA recommends $Q_V = 10$ for automotive transmissions (NORTON, 2010).

The dynamic factor K_V is calculated through the empirical equation Eq. (146):

$$K_V = \left(\frac{A + \sqrt{200v_{et}}}{A} \right)^B \quad (185)$$

where v_{et} is the pitch-line velocity at outside pitch diameter, in m/s, and the factors A and B are defined as:

$$A = 50 + 56(1 - B) \quad (186)$$

$$B = \frac{(12 - Q_V)^{2/3}}{4} \quad (187)$$

The pitch-line velocity is given by:

$$v_{et} = 5.236(10^{-5})d_p n_p \quad (188)$$

where n_p is the pinion speed in rpm.

For gear pair 1-2, v_{et} is calculated considering $d_p = d_2$ and $n_p = s_{a_{opt}}$. For gear pairs 3-5 and 4-5, n_p is computed for the maximum speed the HEV reaches in the

HWFET drive cycle: 96.4 km/h. For gear pair 4-5, $d_p = d_4$ and $n_p = t_d$, in rpm, which is calculated using Eqs. (9) and (10).

For gear pair 3-5, $d_p = d_3$ and $n_p = |t_c|$, in rpm. t_c is obtained via Davies' method from the motion analysis, see Sec. 7.2, and is calculated as:

$$t_c = s_a \frac{r_2 r_3 + r_2 r_4}{r_1 r_3} - t_d \frac{r_4}{r_3} \quad (189)$$

Size factor for bending strength Y_x

The size factor Y_x depends on the module m :

$$Y_x = \begin{cases} 0.5 & \text{for } m < 1.6 \text{ mm} \\ 0.4867 + 0.008339m & \text{for } 1.6 \leq m \leq 50 \text{ mm} \end{cases} \quad (190)$$

Load distribution factor $K_{H\beta}$

The load distribution factor $K_{H\beta}$ depends on the tooth width b and if the members of the gear pair are straddle-mounted. $K_{H\beta}$ is defined as:

$$K_{H\beta} = K_{mb} + 5.6(10^{-6})b^2 \quad (191)$$

where

$$K_{mb} = \begin{cases} 1.00 & \text{both members straddle-mounted} \\ 1.10 & \text{one member straddle-mounted} \\ 1.25 & \text{neither member straddle-mounted} \end{cases}$$

No gear in the DHS is straddle-mounted, so $K_{mb} = 1.25$.

Lengthwise curvature factor for bending strength Y_β

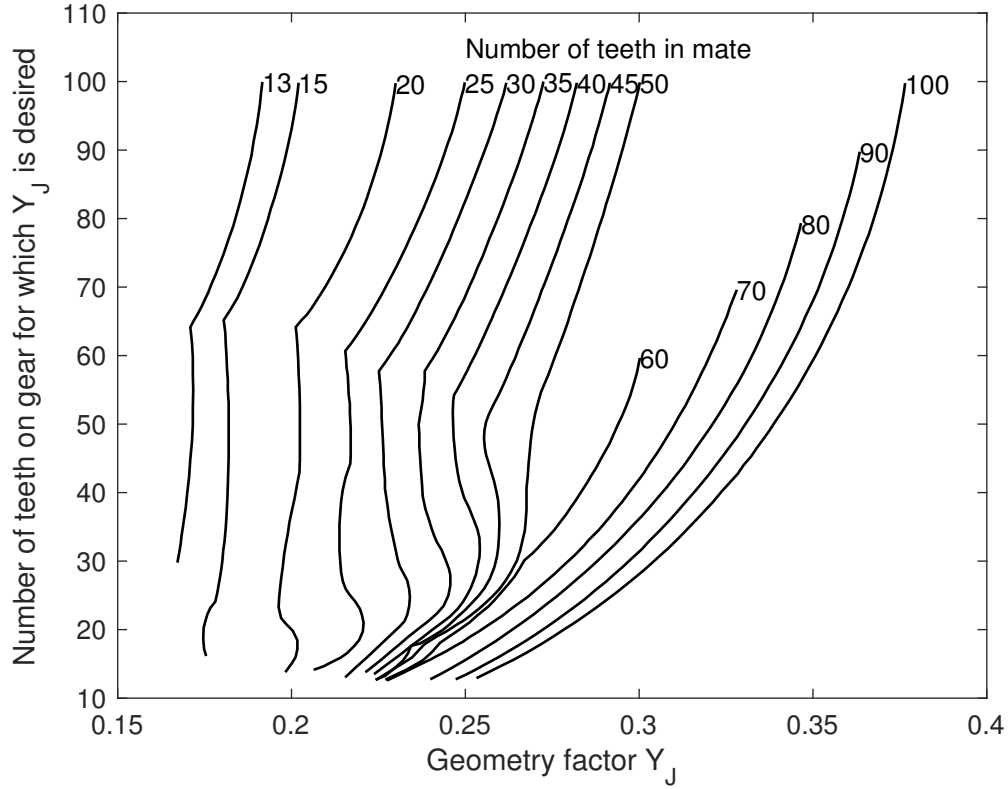
The lengthwise curvature factor for bending strength for straight bevel gears is $Y_\beta = 1$.

Bending strength geometry factor Y_J

The bending strength geometry factor depends on the number of teeth on gear for which Y_J is desired and the number of teeth of the gear in mate. Fig. 77 shows Y_J for straight bevel gears with a 20° normal pressure angle and 90° shaft angle.

When the number of teeth does not belong to any curve of Fig. 77, a linear interpolation between two consecutive curves is made to obtain Y_j .

Figure 77 – Y_J for coniflex straight bevel gears with 20° normal pressure angle and 90° shaft angle.



Source: Adapted from Budynas and Nisbett (2020).

The peak of the bending stress σ_b must be lesser than the permissible bending stress S_{fb} , calculated as (BUDYNAS; NISBETT, 2020):

$$S_{fb} = \frac{Y_{NT}}{K_T Y_Z} S'_{fb} \quad (192)$$

where Y_{NT} is the stress-cycle factor for bending strength, K_T is the temperature factor, Y_Z is the reliability factor for bending strength and S'_{fb} is the allowable bending stress number.

The stress-cycle factor for bending strength depends on the number of load cycles n_L and has a general formula, valid for $3(10^6) \leq n_L \leq 10^{10}$, defined as:

$$Y_{NT} = 1.3558 n_L^{-0.0178} \quad (193)$$

In Chapter 7, $n_L = 10^{10}$.

The temperature factor K_T depends on the temperature θ and is calculated as:

$$K_T = \begin{cases} 1 & \text{for } 0^\circ \text{C} \leq \theta \leq 120^\circ \text{C} \\ \frac{273+\theta}{393} & \text{for } \theta > 120^\circ \text{C} \end{cases} \quad (194)$$

Since the temperature during the DHS operation is unknown, $K_T = 1$ is assumed.

The reliability factor for bending strength Y_Z depends on the acceptable maximum number of failures for a given number of applications. It is assumed that fewer than one failure in 10000 applications is acceptable, so $Y_Z = 1.5$.

H.2 CONTACT STRESSES OR PITTING RESISTANCE

The contact stress σ_c must be calculated for each gear pair of the DHS. The AGMA pitting resistance σ_c formula for straight bevel gears is given by (BUDYNAS; NISBETT, 2020):

$$\sigma_c = C_p \sqrt{\frac{W_t}{bd_p Z_I} K_A K_V K_{H\beta} Z_X Z_{xc}} \quad (195)$$

where C_p is the elastic coefficient for pitting resistance, d_p is the pitch diameter of the smallest gear of the pair, Z_I is the pitting resistance geometry factor, Z_X is the size factor for pitting resistance and Z_{xc} is the crowning factor for pitting resistance.

Elastic coefficient for pitting resistance C_p

It is assumed that all gears of the DHS are made of steel, so $C_p = 190 \sqrt{\text{N/mm}^2}$.

Pitting resistance geometry factor Z_I

The pitting geometry factor depends on the number of teeth of the pinion gear and the number of teeth of the gear in mate. Fig. 78 shows Z_I for straight bevel gears with a 20° normal pressure angle and 90° shaft angle.

When the number of teeth does not belong to any curve of Fig. 78, a linear interpolation between two consecutive curves is made to obtain Z_I .

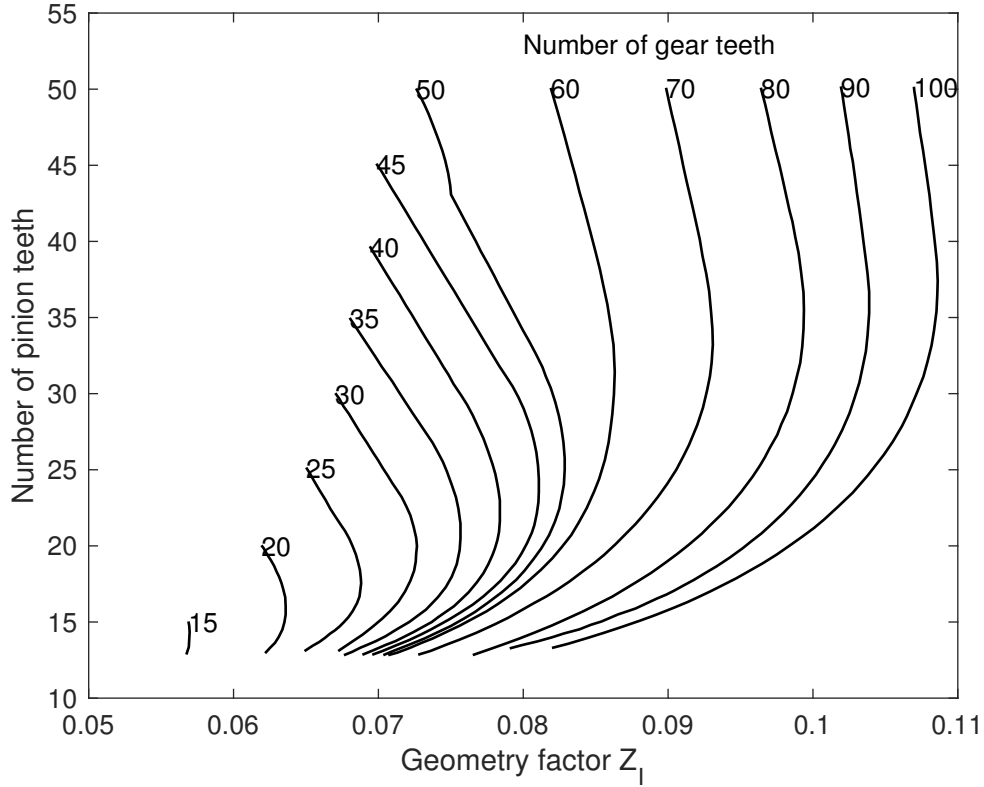
Size factor for pitting resistance Z_X

The size factor for pitting resistance depends on the face width b :

$$Z_X = \begin{cases} 0.5 & b < \text{for } 12.7 \text{ mm} \\ 0.00492b + 0.4375 & \text{for } 12.7 \leq b \leq 114.3 \text{ mm} \\ 1 & \text{for } b > 114.3 \text{ mm} \end{cases} \quad (196)$$

Crowning factor for pitting resistance Z_{xc}

Figure 78 – Z_I for coniflex straight bevel gears with 20° normal pressure angle and 90° shaft angle.



Source: Adapted from Budynas and Nisbett (2020).

The teeth of most bevel gears are crowned in the lengthwise direction during manufacture to accommodate the deflection of the mountings. The crowning factor for pitting resistance takes into account the crowning and is defined as:

$$Z_{xc} = \begin{cases} 1.5 & \text{properly crowned teeth} \\ 2.0 & \text{larger uncrowned teeth} \end{cases} \quad (197)$$

The DHS gears are assumed to be properly crowned, so $Z_{xc} = 1.5$.

The pitting resistance σ_c must be lesser than the permissible contact stress number S_{fc} , defined as (BUDYNAS; NISBETT, 2020)

$$S_{fc} = \frac{Z_{NT} Z_W}{K_T Z_Z} S'_{fc} \quad (198)$$

where Z_{NT} is the stress-cycle factor for pitting resistance, Z_W is hardness ratio factor for pitting resistance, Z_Z is the reliability factor for pitting resistance and S'_{fc} is the allowable contact stress number.

The stress-cycle factor for pitting resistance Z_{NT} depends on the number of load cycles n_L and is defined as:

$$Z_{NT} = \begin{cases} 2 & \text{for } 10^3 \leq n_L < 10^4 \\ 3.4822n_L^{-0.0602} & \text{for } 10^4 \leq n_L \leq 10^{10} \end{cases} \quad (199)$$

In Chapter 7, $n_L = 10^{10}$.

The hardness ratio factor for pitting resistance Z_W is equal to 1 for straight bevel gears pairs whose gears are made of carburized steel and have approximately equal hardness.

The reliability factor for pitting resistance Z_Z depends on the acceptable maximum number of failures for a given number of applications. It is assumed that fewer than one failure in 10000 applications is acceptable, so $Z_Z = 1.22$.

APPENDIX I – ANALYSIS OF THE FUEL ECONOMY OPTIMIZATION OF THE HEV EQUIPPED WITH THE DHS

Chapter 7 presents an optimization of the fuel economy of a HEV equipped with the DHS. The objective function to be minimized is the composite fuel economy $FC_{C_{DHS}}$ and the design variables are the module m , the number of teeth z_1 , z_2 , z_3 and z_5 , and the gear ratio i_G of the reducer connecting the generator MG1 to the DHS.

The new module, number of teeth and i_G obtained via the optimization described in Sec. 7.4 are presented in Tab. 34, side-by-side with the DHS original dimensions.

Table 34 – Original and optimized DHS design variables

	Original DHS	Optimized DHS
m [mm]	5.4	3
z_1	34	48
z_2	34	48
z_3	14	40
z_5	10	20
i_G	1.8	2.88

Source: From the author.

The design variables found through the optimization lead to a composite fuel economy of $FC_{C_{DHS}}^{opt} = 4.2$ L/100 km, a reduction of 2.2% in relation to the result obtained in Sec. 7.3: $FC_{C_{DHS}} = 4.3$ L/100 km.

The fuel economy results obtained by running Algorithm 2 for the FTP75 and HWFET cycles, using the original and optimized design variables shown in Tab. 25, are presented in Tab. 35, along with the composite fuel economy $FC_{C_{DHS}}$.

Table 35 – Original and optimized fuel economy

	Original DHS	Optimized DHS	Reduction [%]
FC_{DHS}^{FTP} [L/100 km]	4.25	4.10	3.33
FC_{DHS}^{HW} [L/100 km]	4.36	4.33	0.81
$FC_{C_{DHS}}$ [L/100 km]	4.3	4.2	2.2

Source: From the author.

According to Tab. 35, the drive cycle which presented the highest fuel economy reduction is the FTP75 cycle. So the FTP75 results are used for analyzing the causes of the consumption reduction. The first parameter analyzed is the DHS efficiency η_{DHS} before and after optimization.

On average, the DHS efficiency η_{DHS} of the optimized DHS, during the FTP75 cycle, presents a value of 97.55%, with a standard deviation of 0.213%, when the cycle intervals where the HEV stops are neglected. Under the same circumstances, the mean value of the η_{DHS} for the original DHS is 97.29%, with a standard deviation of 0.228%.

The minimum value of η_{DHS} during the FTP75 drive cycle before and after optimization can also be compared. The minimum η_{DHS} for the original DHS is 96.5%, while for the optimized DHS is 96.81%. The statical analysis of η_{DHS} is summarized in Tab. 36.

Table 36 – Statistical analysis of η_{DHS} before and after optimization

	Original DHS	Optimized DHS
Mean	97.55%	97.29%
Standard deviation	0.213%	0.228%
Minimum value	96.81%	96.50%

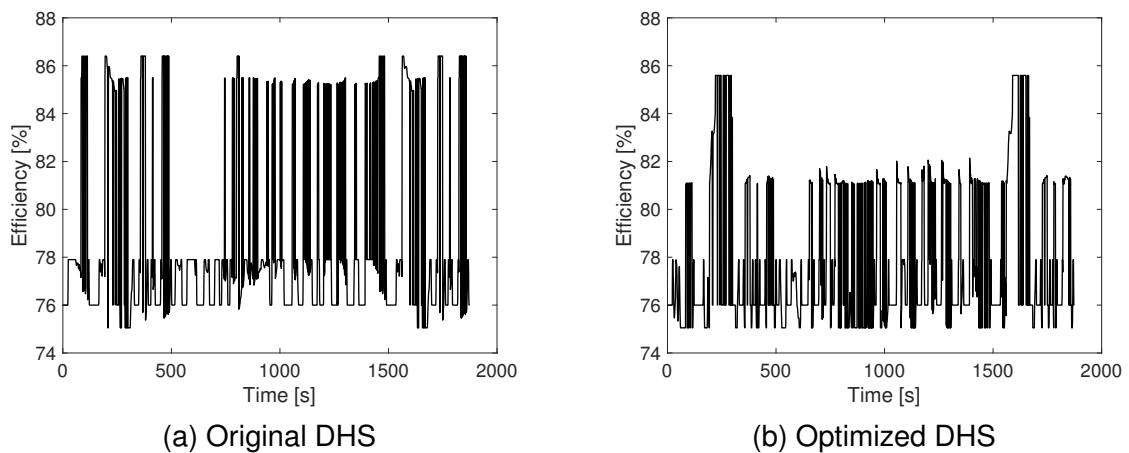
Source: From the author.

A low standard deviation indicates that the values tend to be close to the mean. So the values of η_{DHS} after optimization are closer to a higher mean than the efficiency values of the original DHS. Thus, this is an indication that η_{DHS} is, on average, higher for the optimized DHS during the FTP75 cycle.

The difference of η_{DHS} may seem insignificant, but perhaps the cumulated effect of a slightly higher average for the whole drive cycle impacts the fuel economy at the end.

Moreover, η_{DHS} is not an isolated parameter affecting the fuel economy. By altering the DHS dimensions and i_G , the operation point of MG1 is also altered when the design variables values obtained through the optimization, specially i_G , shown in the right column of Tab 25, are applied in Algorithm 2. One way to see the changing of MG1 operation point is through the values that MG1 efficiency η_{MG1} assume during the FTP75 cycle, for the original and optimized DHS, shown in Fig. 79.

Figure 79 – MG1 efficiency during the FTP75 drive cycle for (a) the HEV equipped with the original DHS and (b) the HEV equipped with the optimized DHS



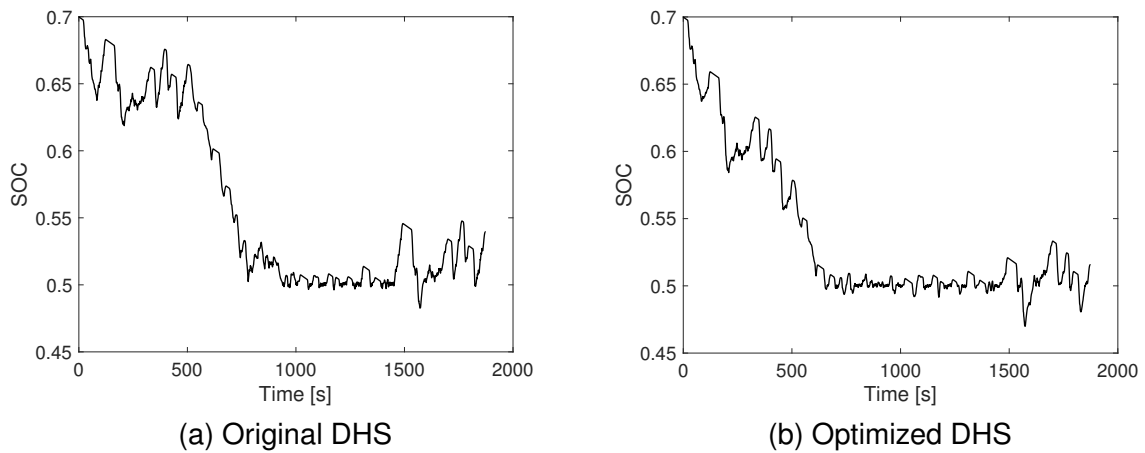
Source: From the author.

Note that, in the middle of the FTP75 cycle, η_{MG1} is lower in Fig. 79(b),

corresponding to η_{MG1} obtained via the optimized DHS, when compared to Fig. 79(a), corresponding to η_{MG1} obtained via the original DHS.

The generator efficiency η_{MG1} directly impacts the battery pack SOC, as it can be seen in Fig. 80, which shows the SOC variation along the FTP75 drive cycle, for the original and the optimized DHS.

Figure 80 – Battery pack SOC during the FTP75 drive cycle for (a) the HEV equipped with the original DHS and (b) the HEV equipped with the optimized DHS



Source: From the author.

Note that the SOC decreases faster in the first 500 s of Fig. 80(b), corresponding to the optimized DHS, than in Fig. 80(a), corresponding to the original DHS. The rapid SOC decrease in Fig. 80(b) is due to the lower η_{MG1} resulting from the optimized DHS. As seen in Algorithm 2, the SOC value determines when the ICE turns on, what affects directly the fuel consumption. Thus, the composite fuel economy reduction obtained through the optimization can also come from the changing of the MG1 operation point on the efficiency map shown in Fig. 29.